



Data Mining

Dynamic Time Warping

Richard Dirauf, M.Sc. Machine Learning and Data Analytics (MaD) Lab Friedrich-Alexander-Universität Erlangen-Nürnberg (FAU) MLTS Exercise, 05.12.2024

MLTS Exercise – Organization



- - - Holiday

Introduction (31.10.2024) Dynamic Time Warping (12.12.2024)

Bayesian Linear Regression (07.11.2024) No exercise planned (19.12.2024)

Bayesian Linear Regression (14.11.2024) RNN + LSTM (09.01.2025)

Kalman Filter (21.11.2024) RNN + LSTM (16.01.2025)

Kalman Filter (28.11.2024) Transformers (23.01.2025)

Dynamic Time Warping (05.12.2024)Transformers (30.01.2025)

Data Mining Lecture





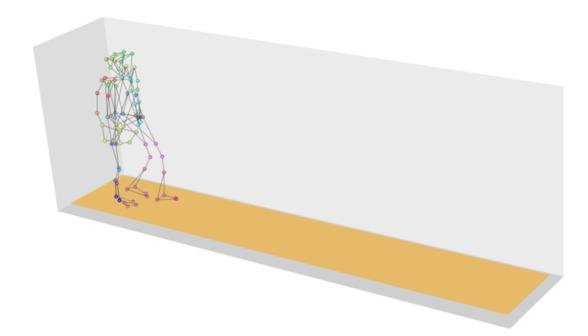
- Data characteristics, reduction, transformation
- Frequency analysis
 - Spectral analysis
 - Continuous Wavelet Transform
- Dynamic Time Warping
- Similarity join, Matrix profile, Signature method





In many application, there is the need to analyze multiple time series at the same time to find similarities between time series.

• E.g., speech recognition, signature recognition, similarity in walking, ...



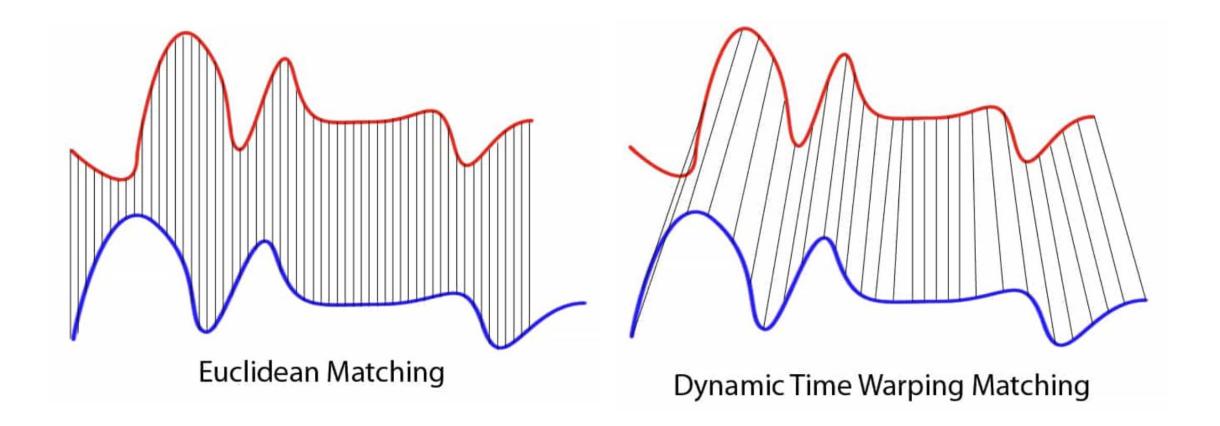
https://en.wikipedia.org/wiki/Motion_capture





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Euclidean distance does not work for time series that are not perfectly synchronized.

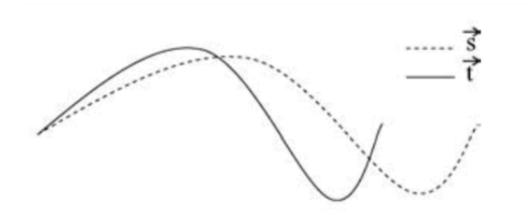






Dynamic time warping (DTW) is an algorithm to measure similarity between two timeseries that may vary in speed and time.

DTW determines the optimal global alignment between two time series.





Use cases

- Financial markets comparing stock trading data over similar time frames. For example, comparing monthly trading data for February (28 days) and March (31 days).
- Wearable fitness trackers more accurately calculating a walker's speed and the number of steps, even if their speed varied over time.
- Route calculation calculating more accurate information about a driver's ETA, if we
 know something about their driving habits (for example, they drive quickly on
 straight paths but take more time than average to make left turns).

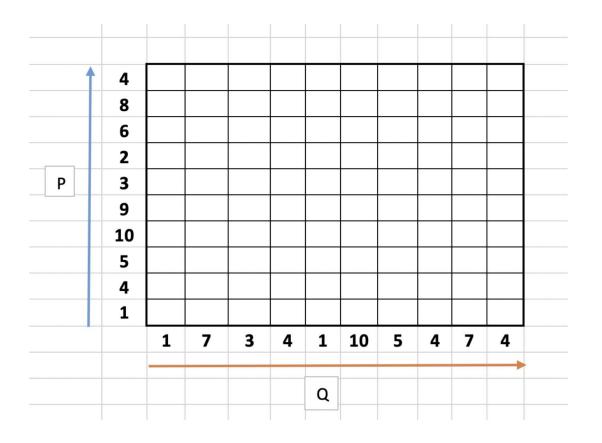
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$$M(i,j) = dist(x_i, y_j) + min(M(i-1, j-1), M(i, j-1), M(i-1, j))$$

5 7 5 5 5	9 24 10	P	P 3 26 14
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3. Identify the warping path d, starting from the top right corner

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2	27	19	11	12	12	20	14	13	17	18
3	26	14	10	11	13	16	11	12	16	17
9	24	10	14	16	19	9	12	17	18	21
10	16	8	12	11	14	8	13	18	16	21
5	7	5	5	5	8	12	12	13	15	16
4	3	3	4	4	7	13	14	14	17	17
1	0	6	8	11	11	20	24	27	33	36
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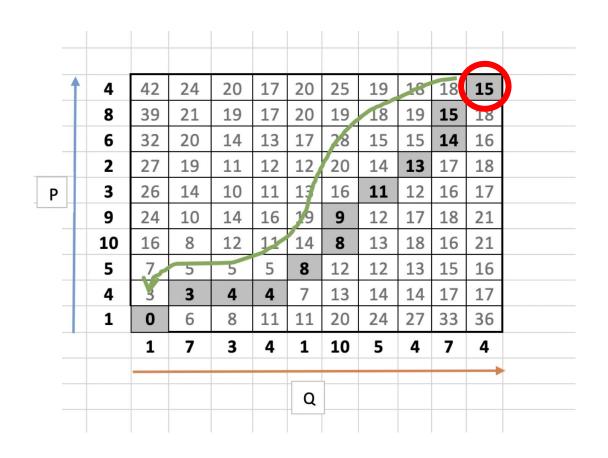
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- 3. Identify the warping path d, starting from the top right corner
- 4. Calculate the final distance *D* as

$$D = \sum_{(i,j)\in d} dist(x_i, y_j)$$



Dynamic time warping: Pros and Cons



Pros:

Exploit a non-linear distortion (in time) to find non-trivial similarity

Cons:

- High computational cost. Alternatives for computing the alignment path more efficiently have been presented.
- It needs the preparation of reliable reference templates for the set of words to be recognized.

Practice Question





Dynamic Time Warping Task

- Given the two time-series
 P = [1, 9, 5, 7] and Q = [3, 8, 2, 4]
- Fill the distance matrix M
- Identify the warping path d
- Calculate the final distance D along the warping path

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