Math

Check Your Work - How Much Do You Know: Advanced Math

1. D

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: With absolute value equations, you must solve twice: once for the positive version of the given value and once for the negative version of that value. Start by solving for the positive version:

$$\begin{aligned} 3x - 14 &= x + 4 \\ 3x &= x + 18 \\ 2x &= 18 \\ x &= 9 \end{aligned}$$

Next, solve for the negative version:

$$\begin{aligned} 3x - 14 &= -(x + 4) \\ 3x - 14 &= -x - 4 \\ 3x &= -x + 10 \\ 4x &= 10 \\ x &= 2.5 \end{aligned}$$

So both 2.5 and 9 are the values of x. **(D)** is correct.

2. 87

Difficulty: Easy

Category: Advanced Math

Getting to the Answer: Use FOIL to expand the factored form of the quadratic into standard form:

$$\begin{aligned} y &= 3(x + 5)^2 + 12 \\ &= 3(x + 5)(x + 5) + 12 \\ &= 3(x^2 + 5x + 5x + 25) + 12 \\ &= 3(x^2 + 10x + 25) + 12 \\ &= 3x^2 + 30x + 75 + 12 \\ &= 3x^2 + 30x + 87 \end{aligned}$$

The question asks for the value of c, so enter **87**.

3. A

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: Write each factor in the expression in exponential form and use exponent rules to simplify the expression. The number 4 is being multiplied by the variables, which will not go into the denominator. Eliminate (D). The power of a under the radical is 1 and the root is 3, so a is raised to the power of $\frac{1}{3}$. Eliminate (B) and (C). Only **(A)** is left and is correct. On test day, you would move on, but for the record: the power of b is 9 and the root is 3, so the exponent on b is $\frac{9}{3} = 3$. Therefore, $4\sqrt[3]{ab^9} = 4a^{\frac{1}{3}}b^3$. **(A)** is indeed correct.

4. B

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: When you see decimals that do not simplify, change them to an equivalent form, such as fractions. Then simplify:

$$\sqrt{0.75} \times \sqrt{0.8} = \sqrt{\frac{3}{4}} \times \sqrt{\frac{4}{5}} = \sqrt{\frac{3}{\cancel{4}} \times \frac{\cancel{4}}{5}} = \sqrt{\frac{3}{5}} = \frac{\sqrt{3}}{\sqrt{5}}$$

A radical is not allowed in the denominator of a fraction, so rationalize it by multiplying the numerator and the denominator by the same radical that you are trying to rationalize: $\frac{\sqrt{3}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{15}}{5}$. Alternatively, you can evaluate the expression in the question on your calculator, 0.77459..., and then evaluate the expressions in the answer choices until you find a match. This might, however, consume time that could be better spent on other questions. Either way, **(B)** is correct.

5. −2

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: The axis of symmetry of a parabola always passes through the x-coordinate of the parabola's vertex. The trick for finding the x-coordinate of the vertex is to calculate $\frac{-b}{2a}$ (the quadratic formula without the radical part). In the equation, $a = 3$ and $b = 12$, so the equation of the axis of symmetry is $x = \frac{-(12)}{2(3)} = \frac{-12}{6} = -2$. In the question, the equation is $x = m$, so m must be -2. Enter **−2**.

6. −27

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: Since $f(x) = \sqrt[3]{x} + 3$, and the question states that this value can be no greater than 0, set up an inequality to solve for the maximum value of x:

$$\begin{aligned} \sqrt[3]{x} + 3 &\leq 0 \\ \sqrt[3]{x} &\leq -3 \\ (\sqrt[3]{x})^3 &\leq (-3)^3 \\ x &\leq -27 \end{aligned}$$

Enter **−27**.

7. D

Difficulty: Hard

Category: Rational Expressions and Equations

Getting to the Answer: Factor the denominator in the second term to find that the common denominator for all three terms is $(x-2)(x+2)$. Multiply each term in the equation by the common denominator (in factored form or the original form, whichever is more convenient) to clear the fractions. Then solve the resulting equation for x:

$$^{(x+2)}\cancel{(x^2-4)}\left(\frac{3}{\cancel{x-2}}\right) - \cancel{(x^2-4)}\left(\frac{12}{\cancel{x^2-4}}\right) = 1(x^2-4)$$

$$\begin{aligned} 3(x+2) - 12 &= x^2 - 4 \\ 3x + 6 - 12 &= x^2 - 4 \\ 3x - 6 &= x^2 - 4 \\ \\ 0 &= x^2 - 3x + 2 \\ 0 &= (x-1)(x-2) \end{aligned}$$

Set each factor equal to 0 to find that the potential solutions are $x = 1$ and $x = 2$. Note that these are only *potential* solutions because the original equation was a rational equation. When $x = 2$, the denominators in both terms on the left side are equal to 0, so 2 is an extraneous solution, which means **(D)** is correct.

8. C

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: According to the graph, one x-intercept is to the left of the y-axis and the other is to the right. Therefore, one value of x is positive, while the other is negative. Eliminate (D) because both factors have the same sign. To evaluate the remaining equations, find the x-intercepts by setting each factor equal to 0 and solving for x. In (A), the x-intercepts are $\frac{1}{2}$ and -1, but that would mean that e (the negative intercept) is twice as far from the origin as f, not half as far, so eliminate (A). In (B), the x-intercepts are 1 and -2. Again, e is twice as far from the origin as f, not half, so eliminate (B). Only **(C)** is left and must be correct. The x-intercepts are 1 and $-\frac{1}{2}$, which fits the criterion that e is half as far from the origin as f.

9. B

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: Work from the inside out. Find $g(2)$, multiply it by 2 to find $2g(2)$ and then use the result as the input for $f(x)$: $g(2) = 2^2 - 7(2) + 8 = 4 - 14 + 8 = -2$. Next, $2g(2) = 2(-2) = -4$. Now, find $f(-4)$: $\frac{(-4)^2 - (-4) - 12}{3(-4) - 4} = \frac{16 + 4 - 12}{-12 - 4} = \frac{8}{-16} = -\frac{1}{2}$

(B) is correct.

10. C

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: There are two ways to approach this question. After determining the value of x when $z = 8$, you can either plug that value into both functions or you can write out the equations for $j(x)$ and $k(x)$ and see if there are any common factors that can be canceled out. Both methods begin the same way: $x = 8 - 5 = 3$.

The function $j(x)$ is $3(3)^2 + 6(3) - 24 = 27 + 18 - 24 = 21$ when $x = 3$. Similarly, $k(x) = 3 + 4 = 7$. So, $\frac{j(x)}{k(x)} = \frac{21}{7} = 3$. **(C)** is correct.

Alternatively, factoring an $(x + 4)$ out of $j(x)$ and canceling with $k(x)$ in the denominator will leave you with just $3(x - 2)$. At $x = 3$, this also evaluates to **3**.

Check Your Work - Chapter 11

1. D
Difficulty: Easy

Category: Advanced Math

Getting to the Answer: The function graphed is the absolute value function. The range (y-values) of this function is all positive numbers and zero. Eliminate (A) and (C). Also eliminate (B); it is the range, not the domain, that fits this; all real numbers can be inputted as the x-values (domain) for this function. Choice **(D)** is correct.

2. D
Difficulty: Easy

Category: Advanced Math

Getting to the Answer: First set $|2x - 8| + 1$ equal to 3 and then subtract 1 from both sides.

$$|2x - 8| + 1 = 3$$
$$|2x - 8| = 2$$

Since absolute value gives a non-negative value, the expression inside the absolute value may be equal to 2 or -2. Solving for each case gives the values of x.

$$\begin{aligned} 2x - 8 &= 2 \qquad & 2x - 8 &= -2 \\ 2x &= 10 & 2x &= 6 \\ x &= 5 & x &= 3 \end{aligned}$$

(D) is correct.

3. 3/2 or 1.5
Difficulty: Hard

Category: Advanced Math

Getting to the Answer: Since absolute value gives a non-negative value, the expression inside the absolute value may be equal to $4x$ or $-4x$. Solving for each case gives the solutions to the equation.

$$\begin{aligned} 4x &= 9 - 2x \qquad & -4x &= 9 - 2x \\ 6x &= 9 & -2x &= 9 \\ x &= \frac{9}{6} & x &= -\frac{9}{2} \\ x &= \frac{3}{2} \end{aligned}$$

Check to see if these are valid solutions. Substituting $x = \frac{3}{2}$ back into the equation gives us $6 = |9 - 3|$. That works. However, substituting $x = -\frac{9}{2}$ back into the equation gives us $-18 = |9 + 9|$. It is impossible for an absolute value to be negative, so this is not a solution. The only solution is $x = \frac{3}{2}$. Enter **3/2** or **1.5** into the box.

4. B
Difficulty: Hard

Category: Advanced Math

Getting to the Answer: By definition, $|x|$ indicates the distance of x from 0. Thus, $|x| = 4$ says that x is 4 units from 0. That is, $x = 4$ or $x = -4$. Since c and d are 4 units from a, then the coordinates of c and d are $x = a + 4$ and $x = a - 4$. Subtracting a from both sides gives $x - a = 4$ and $x - a = -4$, which can be written more concisely as $|x - a| = 4$. **(B)** is correct.

5. C
Difficulty: Easy

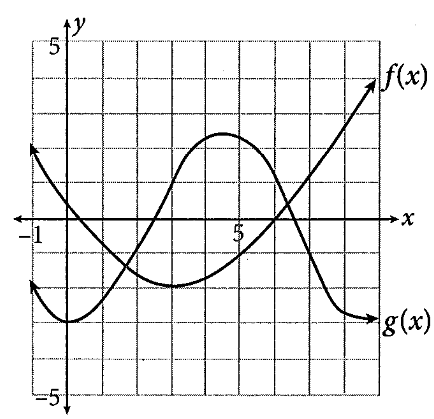
Category: Advanced Math

Getting to the Answer: The absolute value of a number is always non-negative. Thus, for the expression to equal 0, the result of the absolute value must be combined with a negative number. Only (C) subtracts 3 from the absolute value. Thus, **(C)** is correct. If you isolated the absolute value term in (A), (B), and (D), you would find that the absolute value term equals -3, which can never be true.

6. A
Difficulty: Medium

Category: Advanced Math

Getting to the Answer: Graphically, the notation $f(3)$ means the y-value when $x = 3$. Find $x = 3$ on the x-axis. Then, read the y-coordinates from the graph, paying close attention to which function is which:



$f(3) = -2$ and $g(3) = 1$, so $f(3) - g(3) = -2 - 1 = -3$. **(A)** is correct.

Math

7. 8

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: The maximum of $f(x)$ occurs at the point where the y-value is the greatest, which in this case is (2, 4). So, $a = 2$ and $b = 4$. The point with the smallest y-value is (4, −2). Thus, $c = 4$ and $d = -2$. The total of the four values is $2 + 4 + 4 + (-2) = 8$. Enter **8**.

8. D

Difficulty: Easy

Category: Advanced Math

Getting to the Answer: To determine the domain, look at the x-values. Since the domain is the set of inputs, not outputs, it will not include $f(x)$. This means you can eliminate (A) and (B). Note that the graph is continuous and has arrows on both sides, so the domain is all real numbers.

To determine the range, look at the y-values. For the range, the function's maximum is located at (0, 5), which means the highest possible value of $f(x)$ is 5. The graph is continuous and opens downward, so the range of the function is $f(x) \leq 5$, making **(D)** correct.

9. D

Difficulty: Easy

Category: Advanced Math

Getting to the Answer: Evaluate this function at $y = 5$ and $y = 1$. At $y = 5$, $f(5) = (5)^3 - 7(5) + 5 = 125 - 35 + 5 = 95$. At $y = 1$, $f(1) = (1)^3 - 7(1) + 5 = 1 - 7 + 5 = -1$. Your final answer is $95 - (-1) = 96$. Choice **(D)** is correct.

10. C

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: A factor of a polynomial is equal to 0. So if $x - a$ is a factor of $f(x)$, then $x - a = 0$ and $x = a$, where a is an x-intercept of the polynomial. In other words, look for when $f(a) = 0$ in the table. According to the table, $f(-1) = 0$, so $x + 1$ is a factor of $f(x)$. **(C)** is correct.

If you missed this question, there's more on polynomials in the next chapter.

Check Your Work - Chapter 12

1. 7

Difficulty: Easy

Category: Advanced Math

Getting to the Answer: Simplify the fraction on the left side of the equation using exponent rules. Start by distributing the power of 2 outside the parentheses to both the 3 and the x inside:

$$\frac{x^c(3x)^2}{9x^3} = x^6$$
$$\frac{x^c(3)^2(x^2)}{9x^3} = x^6$$
$$\frac{9(x^c)(x^2)}{9x^3} = x^6$$

Now, cancel the 9s in the numerator and denominator, and add the exponents of the two x terms, since it's multiplication of the same base to different exponents:

$$\frac{\cancel{9}(x^{c+2})}{\cancel{9}x^3} = x^6$$

There's now an x in both the numerator and the denominator raised to different exponents, so subtract them:

$$x^{(c+2)-3} = x^6$$
$$x^{c-1} = x^6$$

Since, according to the question, $x \neq 0$, this means that $c - 1 = 6$, or $c = 7$. Enter **7**.

2. A

Difficulty: Easy

Category: Advanced Math

Getting to the Answer: Find the greatest common factor (GCF) of both the numerator and the denominator, which in this question happens to be the denominator. Factor out the GCF, $9x^2$, from the numerator and denominator and then cancel what you can:

$$\frac{18x^4 + 27x^3 - 36x^2}{9x^2} = \frac{\cancel{9x^2}(2x^2 + 3x - 4)}{\cancel{9x^2}}$$
$$= 2x^2 + 3x - 4$$

This matches **(A)**. As an alternate method, you could split the expression up and reduce each term, one at a time:

$$\frac{18x^4 + 27x^3 - 36x^2}{9x^2} = \frac{18x^4}{9x^2} + \frac{27x^3}{9x^2} - \frac{36x^2}{9x^2}$$
$$= 2x^2 + 3x - 4$$

3. D

Difficulty: Hard

Category: Advanced Math

Getting to the Answer: If the product of x^a and y^b is negative, then either x^a is positive and y^b is negative, or vice versa. A negative number raised to an even exponent is positive, and a negative number raised to an odd exponent is negative. Since x and y are both negative, then either a is even and b odd, or vice versa.

Now, evaluate the choices to see what must be true. (A) says that a is even; this is possibly true, but it's also possible that b is even and a is odd. Thus, (A) can be eliminated. (B) can be eliminated for the same reason. (C) says that ab is odd. The only way for the product of two integers to be odd is if both of the integers are odd; since one of either a or b must be even, ab must be even. Eliminate (C); **(D)** is correct.

If you are ever unsure of even and odd rules, you can pick numbers to test them out: Odd $\times$ Even = Even; $3 \times 4 = 12$.

4. 256

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: If $n^3 = -8$, then $n = -2$. Plug -2 in for n and simplify the given expression via exponent rules:

$$\frac{((-2)^2)^3}{\frac{1}{(-2)^2}} = \frac{4^3}{\frac{1}{4}} = 4^3 \times 4 = 4^4 = 256$$

Enter **256**.

5. B

Difficulty: Hard

Category: Advanced Math

Getting to the Answer: Because the bases are the same, to simplify the fraction on the left side of the equation just subtract the powers and combine:

$$\frac{x^{5r}}{x^{3r-2s}} = x^{5r-(3r-2s)}$$
$$= x^{5r-3r+2s}$$
$$= x^{2r+2s}$$

Note that in the expression $2r + 2s$, it is possible to factor out a 2. Thus, $x^{2r+2s} = x^{2(r+s)}$. The question indicates that $r + s = 6$, so $x^{2(r+s)} = x^{2(6)} = x^{12}$. This is equal to x^t, so $t = 12$. The answer is **(B)**.

6. A

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: Solve equations containing radical expressions the same way you solve any other equation: isolate the variable using inverse operations. Start by subtracting 8 from both sides of the equation and then multiply by 3. Then, square both sides to remove the radical:

$$\begin{aligned} 8+\frac{\sqrt{2x+29}}{3} &= 9 \\ \frac{\sqrt{2x+29}}{3} &= 1 \\ \sqrt{2x+29} &= 3 \\ 2x+29 &= 9 \end{aligned}$$

Now you have a simple linear equation that you can solve using more inverse operations: subtract 29 and divide by 2 to find that $x=-10$. Be careful—just because the equation started with a radical and the answer is negative, it does not follow that *No solution* is the correct answer. If you plug -10 into the expression under the radical, the result is a positive number, which means -10 is a perfectly valid solution. Therefore, **(A)** is correct.

7. C

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: Subtract x from both sides of the first equation to get $2x=14$. You could solve for x, but since $2x$ appears in the second equation, plug it in to get $\sqrt{3z^2-11}+14=22$. Thus, $\sqrt{3z^2-11}=8$. Square both sides of this equation and solve for z:

$$\begin{aligned} 3z^2-11 &= 64 \\ 3z^2 &= 75 \\ z^2 &= 25 \\ z &= \pm 5 \end{aligned}$$

Since the question specifies that $z>0$, **(C)** is correct.

8. C

Difficulty: Easy

Category: Advanced Math

Getting to the Answer: Follow the standard order of operations—deal with the exponent first and then attach the negative sign (because a negative in front of an expression means multiplication by -1). The variable x is being raised to the $\frac{1}{4}$ power, so rewrite the term as a radical expression with 4 as the degree of the root and 1 as the power to which the radicand, x, is being raised:

$$x^{\frac{1}{4}}=\sqrt[4]{x^1}=\sqrt[4]{x}$$

Now attach the negative to arrive at the correct answer, $-\sqrt[4]{x}$, which is **(C)**.

9. 16

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: While this could be solved for with a calculator, learning exponent rules may save you time in the long run. The following calculation could also be done quickly with mental math after some practice. Rewrite the exponent in a way that makes it easier to evaluate: use exponent rules to rewrite $\frac{4}{3}$ as a unit fraction raised to a power. Then write the expression in radical form and simplify:

$$\begin{aligned} 8^{\frac{4}{3}} &= \left(8^{\frac{1}{3}}\right)^4 \\ &= (\sqrt[3]{8})^4 \\ &= 2^4 \\ &= 2\times 2\times 2\times 2 \\ &= 16 \end{aligned}$$

Enter **16**.

10. A

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: This is a question that lends itself to backsolving. Each answer is easily plugged into the equation and checked:

Choice (A):

$$\begin{aligned}\sqrt{3a+16}-3&=a-1\\ \sqrt{3(3)+16}-3&=3-1\\ \sqrt{25}-3&=2\\ 5-3&=2\end{aligned}$$

Choice **(A)** is correct. For the record:

Choice (B) would yield the false statement $\sqrt{22}-3=1$ and would be eliminated.

Choice (C) would yield the false statement $\sqrt{19}-3=0$ and would be eliminated.

Choice (D) would yield the false statement $4-3=-1$ and would be eliminated.

You could also solve this with quadratics (which is covered in a later chapter). Start by isolating the radical on the left side of the equation by adding 3 to both sides to get $\sqrt{3a+16}=a+2$. Now you can square both sides to get rid of the radical: $3a+16=(a+2)^2=a^2+4a+4$. Since the right side of this equation is a quadratic, set it equal to 0 in order to determine the solutions: $0=a^2+4a+4-(3a+16)=a^2+a-12$. Next, factor the quadratic using reverse FOIL. The two factors of -12 that add up to 1 are -3 and 4, so $(a-3)(a+4)=0$. Thus, a can be either 3 or -4, but the question says $a\geq 0$, so the only permissible value is 3. **(A)** is correct.

11. A

Difficulty: Easy

Category: Advanced Math

Getting to the Answer: Add polynomial expressions by combining like terms. Be careful of the signs of each term. It may help to write the sum vertically, lining up the like terms:

$$\begin{array}{r}6a^2-17a-9\\ \underline{+(-5a^2+8a-2)}\\ a^2-9a-11\end{array}$$

The correct choice is **(A)**.

12. D

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: First, write the question as a subtraction problem. Pay careful attention to which expression is being subtracted so that you distribute the negative sign correctly, and make sure you only subtract terms with the same base and exponent.

$$8x^2+4x+10-(3x^3+7x-5)=-3x^3+8x^2-3x+15$$

This expression matches **(D)**.

13. D

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: Multiply each term in the first expression by $\frac{3}{2}$ and each term in the second expression by 2. Then, subtract the two polynomials by writing them vertically and combining like terms. You'll have to find a common denominator to combine the x-coefficients and to combine the constant terms:

$$\frac{3}{2}A=\frac{3}{2}(4x^2+7x-1)=6x^2+\frac{21}{2}x-\frac{3}{2}$$
$$2B=2(-x^2-5x+3)=-2x^2-10x+6$$

$$\begin{array}{r}6x^2+\frac{21}{2}x-\frac{3}{2}\\ \underline{-\left(-2x^2-\frac{20}{2}x+\frac{12}{2}\right)}\\ 8x^2+\frac{41}{2}x-\frac{15}{2}\end{array}$$

This means **(D)** is correct. Notice that if you are simplifying the expression from left to right, after you find the x^2-coefficient, you can eliminate (A) and (B). After you find the x-coefficient, you can eliminate (C) and stop your work.

14. C

Difficulty: Hard

Category: Advanced Math

Getting to the Answer: In order to solve the equation, move all the terms to one side of the equation to set them equal to 0, then factor the expression. Thus, the given equation becomes $x^3 + x^2 - 9x - 9 = 0$. Think of this as two pairs of terms, $(x^3 + x^2)$ and $(-9x - 9)$. The first pair of terms share a common factor of x^2, so they can be written as $x^2(x + 1)$. The second pair share the common factor of -9, so they are equivalent to $-9(x + 1)$. So, the equation becomes $x^2(x + 1) - 9(x + 1) = 0$. Now, factor out the $(x + 1)$ term: $(x^2 - 9)(x + 1) = 0$.

In order for the product of two terms to be 0, either one or both must be 0. If $x^2 - 9 = 0$, then $x^2 = 9$ and $x = \pm 3$. Eliminate (A) and (D). If $x + 1 = 0$, then $x = -1$. Eliminate (B), so **(C)** is correct. You could also answer the question using Backsolving by plugging in each answer choice until you found the value for x that did *not* satisfy the equation.

15. C

Difficulty: Medium

Category: Advanced Math

Strategic Advice: To multiply two polynomials, multiply each term in the first factor by each term in the second factor, then combine like terms.

Getting to the Answer: Multiply each part of the trinomial expression by each part of the binomial one piece at a time and then combine like terms:

$$\begin{aligned} &(2x^2 + 3x - 4)(3x + 2) \\ &= 2x^2(3x + 2) + 3x(3x + 2) - 4(3x + 2) \\ &= 6x^3 + 4x^2 + 9x^2 + 6x - 12x - 8 \\ &= 6x^3 + 13x^2 - 6x - 8 \end{aligned}$$

Because a represents the coefficient of x^2, $a = 13$. Hence, **(C)** is correct.

16. D

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: Use polynomial long division to simplify the expression:

$$\begin{array}{r} 2a + 1 \\ a - 3 \overline{\smash{)}\, 2a^2 - 5a - 1} \\ \underline{-(2a^2 - 6a)} \\ a - 1 \\ \underline{-(a - 3)} \\ 2 \end{array}$$

The quotient is $2a + 1$ and the remainder is 2, which will be divided by the divisor in the final answer: $2a + 1 + \frac{2}{a - 3}$. Thus, **(D)** is correct.

17. C

Difficulty: Hard

Category: Advanced Math

Getting to the Answer: A fraction is the same as division, so you can use polynomial long division to simplify the expression:

$$\begin{array}{r} 3x + 2 \\ 2x + 5 \overline{\smash{)}\, 6x^2 + 19x + 10} \\ \underline{-(6x^2 + 15x)} \\ 4x + 10 \\ \underline{-(4x + 10)} \\ 0 \end{array}$$

The simplified expression is $3x + 2$, so $a = 3$ and $b = 2$, and $a + b = 3 + 2 = 5$, which is **(C)**. As an alternate approach, you could factor the numerator of the expression and cancel common factors:

$$\frac{6x^2 + 19x + 10}{2x + 5} = \frac{\cancel{(2x + 5)}(3x + 2)}{\cancel{(2x + 5)}} = 3x + 2$$

18. B

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: Use polynomial long division to simplify the expression:

$$\begin{array}{r} 2x - 5 \\ 2x+2\overline{\smash{)}4x^2 - 6x} \\ \underline{-(4x^2 + 4x)} \\ -10x \\ \underline{-(-10x - 10)} \\ +10 \end{array}$$

The quotient is $2x - 5$ and the remainder is 10. Put the remainder over the divisor and add this to the quotient: $2x - 5 + \frac{10}{2x+2}$ **(B)** is correct.

19. B

Difficulty: Hard

Category: Advanced Math

Getting to the Answer: The question provides the quotient of $-9x + 5$ of a division problem and asks you to find the coefficient of the first term of the divisor $tx - 4$. Set this up in polynomial long division form to better understand the relationship between t and the other terms:

$$\begin{array}{r} -9x + 5 \\ tx - 4\overline{\smash{)}36x^2 + 16x - 21} \end{array}$$

Viewed this way, it becomes apparent that $36x^2 \div tx = -9x$. Multiplying both sides by tx gives you $tx(-9x) = 36x^2$; therefore, $t(-9) = 36$, so $t = -4$. **(B)** is correct.

20. C

Difficulty: Hard

Category: Advanced Math

Getting to the Answer: Because $f(x)$ is divisible by $x - 5$, the value $x - 5$ must be a factor of $f(x)$. Therefore, you can define $f(x)$ as $(x - 5)(n)$, where n is some unknown polynomial. Since $g(x)$ is $f(x) + 4$, you can say that $g(x)$ must be $(x - 5)(n) + 4$.

Thus, $g(5)$ will be $(5 - 5)(n) + 4 = 0(n) + 4 = 0 + 4 = 4$. Therefore, **(C)** is correct.

21. B

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: The solutions, or x-intercepts, of a polynomial are the factors of that polynomial. This polynomial has x-intercepts of -4, 0, and 6. The factors that generate those solutions are $(x + 4)$, x, and $(x - 6)$. Eliminate (C) and (D) because they do not include those three factors. Because the graph *crosses* the x-axis at each x-intercept (rather than merely touching the x-axis), none of the factors can be raised to an even exponent. Therefore, eliminate (A) because of the x^2 term. **(B)** is correct.

22. C

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: To find the solutions to a polynomial function, factor the polynomial and set each factor equal to 0. The solutions of a function are the x-intercepts, so $h(x)$ or the y-coordinate of the solution must equal 0. From the chart, the only point with $h(x) = 0$ is at $x = -1$. If $x = -1$, the factor that generates that solution is $x + 1 = 0$ because $(-1) + 1 = 0$. **(C)** is correct.

23. C

Difficulty: Hard

Category: Advanced Math

Getting to the Answer: Translate the notation: $b(a(x))$ means b of $a(x)$. This tells you to use $a(x)$ as the input for $b(x)$. You can rewrite this as $\frac{1}{a(x)}$, which is the reciprocal of $a(x)$. This new function will be undefined anywhere that $a(x) = 0$ because a denominator of 0 is not permitted. Looking at the graph, you can see that $a(x)$ crosses the x-axis four times, at which point the value of $a(x)$ is 0. Since division by 0 is undefined, $b(a(x))$ will be undefined for at least these four points, so **(C)** is correct.

24. A

Difficulty: Easy

Category: Advanced Math

Getting to the Answer: The phrase "exactly 2 distinct real zeros" means that the graph must have exactly two different x-intercepts on the graph. An x-intercept is indicated any time that the graph either crosses or touches the x-axis. (B) and (D) have three distinct zeros, and (C) has two zeros, but because the graph only touches the x-axis, they are the same, not distinct. The only graph with exactly two distinct zeros is **(A)**.

Math

25. B

Difficulty: Easy

Category: Advanced Math

Getting to the Answer: The keyword "root" in the question stem means that you should examine the places at which the graph intersects the x-axis. Thus, this graph has roots at (0, 0) and (6, 0). The x-axis, according to the graph, represents the horizontal distance traveled by the balloon. When $x = 0$, the distance the water balloon has traveled is 0, which is the balloon's starting position. The initial location of the balloon is not an answer choice, so the correct answer must be what the other root represents. When $x = 6$, the balloon's height is 0, which is the end point of the balloon's trajectory. This value, 6, is a root that represents the total horizontal distance traveled. **(B)** is correct.

26. 74

Difficulty: Medium

Category: Advanced Math

Strategic Advice: The goal is to find the number of applicants *eliminated* after three days, not the number remaining.

Getting to the Answer: The question describes the decay as the result of removing a certain fraction of the remaining applicants each day. The situation involves repeated division, so this is an example of exponential decay. You could use the exponential decay formula for a given rate, but it may be more straightforward to determine how many applicants are eliminated each day and tally them up.

After the first day, the judges eliminate one-fourth of 128, or 32, applicants. This leaves $128 - 32 = 96$ applicants. On the second day, one-fourth of 96, or 24, applicants are eliminated, leaving $96 - 24 = 72$. Finally, on the third day, one-fourth are eliminated again; one-fourth of 72 is 18, so there are $72 - 18 = 54$ applicants remaining. If 54 applicants remain, then $128 - 54 = 74$ applicants have been eliminated. Enter **74**.

27. C

Difficulty: Medium

Category: Advanced Math

Strategic Advice: This question gives you a percent increase per year, so use the exponential growth equation to solve for the number of years.

Getting to the Answer: Use the formula for exponential growth and plug in the values from the question. The rate is 16%, which as a decimal is 0.16. The rate will remain positive because the question asks about increase, or growth; therefore, $r = 0.16$. The current number of members is 42, so this will be $f(0)$. The goal is at least 100 members, so that will be the output, or $f(t)$. Put it all together:

$$f(t) = f(0)(1 + r)^t$$
$$100 = 42(1 + 0.16)^t$$
$$100 = 42(1.16)^t$$

At this point, backsolving is the best approach. Plug in the number of years for t. Because the answer choices are in ascending order, try one of the middle options first. You might be able to eliminate more than one choice at a time. Choice (B) is $t = 5$:

$$42(1.16)^5 \approx 88$$

Since (B) is too small, (A) must be as well. Eliminate them both. Unfortunately, 88 is not close enough to 100 to be certain that **(C)** is the correct answer, so test it:

$$42(1.16)^6 \approx 102$$

Six years is enough to put the club over 100 members. **(C)** is correct.

28. A

Difficulty: Hard

Category: Advanced Math

Strategy Advice: The term "half-life" signals exponential decay because it implies repeated division by 2. Using the exponential decay formula here could be complicated. Instead, you can use the percentage given in the question, along with the picking numbers strategy, to figure out how many half-lives have elapsed.

Getting to the Answer: Instead of providing an actual amount of ^{14}C, this question tells you what percent is left. For questions involving percentages of unknown values, it is often a good idea to pick 100. So, assume that the amount of ^{14}C in the sample when the tree died is 100. (Fortunately, there is no need to worry about the units here.) After one half-life, the amount of ^{14}C is halved to 50. A second half-life leaves 25, a third leaves 12.5, and a fourth leaves 6.25, which is 6.25% of 100. So four half-lives have elapsed. Since each half-life is 5,600 years, the tree died $4 \times 5{,}600$ or 22,400 years ago. Choice **(A)** is correct.

29. 17

Difficulty: Medium

Category: Advanced Math

Strategic Advice: The question describes a situation with linear growth since Penelope is adding the same amount of money to her piggy bank each month. Note: the question is asking for her monthly allowance, but she puts in only half that amount each month.

Getting to the Answer: Use the linear growth equation $y = mx + b$. The question gives you the starting amount b (\$40), the final amount y (\$244), and the amount of time x (2 years, which is 24 months). Plug these values into the equation and solve for m, which is the slope, or the rate of change—or in this case, how much Penelope puts in her piggy bank each month:

$$\begin{aligned} y &= mx + b \\ 244 &= m(24) + 40 \\ 24m &= 204 \\ m &= 8.5 \end{aligned}$$

Remember that what she puts in the piggy bank is only half of her allowance, so her total monthly allowance is twice \$8.50. Enter **17**.

30. 160

Difficulty: Hard

Category: Advanced Math

Strategic Advice: This question describes both types of growth. Account X adds a percentage of the original amount, which never changes, so the same amount of money is added each month. Account X grows linearly. Account Y, however, adds a percentage of the current balance, which grows monthly, so account Y grows exponentially.

Getting to the Answer: Account X begins with \$500 (the y–intercept, or b) and adds 2% of \$500, or $\$500 \times 0.02 = \$10$ (the rate of change, or m), each month for 3 years, which is 36 months (the input, or x). Plug these values into the linear growth equation to solve for the final value of the account:

$$\begin{aligned} y &= mx + b \\ y &= 10(36) + 500 \\ y &= 360 + 500 = \$860 \end{aligned}$$

Account Y begins with \$500 ($f(0)$) and adds 2%, or 0.02, (r) each month for 36 months (t). Plug these values into the exponential growth equation to solve for the final value of the account:

$$\begin{aligned} f(t) &= f(0)\,(1 + r)^t \\ f(t) &= 500\,(1 + 0.02)^{36} \\ f(t) &= 500\,(1.02)^{36} \approx \$1{,}019.94 \end{aligned}$$

The positive difference between the two accounts is therefore $\$1{,}019.94 - \$860 = \$159.94$. Round up to the nearest dollar, and enter **160**.

31. A

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: There are two variables and only one equation, but because you're asked to solve for one of the variables *in terms of* the other, you solve the same way you would any other equation, by isolating x on one side of the equation. Cross-multiplying is a quick route to the solution:

$$\begin{aligned} \frac{6}{x} &= \frac{3}{k+2} \\ 6(k+2) &= 3x \\ 6k + 12 &= 3x \\ \frac{6k}{3} + \frac{12}{3} &= \frac{3x}{3} \\ 2k + 4 &= x \end{aligned}$$

Switch x to the left side of the equation and the result matches **(A)**.

Math

32. 3

Difficulty: Hard

Category: Advanced Math

Getting to the Answer: Because the expression is adding fractions with different denominators, you'll need to establish a common denominator. Note that the second fraction is divisible by 3, so you can simplify the expression and then create the common denominator. Since both fractions now have denominators involving $(a-3)$, wait to substitute 6 for $(a-3)^2$ until you've added the two fractions.

$$\begin{aligned}&\frac{3a+9}{(a-3)^2}+\frac{-3}{a-3}\\&=\frac{3a+9}{(a-3)^2}+\frac{-3}{a-3}\times\frac{a-3}{a-3}\\&=\frac{3a+9}{(a-3)^2}+\frac{-3a+9}{(a-3)^2}\\&=\frac{18}{(a-3)^2}\end{aligned}$$

The question specifies that $(a-3)^2=6$, so $\frac{18}{(a-3)^2}=\frac{18}{6}=3$. Therefore, the expression equals 3. Enter **3**.

33. C

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: The denominator of the expression contains the sum of two fractions that themselves have different denominators, so start by finding a common denominator:

$$\frac{\frac{2}{a}}{\frac{a-6}{(a-2)(a-6)}+\frac{a-2}{(a-2)(a-6)}}=\frac{\frac{2}{a}}{\frac{2a-8}{a^2-8a+12}}$$

Next, multiply the numerator of the expression by the reciprocal of the denominator and simplify:

$$\begin{aligned}&\frac{2}{a}\times\frac{a^2-8a+12}{2a-8}\\&=\frac{2(a^2-8a+12)}{2a^2-8a}\\&=\frac{a^2-8a+12}{a^2-4a}\end{aligned}$$

This expression matches **(C)**.

34. A

Difficulty: Easy

Category: Advanced Math

Getting to the Answer: Both terms in the numerator share a common x^2 term, so factor that out:

$$\frac{x^2(x-3)}{x-3}=9$$

Now, cancel $(x-3)$ in the numerator and the denominator to get $x^2=9$. Normally, this would mean that x could be 3 or -3. However, remember to check for extraneous solutions. If $x=3$, then the denominator in the original equation would be equal to 0; thus, $x=-3$, and **(A)** is correct.

35. B

Difficulty: Hard

Category: Advanced Math

Getting to the Answer: Because the question states that the expressions are equivalent, set up the equation $\frac{16}{7x+4}+A=\frac{49x^2}{7x+4}$ and solve for A. Start by subtracting the first term from both sides of the equation to isolate A. Then, simplify. The denominators of the rational terms are the same, so they can be combined. Then, cancel common factors.

$$\begin{aligned}\frac{16}{7x+4}+A&=\frac{49x^2}{7x+4}\\A&=\frac{49x^2}{7x+4}-\frac{16}{7x+4}\\A&=\frac{49x^2-16}{7x+4}\\A&=\frac{\cancel{(7x+4)}(7x-4)}{\cancel{7x+4}}\\A&=7x-4\end{aligned}$$

The correct choice is **(B)**.

36. B

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: Move the second fraction over to the other side of the equation by subtracting it from both sides, then cross-multiply to simplify:

$$\begin{aligned}\frac{1-2c}{3c}&=\frac{c-8}{12}\\12(1-2c)&=3c(c-8)\\12-24c&=3c^2-24c\\12&=3c^2\\c^2&=4\end{aligned}$$

Therefore, either $c=-2$ or $c=2$. The question specifies that $c<0$, so c must equal -2. **(B)** is correct.

Check Your Work - Chapter 13

1. B

Difficulty: Easy

Category: Advanced Math

Getting to the Answer: FOIL the binomials $(6 - 5x)(15x - 11)$. First: $(6)(15x) = 90x$. Outer: $(6)(-11) = -66$. Inner: $(-5x)(15x) = -75x^2$. Last: $(-5x)(-11) = 55x$. Combining like terms gives $90x - 66 - 75x^2 + 55x = -75x^2 + 145x - 66$. The correct answer is **(B)**.

2. C

Difficulty: Easy

Category: Advanced Math

Getting to the Answer: First, factor out a 3 in the denominator to make that quadratic a bit simpler. Next, factor the numerator and denominator using reverse-FOIL to reveal an $x - 5$ term that will cancel out.

$$\frac{x^2 - 10x + 25}{3x^2 - 9x - 30} = \frac{x^2 - 10x + 25}{3(x^2 - 3x - 10)}$$
$$= \frac{\cancel{(x-5)}(x-5)}{3\cancel{(x-5)}(x+2)} = \frac{x-5}{3(x+2)}$$

The correct answer is **(C)**.

3. 1

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: An expression is undefined when it involves division by 0, so the key to the question is to recognize that the denominator will be 0 if either of the factors of the quadratic are 0. Factoring 2 out of the denominator leaves a relatively easy-to-factor quadratic:

$$\frac{3}{2x^2 + 4x - 6} = 0$$
$$\frac{3}{2(x^2 + 2x - 3)} = 0$$
$$\frac{3}{2(x+3)(x-1)} = 0$$

The denominator will be 0 if the value of x is either 1 or -3. Because the question asks for a positive value of x, enter **1**.

4. B

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: Set the equation equal to zero and then divide by 3 to remove the x^2 coefficient:

$$3x^2 + 9x - 54 = 0$$
$$x^2 + 3x - 18 = 0$$
$$(x - 3)(x + 6) = 0$$
$$x = 3 \text{ or } -6$$

The question asks for the sum of the roots, which is $3 + (-6) = -3$. The correct answer is **(B)**.

5. A

Difficulty: Hard

Category: Advanced Math

Strategic Advice: The question asks for an equivalent expression, so ignore the function notation and focus on simplifying the polynomial so that it looks more like the answer choices.

Getting to the Answer: Expand the polynomial and distribute as necessary so that all of the parentheses are eliminated:

$$(1.3x - 3.9)^2 - (0.69x^2 - 0.14x - 9.79)$$
$$(1.3x - 3.9)(1.3x - 3.9) - 0.69x^2 + 0.14x + 9.79$$
$$1.69x^2 - 10.14x + 15.21 - 0.69x^2 + 0.14x + 9.79$$

Combine like terms:

$$x^2 - 10x + 25$$

Then factor the polynomial by finding two integers that multiply to 25 and add up to -10: $x^2 - 10x + 25 = (x - 5)(x - 5) = (x - 5)^2$. **(A)** is correct.

Math

6. C

Difficulty: Easy

Category: Advanced Math

Getting to the Answer: Expand both classic quadratics and combine like terms to find the sum:

$$\begin{aligned}&(a-b)^2+(a+b)^2\\&=(a^2-2ab+b^2)+(a^2+2ab+b^2)\\&=2a^2+2b^2\end{aligned}$$

This matches **(C)**.

7. 0

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: To find the roots, set the equation equal to 0, factor it, and then solve. Clear the fraction the same way you do when solving equations, multiplying both sides of the equation by the denominator of the fraction:

$$\begin{aligned}0&=\frac{1}{3}x^2-2x+3\\3(0)&=3\left(\frac{1}{3}x^2-2x+3\right)\\0&=x^2-6x+9\\0&=(x-3)(x-3)\end{aligned}$$

The equation has only one unique solution ($x=3$), so the positive difference between the roots is $3-3=0$. Enter **0**.

8. 4

Difficulty: Hard

Category: Advanced Math

Getting to the Answer: A fraction is undefined when the denominator equals 0. To find the value of x where $f(x)$ is undefined, set the denominator equal to 0 and solve for x.

The equation $(x-7)^2+6(x-7)+9=0$ is the expansion of the classic quadratic $a^2+2ab+b^2=(a+b)^2$, where $a=(x-7)$ and $b=3$, so the denominator will factor as $[(x-7)+3]^2$. That's equivalent to $(x-4)^2$. Set this expression equal to 0 to find that the function is undefined when $x-4=0$, or $x=4$. Enter **4**.

9. B

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: Start by noticing that x^4-196 is a difference of perfect squares. Use the pattern for difference of squares $a^2-b^2=(a+b)(a-b)$ where $x^4-196=(x^2+14)(x^2-14)$. Because area is length times width ($A=lw$) and the width is x^2-14, the length must be x^2+14. Choice **(B)** is correct.

10. B

Difficulty: Medium

Category: Advanced Math

Strategic Advice: Recognizing the classic quadratic $(x-y)^2=x^2-2xy+y^2$ will save you time when factoring.

Getting to the Answer: In this question, the goal is to manipulate the polynomial so that it matches the factored form given. First, recognize that 2 can be factored out. The resulting expression is then $2(x^2-14x+49)$. Notice that $\sqrt{49}=7$ and factor the quadratic to get $2(x-7)(x-7)=2(x-7)^2$. Now the expression is in the same form as $a(x-b)^2$. Therefore, $b=7$, so **(B)** is correct.

11. D

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: Factoring won't work here because no two factors of -5 sum to 2. However, the coefficient of x^2 is 1, so try completing the square:

$$\begin{aligned}x^2+2x-5&=0\\x^2+2x&=5\\\left(\frac{b}{2}\right)^2=\left(\frac{2}{2}\right)^2&=1^2=1\\x^2+2x+1&=5+1\\(x+1)^2&=6\\x+1&=\pm\sqrt{6}\\x&=-1\pm\sqrt{6}\end{aligned}$$

(D) matches one of the two possible values of x, so it's correct.

12. D

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: To complete the square, restate this as $a^2 - 12a = 72$. One-half of the x-coefficient is -6, which, when squared, becomes 36. So, $a^2 - 12a + 36 = 108$. Factor to find that $(a - 6)^2 = 108$ and then take the square root of both sides to get $a - 6 = \pm\sqrt{108}$. Since $108 = 36 \times 3$, the radical simplifies to $6\sqrt{3}$.

Since the question asks for the root with the greatest value, you can ignore the root with the minus sign, so $a = 6 + 6\sqrt{3} = 6(1 + \sqrt{3})$. **(D)** is correct.

13. C

Difficulty: Hard

Category: Advanced Math

Getting to the Answer: The radical looks as if it will make the calculation difficult, but it will drop out when you complete the square. The coefficient, b, is $6\sqrt{5}$, so $\left(\frac{6\sqrt{5}}{2}\right)^2 = \left(\frac{36 \times 5}{4}\right) = 45$. Adding 45 to both sides of the equation gives you $x^2 - (6\sqrt{5})x + 45 = 5$, so the factored form is $(x - 3\sqrt{5})^2 = 5$. Take the square root of both sides to get $x - 3\sqrt{5} = \pm\sqrt{5}$. The two possible values of x are $3\sqrt{5} + \sqrt{5} = 4\sqrt{5}$ and $3\sqrt{5} - \sqrt{5} = 2\sqrt{5}$. The question asks for the sum of these values, which is $4\sqrt{5} + 2\sqrt{5} = 6\sqrt{5}$. **(C)** is correct.

14. B

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: Rewrite the equation of the graph by completing the square. The coefficient, b, is 4, so $\left(\frac{4}{2}\right)^2 = 2^2 = 4$. Completing the square gives you $y + 4 = x^2 + 4x + 4 - 4$. Isolate y and then factor.

$$y = x^2 + 4x + 4 - 4 - 4$$
$$y = (x + 2)^2 - 8$$

(B) is correct.

In the upcoming lesson, Graphs of Quadratics, you'll see how to solve this question by noting that in this form, the vertex of the parabola can be read: $(-2, -8)$.

15. 2

Difficulty: Hard

Category: Advanced Math

Strategic Advice: Recall that when the value of the discriminant, $b^2 - 4ac$, is 0, there is exactly one solution to the quadratic equation.

Getting to the Answer: The given equation is $2x^2 + 8x + 4 + 2z = 0$, but there is a common factor of 2 in all the terms, so this becomes $x^2 + 4x + 2 + z = 0$. Thus, $a = 1$, $b = 4$, and $c = 2 + z$. Set the discriminant $4^2 - 4(1)(2 + z)$ equal to 0 so that there is only one solution. Expand the equation to $16 - 8 - 4z = 0$. Thus, $8 = 4z$, and $z = 2$. Enter **2**.

16. B

Difficulty: Hard

Category: Advanced Math

Getting to the Answer: The question presents a quadratic equation that cannot be easily factored. Therefore, use the quadratic formula to solve. The quadratic formula states that $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

In this case, $a = 3$, $b = 4$, and $c = -2$. Plug in these values to get:

$$\begin{aligned} x &= \frac{-4 \pm \sqrt{4^2 - 4(3)(-2)}}{2(3)} \\ &= \frac{-4 \pm \sqrt{16 - (-24)}}{6} \\ &= \frac{-4 \pm \sqrt{40}}{6} \end{aligned}$$

Thus, the solutions to the equation are $\frac{-4 + \sqrt{40}}{6}$ and $\frac{-4 - \sqrt{40}}{6}$. The question asks for their product, so multiply the solutions:

$$\begin{aligned} &\left(\frac{-4 + \sqrt{40}}{6}\right)\left(\frac{-4 - \sqrt{40}}{6}\right) \\ &= \frac{16 + 4\sqrt{40} - 4\sqrt{40} - 40}{36} \\ &= \frac{-24}{36} \\ &= -\frac{2}{3} \end{aligned}$$

(B) is correct.

Math

17. A

Difficulty: Medium

Category: Advanced Math

Strategic Advice: When all of the coefficients in a quadratic equation are divisible by a common factor, simplify the equation by dividing all terms by that factor before solving.

Getting to the Answer: The given equation is $4x^2 - 24x + 16 = 0$, but there is a common factor of 4 in all the terms, so this becomes $x^2 - 6x + 4 = 0$.

The radicals in the answer choices are a strong clue that the quadratic formula is the way to solve this equation. The quadratic formula is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, and after you plug in the coefficients, $a = 1$, $b = -6$, and $c = 4$, you get:

$$\begin{aligned} x &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(4)}}{2(1)} \\ &= \frac{6 \pm \sqrt{36 - 16}}{2} \\ &= \frac{6 \pm \sqrt{20}}{2} \end{aligned}$$

This doesn't resemble any of the answer choices, so continue simplifying:

$$\begin{aligned} &\frac{6 \pm \sqrt{20}}{2} \\ &= \frac{6 \pm \sqrt{4}\sqrt{5}}{2} \\ &= \frac{6 \pm 2\sqrt{5}}{2} \\ &= \frac{6}{2} \pm \frac{2\sqrt{5}}{2} \\ &= 3 \pm \sqrt{5} \end{aligned}$$

Hence, **(A)** is correct.

18. B

Difficulty: Hard

Category: Advanced Math

Getting to the Answer: A glance at the radicals in the answer choices suggests that using the quadratic formula to solve is appropriate. Because there are so many variables, it might help to write down the quadratic formula on your scratch paper as a guide: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Begin by reorganizing the quadratic into the standard form $ax^2 + bx + c = 0$:

$$\begin{aligned} 3x^2 &= m(5x + v) \\ 3x^2 &= 5mx + mv \\ 3x^2 - 5mx - mv &= 0 \end{aligned}$$

In this case, $a = 3$, $b = -5m$, and $c = -mv$. Now solve:

$$\begin{aligned} x &= \frac{-(-5m) \pm \sqrt{(-5m)^2 - 4(3)(-mv)}}{2(3)} \\ &= \frac{5m \pm \sqrt{25m^2 - (-12mv)}}{6} \\ &= \frac{5m \pm \sqrt{25m^2 + 12mv}}{6} \\ &= \frac{5m}{6} \pm \frac{\sqrt{25m^2 + 12mv}}{6} \end{aligned}$$

Therefore, **(B)** is correct.

19. D

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: Get the equation $x(dx + 10) = -3$ into the form $ax^2 + bx + c = 0$. Multiply out the left side of the equation $x(dx + 10) = -3$ to get $dx^2 + 10x = -3$. Add 3 to both sides to obtain $dx^2 + 10x + 3 = 0$.

The equation $ax^2 + bx + c = 0$ (when $a \neq 0$) does not have real solutions if the discriminant, which is $b^2 - 4ac$, is negative. In the equation $dx^2 + 10x + 3 = 0$, $a = d$, $b = 10$, and $c = 3$. The discriminant in this question is $10^2 - 4(d)(3) = 100 - 12d$.

Since you're looking for a negative discriminant, that is, $b^2 - 4ac < 0$, you need $100 - 12d < 0$. Solve the inequality $100 - 12d < 0$ for d:

$$\begin{aligned} 100 - 12d &< 0 \\ 100 &< 12d \\ \frac{100}{12} &< d \\ \frac{25}{3} &< d \\ 8\tfrac{1}{3} &< d \end{aligned}$$

Among the answer choices, only 10 is greater than $8\frac{1}{3}$, so **(D)** is correct.

20. C

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: Recall that when a quadratic equation has no real solutions, its discriminant, which is $b^2 - 4ac$, will be less than 0. Calculate the discriminant of each answer choice and pick the one that's negative. You don't need to actually solve for x.

(A): $8^2 - 4(1)(-12) = 64 + 48 > 0$. Eliminate.

(B): $(-8)^2 - 4(1)(12) = 64 - 48 > 0$. Eliminate.

(C): $(-9)^2 - 4(1)(21) = 81 - 84 = -3 < 0$. Pick **(C)** and move on. For the record:

(D): $(100)^2 - 4(1)(-1) = 10{,}000 + 4 > 0$. Eliminate.

21. B

Difficulty: Easy

Category: Advanced Math

Getting to the Answer: The factored form of a quadratic equation makes it easy to find the solutions to the equation, which graphically represent the x-intercepts. The graph shows x-intercepts at $x = -\frac{3}{4}$ and $x = 1$. For each answer choice, set each factor equal to 0 and quickly solve to find the x-intercepts and see which ones agree with the graph.

(A): $x = \frac{3}{4}$ and $x = -1$. This does not match the graph; eliminate.

(B): $x = -\frac{3}{4}$ and $x = 1$. This matches the graph, so **(B)** is correct. You do not need to check the remaining choices.

22. C

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: An axis of symmetry splits a parabola in half and travels through the vertex. Use the formula to find h, plug in the correct values from the equation, and simplify:

$$\begin{aligned} x &= -\frac{b}{2a} \\ &= -\frac{17}{2\left(\frac{-11}{3}\right)} \\ &= -\frac{17}{\left(\frac{-22}{3}\right)} \\ &= -17 \cdot \frac{-3}{22} \\ &= \frac{51}{22} \end{aligned}$$

The correct answer is **(C)**. Note that you could have also graphed the function on your calculator to determine the axis of symmetry. Use the approach that is best for you.

23. C

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: The given function is in vertex form, $y = a(x - h)^2 + k$, where (h, k) is the vertex and the sign of a indicates whether the parabola opens up or down. Since $a = -1$ for $f(x) = -(x - p)^2 + q$, the parabola opens downward. Eliminate (A) and (B). The vertex is (p, q), and the question tells you that p is negative, so **(C)** is correct.

24. 5/2 or 2.5

Difficulty: Easy

Category: Advanced Math

Getting to the Answer: The factored form of the equation $y = a(x + 1)(x - 6)$ tells you that the x-intercepts of the parabola are -1 and 6. The x-coordinate of the vertex (h) is the axis of symmetry, which is halfway between the x-intercepts. Thus, $h = \frac{-1 + 6}{2} = \frac{5}{2}$. Enter **5/2** or **2.5**.

Math

25. A
Difficulty: Hard

Category: Advanced Math

Getting to the Answer: The answer choices are all similar, so pay careful attention to their differences and see if you can eliminate any choices logically. A rocket goes up and then comes down, which means that the graph will be a parabola opening downward. The equation, therefore, should have a negative sign in front. Eliminate (C) and (D).

To evaluate the two remaining choices, recall the *vertex form* of a quadratic, $y = a(x - h)^2 + k$, and what it tells you: the vertex of the graph is (h, k). The h is the x-coordinate of the maximum (or minimum) and k is the y-coordinate of the maximum (or minimum). In this situation, x has been replaced by t, or time, and y is now $h(t)$, or height. The question says that the maximum height occurs at 3 seconds and is 34 feet, so h is 3 and k is 34. Substitute these values into vertex form to find that the correct equation is $y = -16(x - 3)^2 + 34$. The function that matches is **(A)**.

26. B
Difficulty: Medium

Category: Advanced Math

Strategic Advice: Because each of the two expressions containing b is equal to a, the two expressions must be equal to each other.

Getting to the Answer: Set the two expressions equal to each other and then solve for b:

$$\begin{aligned} b^2 + 4b - 12 &= -12 + b \\ b^2 + 4b &= b \\ b^2 + 3b &= 0 \\ b(b + 3) &= 0 \end{aligned}$$

If $b(b + 3) = 0$, then $b = 0$ or $b = -3$. Of these two values, only -3 is among the answer choices, so **(B)** is correct.

27. 2
Difficulty: Medium

Category: Advanced Math

Getting to the Answer: The points of intersection of the graphs are the points at which the equations are equal. Since (a, b) is the label for an (x, y) point, set the two equations equal to each other and solve for the value of x to find the value of a:

$$\begin{aligned} -2x &= 5x^2 - 12x \\ 0 &= 5x^2 - 10x \\ 0 &= 5x(x - 2) \end{aligned}$$

Thus, $x = 0$ or $x = 2$. The question states that the intersection points are $(0, 0)$ and (a, b), so a must equal 2. Enter **2**.

Alternatively, you could graph the two equations on the calculator to determine the point of intersection.

28. −1/4, −0.25, or −.25
Difficulty: Hard

Category: Advanced Math

Getting to the Answer: Since $f(x)$ and $g(x)$ intersect, set the equations equal to each other: $x^2 + x = d$. Then subtract d from both sides to set the quadratic equal to 0: $x^2 + x - d = 0$.

Recall that when there is exactly one solution, the discriminant, $b^2 - 4ac$, equals 0. In the equation $x^2 + x - d = 0$, $a = 1$, $b = 1$, and $c = -d$. Therefore, $1^2 - 4(1)(-d) = 0$. Solving for d gives $d = -\frac{1}{4}$. Enter **−1/4, −0.25,** or **−.25**

29. C
Difficulty: Hard

Category: Advanced Math

Getting to the Answer: Because the question states that $f(c) = g(c)$, set the two functions equal to each other and solve for x. To make calculations easier, begin by converting $f(x)$ into standard form:

$$\begin{aligned} f(x) &= -2(x - 3)^2 - 4 \\ &= -2(x - 3)(x - 3) - 4 \\ &= -2(x^2 - 6x + 9) - 4 \\ &= -2x^2 + 12x - 18 - 4 \\ &= -2x^2 + 12x - 22 \end{aligned}$$

Now set the two functions equal to each other:

$$-2x^2 + 12x - 22 = 2x - 10$$

Simplify by dividing all terms by -2:

$$x^2 - 6x + 11 = -x + 5$$

Next, combine like terms and solve for x:

$$x^2 - 6x + 11 = -x + 5$$
$$x^2 - 5x + 6 = 0$$
$$(x - 2)(x - 3) = 0$$

Therefore, $x = 2$ or $x = 3$, which means that c could also be either 2 or 3. Because 3 is not an answer choice, the answer must be 2. **(C)** is correct.

If you find it faster, you could graph the two functions to determine the points of intersection.

30. C

Difficulty: Hard

Category: Advanced Math

Strategic Advice: When you need to find the points of intersection of two equations, set the equations equal to each other.

Getting to the Answer: The question indicates that points P and Q are the points of intersection of the two equations, so set the two equations equal to each other and consolidate terms to get a single quadratic equation equal to 0:

$$3x^2 + \frac{14}{3}x - \frac{73}{3} = -\frac{4}{3}x - \frac{1}{3}$$
$$3x^2 + \frac{18}{3}x - \frac{72}{3} = 0$$
$$3x^2 + 6x - 24 = 0$$
$$x^2 + 2x - 8 = 0$$

Factor the equation to find the values of x:

$$x^2 + 2x - 8 = 0$$
$$(x + 4)(x - 2) = 0$$
$$x = -4 \text{ or } 2$$

You can plug each value of x into either of the original equations to find the corresponding values of y, but the linear equation is easier to work with. For $x = -4$:

$$y = -\frac{4}{3}(-4) - \frac{1}{3}$$
$$y = \frac{16}{3} - \frac{1}{3}$$
$$y = \frac{15}{3} = 5$$

Therefore, one of the points of intersection is $(-4, 5)$. Find the other point of intersection by plugging $x = 2$ into the linear equation:

$$y = -\frac{4}{3}(2) - \frac{1}{3}$$
$$y = -\frac{8}{3} - \frac{1}{3}$$
$$y = -\frac{9}{3} = -3$$

Thus, the other point of intersection is $(2, -3)$.

Note that you could have also found the points of intersection by graphing the two equations.

The question asks for the distance between these two points. The formula for the distance, d, between the points (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Find the distance between points P and Q:

$$d = \sqrt{(-4 - 2)^2 + (5 - (-3))^2}$$
$$= \sqrt{(-6)^2 + 8^2}$$
$$= \sqrt{36 + 64}$$
$$= \sqrt{100}$$
$$= 10$$

Therefore, the distance between points P and Q is 10. **(C)** is correct.

Check Your Work – How Much Have You Learned: Advanced Math

1. D

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: With absolute value equations, you must solve twice: once for the positive version of the given value and once for the negative version of that value. Start by solving for the positive version:

$$5a = 2a - 15$$
$$3a = -15$$
$$a = -5$$

However, plugging -5 back into the right side of the original equation yields $2(-5) - 15 = -25$. Since an absolute value can never be negative, -5 is not a solution; therefore, you can eliminate (A) and (C). Next, solve for the negative version:

$$5a = -2a + 15$$
$$7a = 15$$
$$a = \frac{15}{7}$$

Again, when you plug $\frac{15}{7}$ into the right side of the original equation, you get

$$2\left(\frac{15}{7}\right) - 15 = \frac{30}{7} - 15 = \frac{30}{7} - \frac{105}{7} = -\frac{75}{7}.$$

That's a negative value, so $\frac{15}{7}$ is not a solution either.

There is no solution, so **(D)** is correct.

2. 67

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: Use FOIL to expand the factored form of the quadratic into standard form:

$$\begin{aligned} y &= 4(x+3)^2 + 7 \\ &= 4(x+3)(x+3) + 7 \\ &= 4(x^2 + 3x + 3x + 9) + 7 \\ &= 4(x^2 + 6x + 9) + 7 \\ &= 4x^2 + 24x + 36 + 7 \\ &= 4x^2 + 24x + 43 \end{aligned}$$

The question asks for the value of $b + c$, which is $24 + 43 = 67$, so enter **67**.

3. B

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: Start by multiplying and combining the values under each root:

$$\sqrt{0.5} \times \sqrt{0.4} = \sqrt{0.5 \times 0.4} = \sqrt{0.2}$$

When you have a root of a decimal, you can simplify by changing the decimal to a fraction. Then simplify the root of the fraction form:

$$\sqrt{0.2} = \sqrt{\frac{2}{10}} = \sqrt{\frac{1}{5}} = \frac{\sqrt{1}}{\sqrt{5}} = \frac{1}{\sqrt{5}}$$

A radical is not allowed in the denominator of a fraction, so rationalize it by multiplying the numerator and the denominator by the radical that you are trying to rationalize: $\frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$. Alternatively, you can evaluate the expression in the question on your calculator, 0.44721..., and then evaluate the expressions in the answer choices until you find a match. This might, however, consume time that could be better spent on other questions. Choice **(B)** is correct.

4. A

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: Cancel the root on top by separating each of the terms underneath it. First, $\sqrt[3]{8} = 2$, so you can pull that to the outside of the expression: $\frac{2\left(\sqrt[3]{a^9b^6}\right)}{ab^2}$. Next, convert the cube root of the variable terms using fractional exponents and simplify step-by-step:

$$\begin{aligned} &\frac{2a^{\frac{9}{3}}b^{\frac{6}{3}}}{ab^2} \\ &= \frac{2a^3b^2}{ab^2} \\ &= \frac{2a^3\cancel{b^2}}{a\cancel{b^2}} \\ &= \frac{2a^3}{a} \\ &= 2a^2 \end{aligned}$$

Choice **(A)** is correct.

5. 14

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: You could graph this function on a calculator to see where its minimum value falls. Without graphing, the key to answering this question is to determine the minimum value of x^2 and then add 14 to that value. Numbers that have been squared can never be negative, so the smallest possible value of x^2 is $0^2 = 0$. Therefore, the minimum value of the function is $0 + 14 = 14$. Enter **14**.

6. −11

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: You could graph this on a calculator to locate the vertex. Otherwise, the first step is to find h, the x-coordinate of the vertex. The formula for finding the x-coordinate of the vertex is $\frac{-b}{2a}$ (the quadratic formula without the radical part). In the equation, $a = 2$ and $b = -8$, so the equation is $h = \frac{-(-8)}{2(2)} = \frac{8}{4} = 2$.

To find the value of k, which is the y-coordinate of the vertex, plug 2 in for x in the original quadratic equation: $k = 2(2)^2 - 8(2) - 3 = 8 - 16 - 3 = -11$. Enter **−11**.

7. A

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: According to the graph, both x-intercepts are to the left of the y-axis. Therefore, both values of x are negative. Eliminate (B) because both x values are positive when y equals 0. To evaluate the remaining equations, find the x-intercepts by setting each factor equal to 0 and solving for x. In (A), the x-intercepts are −2 and −6, which means that one intercept is 3 times farther from the origin than the other, which matches the description in the question, so this must be the correct answer. For the record, in (C), the x-intercepts are −1 and −4, making one intercept 4 times farther from the origin than the other; in (D), the x-intercepts are −2 and −4, making one intercept 2 times farther from the origin than the other. Thus, **(A)** is correct.

8. B

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: Work from the inside out. First find $g(1)$ and then plug the result into $f(x)$: $g(1) = 1 + 4 = 5$. Now, find $f(5)$: $\frac{5^2 + 5(5) + 10}{3(5)} = \frac{25 + 25 + 10}{15} = \frac{60}{15} = 4$. **(B)** is correct.

9. D

Difficulty: Hard

Category: Advanced Math

Getting to the Answer: Factor the denominator in the second term to find that the common denominator for all three terms is $(x - 3)(x + 3)$. Multiply each term in the equation by the common denominator (in factored form or the original form, whichever is more convenient) to clear the fractions. Then solve the resulting equation for x:

$$(x-3)\cancel{(x^2-9)}\left(\frac{2}{\cancel{x+3}}\right) - \cancel{(x^2-9)}\left(\frac{-12}{\cancel{x^2-9}}\right) = 1(x^2 - 9)$$

$$\begin{aligned} 2(x-3) - (-12) &= x^2 - 9 \\ 2x - 6 + 12 &= x^2 - 9 \\ 2x + 6 &= x^2 - 9 \end{aligned}$$

Subtract the left side of the equation to yield a quadratic equation, set equal to zero, and then factor the equation to find the values of x:

$$\begin{aligned} 0 &= x^2 - 2x - 15 \\ 0 &= (x + 3)(x - 5) \end{aligned}$$

Set each factor equal to 0 to find that the potential solutions are $x = -3$ and $x = 5$. Note that these are only *potential* solutions because the original equation was a rational equation. When $x = -3$, both denominators on the left side of the original equation are equal to 0, so −3 is an extraneous solution, which means **(D)** is correct.

Math

10. B

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: There are two ways to approach this question. One is to plug the value of x, 10, into both functions and then calculate the value of the fraction:

The function $f(x)$ is $2(10)^2 - 14(10) + 20 = 200 - 140 + 20 = 80$ when $x = 10$. The function $g(x) = 10 - 5 = 5$.

So $\frac{f(x)}{g(x)} = \frac{80}{5} = 16$.

Alternatively, you could factor the $f(x)$ equation to see if there are any common factors with the $g(x)$ equation that can be canceled out: $2x^2 - 14x + 20 = 2(x^2 - 7x + 10) = 2(x - 2)(x - 5)$. So the fraction becomes

$\frac{2(x-2)\cancel{(x-5)}}{\cancel{(x-5)}} = 2(x - 2) = 2(10 - 2) = 2(8) = 16$.

(B) is correct.