SECTION –A (30 M)

Choose the correct answer from the given options for each question given below and each carries 2 marks.

1. The domain of the function $f(x) = log(x^2 - 4x + 3)$ is

A. $(-\infty, 1) \cup (3, \infty)$ B. $(-\infty, -1) \cup (3, \infty)$ C. $(1,3)$ D. $[-1,3]$

2. If $f(x) = \frac{\cos^2 x + \sin^4 x}{\sin^2 x + \cos^4 x}$, $\forall x \in \boldsymbol{R}$ then $f(2022) =$

A. 0 B. 1 C. 2 D. 3

3. $3n^5 + 5n^3 + 7n, \forall n \in \boldsymbol{N}$ is divisible by

A. 2 B. 4 C. 15 D.10

4. If $\begin{vmatrix} x & y & z \\ -x & y & z \\ x & -y & z \end{vmatrix} = kxyz$, then $k =$

A. 1 B. 2 C. 3 D. 4

5. If the matrix $\begin{pmatrix} 1 & x & 2x \\ 1 & 3x & 5x \\ 1 & 3 & 4 \end{pmatrix}$ is singular then $x =$

A. -1 B. 1 C. 2 D. 3

6. The rank of the matrix $\begin{pmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 6 & 2 \\ 1 & 2 & 3 & 2 \end{pmatrix}$ is

A. 1 B. 2 C. 3 D. 4

7. If A lies in III quadrant and $3tanA - 4 = 0$, then $5sin2A + 3sinA + 4cosA =$

A. $-\frac{24}{5}$ B. $\frac{5}{24}$ C. 0 D. $\frac{24}{5}$

8. $cos^2\frac{\pi}{10} + cos^2\frac{2\pi}{5} + cos^2\frac{3\pi}{5} + cos^2\frac{9\pi}{10} =$

A. 8 B. 6 C. 4 D. 2

9. If $A + B + C = \pi$ and $cosA = cosB.cosC$ then $cotB.cotC =$

A. $\frac{1}{2}$ B. 1 C. 2 D. $-\frac{1}{2}$

10. If $\sqrt{3}\,cos\theta - sin\theta = 1$, then $\theta =$

A. $\frac{\pi}{3}$ B. $\frac{\pi}{6}$ C. $\frac{\pi}{4}$ D. $\frac{\pi}{2}$

11. The general solution of $tan\theta.tan2\theta = 1$ is

A. $n\pi + \frac{\pi}{4}$ B. $n\pi + \frac{\pi}{2}$ C. $n\frac{\pi}{3} + \frac{\pi}{2}$ D. $n\pi + \frac{\pi}{6}$

12. $cos\left[\text{Cos}^{-1}\left(-\frac{2}{3}\right) - \text{Sin}^{-1}\left(\frac{2}{3}\right)\right] =$

A. $\frac{1}{2}$ B. -1 C. 1 D. 0

13. If $Tan^{-1}x + Tan^{-1}y + Tan^{-1}z = \frac{\pi}{2}$, then $1 - xy - yz - zx =$

A. 0 B. 1 C. 2 D. -1

14. If $Cos^{-1}\frac{5}{13} + Cos^{-1}\frac{3}{5} = Cos^{-1}\,x$, then $x =$

A. $\frac{3}{65}$ B. $-\frac{33}{65}$ C. $-\frac{36}{65}$ D. -1

15. The value of $\cosh(2) + \sinh(2) =$

A. e B. $\frac{e}{2}$ C. e^2 D. $\frac{e^2}{2}$

Section B $(3 \times 10 = 30\ M)$

Answer any *Three* of the following questions

16.

a. Let $f: A \longrightarrow B$, $g: B \longrightarrow C$ be bijections then prove that $g \circ f: A \longrightarrow C$ is a bijection.

b. Prove that the real valued function $f(x) = \frac{x}{e^x - 1} + \frac{x}{2} + 1$ is an even function on $R \backslash \{0\}$.

17.

a. Use mathematical induction to prove that $49^n + 16n - 1$ is divisible by 64 for all positive integers n.

b. Show that $\begin{vmatrix} b+c & c+a & a+b \\ a+b & b+c & c+a \\ a & b & c \end{vmatrix} = a^3 + b^3 + c^3 - 3abc.$.

18.

a. Solve the system of equation by Matrix Inversion Method:

$$2x - y + 3z = 8$$
$$-x + 2y + z = 4$$
$$3x + y - 4z = 0$$

b. If $A + B + C = \pi$, then prove that

$$\cos^2\frac{A}{2} + \cos^2\frac{B}{2} + \cos^2\frac{C}{2} = 2\left(1 + \sin\frac{A}{2}.\sin\frac{B}{2}.\sin\frac{C}{2}\right)$$

19.

a. Prove that $sin^4\frac{\pi}{8} + sin^4\frac{3\pi}{8} + sin^4\frac{5\pi}{8} + sin^4\frac{7\pi}{8} = \frac{3}{2}$.

b. Solve $sinx + sin2x + sin3x = cosx + cos2x + cos3x$.

20.

a. Show that the solutions of $cosp\theta + cosq\theta = 0, p \neq \pm q$ forms two series and each of which is in arithmetic progression and also find the common difference of each arithmetic progression.

b. If $Sin^{-1}x + Sin^{-1}y + Sin^{-1}z = \pi$ then prove that
$$x^4 + y^4 + z^4 + 4x^2y^2z^2 = 2(x^2y^2 + y^2z^2 + z^2x^2)$$

21.

a. Prove that $tan\left[\frac{\pi}{4} + \frac{1}{2}cos^{-1}\left(\frac{a}{b}\right)\right] + tan\left[\frac{\pi}{4} - \frac{1}{2}cos^{-1}\left(\frac{a}{b}\right)\right] = \frac{2b}{a}$.

b. For any $x \in [1, \infty)$ prove that $\cosh^{-1}x = log_e(x + \sqrt{x^2 - 1})$