Numbering Systems: Numeric weighted representations $b^{n-1}\, b^{n-2} \ldots b^4\, b^3\, b^2\, b^1\, b^0$

Decimal (base 10)	Binary (base 2)	Hexadecimal (base 16)	Octal (base 8)
Digits: 0123456789	01	0123456789ABCDEF	01234567

5-bit: $2^4 2^3 2^2 2^1 2^0$

Decimal	Binary	Hexadecimal	Octal	Decimal	Binary	Hexadecimal	Octal
0	00000	00	000	16	10000	10	020
1	00001	01	001	17	10001	11	021
2	00010	02	002	18	10010	12	022
3	00011	03	003	19	10011	13	023
4	00100	04	004	20	10100	14	024
5	00101	05	005	21	10101	15	025
6	00110	06	006	22	10110	16	026
7	00111	07	007	23	10111	17	027
8	01000	08	010	24	11000	18	030
9	01001	09	011	25	11001	19	031
10	01010	0A or 0a	012	26	11010	1A or 1a	032
11	01011	0B or 0b	013	27	11011	1B or 1b	033
12	01100	0C or 0c	014	28	11100	1C or 1c	034
13	01101	0D or 0d	015	29	11101	1D or 1d	035
14	01110	0E or 0e	016	30	11110	1E or 1e	036
15	01111	0F or of	017	31	11111	1F or 1f	037
				32	**100000**	**20**	**040**

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Binary-Decimal Conversions

- Convert from binary to decimal

Each binary bit has a power-of-2 weighting. Add 'em up!

$2^6 \quad 2^3 \quad 2^0$

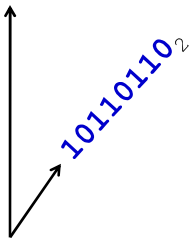
$10110110_2 = 2^7+0+2^5+2^4+0+2^2+2^1+0 = 128+32+16+4+2 = 182_{10}$

- Convert from decimal to binary

Repeatedly divide by 2.

Sequence of remainders (0 or 1) is binary number.

```
182 ÷ 2 = 91 r. 0
 91 ÷ 2 = 45 r. 1
 45 ÷ 2 = 22 r. 1
 22 ÷ 2 = 11 r. 0
 11 ÷ 2 =  5 r. 1
  5 ÷ 2 =  2 r. 1
  2 ÷ 2 =  1 r. 0
  1 ÷ 2 =  0 r. 1
```



10110110_2

2

Powers of 2

2^n	Decimal	Notes	HEX	BINARY
2^0	1		0001	00000001
2^1	2		0002	00000010
2^2	4		0004	00000100
2^3	8		0008	00001000
2^4	16	nybble	0010	00010000
2^5	32		0020	00100000
2^6	64		0040	01000000
2^7	128		0080	10000000
2^8	256	byte	0100	00000001-00000000
2^9	512		0200	00000010-00000000
2^{10}	1,024	**1KB**	0400	00000100-00000000
2^{12}	4,096	4K	1000	00010000-00000000
2^{16}	65,536	**64K**	00010000	00000001-00000000-00000000
2^{20}	1,048,576	**1MB**	00100000	00000000-00001000-00000000-00000000
2^{24}	16,777,216		01000000	00000001-00000000-00000000-00000000
2^{30}	1,073,741,824	**1GB**	40000000	01000000-00000000-00000000-00000000
2^{32}	4,294,967,296	4GB	100000000	1-00000000-00000000-00000000-00000000

2^{48}	70,368,744,177,664	10^{13}	Hex: 0000-4000-0000-0000
2^{64}	1,152,921,504,606,846,976	10^{18}	Hex: 1-0000-0000-0000-0000

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Powers of 2 - KB, MB, GB

2^n	Decimal	Names	HEX
2^0	1		0001
2^1	2		0002
2^2	4		0004
2^3	8		0008
2^4	16	nybble	0010
2^5	32		0020
2^6	64		0040
2^7	128		0080
2^8	256	byte	0100
2^9	512		0200
2^{10}	1,024	**1KB**	0400
2^{12}	4,096	4K	1000
2^{16}	65,536	**64K**	00010000
2^{20}	1,048,576	**1MB**	00100000
2^{24}	16,777,216		01000000
2^{30}	1,073,741,824	**1GB**	40000000
2^{32}	4,294,967,296	4GB	100000000

Observations

2^4 = 1 nybble → 16 → 4 bits → 1 hex digit

2^8 = 1 byte → 256 → 16^2 → 2 hex digits

2^{16} = 2 bytes → 64K → $2^8 * 2^8$ → 256^2

2^{24} = 3 bytes → 16M

2^{32} = 4 bytes → 4GB

2^{64} = 8 bytes → $(4GB)^2$ = 16 "billion-billion " = really huge

2^{128} = really-really-huge → WIFI encryption key → 10^{36} possible keys

1,000,000 = 1000 * 1000 → 1 million

2^{10} = 1024 → 1KB

$2^{20} = 2^{10} * 2^{10}$ = 1024 * 1024 = 1,048,576

1MB = 1KB * 1KB

$2^{30} = 2^{10} * 2^{10} * 2^{10}$

1GB = 1KB * 1KB * 1KB

2^{48}	70,368,744,177,664	10^{13}	Hex: 0000-4000-0000-0000
2^{64}	1,152,921,504,606,846,976	10^{18}	Hex: 1-0000-0000-0000-0000

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