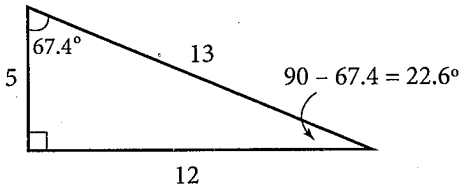
Check Your Work – Chapter 15

1. C

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: Find the unknown leg length and angle measure. The triangle is a right triangle with one leg length of 5 and a hypotenuse of 13, so the other leg is length 12. (If you didn't see the Pythagorean triple 5:12:13, you could have used the Pythagorean theorem to find the missing leg length.) Use the measures of the internal angles to find the missing angle. It may be helpful to sketch this figure out:



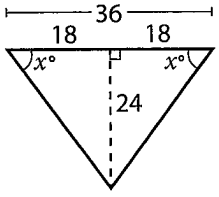
Sine and cosine both involve the hypotenuse, 13, so you can eliminate (A) and (D). Compare the remaining answer choices to the trig ratios given by SOHCAHTOA. Sine is opposite over hypotenuse, but the side opposite the 67.4° angle has length 12 (not 5), so eliminate (B). Only **(C)** is left and must be correct. For the record, the side adjacent to the 22.6° angle has length 12 and the hypotenuse has length 13, so $\cos 22.6° = \frac{5}{13}$.

2. 18/30, 3/5, 0.6, or .6

Difficulty: Hard

Category: Geometry and Trigonometry

Getting to the Answer: Because trig functions typically apply to right triangles, draw in an altitude and label what you know. You know the trough is 24 inches deep and 36 inches across the top. Because the given angles have equal measures, $x°$, the triangle is isosceles and the altitude bisects the top. Draw a figure:



You're given that $B = \cos x$, and the cosine of an angle involves the hypotenuse, so you need to find the length of the hypotenuse. The triangle is a 3:4:5 triple multiplied by 6: $3 \times 6 = 18$ and $4 \times 6 = 24$, so the hypotenuse must be $5 \times 6 = 30$. If you didn't notice the triple, you could find the hypotenuse using the Pythagorean theorem:

$$18^2 + 24^2 = c^2$$
$$324 + 576 = c^2$$
$$\sqrt{900} = \sqrt{c^2}$$
$$30 = c$$

Thus, $\cos x = \frac{\text{adj}}{\text{hyp}} = \frac{18}{30} = \frac{3}{5}$. Enter **18/30, 3/5, 0.6,** or **.6**.

3. 12/5 or 2.4

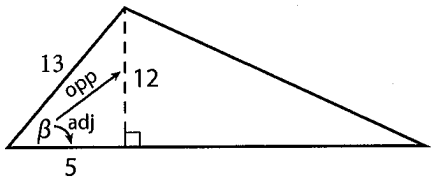
Difficulty: Hard

Category: Geometry and Trigonometry

Getting to the Answer: Find the height of the triangle using the information given about the area and add it to the figure:

$$A = \frac{1}{2}bh$$
$$240 = \frac{1}{2}(40)h$$
$$240 = 20h$$
$$12 = h$$

After you find the height, you might recognize the 5–12–13 Pythagorean triple, which gives you another side of the triangle that contains β:



Now use SOHCAHTOA: $\tan \beta = \frac{\text{opp}}{\text{adj}} = \frac{12}{5}$. Enter **12/5** or **2.4**.

4. 12

Difficulty: Hard

Category: Geometry and Trigonometry

Getting to the Answer: The fact that DE is parallel to BC means that triangles ABC and ADE are similar. Convert $\sin C$, 0.6, to a fraction, $\frac{3}{5}$. Because $\sin x$ is $\frac{\text{opposite}}{\text{hypotenuse}}$, both triangles have the side ratio 3:4:5. The question states that $AC = 16$. This is the long leg of a 3:4:5 right triangle multiplied by 4 (4×4), so $AB = 3 \times 4 = 12$ and $BC = 5 \times 4 = 20$.

The other known dimension is $BD = 3$. Since the length of AB is 12, the length of AD is $12 - 3 = 9$. Thus, the ratio of the sides of triangle ADE to those of triangle ABC is $\frac{9}{12} = \frac{3}{4}$. Therefore, AE is $\frac{3}{4}$ of AC, which is $\frac{3}{4} \times 16 = 12$. Enter **12**.

5. A

Difficulty: Hard

Category: Geometry and Trigonometry

Getting to the Answer: The sine of an angle is equal to the cosine of its complementary angle, so $\angle A + \angle C = 90°$. Since $\angle B$ is the third interior angle of the triangle ABC, $\angle B = 180° - 90° = 90°$. Therefore, the measures of angles ABD and DBC must total 90°, which means they are complementary angles. Thus, $\sin \angle ABD = \cos \angle DBC$, and $\sin \angle ABD - \cos \angle DBC = 0$. **(A)** is correct.

Check Your Work – How Much Have You Learned: Geometry and Trigonometry

1. B

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: Start by translating from English into math. Because the base of the triangle is $\frac{2}{3}$ the height of the triangle, $b = \frac{2}{3}h$. Use the Pythagorean theorem to find the hypotenuse:

$$h^2 + b^2 = c^2$$
$$h^2 + \left(\frac{2}{3}h\right)^2 = c^2$$
$$\frac{9}{9}h^2 + \frac{4}{9}h^2 = c^2$$
$$\frac{13}{9}h^2 = c^2$$
$$\sqrt{\frac{13}{9}h^2} = c$$
$$\frac{\sqrt{13}}{3}h = c$$

(B) is correct.

2. A

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: Because one angle of the triangle measures 90° and the two legs are congruent (notice the tick marks), this is a 45-45-90 triangle. The side lengths of a 45-45-90 triangle are in the ratio $x{:}x{:}x\sqrt{2}$, where x represents the length of a leg and $x\sqrt{2}$ represents the length of the hypotenuse. Set up an equation using the ratio and the length of the hypotenuse, $\sqrt{18}$, to find the length of each leg:

$$x\sqrt{2} = \sqrt{18}$$
$$x\sqrt{2} = \sqrt{9}\sqrt{2}$$
$$x\sqrt{2} = 3\sqrt{2}$$
$$x = 3$$

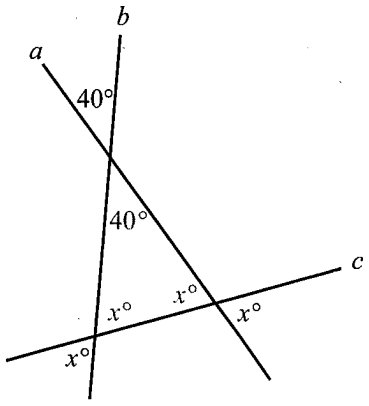
The length of each leg is 3, so **(A)** is correct.

3. 70

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: The angle in the triangle that is vertical to 40° is also 40°, and the angles in the triangle vertical to $x°$ are also $x°$. Thus, the triangle formed by lines a, b, and c is an isosceles triangle. Since angles in a triangle sum to 180°, $40° + x° + x° = 180°$, and $x° = 70°$.



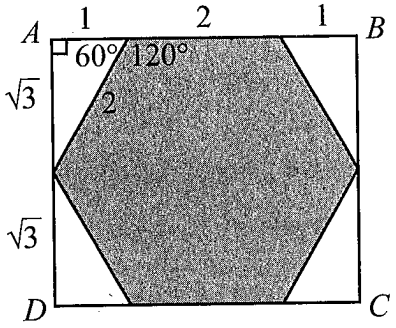
Enter **70**.

4. C

Difficulty: Hard

Category: Geometry and Trigonometry

Getting to the Answer: Start with the shaded hexagon. One way to determine the sum of the interior angles of the hexagon is to use $180°(n - 2)$, where n represents the number of sides of the polygon. Since the hexagon has six sides, the sum of the interior angles equals $180°(6 - 2) = 720°$. Thus, each angle is $720° \div 6 = 120°$. (You could also note that a hexagon can be thought of as made of 6 smaller equilateral triangles whose angles are all 60°. Two of those angles form the same 120°.) The angle in the white right triangle supplementary to 120° is then $180° - 120° = 60°$, indicating the white triangles are 30-60-90 triangles with ratio side lengths of $x{:}x\sqrt{3}{:}2x$. The question tells you that each side length of the hexagon, which is also the hypotenuse of the triangles, is 2 units. Thus, the leg lengths of the triangle are 1 and $\sqrt{3}$.



You now have the length of the rectangle, $1 + 2 + 1 = 4$, and the width of the rectangle: $\sqrt{3} + \sqrt{3} = 2\sqrt{3}$. The area of the rectangle is therefore, $l \times w = 4 \times 2\sqrt{3} = 8\sqrt{3}$ square units, making **(C)** correct.

5. D

Difficulty: Easy

Category: Geometry and Trigonometry

Getting to the Answer: Corresponding angles of similar triangles are congruent. Therefore, the similar triangle will also have angles measuring 23° and 48° and $180° - (23° + 48°) = 109°$. **(D)** is correct.

6. D

Difficulty: Easy

Category: Geometry and Trigonometry

Strategic Advice: Whenever you're given a ratio, you can set up an equation. Sometimes the equation takes the form of a proportion and sometimes it takes the form of the sum of the parts equals the whole. In this question, the *whole* is the total number of degrees in a circle, which is 360.

Getting to the Answer: You know the relative size of each of the parts in this question. You don't know the exact size of one part, so call it x. Now, set up an equation:

$$4x + 3x + 2x = 360$$
$$9x = 360$$
$$x = 40$$

Note that the question asks for the measure of the smallest angle, which is represented by $2x$. The correct answer is $2(40) = 80$, which is **(D)**.

7. D

Difficulty: Hard

Category: Geometry and Trigonometry

Strategic Advice: Note that the graph is not labeled with any units, which means that one square does not necessarily equal one unit.

Getting to the Answer: To find the equation of a circle, you need the radius and the x- and y-coordinates of the center point. Then, you can use the standard equation: $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center of the circle and r is the length of the radius.

Be careful—choice (A) is incorrect, but might appear correct if you assume that one square represents one unit. The graph has no number labels on it, so you'll need to use the information given in the question about the smaller circle to determine the value of each grid line:

$$A = \pi r^2$$
$$144\pi = \pi r^2$$
$$144 = r^2$$
$$\pm 12 = r$$

The radius can't be negative, so it must be 12. There are only 3 grid lines between the center of the smaller circle and its edge, so each grid line must be equal to $\frac{12}{3} = 4$ units. The center of the larger circle, therefore, is $(-8, 0)$, and its radius is $6 \times 4 = 24$. Thus, the equation must be $(x - (-8))^2 + (y - 0)^2 = 24^2$, or written in simplified form, $(x + 8)^2 + y^2 = 576$. **(D)** is the correct answer.

8. 3

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: Start by determining whether there is a relationship between triangles ABC and EDC. The two triangles share a common angle, C. It is given that $\overline{DE} \perp \overline{AC}$, so angle DEC is a right angle. Because $\overline{DE} \parallel \overline{AB}$, $\overline{BA}$ is also perpendicular to $\overline{AC}$, making angle BAC another right angle. Triangles that have two congruent interior angles are similar, so triangle ABC is similar to triangle EDC.

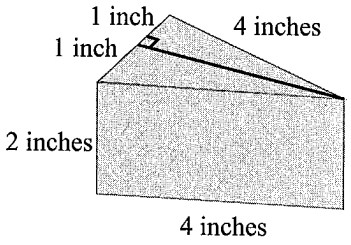
The length of AE is 28, which means the length of AC is $28 + 4 = 32$. The length of EC is 4, which means that the side lengths of triangle ABC are 8 times the side lengths of triangle EDC. Use this ratio to find DE: $\frac{24}{8} = 3$. Enter **3**.

9. D

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: To calculate the amount of wax needed to cover the piece of cheese, first determine the surface area of the cheese by summing the areas of the faces (one square, two rectangles, and two triangles). The square side of the cheese has an area of $s^2 = 2^2 = 4$, and the two rectangular sides of the cheese have an area of $2A = 2(lw) = 2(2)(4) = 16$. The bottom and top of the cheese are triangles; each can be split into two right triangles.



The height of the triangles can be found by splitting the triangle into two right triangles and then using the Pythagorean theorem: height $= \sqrt{4^2 - 1^2} = \sqrt{15}$. Thus, the surface area of the top and bottom triangles, which have a base of 2, is $2A_{\text{triangle}} = 2\left(\frac{1}{2}bh\right) = bh = (2)(\sqrt{15}) = 2\sqrt{15}$

The total surface area of the cheese is $4 + 16 + 2\sqrt{15} = 20 + 2\sqrt{15}$ square inches. Note that the question asks for the amount of wax needed to coat the piece of cheese. Use unit conversion to find the number of grams of wax needed:

$$20 + 2\sqrt{15}\ \cancel{\text{square inches}} \times \frac{2 \text{ grams}}{1\ \cancel{\text{square inches}}} = 40 + 4\sqrt{15} \text{ grams}$$

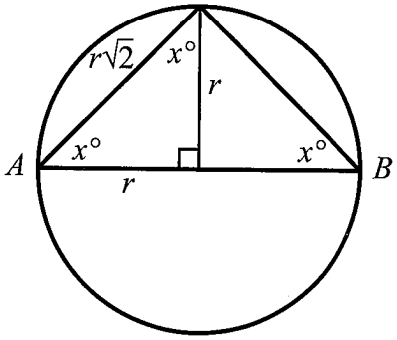
(D) is correct.

10. B

Difficulty: Hard

Category: Geometry and Trigonometry

Getting to the Answer: Although the triangle containing $x°$ does not have a right angle, it can be split into two 45-45-90 triangles. So, $x° = 45°$. You can use the side ratios of a 45-45-90 triangle to find cos 45°. That ratio is $r{:}r{:}r\sqrt{2}$.



Because $\cos x = \frac{\text{adjacent}}{\text{hypotenuse}}$, the $\cos x$ is $\frac{1}{\sqrt{2}}$.

(B) is correct.