

Check Your Work – How Much Do You Know: Geometry and Trigonometry

1. A

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: Start by translating from English into math. Because one leg of the triangle is three times as long as the other, let x and $3x$ represent the lengths. Use the Pythagorean theorem to find the hypotenuse:

$$\begin{aligned}a^2 + b^2 &= c^2 \\x^2 + (3x)^2 &= c^2 \\x^2 + 9x^2 &= c^2 \\10x^2 &= c^2 \\\sqrt{10x^2} &= c\end{aligned}$$

Although you can't find a numerical value for c , $\sqrt{10x^2}$ tells you that the number under the radical is a multiple of 10. The only choice with a multiple of 10 under the radical is (A), so (A) is correct.

Note that when $\sqrt{10x^2} = \sqrt{40}$, $x = 2$.

2. D

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: Because one angle of the triangle measures 90° and the two legs are congruent (notice the tick marks), this is a 45-45-90 triangle. The side lengths of a 45-45-90 triangle are in the ratio $x:x:x\sqrt{2}$, where x represents the length of a leg and $x\sqrt{2}$ represents the length of the hypotenuse. Set up an equation using $x\sqrt{2}$ from the ratio and the length of the leg, $7\sqrt{2}$, to find h :

$$\begin{aligned}h &= x\sqrt{2} \\&= 7\sqrt{2} \times \sqrt{2} \\&= 7\sqrt{4} \\&= 7(2) \\&= 14\end{aligned}$$

The length of the hypotenuse is 14, so (D) is correct.

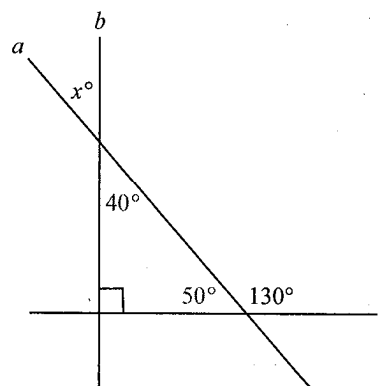
3. 40

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: Since line b is perpendicular to line c , the triangle formed by lines a , b , and c is a right triangle. The angle supplementary to 130° is $180^\circ - 130^\circ = 50^\circ$, so the remaining angle in the

triangle is $180^\circ - (90^\circ + 50^\circ) = 40^\circ$. Since x° is vertical to that angle, x° is also 40° .



Enter 40.

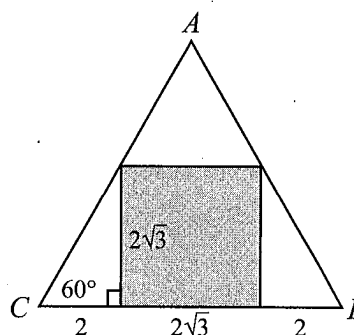
4. B

Difficulty: Hard

Category: Geometry and Trigonometry

Getting to the Answer: Start with what you know about the shaded square. Because its area is 12, each side must be $\sqrt{12} = 2\sqrt{3}$.

Triangle ABC is an equilateral triangle, so each of its interior angles measures 60° . The two vertical sides of the square, therefore, each represent the longer leg of a 30-60-90 triangle (the small white triangles on the sides). The side lengths of a 30-60-90 triangle are in the ratio $x:x\sqrt{3}:2x$, where x represents the length of the short leg. For the small white triangles, the long leg has a length of $2\sqrt{3}$, which means the length of each short leg is 2.



You now have the length of the base of the large equilateral triangle: $2 + 2\sqrt{3} + 2 = 4 + 2\sqrt{3}$. Thus, each side of the large equilateral triangle has length $4 + 2\sqrt{3}$. The perimeter is the sum of all three sides, so multiply by 3 to get $12 + 6\sqrt{3}$, making (B) correct.

5. D

Difficulty: Easy

Category: Geometry and Trigonometry

Getting to the Answer: Corresponding sides of similar triangles are proportional. In other words, the larger triangle is a “scaled-up” version of the smaller triangle. Therefore, you’re looking for the same ratio of sides (8:15:17), multiplied by a scale factor. This means **(D)** is correct because each side length of ABC has been scaled up by a factor of 3.

6. C

Difficulty: Easy

Category: Geometry and Trigonometry

Getting to the Answer: You are given the relative size of each of the parts in this question, and the *whole* is the total number of degrees in a circle, which is 360. Now, set up an equation:

$$\begin{aligned} 25x + 11x &= 360 \\ 36x &= 360 \\ x &= 10 \end{aligned}$$

Note that the question asks for the measure of the smallest angle, which is represented by $11x$. The correct answer is $11(10) = 110$, which is **(C)**.

7. C

Difficulty: Hard

Category: Geometry and Trigonometry

Getting to the Answer: To find the equation of a circle, you need the radius and the x - and y -coordinates of the center point. Then, you can use the standard equation: $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center of the circle and r is the length of the radius.

Since each grid line is 3 units, the center of the circle is $(3 \times 4, 3 \times 1)$, or $(12, 3)$ and its radius is $3 \times 5 = 15$. Thus, the equation must be $(x - 12)^2 + (y - 3)^2 = 15^2$, or $(x - 12)^2 + (y - 3)^2 = 225$. **(C)** is the correct answer.

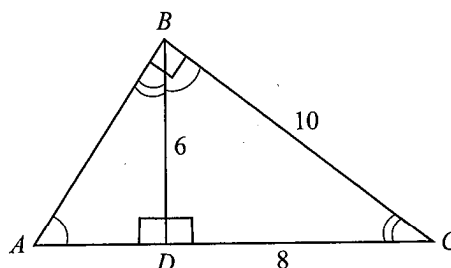
8. 60/8 or 7.5

Difficulty: Hard

Category: Geometry and Trigonometry

Getting to the Answer: Triangles ABC , BDC , and ADB are similar triangles because their corresponding angles are congruent.

A right triangle with side length 6 and hypotenuse 10 means a 3:4:5 triangle, but you could also use the Pythagorean theorem to find that $DC = 8$.



To find the length of AB , set up a proportion using the lengths of triangle ADB and triangle BDC and solve:

$$\begin{aligned} \frac{AB}{10} &= \frac{6}{8} \\ 8AB &= 60 \\ AB &= \frac{60}{8} \\ AB &= 7.5 \end{aligned}$$

Enter **60/8** or **7.5**.

Alternatively, you could set up a proportion using the lengths of triangle ABC and triangle BDC

$$\begin{aligned} \frac{AB}{6} &= \frac{10}{8} \\ 8AB &= 60 \\ AB &= \frac{60}{8} \\ AB &= 7.5 \end{aligned}$$

9. A

Difficulty: Medium

Category: Geometry and Trigonometry

Strategic Advice: You’re looking for surface area because you need to spray all of the faces of the tent, *excluding* the bottom. Decompose the figure into 2-D shapes, and add their areas together. Lastly, find the number of ounces of waterproofing agent using unit conversion.

Getting to the Answer: The two sides of the tent are rectangles: $2A = 2(lw) = 2(9)(5) = 90$. The front and back of the tent are triangles:

$$2A_{\text{triangle}} = 2\left(\frac{1}{2}bh\right) = bh = (6)(4) = 24$$

Thus, the total surface area of the tent is $90 + 24 = 114$ square feet. Use unit conversion to find the number of ounces of waterproofing agent needed:

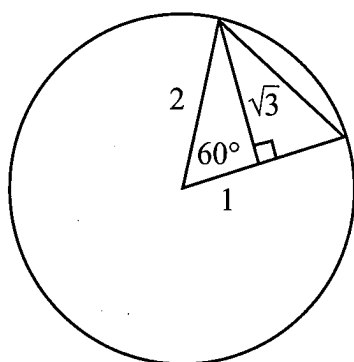
$$114 \text{ square feet} \times \frac{1 \text{ ounce}}{3 \text{ square feet}} = 38 \text{ ounces}$$

(A) is correct.

10. D

Difficulty: Hard**Category:** Geometry and Trigonometry

Getting to the Answer: The chord (the line segment that connects two points on the circle's circumference) forms a triangle with the two radii of the central angle. Since the chord is the same length as the radii, this is an equilateral triangle. Therefore, each angle, including the central angle, is 60° . Although this triangle does not have a right angle, it can be split into two 30-60-90 triangles. So, you can use the side ratios of a 30-60-90 triangle to find $\sin 60^\circ$. That ratio is $1:\sqrt{3}:2$.



Because $\sin x = \frac{\text{opposite}}{\text{hypotenuse}}$, the sine of 60° is $\frac{\sqrt{3}}{2}$. **(D)** is correct.

Check Your Work – Chapter 14

1. C

Difficulty: Easy

Category: Geometry and Trigonometry

Getting to the Answer: When a line intersects two parallel lines, it intersects each of the parallel lines at the same angle. Therefore, the value of x will equal the value of the angle supplementary to 76.

Since $180 - 76 = 104$, $x = 104$, so choice (C) is correct.

2. 70

Difficulty: Easy

Category: Geometry and Trigonometry

Getting to the Answer: When two or more angles combine to form a straight line, the sum of the angles is 180° . Therefore, $v + w + x = 180$.

The question states that $x = 45$ and $v = 65$, so you can say that $65 + w + 45 = 180$. That means $180 - 65 - 45 = w$. Thus, $w = 70$.

Angle w forms a vertical angle with angle y . Therefore, $y = w$, so $y = 70$. Enter **70**.

3. A

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: When a transversal crosses two or more parallel lines, all acute angles formed will be equal and all obtuse angles formed will be equal.

The question asks for the answer choice that does NOT necessarily correspond to the value of angle a , which is an obtuse angle. Check each answer and eliminate any choice that is equal to the value of an obtuse angle in the diagram.

Angles b and f are both acute angles in the diagram; while it's possible that the sum of two acute angles is equal to one obtuse angle, this is not *necessarily* true. Therefore, choice (A) is correct.

For the record, d forms a vertical angle with a , so $d = a$. Eliminate (B). Since any obtuse and acute angle pair in the diagram form a straight line (that is, the pair are supplementary angles) $180 - b = a$. Eliminate (C). Finally, e and h are both obtuse angles, so they are each equal to a . So

$$\frac{e + h}{2} = \frac{a + a}{2} = \frac{2a}{2} = a$$

This allows you to eliminate (D). Thus, every other answer choice is a match in value for angle a and can be eliminated. Choose (A).

4. B

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: Since lines j and k are parallel, and line l is a transversal passing through j and k , all obtuse angles formed by the transversal will be identical, and all acute angles formed will also be identical. Therefore, acute angles b and d are equal.

The question specifies that $f = 9b$. Therefore, you can say that the value of each obtuse angle in the diagram is 9 times the value of each acute angle. Since an acute angle and an obtuse angle in the figure are supplementary—they sum to 180° —you can say that $f + b = 180^\circ$. Substituting $9b$ for f , the equation becomes $9b + b = 180^\circ$, so $10b = 180^\circ$. Now isolate b :

$$\begin{aligned} 10b &= 180^\circ \\ \frac{10b}{10} &= \frac{180^\circ}{10} \\ b &= 18^\circ \end{aligned}$$

The question asks for the value of $2d$. Since $d = b$, $2d = 2b$. That is $2 \times 18 = 36$. Choice (B) is correct.

5. 330

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: When a transversal crosses parallel lines, all the acute angles are equal, and all the obtuse angles are equal.

Since $b = 30$ and $a + b = 180$, you know that $a = 180 - 30 = 150$. The angles represented by b and c are both acute angles formed by the same transversal, so $b = c$. Since $b = 30$, $c = 30$. Similarly, the angles represented by a and e are both obtuse angles formed by the same transversal, so $a = e$.

Since $a = 150$, you know that $e = 150$. The question asks for the sum of a , c , and e , which is $a + c + e = 150 + 30 + 150 = 330$.

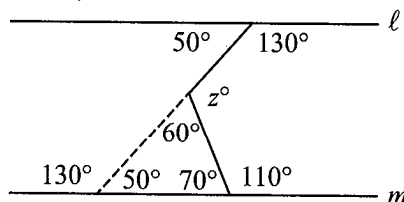
Enter **330**.

6. D

Difficulty: Hard

Category: Geometry and Trigonometry

Getting to the Answer:



Continue the transversal as shown until it intersects line m to create a triangle. The base interior angles of the triangle are $180^\circ - 130^\circ = 50^\circ$ and $180^\circ - 110^\circ = 70^\circ$ (from alternate interior angles of the transversals). Since the sum of the interior angles in a triangle is always 180° , the remaining angle in the triangle is $180^\circ - 50^\circ - 70^\circ = 60^\circ$. Therefore, z equals $180^\circ - 60^\circ = 120^\circ$. Choice **(D)** is correct.

7. B

Difficulty: Easy

Category: Geometry and Trigonometry

Getting to the Answer: This is a matter of plugging into the formulas and subtracting the results. The area of a parallelogram is base times height, or $A = 8 \times 9 = 72$. The area of a triangle is $A = \frac{1}{2}bh$, or $A = \frac{1}{2}(6)(7) = 21$. Subtract to find the difference of $72 - 21 = 51$. **(B)** is correct.

8. C

Difficulty: Easy

Category: Geometry and Trigonometry

Getting to the Answer: The perimeter of a rectangle is $2l + 2w$, where l is the length and w is the width. Since the length is 50 times its width, or $l = 50w$, the perimeter of the beam is $2(50w) + 2w = 102w$ centimeters. **(C)** is correct.

9. 14

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: Use the formula for the area of a rectangle: $A = l \times w$. In this case, l is height and equals

$w - 34$. Plugging that and the area into the formula and solving for w by factoring gives:

$$672 = (w - 34)(w)$$

$$672 = w^2 - 34w$$

$$0 = w^2 - 34w - 672$$

$$0 = (w - 48)(w + 14)$$

$$w = 48 \text{ and } -14$$

Because the width cannot be negative, the width is 48 ft. The height is $48 - 34 = 14$ ft. Enter **14**.

10. B

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: The formula for the area of a triangle is $A = \frac{1}{2}bh$, so $x = \frac{1}{2}bh$. Since both the base and height of the triangle are doubled, the base of the new triangle is $2b$ and the height of the new triangle is $2h$. This gives an area of $\frac{1}{2}(2b)(2h)$ or $\frac{1}{2}bh \times 4$. Given that $x = \frac{1}{2}bh$, the area of the new triangle in terms of x is $4x$. **(B)** is correct.

11. A

Difficulty: Hard

Category: Geometry and Trigonometry

Getting to the Answer: Let x be a side of Square B. The area of a square is equal to its side length squared, so the area of Square B is x^2 . Since the area of Square A is 4 times the area of Square B, the area of Square A is $4x^2$. Taking the square root gives $2x$ as the side of Square A. The perimeter of a square is 4 times its side length. Thus, the perimeter of Square A is $4(2x) = 8x$ and the perimeter of Square B is $4x$. The perimeter of Square A is $\frac{8x}{4x} = 2$ times greater than the perimeter of Square B. **(A)** is correct.

12. A

Difficulty: Hard

Category: Geometry and Trigonometry

Getting to the Answer: Let h represent the height of the original triangle and b the base. By definition, x percent is $\frac{x}{100}$. Thus, the new triangle has a height of $1.5h$ and a base of $(1 - \frac{x}{100})b$. The new area is $1.2A$. Plugging

GEOMETRY AND TRIGONOMETRY

these into the formula for the area of a triangle, $A = \frac{1}{2}bh$, gives:

$$\begin{aligned} 1.2A &= \frac{1}{2}\left(1 - \frac{x}{100}\right)b(1.5h) \\ 1.2\left(\frac{1}{2}bh\right) &= \frac{1}{2}\left(1 - \frac{x}{100}\right)b(1.5h) \\ 1.2 &= \left(1 - \frac{x}{100}\right)(1.5) \\ 0.8 &= 1 - \frac{x}{100} \\ -0.2 &= -\frac{x}{100} \\ 20 &= x \end{aligned}$$

(A) is correct.

13. 426

Difficulty: Medium

Category: Geometry and Trigonometry

Strategic Advice: The figure contains a pair of similar triangles. Use the fact that their sides are proportional to find the required length.

Getting to the Answer: The question asks for the length of DE . First, find the length of EC by setting up a proportion where BD is related to AE in the same way that DC is related to the unknown EC . AE is 324 and DC is 50% longer than segment BD , or $1.5 \times 40 = 60$ feet:

$$\begin{aligned} \frac{BD}{AE} &= \frac{DC}{EC} \\ \frac{40}{324} &= \frac{60}{EC} \\ 40(EC) &= 19,440 \\ EC &= 486 \end{aligned}$$

Note that $DE + DC = EC$, which means $DE = EC - DC$. Subtract the length of DC , 60, from the length of EC , 486, to obtain 426, the length of DE . Enter **426**.

14. B

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: Given the area of $\triangle ABC$ and the length of the base AB , you can find BC , its height. Use the triangle area formula:

$$\begin{aligned} \text{Area} &= \frac{1}{2}b \times h \\ 150 &= \frac{1}{2}(20)(BC) \\ 150 &= 10(BC) \\ 15 &= BC \end{aligned}$$

Because lengths $BC = 15$ and $AB = 20$, $\triangle ABC$ is a 3:4:5 triangle with dimensions scaled up by a factor of 5. The hypotenuse, AC , must be $5 \times 5 = 25$. $\triangle ABC$ and $\triangle AGH$ are similar triangles because they share an angle at vertex A and they each have a right angle. Therefore, their corresponding sides must be proportional. The question says that the hypotenuse of $\triangle AGH$, AH , is 20, so use this information to create a proportion:

$$\begin{aligned} \frac{AH}{AC} &= \frac{HG}{BC} \\ \frac{20}{25} &= \frac{HG}{15} \\ 300 &= 25(HG) \\ HG &= 12 \end{aligned}$$

Choice (B) is the correct answer.

15. B

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: The question says that segment HI is the bisector of segments LO and OW . This tells you two things: 1) HI divides both LO and OW exactly in half and 2) HI is parallel to LW .

Because HI is parallel to LW , angles L and H must be congruent (they are corresponding angles), and angles W and I must be congruent (they are also corresponding angles). Angle O is shared by both triangles. The triangles, therefore, are similar. Side lengths of similar triangles are in proportion to one another. Because I is the midpoint of OW , OI is half as long as OW . The same is true for the other side: OH is half as long as OL . Thus, the sides are in the ratio 1:2. The question gives the side lengths of LW and HI . Use this ratio and these side lengths to set up a proportion and solve for x :

$$\begin{aligned} \frac{1}{2} &= \frac{HI}{LW} \\ \frac{1}{2} &= \frac{4x - 1}{30} \\ 30 &= 2(4x - 1) \\ 30 &= 8x - 2 \\ 32 &= 8x \\ 4 &= x \end{aligned}$$

The correct answer is (B).

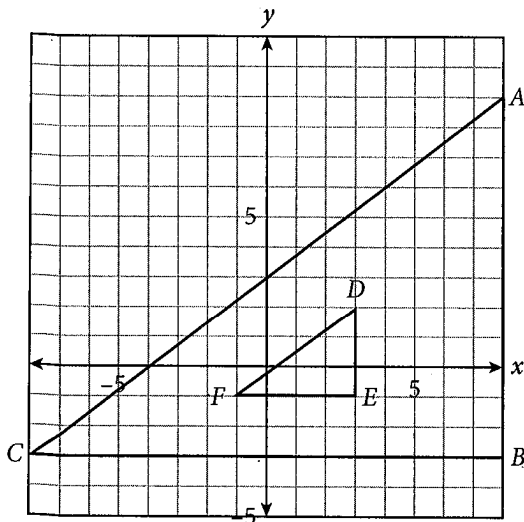
16. 9

Difficulty: Hard

Category: Geometry and Trigonometry

Getting to the Answer: Corresponding sides of similar triangles are proportional. Draw a quick sketch to find as many side lengths as you can, determine the ratio of the sides between the two triangles, and use that ratio to obtain the missing vertex.

On either the included graphing calculator or your scratch paper, plot all the points given in the question, labeling them as you go so you don't get confused. You know that D and A are in the same quadrant, which means the triangles are both oriented the same way, so make your sketch accordingly:



Once you have plotted triangle DEF and the base of triangle ABC , you can determine that the ratio of the triangles is 1:4 (the base of DEF has a length of 4 and the base of ABC has a length of 16). To determine where you should put A , find the length of side DE and then multiply by 4. The length of the vertical side of triangle ABC is $3 \times 4 = 12$. Because one vertex is at $(8, -3)$, vertex A must be 12 vertical units above that point, or $(8, 9)$. The y -coordinate of A is 9. Enter **9**.

17. B

Difficulty: Hard

Category: Geometry and Trigonometry

Getting to the Answer: The figure gives two nested triangles that share angle C . And, because BD is parallel to AE , angles CAE and CBD are congruent—they are corresponding angles. Two pairs of congruent angles

means that triangles ACE and BCD are similar. (You could also have analyzed angles CDB and CEA , but you need only two pairs of congruent angles to conclude that two triangles are similar.)

Set up a proportion using the triangles' side lengths. You'll need to translate from English into math as you go: $AB = 5$ and BC is three times that, or 15. This means $AC = 5 + 15 = 20$. CD is 2 more than half AC , $\frac{20}{2} + 2 = 10 + 2 = 12$. Use the three known side lengths to create a proportion and solve for EC :

$$\begin{aligned}\frac{BC}{AC} &= \frac{CD}{EC} \\ \frac{15}{20} &= \frac{12}{EC} \\ 15(EC) &= 240 \\ EC &= 16\end{aligned}$$

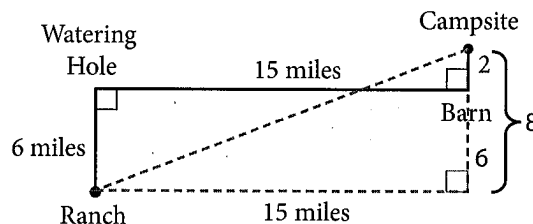
The question asks for the length of segment DE , which is $EC - CD$, or $16 - 12 = 4$. **(B)** is correct.

18. B

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: Start by connecting the ranch to the campsite. Then draw in a horizontal line and a vertical line to form a right triangle.



The length of one leg of the triangle is 15 miles, the distance from the watering hole to the barn. The length of the other leg is $6 + 2 = 8$ miles, the distance from the ranch to the watering hole plus the distance from the barn to the campsite. The two legs of the right triangle are 8 and 15. You may recognize the Pythagorean triple 8:15:17, but if you don't, you can always use the Pythagorean theorem:

$$\begin{aligned}8^2 + 15^2 &= c^2 \\ 64 + 225 &= c^2 \\ 289 &= c^2 \\ 17 &= c\end{aligned}$$

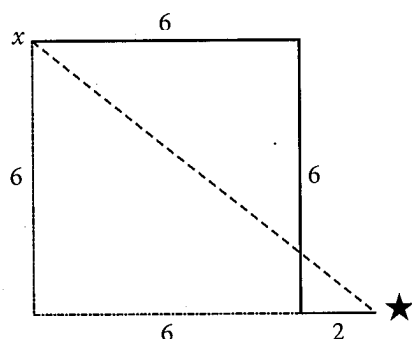
The direct route is 17 miles, so **(B)** is correct.

19. 4

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: Draw Ted's car and bike routes and label your diagram with the distances that you know. To find the distance of Ted's bike route, create a right triangle:



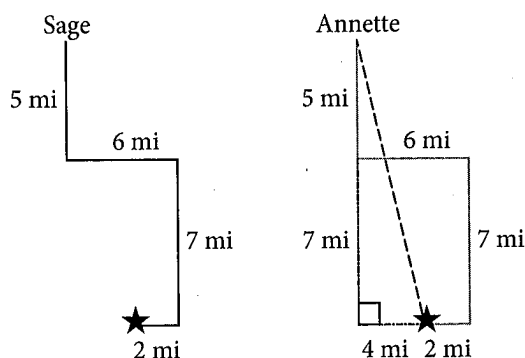
You can use the Pythagorean theorem to find the hypotenuse (bike route), but you can save time if you recognize the 6:8:10 Pythagorean triple. Ted's car route is $6 + 6 + 2 = 14$ miles, and his bike route is 10 miles. The difference between the two is 4.

20. A

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: The question asks for the total distance that Sage and Annette traveled. Sage hiked $5 + 6 + 7 + 2 = 20$ miles. To find the distance that Annette traveled, draw a diagram of the routes and make Annette's direct route the hypotenuse of a right triangle:



Use the Pythagorean theorem to calculate the distance that Annette traveled:

$$\begin{aligned} c^2 &= (5 + 7)^2 + 4^2 \\ c^2 &= 144 + 16 \\ c^2 &= 160 \\ c &= \sqrt{160} \\ c &\approx 12.65 \end{aligned}$$

Add this to Sage's distance to find the total distance:
 $20 + 12.65 = 32.65$ miles. Choice (A) is correct.

21. 4

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: Use the Pythagorean theorem to solve for x :

$$\begin{aligned} (3x)^2 + (x + 1)^2 &= (3x + 1)^2 \\ 9x^2 + x^2 + 2x + 1 &= 9x^2 + 6x + 1 \\ x^2 + 2x + 1 &= 6x + 1 \\ x^2 - 4x &= 0 \\ x(x - 4) &= 0 \end{aligned}$$

Therefore, $x = 0$ or $x = 4$. Since $3x$ represents a side of the triangle and a triangle cannot have a side length of 0, $x = 0$ is not a possible solution. This means that $x = 4$ must be correct. Enter 4.

22. B

Difficulty: Hard

Category: Geometry and Trigonometry

Getting to the Answer: The word "isosceles" means that the triangle has two equal sides, so you can call them both x . The hypotenuse is given as 4. Plug these values into the Pythagorean theorem and solve for x :

$$\begin{aligned} a^2 + b^2 &= c^2 \\ x^2 + x^2 &= 4^2 \\ 2x^2 &= 16 \\ x^2 &= 8 \\ x &= \sqrt{8} \\ x &= \sqrt{4} \sqrt{2} \\ x &= 2\sqrt{2} \end{aligned}$$

The question asks for the perimeter, which is the sum of all of the sides. Use this value of x to find the perimeter:
 $4 + 2\sqrt{2} + 2\sqrt{2} = 4 + 4\sqrt{2}$. Choice (B) is correct.

23. C

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: Triangle BCD is an isosceles right triangle, which means that it is a 45-45-90 right triangle with side length ratios of $x:x:x\sqrt{2}$. Because the two legs are each 4 units long, the hypotenuse BD is $4\sqrt{2}$. BD also acts as one leg of right triangle ABD . Since this leg is $4\sqrt{2}$ and the hypotenuse is $5\sqrt{2}$, triangle ABD is a 3:4:5 right triangle. Thus, the length of AD is $3\sqrt{2}$. The legs of a right triangle are also its base and height, so plug the lengths of the legs into the formula for the area of a triangle, $\frac{1}{2}bh$, to get $\frac{1}{2} \times 4\sqrt{2} \times 3\sqrt{2} = \frac{1}{2} \times 12 \times \sqrt{2} \times \sqrt{2} = 6 \times 2 = 12$. The correct answer is (C).

24. B

Difficulty: Easy

Category: Geometry and Trigonometry

Getting to the Answer: The height of the amusement ride at point M is perpendicular to the ground, and the ride's angle of ascent is 30° , which creates a 30-60-90 triangle. Use the ratio of the sides, $x:x\sqrt{3}:2x$, to find the height at point M . Because the distance from point O to point M is $200\sqrt{3}$ and corresponds to the hypotenuse of the triangle, you can conclude that $2x = 200\sqrt{3}$. Solving for x by dividing both sides by 2 gives the length of the side opposite the 30° angle. This corresponds to the height of the amusement ride at point M : $x = 100\sqrt{3}$ feet. (B) is correct.

25. B

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: The two given 45° angles mean that the triangle must be a 45-45-90 triangle. Define the length of each of the legs as x and use the ratio of the sides $x:x:x\sqrt{2}$ to solve for x :

$$\begin{aligned} x\sqrt{2} &= 16 \\ \frac{x\sqrt{2}}{\sqrt{2}} &= \frac{16}{\sqrt{2}} \\ x &= \frac{16}{\sqrt{2}} \end{aligned}$$

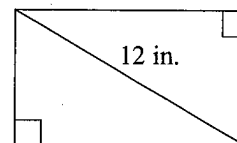
Next, determine the triangle's area using $\frac{1}{2}bh$. Note that in a right triangle, the sides that form the 90° angle can form the base and height. Thus, the area is $\frac{1}{2} \left(\frac{16}{\sqrt{2}} \right) \left(\frac{16}{\sqrt{2}} \right) = \frac{1}{2} \left(\frac{256}{2} \right) = \frac{256}{4} = 64$ square inches. Choice (B) is correct.

26. D

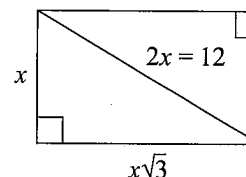
Difficulty: Hard

Category: Geometry and Trigonometry

Getting to the Answer: To find the area of the rectangle, you need to determine its length and width. On scratch paper, sketch a rectangle with a diagonal of 12. Note that the diagonal splits the rectangle into two right triangles and is the hypotenuse of each. The length and width of the rectangle correspond to the base and height of the right triangles.



The question says that the length is $\sqrt{3}$ times the width, so the ratio of the triangle's short leg to its long leg is $x:x\sqrt{3}$, where x represents the shorter leg and $x\sqrt{3}$ represents the longer leg. This relationship is part of the ratio of the side lengths of a 30-60-90 triangle: $x:x\sqrt{3}:2x$.



The hypotenuse is 12, which corresponds to $2x$, so $x = 6$. Therefore, the width is 6, and the length is $\sqrt{3} \times 6 = 6\sqrt{3}$. Use $l \times w$ to calculate the area: $6\sqrt{3} \times 6 = 36\sqrt{3}$ square inches. The correct answer is (D).

27. 30

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: Note the right triangle with a height of 2 and hypotenuse of 4 formed by the ramp to the stage. The ratio of the short side to the hypotenuse is 2:4, or 1:2. Notice that this matches the ratio of the short side to the hypotenuse for a 30-60-90 triangle: $x:x\sqrt{3}:2x$. The angle of incline is opposite the short side, so that is 30° . Enter 30.

28. D

Difficulty: Easy

Category: Geometry and Trigonometry

Getting to the Answer: The standard form of the equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center of the circle and r is the length of the radius.

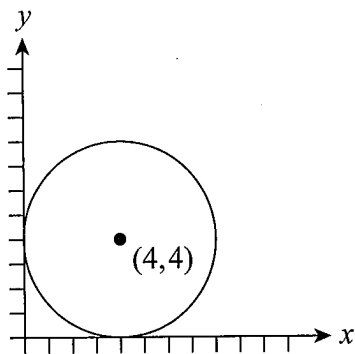
The answer choices have many similarities, which will make them easy to eliminate piece by piece. Use the graph to find the radius. From the center, you can count horizontally or vertically to the edge of the circle to find that its radius is 6. If $r = 6$, then $r^2 = 36$. Eliminate (A) and (B). Now find the x -coordinate of the center of the circle, -1 . This means the $(x - h)^2$ part of the equation is $(x - (-1))^2 = (x + 1)^2$. Eliminate (C). Only (D) is left and is correct. Note that you do not even need to find the $(y - k)^2$ part of the equation, but for the record: because the y -coordinate of the center of the circle is 2, then $(y - k)^2$ becomes $(y - 2)^2$, and the full equation is $(x + 1)^2 + (y - 2)^2 = 36$. (D) is correct.

29. D

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: Draw a quick sketch on your scratch paper to help visualize the circle:



You can see from the diagram that the center of the circle is at $(4, 4)$, and the radius is 4. In the equation of a circle, $(x - h)^2 + (y - k)^2 = r^2$, the center is at (h, k) , and the radius is r . Plug in the information you know to get $(x - 4)^2 + (y - 4)^2 = 4^2$, which matches choice (D).

Alternatively, you can input each of the equations in the answer choices into the graphing calculator to see which matches the description of the circle given the question. This will also lead you to the correct answer of (D).

30. B

Difficulty: Hard

Category: Geometry and Trigonometry

Getting to the Answer: The question asks for the diameter, which is twice the radius. When the equation of a circle is in the form $(x - h)^2 + (y - k)^2 = r^2$, the r represents the length of the radius. The question gives the equation in general form, so you need to complete the square to put the equation into standard form.

You already have an x^2 and a y^2 in the given equation and the coefficients of x and y are even, so completing the square is fairly straightforward—there are just a lot of steps. Start by grouping the x 's and y 's together. Then, take the coefficient of the x term and divide it by 2, square it, and add it to the two terms with x variables.

Do the same with the y term. Remember to add these amounts to the other side of the equation as well. Then factor the perfect squares and simplify:

$$x^2 + y^2 + 8x - 20y = 28$$

$$x^2 + 8x + y^2 - 20y = 28$$

$$(x^2 + 8x + 16) + (y^2 - 20y + 100) = 28 + 16 + 100$$

$$(x + 4)^2 + (y - 10)^2 = 144$$

This equation tells you that r^2 is 144, which means that the radius is 12, and the diameter is twice that, or 24, which is (B).

31. D

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: The given equation defines a circle with its center at $(4, -2)$ and a radius of 10. Any point on the circumference of the circle must satisfy that equation. Eliminate (C) because it is the center of the circle. Plug the other choices into the equation to see if they satisfy the given equation:

(A): $(-3 - 4)^2 + (5 + 2)^2 = (-7)^2 + (7)^2 = 49 + 49 = 98$. Since $98 < 100$, this is inside the circle. Eliminate (A).

(B): $(0 - 4)^2 + (9 + 2)^2 = (-4)^2 + (11)^2$. Since the second term alone is greater than 100, this is outside the circle. Eliminate (B).

Only (D) is left and is correct. For the record:

(D): $(4 - 4)^2 + (8 + 2)^2 = (0)^2 + (10)^2 = 100$. This point satisfies the equation and is, therefore, on the circumference. (D) is indeed correct.

32. A

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: To find a central angle based on a known arc length, use the relationship $\frac{\text{arc length}}{\text{circumference}} = \frac{\text{central angle}}{360^\circ}$. The unknown in the relationship is the central angle, so call it a . Before you can fill in the rest of the equation, you need to find the circumference of the circle: $C = 2\pi r = 2\pi(120) = 240\pi$. Now, you're ready to solve for a :

$$\begin{aligned}\frac{\text{arc length}}{\text{circumference}} &= \frac{\text{central angle}}{360^\circ} \\ \frac{200}{240\pi} &= \frac{a}{360^\circ} \\ \frac{200(360)}{240\pi} &= a \\ 95.5^\circ &\approx a\end{aligned}$$

(A) is correct.

33. C

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: Because the ratio of the shaded area to the non-shaded area is 4:5, the ratio of the shaded area to the entire circle is $4:(4+5) = 4:9$. This ratio is the same as the ratio of the interior angle of the shaded sector to 360° , or $x:360$. Set up a proportion using these ratios:

$$\begin{aligned}\frac{4}{9} &= \frac{x}{360} \\ 360(4) &= 9x \\ 1,440 &= 9x \\ 160 &= x\end{aligned}$$

Choice (C) is correct.

34. B

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: The measure of an arc is directly related to the degree measure of its central angle. The measure of an inscribed angle is half of that of the central angle. Because the measure of arc KL is 150° , the degree measure of the inscribed angle is half of that. The

inscribed angle for arc KL can be written as $3x + 2x$. Set up an equation to solve for x :

$$\begin{aligned}3x + 2x &= \frac{1}{2}(150) \\ 5x &= 75 \\ x &= 15\end{aligned}$$

Choice (B) is correct.

35. C

Difficulty: Hard

Category: Geometry and Trigonometry

Getting to the Answer: Note that the figure is composed of two triangles inscribed in a circle. If you have enough information about the triangles, you can find the length of WX . Arcs XY and YZ are each 60° . Because an inscribed angle is half of its corresponding arc, the inscribed angles XWY and ZWY are each 30° . Similarly, since arc YXW is 180° , the inscribed angle YZW is 90° . Therefore, both triangles are 30-60-90 special right triangles. Use the ratio of the sides $x:x\sqrt{3}:2x$ to find the length of WX . Given $YZ = 3$, XY also equals 3 and $\frac{3}{WX} = \frac{1}{\sqrt{3}}$, so $WX = 3\sqrt{3}$. Choice (C) is correct.

36. D

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: Since BC is the diameter, the measure of arc BAC is 180° . Therefore, the measure of arc BA is the measure of arc BAC minus the measure of arc AC : $180^\circ - 30^\circ = 150^\circ$. Use $180^\circ = \pi$ to convert to radians: $150^\circ \times \frac{\pi}{180^\circ} = \frac{5\pi}{6}$. Thus, (D) is correct.

37. B

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: Begin by finding the volume of each sphere using the volume formula for a sphere, remembering to halve the diameters first:

$$\begin{aligned}V_1 &= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3)^3 = \frac{4}{3}\pi(27) = 36\pi \text{ cm}^3 \\ V_2 &= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6)^3 = \frac{4}{3}\pi(216) = 288\pi \text{ cm}^3\end{aligned}$$

The positive difference is $288\pi \text{ cm}^3 - 36\pi \text{ cm}^3 = 252\pi \text{ cm}^3$, which is (B).

38. C

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: After the water is poured into the larger glass, the volume of the water in the glass will be the same as the volume when it was in the smaller glass. Find the volume of the water in the smaller glass, whose height is 6 inches and diameter is 3 inches. Then, substitute this volume into a second equation where the height is unknown and the radius is 2 inches (the radius of the larger glass) and solve for h . The volume of a cylinder equals the area of its base times its height, or $V = \pi r^2 h$:

$$V = \pi r^2 h$$

$$V = \pi(1.5)^2(6)$$

$$V = \pi(2.25)(6)$$

$$V = 13.5\pi$$

$$13.5\pi = \pi(2)^2 h$$

$$13.5\pi = 4\pi h$$

$$3.375 = h$$

The water will be 3.375 inches high in the larger glass. (C) is correct.

39. 68

Difficulty: Hard

Category: Geometry and Trigonometry

Getting to the Answer: When a question involves many steps, as this one does, plan out the order of your calculations and conversions. In this case, you can go from the volume of sand in 1 tank to the volume of sand in 50 tanks, to the weight of sand in pounds, to the number of bags of sand.

The volume of sand in one tank (only 2 inches of the height) will be $V = 24 \times 9 \times 2 = 432$ cubic inches, which means the volume of sand in all 50 tanks will be $50 \times 432 = 21,600$ cubic inches. Each cubic inch of sand weighs 0.125 pounds, so the weight of all the sand will be $0.125 \times 21,600 = 2,700$ pounds. Finally, each bag contains 40 pounds of sand, so the pet store needs to buy $2,700 \text{ pounds} \times \frac{1 \text{ bag}}{40 \text{ pounds}} = 67.5$ bags. Because the store cannot buy one-half of one 40-pound bag of sand, it will need to buy 68 bags of sand. Enter **68**.

40. A

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: Determine the volumes of the rectangular box and of the cylindrical candle. Then calculate the difference between the two to find the volume of space between them. The volume of the box is the area of the base times its height: $15 \times 8 = 120$ cubic inches. Eliminate (C) and (D). Next, use $\pi r^2 h$ to calculate the volume of the cylinder. The diameter is 2 inches, so the radius is 1 inch: $\pi(1)^2(8) = 8\pi$. Felipe needs $120 - 8\pi$ cubic inches of wax, so (A) is correct.

41. C

Difficulty: Medium

Category: Geometry and Trigonometry

Getting to the Answer: A cube consists of 6 square faces. If the perimeter of one square face of the cube is $4x$, the length of one side is $\frac{4x}{4} = x$. The surface area of a cube is the sum of the areas of all 6 square faces or $6 \times \text{side}^2$. Thus, the surface area of the cube in terms of x is $6x^2$. (C) is correct.

42. C

Difficulty: Hard

Category: Geometry and Trigonometry

Getting to the Answer: The formula for finding the volume of a pyramid with a rectangular base is $V = \frac{1}{3}lwh$. Start by substituting what you know into the formula. The volume is represented by $x^3 - x$, the length is $x + 1$, and the width is $3x$:

$$V = \frac{1}{3}lwh$$

$$x^3 - x = \frac{1}{3}(x + 1)(3x)h$$

$$x(x^2 - 1) = (x + 1)xh$$

Notice that if you divide both sides of the equation by x , you'll be left with $x^2 - 1$ on the left side and $(x + 1)$ times h on the right side. Note that this is a classic quadratic: the factors of $x^2 - 1$ are $x + 1$ and $x - 1$. So:

$$(x + 1)(x - 1) = (x + 1)h$$

$$x - 1 = h$$

This means the height of the pyramid must be represented by $x - 1$. Therefore, (C) is correct.