Reinforcement Learning HW1 Part 2

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(1) Value Iteration

The optimal value function is:

[379. 273.301 304.988 258.636 195.604 273.301 174.582 206.951
244.705 195.604 304.988 206.951 174.582 195.604 174.582 258.636
400. 288.738 322.092 273.301 359.05 258.636 288.738 244.705
206.951 288.738 184.823 218.896 231.469 184.823 288.738 195.604
184.823 206.951 184.823 273.301 379. 304.988 304.988 288.738
304.988 218.896 244.705 206.951 244.705 340.097 218.896 258.636
218.896 174.582 273.301 184.823 195.604 218.896 195.604 288.738
322.092 359.05 288.738 304.988 288.738 206.951 231.469 195.604
258.636 359.05 231.469 273.301 206.951 164.853 258.636 174.582
206.951 231.469 206.951 304.988 304.988 379. 273.302 322.092
273.302 195.604 218.896 184.823 273.302 379. 244.705 288.738
195.604 155.61 244.705 164.853 195.604 218.896 195.604 288.738
288.738 400. 258.636 304.988 359.05 258.636 288.738 244.705
206.951 288.738 184.823 218.896 258.636 206.951 322.092 218.896
184.823 206.951 184.823 273.301 379. 304.988 340.097 288.738
340.097 244.705 273.301 231.469 218.896 304.988 195.604 231.469
244.705 195.604 304.988 206.951 195.604 218.896 195.604 288.738
359.05 322.092 322.092 304.988 322.092 231.469 258.636 218.896
231.469 322.092 206.951 244.705 231.469 184.823 288.738 195.604
206.951 231.469 206.951 304.988 340.097 340.097 304.988 322.092
304.988 218.896 244.705 206.951 244.705 340.097 218.896 258.636
218.896 174.582 273.302 184.823 218.896 244.705 218.896 322.092
322.092 359.05 288.738 340.097 288.738 206.951 231.469 195.604
258.636 359.05 231.469 273.301 206.951 164.853 258.636 174.582
206.951 231.469 206.951 304.988 304.988 379. 273.302 322.092
340.097 244.705 273.301 231.469 195.604 273.301 174.582 206.951
273.302 218.896 340.097 231.469 195.604 218.896 195.604 288.738
359.05 288.738 359.05 304.988 322.092 231.469 258.636 218.896
206.951 288.738 184.823 218.896 258.636 206.951 322.092 218.896

206.951 231.469 206.951 304.988 340.097 304.988 340.097 322.092
304.988 218.896 244.705 206.951 218.896 304.988 195.604 231.469
244.705 195.604 304.988 206.951 218.896 244.705 218.896 322.092
322.092 322.092 322.092 340.097 288.738 206.951 231.469 195.604
231.469 322.092 206.951 244.705 231.469 184.823 288.738 195.604
231.469 258.636 231.469 340.097 304.988 340.097 304.988 359.05
273.302 195.604 218.896 184.823 244.705 340.097 218.896 258.636
218.896 174.582 273.302 184.823 218.896 244.705 218.896 322.092
288.738 359.05 288.738 340.097 322.092 231.469 258.636 218.896
184.823 258.636 164.853 195.604 288.738 231.469 359.05 244.705
184.823 206.951 184.823 273.301 340.097 273.302 379. 288.738
304.988 218.896 244.705 206.951 195.604 273.302 174.582 206.951
244.705 195.604 304.988 206.951 195.604 218.896 195.604 288.738
322.092 288.738 322.092 304.988 288.738 206.951 231.469 195.604
206.951 288.738 184.823 218.896 231.469 184.823 288.738 195.604
206.951 231.469 206.951 304.988 304.988 304.988 304.988 322.092
273.302 195.604 218.896 184.823 218.896 304.988 195.604 231.469
218.896 174.582 273.302 184.823 244.705 273.302 244.705 359.05
288.738 322.092 288.738 379. 258.636 184.823 206.951 174.582
231.469 322.092 206.951 244.705 206.951 164.853 258.636 174.582
231.469 258.636 231.469 340.097 273.302 340.097 273.302 359.05
304.988 218.896 244.705 206.951 174.582 244.705 155.61 184.823
304.988 244.705 379. 258.636 174.582 195.604 174.582 258.636
322.092 258.636 400. 273.301 288.738 206.951 231.469 195.604
184.823 258.636 164.853 195.604 231.469 184.823 288.738 195.604
184.823 206.951 184.823 273.301 304.988 273.302 304.988 288.738
273.302 195.604 218.896 184.823 195.604 273.302 174.582 206.951
218.896 174.582 273.302 184.823 195.604 218.896 195.604 288.738
288.738 288.738 288.738 304.988 258.636 184.823 206.951 174.582
206.951 288.738 184.823 218.896 206.951 164.853 258.636 174.582
258.636 288.738 258.636 379. 273.302 304.988 273.302 400.
244.705 174.582 195.604 164.853 218.896 304.988 195.604 231.469
195.604 155.61 244.705 164.853 244.705 273.302 244.705 359.05
258.636 322.092 258.636 379.]

The optimal policy function is:

[4 4 4 4 0 0 0 0 0 0 0 0 0 0 0 0 5 0 0 0 3 3 3 3 0 0 0 0 0 0 0 0 0 0 0 3
0 0 0 0 0 0 0 2 2 2 2 0 0 0 0 0 0 0 0 0 2 0 0 0 0 0 0 2 2 2 2 0 0 0 0 0 0
0 0 0 2 0 0 0 0 0 0 4 4 4 4 0 0 0 0 0 0 0 0 5 0 0 1 1 1 1 2 2 2 2 0 0 0
0 0 0 0 0 1 2 0 0 1 1 1 1 2 2 2 2 0 0 0 0 0 0 0 0 1 2 0 0 3 3 3 3 2 2 2 2
0 0 0 0 0 0 0 0 3 2 0 0 3 3 3 3 2 2 2 2 0 0 0 0 0 0 0 0 3 2 0 0 3 3 3 3 1
1 1 1 0 0 0 0 0 0 0 0 0 3 1 0 0 1 1 1 1 2 2 2 2 0 0 0 0 2 2 2 2 1 2 0 2 1 1
1 1 2 2 2 2 3 3 3 3 2 2 2 2 1 2 3 2 1 1 1 1 2 2 2 2 3 3 3 3 2 2 2 2 1 2 3
2 1 1 1 1 2 2 2 2 3 3 3 3 0 0 0 0 1 2 3 0 1 1 1 1 1 1 1 1 3 3 3 3 0 0 0 0

1 1 3 0 1 1 1 1 1 1 1 0 0 0 0 1 1 1 1 1 1 0 1 1 1 1 1 2 2 2 2 1 1 1 1 2
2 2 2 1 2 1 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 1 1
1 1 0 0 0 0 1 2 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 1 1 1 0 1 1 1 1 1 1 1
1 4 4 4 4 1 1 1 1 1 1 5 1 1 1 1 1 2 2 2 2 1 1 1 1 2 2 2 2 1 2 1 2 1 1 1 1
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 1 1 1 1 4 4 4 4 1 2 1 5 1
1 1 1 1 1 1 1 1 1 1 1 3 3 3 3 1 1 1 3]

The average rewards for running 100 episodes using optimal policy is:

8.414141414141413

(2) Policy Iteration

The optimal value function is:

[379. 273.301 304.988 258.636 195.604 273.301 174.582 206.951
244.705 195.604 304.988 206.951 174.582 195.604 174.582 258.636
400. 288.738 322.092 273.301 359.05 258.636 288.738 244.705
206.951 288.738 184.823 218.896 231.469 184.823 288.738 195.604
184.823 206.951 184.823 273.301 379. 304.988 304.988 288.738
304.988 218.896 244.705 206.951 244.705 340.097 218.896 258.636
218.896 174.582 273.301 184.823 195.604 218.896 195.604 288.738
322.092 359.05 288.738 304.988 288.738 206.951 231.469 195.604
258.636 359.05 231.469 273.301 206.951 164.853 258.636 174.582
206.951 231.469 206.951 304.988 304.988 379. 273.302 322.092
273.302 195.604 218.896 184.823 273.302 379. 244.705 288.738
195.604 155.61 244.705 164.853 195.604 218.896 195.604 288.738
288.738 400. 258.636 304.988 359.05 258.636 288.738 244.705
206.951 288.738 184.823 218.896 258.636 206.951 322.092 218.896
184.823 206.951 184.823 273.301 379. 304.988 340.097 288.738
340.097 244.705 273.301 231.469 218.896 304.988 195.604 231.469
244.705 195.604 304.988 206.951 195.604 218.896 195.604 288.738
359.05 322.092 322.092 304.988 322.092 231.469 258.636 218.896
231.469 322.092 206.951 244.705 231.469 184.823 288.738 195.604
206.951 231.469 206.951 304.988 340.097 340.097 304.988 322.092
304.988 218.896 244.705 206.951 244.705 340.097 218.896 258.636
218.896 174.582 273.302 184.823 218.896 244.705 218.896 322.092
322.092 359.05 288.738 340.097 288.738 206.951 231.469 195.604
258.636 359.05 231.469 273.301 206.951 164.853 258.636 174.582
206.951 231.469 206.951 304.988 304.988 379. 273.302 322.092
340.097 244.705 273.301 231.469 195.604 273.301 174.582 206.951
273.302 218.896 340.097 231.469 195.604 218.896 195.604 288.738
359.05 288.738 359.05 304.988 322.092 231.469 258.636 218.896
206.951 288.738 184.823 218.896 258.636 206.951 322.092 218.896
206.951 231.469 206.951 304.988 340.097 304.988 340.097 322.092

304.988 218.896 244.705 206.951 218.896 304.988 195.604 231.469
244.705 195.604 304.988 206.951 218.896 244.705 218.896 322.092
322.092 322.092 322.092 340.097 288.738 206.951 231.469 195.604
231.469 322.092 206.951 244.705 231.469 184.823 288.738 195.604
231.469 258.636 231.469 340.097 304.988 340.097 304.988 359.05
273.302 195.604 218.896 184.823 244.705 340.097 218.896 258.636
218.896 174.582 273.302 184.823 218.896 244.705 218.896 322.092
288.738 359.05 288.738 340.097 322.092 231.469 258.636 218.896
184.823 258.636 164.853 195.604 288.738 231.469 359.05 244.705
184.823 206.951 184.823 273.301 340.097 273.302 379. 288.738
304.988 218.896 244.705 206.951 195.604 273.302 174.582 206.951
244.705 195.604 304.988 206.951 195.604 218.896 195.604 288.738
322.092 288.738 322.092 304.988 288.738 206.951 231.469 195.604
206.951 288.738 184.823 218.896 231.469 184.823 288.738 195.604
206.951 231.469 206.951 304.988 304.988 304.988 304.988 322.092
273.302 195.604 218.896 184.823 218.896 304.988 195.604 231.469
218.896 174.582 273.302 184.823 244.705 273.302 244.705 359.05
288.738 322.092 288.738 379. 258.636 184.823 206.951 174.582
231.469 322.092 206.951 244.705 206.951 164.853 258.636 174.582
231.469 258.636 231.469 340.097 273.302 340.097 273.302 359.05
304.988 218.896 244.705 206.951 174.582 244.705 155.61 184.823
304.988 244.705 379. 258.636 174.582 195.604 174.582 258.636
322.092 258.636 400. 273.301 288.738 206.951 231.469 195.604
184.823 258.636 164.853 195.604 231.469 184.823 288.738 195.604
184.823 206.951 184.823 273.301 304.988 273.302 304.988 288.738
273.302 195.604 218.896 184.823 195.604 273.302 174.582 206.951
218.896 174.582 273.302 184.823 195.604 218.896 195.604 288.738
288.738 288.738 288.738 304.988 258.636 184.823 206.951 174.582
206.951 288.738 184.823 218.896 206.951 164.853 258.636 174.582
258.636 288.738 258.636 379. 273.302 304.988 273.302 400.
244.705 174.582 195.604 164.853 218.896 304.988 195.604 231.469
195.604 155.61 244.705 164.853 244.705 273.302 244.705 359.05
258.636 322.092 258.636 379.]

The optimal policy function is:

[4 4 4 4 0 0 0 0 0 0 0 0 0 0 0 0 5 0 0 0 3 3 3 3 0 0 0 0 0 0 0 0 0 0 0 3
0 0 0 0 0 0 0 2 2 2 2 0 0 0 0 0 0 0 0 0 2 0 0 0 0 0 0 2 2 2 2 0 0 0 0 0 0
0 0 0 2 0 0 0 0 0 0 4 4 4 4 0 0 0 0 0 0 0 0 0 5 0 0 1 1 1 1 2 2 2 2 0 0 0
0 0 0 0 0 1 2 0 0 1 1 1 1 2 2 2 2 0 0 0 0 0 0 0 0 1 2 0 0 3 3 3 3 2 2 2 2
0 0 0 0 0 0 0 0 3 2 0 0 3 3 3 3 2 2 2 2 0 0 0 0 0 0 0 0 3 2 0 0 3 3 3 3 1
1 1 1 0 0 0 0 0 0 0 0 0 3 1 0 0 1 1 1 1 2 2 2 2 0 0 0 0 2 2 2 2 1 2 0 2 1 1
1 1 2 2 2 2 3 3 3 3 2 2 2 2 1 2 3 2 1 1 1 1 2 2 2 2 3 3 3 3 2 2 2 2 1 2 3
2 1 1 1 1 2 2 2 2 3 3 3 3 0 0 0 0 1 2 3 0 1 1 1 1 1 1 1 1 3 3 3 3 0 0 0 0
1 1 3 0 1 1 1 1 1 1 1 1 0 0 0 0 1 1 1 1 1 1 0 1 1 1 1 1 2 2 2 2 1 1 1 1 2

2 2 2 1 2 1 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 1 1
1 1 0 0 0 0 1 2 1 0 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 1 1 1 0 1 1 1 1 1 1 1
1 4 4 4 4 1 1 1 1 1 1 5 1 1 1 1 1 2 2 2 2 1 1 1 1 2 2 2 2 1 2 1 2 1 1 1 1
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 1 1 1 1 4 4 4 4 1 2 1 5 1
1 1 1 1 1 1 1 1 1 1 1 3 3 3 3 1 1 1 3]

The average rewards for running 100 episodes using optimal policy is:

8.676767676767676

(3) Discussion

Through our program, we can find that in policy iteration, the policy evaluation and policy improvement are executed 17 times. And in policy evaluation, except for the first time of execution, every time the function will iterate 284 times. In value iteration, it also cost 284 times to converge.

It is obvious to find that the final value function and optimal policy function of those two algorithms are the same.

2

(4) Q-learning

The Q value array of each (s, a) pair is:

```
[[  1.57695811e+01   1.38977082e+01  -1.97588113e-01   1.22927318e+01
    2.24964869e+02   1.96495207e+01]
 [ -2.01874904e+00  -2.07173833e+00  -2.03768696e+00   3.59134895e-01
    5.55325646e+01  -2.96810310e+00]
 [ -1.27313426e+00   7.66540215e+00  -1.28375812e+00   4.52851756e+00
    1.22922777e+02  -1.97468322e+00]
 ...,
 [ -9.77797391e-01  -8.75268062e-01  -9.77797391e-01  -1.06100615e+00
   -2.85106829e+00  -1.95851084e+00]
 [ -2.26251646e+00  -2.29003948e+00  -2.34404189e+00  -2.33294893e+00
   -2.91536208e+00  -2.97352331e+00]
 [ -1.99500000e-01  -1.99500000e-01  -1.99500000e-01   9.52935511e+01
   -1.00000000e+00  -1.00000000e+00]]
```

(too long to show all of those)

(5) SARSA

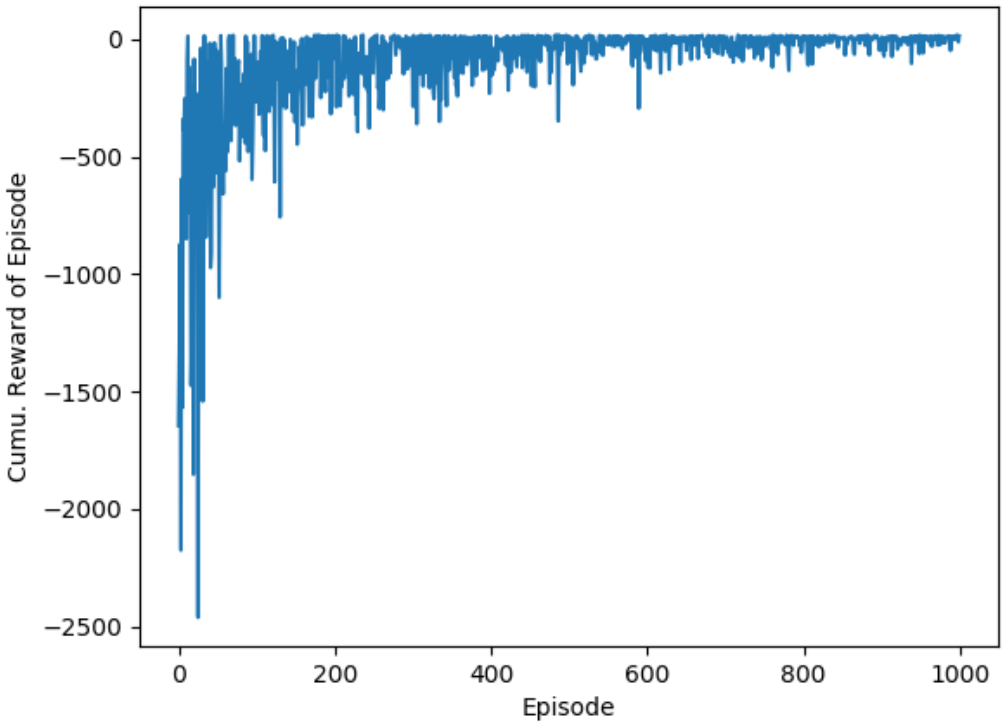
The Q value array of each (s, a) pair is:

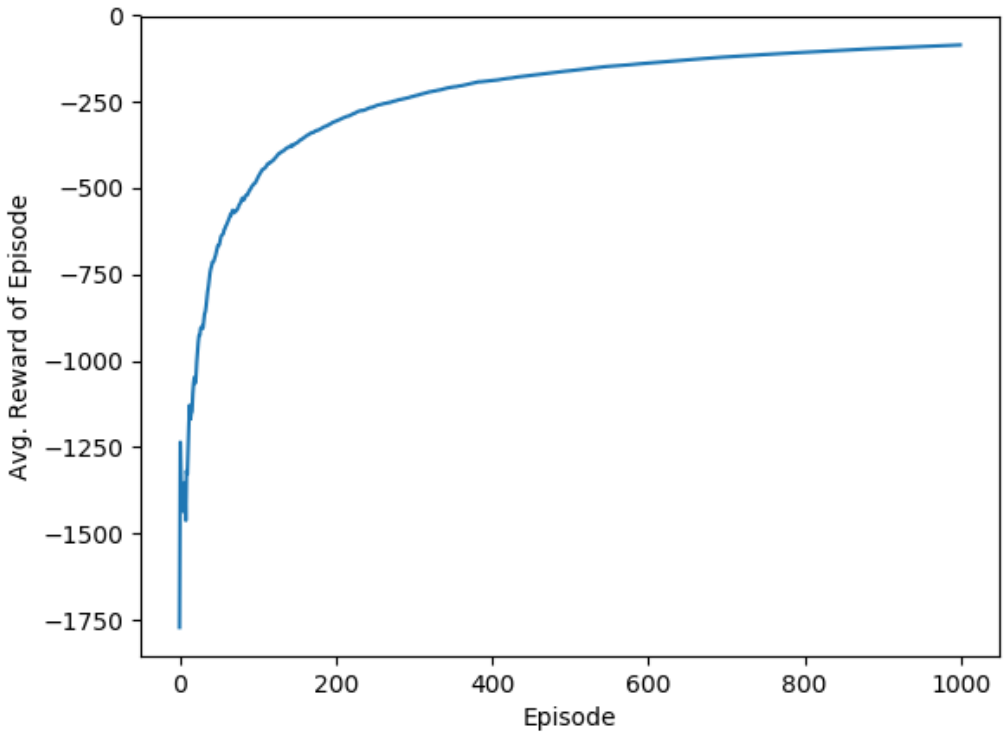
```
[[-1.36657484 -1.29722863 -1.30307844 -1.15618847 -1.09954938 -2.81857346]
 [-5.29191273 -5.25848374 -5.37791735 -5.28929674 -5.26014069 -9.37054361]
 [-4.76142261 -4.79852202 -4.78706306 -4.78450661 -4.69004058 -7.96759454]
 ...,
 [-3.59128127 -3.59226594 -3.66037699 -3.61907009 -6.74396321 -6.26997637]
 [-6.02201626 -6.02959142 -5.98894215 -5.97649343 -8.23659351 -6.61345698]
 [-1.04011687 -1.131997   -1.11934517 -0.28194184 -1.94485187 -2.7366    ]]
```

(too long to show all of those)

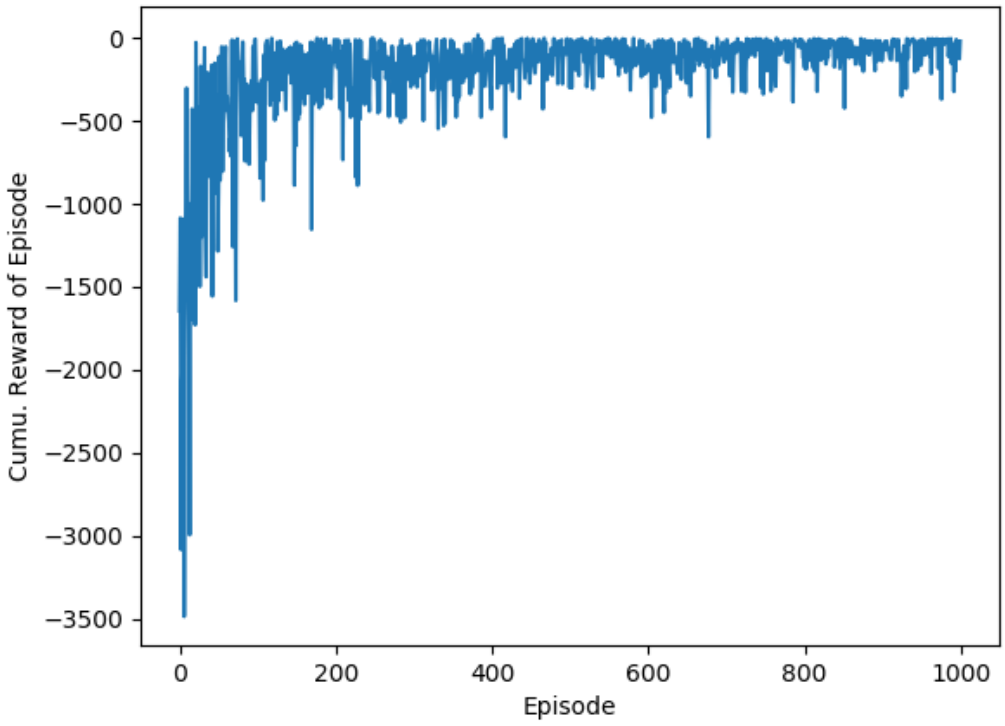
(6) Plot of the learning progress for training 1000 episodes

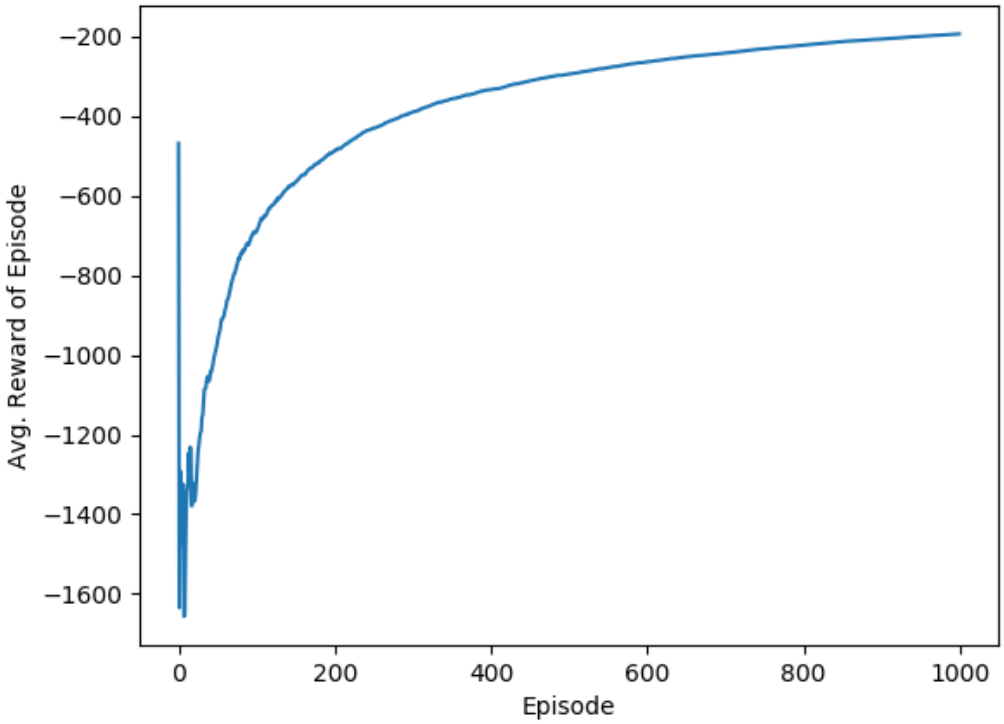
Plot of the learning progress of Q-learning:





Plot of the learning progress of SARSA:

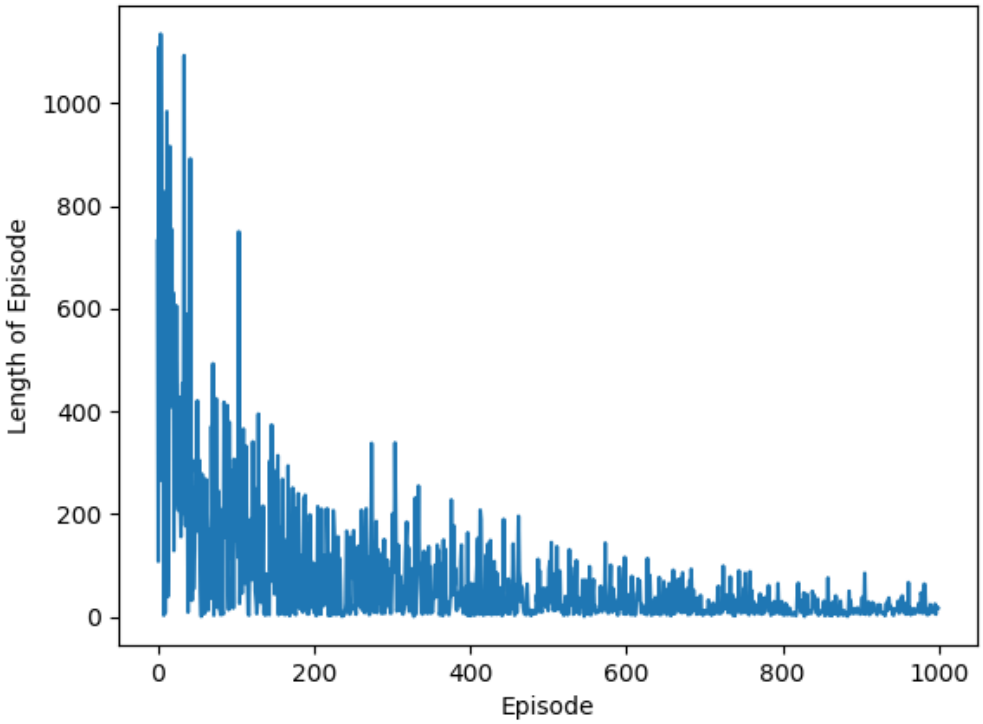




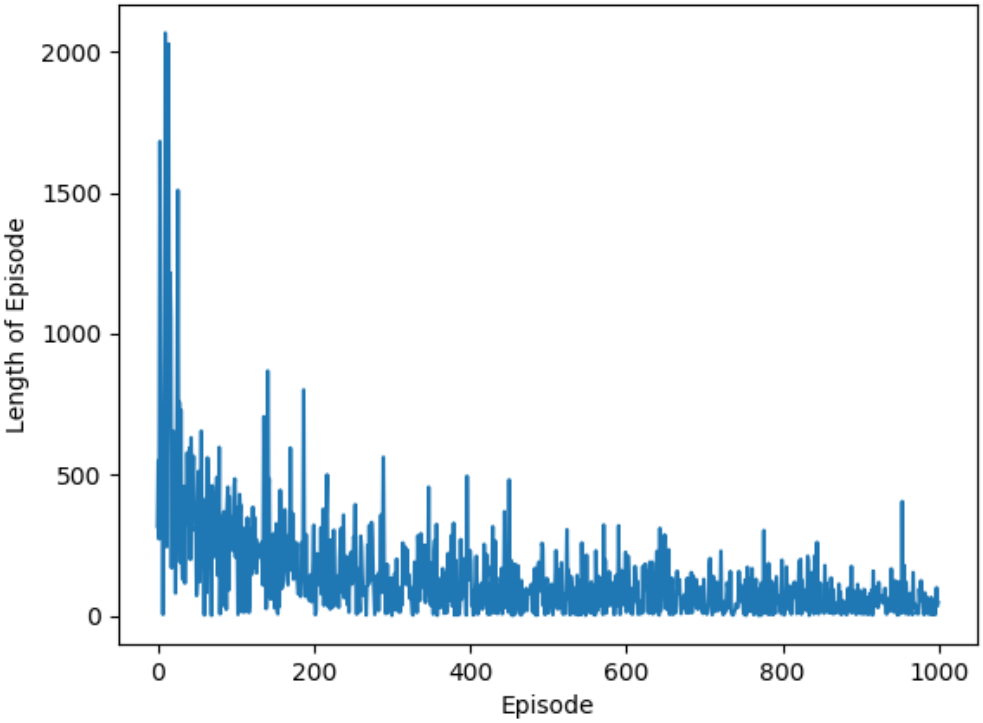
Discussion: From those four plots above, we can find that Q-learning algorithm performs better. In comparison, the Q-learning converges better and more stable in this condition.

(7) Plot of the episode length for training 1000 episodes

Plot of the episode length of Q-learning:



Plot of the episode length of SARSA:



Discussion: From those two plots above, we can find that the episode length of both algorithm also converges. And similar to accumulated reward of episode, the episode length convergence progress of Q-learning is also more stable and have better performance than the progress of SARSA.