

Section-A

Choose the correct answer from the given options for each question given below and each carries 2 marks.

(15X2=30)

1. $\lim_{x \rightarrow 0} \left(\frac{(1+x)^{\frac{1}{2022}} - (1-x)^{\frac{1}{2022}}}{x} \right) =$

A. $\frac{1}{2}$

B. $\frac{1}{1011}$

C. $\frac{1}{2022}$

D. $\frac{1}{4044}$

2. $\lim_{x \rightarrow -\infty} \left(\frac{2x+3}{\sqrt{x^2-1}} \right) =$

A. 2

B. 0

C. -2

D. Undefined

3. If $y = \sin^{-1}(3x - 4x^3) + \cos^{-1}(4x^3 - 3x)$ then $\frac{dy}{dx} =$

A. $3 - 12x^2$

B. $\frac{\pi}{2}$

C. -1

D. 0

4. Which of the following is true ?

A. The derivative of a constant function is not zero.

B. $\lim_{x \rightarrow 0} \left(\frac{1}{1+e^{\frac{1}{x}}} \right)$ does not exist.

C. $y = |x - 1|$ is differentiable everywhere.

D. The 2nd derivative of $y = \cos x + i \sin x$ is iy .

5. $\lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x^2} \right) =$

A. 0

B. 1

C. $\frac{1}{2}$

D. $-\frac{1}{2}$

6. If $x = t^2 + 1$, $y = t^3 + 1$ then $\frac{d^2y}{dx^2} =$

A. 0

B. t

C. \sqrt{t}

D. $\frac{3}{4t}$

7. The function $f(x) = \tan^{-1}(x) - x$ is decreasing on

A. $(-1, 1)$

B. $[-1, 1]$

C. $(-\infty, -1) \cup (1, \infty)$

D. $(-\infty, \infty)$

8. If the curve $ay + x^2 = 7$ and $x^3 = y$ cut orthogonally at $(1,1)$ then the value of a is
A. -6 B. 6 C. 0 D. 1
9. The locus of P such that area of ΔPAB is 12 sq. units where $A(2,2)$ and $B(-4,5)$ is
A. $x^2 + 4xy + 4y^2 - 12x - 24y - 28 = 0$
B. $x^2 - 4xy + 4y^2 - 12x - 24y - 28 = 0$
C. $x^2 + 4xy - 4y^2 - 12x - 24y - 28 = 0$
D. $x^2 - 4xy - 4y^2 - 12x - 24y - 28 = 0$
10. The locus of a point which is at a constant distance from a fixed point is a.....
A. Line B. Point C. Circle D. Parabola
11. The point $P(1,2)$ changes to $Q(3,4)$
I. By translation of axes, the new origin is $(-2, -2)$
II. By rotation of axes, the angle of rotation is $\tan^{-1} \sqrt{2}$
Which of the above statement(s) is/are correct ?
A. Only I B. Only II C. Both I & II D. Neither I nor II
12. The angle through which the axes are to be rotated so as to remove the xy term in the equation $x^2 + 4xy + y^2 - 2x + 2y - 6 = 0$
A. $\frac{\pi}{2}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{6}$ D. $\frac{\pi}{3}$
13. The equation of the straight line passing through the point $(1,2)$ and parallel to the line $y = 3x + 1$ is
A. $y + 2 = x + 1$ B. $y + 2 = 3(x + 1)$
C. $y - 2 = 3(x - 1)$ D. $y - 2 = x - 1$
14. Angle between the lines $2x + y - 4 = 0$ and $y - 3x - 7 = 0$ is
A. $\frac{\pi}{4}$ B. $\frac{\pi}{2}$ C. $\frac{\pi}{6}$ D. $\frac{\pi}{3}$
15. The ratio in which the X -axis divide the line segment joining $A(2, -3)$ and $B(3, -6)$ is
A. -1:2 B. 1:2 C. 2:3 D. -2:3

Section-B

Answer any **THREE** of the following and each carries 10Marks.

3X10=30

16.

a) (i) State Sandwich theorem.

(ii) Find $\lim_{x \rightarrow \infty} \frac{2020 + \sin x}{x^2 + 2022}$.

b) Check the continuity of f given by below at the points 0, 1 and 2

$$f(x) = \begin{cases} 4 - x^2 & \text{if } x \leq 0 \\ x - 5 & \text{if } 0 < x \leq 1 \\ 4x^2 - 9 & \text{if } 1 < x < 2 \\ 3x + 1 & \text{if } x \geq 2 \end{cases}$$

17.

a) Show that Every differentiable function is continuous on R . Is the converse true? Justify your answer.

b) If $x^y = y^x$ then show that $\frac{dy}{dx} = \frac{y(x \log y - y)}{x(y \log x - x)}$.

18.

a) If the tangent at any point on the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ intersects the coordinate axes in A and B, then show that the length AB is a constant.

b) If the curved surface of right circular cylinder inscribed in a sphere of radius r is maximum, show that the height of the cylinder is $\sqrt{2}r$.

19.

a) State **Rolle's Theorem**, and verify the **Lagrange's Mean Value Theorem** for the function $f(x) = \sin x - \sin 2x$ in $[0, \pi]$.

b) Find the equation of the locus of a point, the sum of whose distances from (0, 2) and (0, -2) is 6.

20.

a) Find the point to which the origin is to be shifted by the translation of axes so as to remove the first degree terms from the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0, \text{ where } h^2 \neq ab.$$

b) When the axes are rotated through an angle $\frac{\pi}{6}$, find the transformed equation of $x^2 + 2\sqrt{3}xy - y^2 = 2a^2$.

21.

a) Transform the equation $3x + 4y = 5$ into

(i) Slope-intercept form

(ii) Intercept form and

(iii) Normal form.

b) Find the equations of the straight lines passing through the point (-3, 2) and making an angle of 45° with the straight line $3x - y + 4 = 0$.

*****End of the Paper*****