Check Your Work – How Much Do You Know: Algebra

1. A

Difficulty: Easy

Category: Algebra

Getting to the Answer: To find the cost of potatoes on Friday, multiply their weight p by the sale price. If there is a 30% discount, then the customer actually pays $100\% - 30\% = 70\%$. Therefore, p pounds of potatoes costs $0.90 \times 70\% \times p = 0.90 \times 0.70 \times p = 0.63p$. Now, add the cost of two cantaloupes, $3.50 \times 2 = 7$, to get the total cost: $c = 0.63p + 7$. Choice **(A)** is correct.

2. 0

Difficulty: Medium

Category: Algebra

Getting to the Answer: The first equation contains fractions, which are difficult to work with. Clear the fractions by multiplying both sides of the equation by 15:

$$15\left(\frac{x}{5} + \frac{y}{3} = \frac{21}{15}\right) \rightarrow 3x + 5y = 21$$

Using similar logic, multiply both sides of the second equation by 7:

$$7\left(\frac{3}{7}x + 4y = 3\right) \rightarrow 3x + 28y = 21$$

The question asks for the value of y. You can eliminate the x terms by subtracting the second equation from the first:

$$\begin{array}{r} 3x + 5y = 21 \\ -(3x + 28y = 21) \\ \hline -23y = 0 \\ y = 0 \end{array}$$

Enter **0**.

3. −4.25 or −17/4

Difficulty: Hard

Category: Algebra

Getting to the Answer: Linear functions have a constant rate of change, or slope. Examining the table, you see that every time x is increased by 2 units, $f(x)$ decreases by 0.5 units. This means that the slope is $\frac{-0.5}{2} = -0.25$. The point $(0, -3)$ on the table indicates that the y-intercept is -3. Plugging these values into $y = mx + b$ yields $f(x) = -0.25x - 3$. The question asks for $f(5)$, so plug in 5 for x and simplify: $f(5) = -0.25(5) - 3 = -1.25 - 3 = -4.25$. Enter **−4.25** or **−17/4**.

4. B

Difficulty: Easy

Category: Algebra

Getting to the Answer: Distributing the $\frac{3}{2}$ would create more messy fractions. To avoid this, multiply both sides of the equation by 2:

$$\begin{aligned} 2\left[\frac{3}{2}(x+7) = 6\right] \\ 3(x+7) &= 12 \\ 3x + 21 &= 12 \\ 3x &= -9 \\ x &= -3 \end{aligned}$$

Choice **(B)** is correct. Backsolving is another viable approach. When backsolving, start with either (B) or (C). Try plugging in $x = -3$:

$$\begin{aligned} \frac{3}{2}(x+7) &= 6 \\ \frac{3}{2}(-3+7) &= 6 \\ \frac{3}{2}(4) &= 6 \\ 6 &= 6 \end{aligned}$$

Since this results in a true statement, **(B)** is correct.

5. A

Difficulty: Medium

Category: Algebra

Getting to the Answer: The rate of change (or slope) of a linear relationship is constant, so find the slope and use it to determine the missing value. Examine the points $(-9, 11)$ and $(0, 8)$. When x increased by 9, y decreased by 3. Therefore, if x increases by 9 again, y should decrease by 3 again. This means that the y-value when $x = 9$ is $8 - 3 = 5$. Choice **(A)** is correct.

6. C

Difficulty: Hard

Category: Algebra

Getting to the Answer: Use known points from the graph to determine the equations for the two lines. Then convert them into inequalities.

The solid line passes through the points $(0, 4)$ and $(1, 2)$. Its slope is therefore $\frac{y_2 - y_1}{x_2 - x_1} = \frac{2-4}{1-0} = \frac{-2}{1} = -2$. The graph shows that the y-intercept is 4, so the equation

for the solid line is $y = -2x + 4$. This can be restated as $y + 2x = 4$. Because the shaded region is below this line and the line is solid, the inequality defined by the line is $y + 2x \leq 4$. Eliminate (A) and (D).

The dotted line passes through the points $(-4, 0)$ and $(1, 2)$. Thus, the slope of this line is $\frac{2-0}{1-(-4)} = \frac{2}{5}$. The y-intercept is unclear from the graph, so plug the x-intercept $(-4, 0)$ into $y = \frac{2}{5}x + b$ to get $0 = \frac{2}{5}(-4) + b$. Thus, $b = \frac{8}{5}$. The equation for this line is therefore $y = \frac{2}{5}x + \frac{8}{5}$. Since the line is dotted and the values of the solution are above the line, the inequality is $y > \frac{2}{5}x + \frac{8}{5}$. Simplify this by multiplying all the terms by 5 to get $5y > 2x + 8$, which converts to $5y - 2x > 8$. These two inequalities match the ones in **(C)**.

7. A

Difficulty: Medium

Category: Algebra

Getting to the Answer: Solve the second inequality for y and substitute this result into the first inequality. If $8y > 20 + 3y$, then $5y > 20$ and $y > 4$. Plugging into the second inequality gives $4x > 4 + 8$, or $4x > 12$. This means that $x > 3$. Choice **(A)** is correct.

8. D

Difficulty: Hard

Category: Algebra

Getting to the Answer: Graphically, a "solution" to a system of equations is where the two lines touch each other. For a system of equations to have no solution, they must never touch each other. This means that they are parallel. Parallel lines have the same slope but different y-intercepts. Although you could rearrange each equation to get it into $y = mx + b$ form, it is easier to recognize that the slope of each line is determined by the x- and y-coefficients. If the two lines have the same x- and y-coefficients, then they have the same slope.

Examining the equations, you see that the first one has a $9y$ and the second has a $6y$. If you multiply the second equation by $\frac{3}{2}$, you can make the coefficient of the y term 9:

$$\frac{3}{2}(2x + 6y = 5) \rightarrow 3x + 9y = \frac{15}{2}$$

Since the y-coordinates are now the same, you can simply match up the x-coordinates in both equations. This means that $a = 3$. Choice **(D)** is correct.

9. B

Difficulty: Medium

Category: Algebra

Getting to the Answer: Rather than solving for each variable individually, see if you can solve for the desired expression all at once. Begin by getting the x and y variables together on the left-hand side of the equation and the constants on the right:

$$2x - 3y = -10$$
$$-4x + 7y = -5$$

Look carefully at what the question is asking you to find. You can obtain this expression simply by subtracting the two equations:

$$\begin{array}{r} 2x - 3y = -10 \\ -(-4x + 7y = -5) \\ \hline 6x - 10y = -5 \end{array}$$

Choice **(B)** is correct.

10. −4

Difficulty: Medium

Category: Algebra

Getting to the Answer: To evaluate $g(4)$, replace x with 4 in the g function: $g(4) = 3 - f(4)$. Next, find $f(4)$ by replacing x with 4 in the f function: $f(4) = 2(4) - 1 = 8 - 1 = 7$. Finally, plug this value of $f(4)$ back into the g function: $g(4) = 3 - 7 = -4$. Enter **−4**.

Check Your Work – Chapter 4

1. D

Difficulty: Easy

Category: Algebra

Getting to the Answer: Distribute the factor of 2, combine like terms, multiply both sides of the equation by 2 to clear the fraction, and then solve for y:

$$\begin{aligned} 3y + 2y - 4 &= \frac{3y}{2} + 1 \\ 5y - 4 &= \frac{3y}{2} + 1 \\ 10y - 8 &= 3y + 2 \\ 7y &= 10 \\ y &= \frac{10}{7} \end{aligned}$$

Choice **(D)** is correct.

2. C

Difficulty: Hard

Category: Algebra

Strategic Advice: Complicated-looking equations may appear difficult, but they always succumb to the steps of solving an equation. First, clear the equation of fractions, then collect like terms, and solve for the desired variable.

Getting to the Answer: Clear the equation of the fraction in the numerator by multiplying both sides by 4 to yield:

$$4S = \frac{4C - I}{C + I}$$

Now, multiply both sides by the denominator $C + I$ to clear the equation of fractions:

$$4S(C + I) = 4C - I$$

Distribute the $4S$:

$$4SC + 4SI = 4C - I$$

To solve for I, collect all the terms that include I on one side of the equation:

$$4SI + I = 4C - 4SC$$

Factor out the I:

$$I(4S + 1) = 4C - 4SC$$

Divide to isolate I, and factor out $4C$ from the numerator:

$$I = \frac{4C - 4SC}{4S + 1} = \frac{4C(1 - S)}{4S + 1}$$

(C) is correct.

3. 18

Difficulty: Easy

Category: Algebra

Getting to the Answer: First, clear the fractions by multiplying both sides of the equation by 8. Then, solve for n using inverse operations:

$$\begin{aligned} \frac{7}{8}(n - 6) &= \frac{21}{2} \\ 8\left[\frac{7}{8}(n - 6)\right] &= 8\left[\frac{21}{2}\right] \\ 7(n - 6) &= 4(21) \\ 7n - 42 &= 84 \\ 7n &= 126 \\ n &= 18 \end{aligned}$$

Enter **18**.

4. A

Difficulty: Medium

Category: Algebra

Strategic Advice: Noticing key information about the answer choices and using that information to pick numbers saves time by eliminating algebra.

Getting to the Answer: Rearranging the terms of the equation such that $\frac{a}{b}$ is on one side and the constants are all on the other will work for this question. First, cross-multiply to get rid of the fractions. Then, use inverse operations to isolate $\frac{a}{b}$:

$$\begin{aligned} \frac{3a + b}{b} &\times \frac{11}{2} \\ 2(3a + b) &= 11b \\ 6a + 2b &= 11b \\ 6a &= 9b \\ a &= \frac{9}{6}b \\ \frac{a}{b} &= \frac{3}{2} \end{aligned}$$

The answer is **(A)**.

There is a faster approach. Notice that 2 shows up in the denominator in most of the choices, indicating that it is likely that b equals 2. If $b = 2$, then a must be 3 for the numerator to equal 11. Test this by plugging the numbers into the equation:

$$\frac{3(3) + 2}{2} = \frac{11}{2}$$

Thus, $\frac{a}{b}=\frac{3}{2}$, confirming **(A)** as the answer. You can avoid a lot of work by using key information in the answer choices to pick numbers.

5. D

Difficulty: Medium

Category: Algebra

Getting to the Answer: Simplify the numerators as much as possible, then isolate the variable. Begin by combining like terms on both sides of the equation. Then cross-multiply and solve for z:

$$\frac{4+z-(3+2z)}{6} = \frac{-z-3(5-2)}{7}$$
$$\frac{1-z}{6} = \frac{-z-9}{7}$$
$$7-7z = -6z-54$$
$$-z = -61$$
$$z = 61$$

Choice **(D)** is correct.

6. D

Difficulty: Medium

Category: Algebra

Getting to the Answer: Use the information in the question to write your own expression, then look for the answer choice that matches. Simplify your expression only if you don't find a match. If a couple earns \$50 *per half-hour* that they dance, then they earn $50 \times 2 = \$100$ *per hour*. Multiply this amount by the number of hours (not including the first 3 hours). This can be expressed as $100(h-3)$. This is not one of the answer choices, so simplify by distributing the 100 to get $100h-300$, which is **(D)**.

If you're struggling with the algebra, try picking numbers. Pick a number of hours a couple might dance, such as 5. They don't earn anything for the first 3 hours, but they earn \$50 per half-hour for the last 2 hours, which is 50 times 4 half-hours, or \$200. Now, find the expression that gives you an answer of \$200 when $h=5$ hours: $100(5)-300=500-300=200$. If you use picking numbers, remember that it is possible that the number you choose satisfies more than one answer choice, so plug it in to the other three choices. In this case, $h=5$ does not give a value of 200 in any other expression, which confirms that **(D)** is correct.

7. B

Difficulty: Medium

Category: Algebra

Getting to the Answer: Translate piece by piece to get a final expression for the final value, v. Multiply the second value by 2 to get $2(1)=2$. Subtract that from the first value to get $f-2$. Divide this expression by half of the third value to get $\frac{f-2}{\frac{t}{2}}$, which you can simplify by multiplying by the reciprocal of the denominator: $\left(\frac{f-2}{1}\right)\left(\frac{2}{t}\right)=\left(\frac{2f-4}{t}\right)$. Choice **(B)** matches the final expression. Watch out for (D), the trap answer choice that switches the variables.

8. B

Difficulty: Medium

Category: Algebra

Getting to the Answer: Write expressions to represent the profit generated by selling each type of pizza. You're told Pizza A sells for \$17 each and that its ingredients cost the pizzeria \$450 per week. This means the weekly profit generated by this pizza's sales can be represented by the expression $17x-450$. Do the same for Pizza B: each one sells for \$13, but the pizzeria loses \$310 to pay for ingredients each week. Therefore, the weekly profit from this pizza can be represented by $13x-310$. To determine the value of x at which the profit from the sale of each type of pizza is the same, set the two profit expressions equal to each other and solve:

$$17x-450=13x-310$$
$$4x=140$$
$$x=35$$

Thus, **(B)** is correct.

9. A

Difficulty: Easy

Category: Algebra

Getting to the Answer: When faced with a question that includes abstract expressions, it is often helpful to pick concrete numbers to work with. These numbers don't have to be realistic; just choose numbers that are easy to work with. Suppose the student works 3 hours a week for 2 weeks. She would have worked a total of 6 hours, which would have to be multiplied by the amount

she is paid per hour to get her total pay. In the expression, plugging in $w = 2$ and $n = 3$ demonstrates that the number 10 in the expression must be the amount that she is paid per hour, and thus is the term that would change if the employee got a raise. **(A)** is correct.

10. 7

Difficulty: Medium

Category: Algebra

Getting to the Answer: First, notice that the actual dollar amount of the increase each year is unknown, so assign a variable like d. After 12 increases, his salary will rise by \$25,500, so write a formula $12d = 25{,}500$. Solve for d to find that Malik will receive an increase of \$2,125 per year.

The question asks how many years must go by until Malik's salary is at least \$40,000; the number of years is another unknown, so assign it a variable like n. Multiplying n by the amount of each increase will give you the total increase of dollars over n years, but don't forget to add in the current salary to reflect his total salary amount:

$$25{,}500 + 2{,}125n = 40{,}000$$
$$2{,}125n = 14{,}500$$
$$n \approx 6.8$$

The question asks you to assume that salary increases only take place at the end of a full year. If you are unsure and want to prove that the answer is not 6, check by using $n = 6$ to calculate the total dollar amount: $25{,}500 + 2{,}125(6) = 38{,}250$. Six years is not long enough; enter **7**.

11. B

Difficulty: Easy

Category: Algebra

Getting to the Answer: You're asked to identify the line that the one described in the question stem will never intersect. Lines that never intersect are parallel and therefore have identical slopes, so start by finding the slope of the line whose two coordinate pairs are given. You'll find:

$$m = \frac{1 - 0}{0 - \left(-\frac{2}{5}\right)} = \frac{1}{\frac{2}{5}} = \frac{5}{2}$$

Choices (C) and (D) have negative slopes, so eliminate them. Next, find the slopes of (A) and (B). No need to use the slope formula; counting units on the graphs will be faster. The slope of (A) is $\frac{2}{5}$ because for every 2 units the line rises, it runs 5 units to the right. The slope of (B) is $\frac{5}{2}$ because when the line goes up 5 units, it goes 2 units to the right. Therefore, **(B)** is correct.

12. A

Difficulty: Medium

Category: Algebra

Strategic Advice: Remember that parallel lines have the same slope and perpendicular lines have opposite sign reciprocal slopes.

Getting to the Answer: The first useful piece of information is that the slope of the line perpendicular to line t is $-\frac{4}{3}$. Perpendicular lines have negative reciprocal slopes, so the slope of line t is $\frac{3}{4}$. Eliminate (B) and (D) because they have incorrect slopes.

Plug the values for the slope and the coordinates of point (4, 7) into the slope-intercept equation to solve for b:

$$7 = \frac{3}{4}(4) + b$$
$$7 = 3 + b$$
$$7 - 3 = b$$
$$b = 4$$

Eliminate (C) because it does not have the correct y-intercept. Choice **(A)** is correct.

13. A

Difficulty: Medium

Category: Algebra

Getting to the Answer: Start by finding the slope of the line by picking a pair of points, such as (40, 0) and (65, 100): $m = \frac{100 - 0}{65 - 40} = \frac{100}{25} = 4$. Choices (B) and (C) have slopes other than 4, so eliminate them. Choices (A) and (D) have y-intercepts of -160 and 40, respectively. Now, read the axis labels carefully: the horizontal axis begins at 40 (not 0). The line is trending downward as x-values get smaller, so the y-intercept (when $x = 0$) must be well below 0 on the vertical axis. Therefore, the answer must be **(A)**.

14. 70

Difficulty: Hard

Category: Algebra

Getting to the Answer: Because 40 pounds is not shown on the graph, you need more information. In a real-world scenario, the *y*-intercept of a graph usually represents a flat fee or a starting amount. The slope of the line represents a unit rate, such as the cost per pound to ship the box.

The *y*-intercept of the graph is 10, so the flat fee is \$10. To find the cost per pound (the unit rate), substitute two points from the graph into the slope formula. Using the points (0, 10) and (4, 16), the cost per pound is $\frac{16-10}{4-0}=\frac{6}{4}=1.5$, which means it costs \$1.50 per pound to ship a box. The total cost to ship a 40-pound box is $\$10+1.50(40)=\$10+\$60=\$70$. Enter **70**.

15. A

Difficulty: Hard

Category: Algebra

Getting to the Answer: The question tells you that the relationship is linear, so start by finding the rate of change (the slope, *m*) using any two pairs of values from the table and the slope formula.

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{64-34}{30-10}=\frac{30}{20}=1.5$$

You can stop right there! The next step would be to use the slope and a pair of values from the table to determine *b*; however, only **(A)** has a slope of 1.5, so it must be the correct answer. For the record:

$$34=1.5(10)+b$$
$$34=15+b$$
$$19=b$$

Check Your Work – Chapter 5

1. 1

Difficulty: Medium

Category: Algebra

Getting to the Answer: Start by isolating c in the second equation: $c = 3b - 2$. Then substitute c into the first equation and solve:

$$\begin{aligned} 7(3b-2)+8b &= 15 \\ 21b-14+8b &= 15 \\ 29b-14 &= 15 \\ 29b &= 29 \\ b &= 1 \end{aligned}$$

Enter **1.**

2. A

Difficulty: Easy

Category: Algebra

Getting to the Answer: The quickest way to solve is to realize that you can rearrange the first equation to find that $x = y$. Then substitute y in for x in the second equation: $y = 2y + 5$. Solve to find that $y = -5$. Because $x = y$, x also equals -5, and $x + y = -10$. **(A)** is correct.

3. C

Difficulty: Medium

Category: Algebra

Getting to the Answer: Because x has a coefficient of 1 in the second equation, solve the system using substitution. Before you select your answer, make sure you found the right quantity (the difference between x and y).

First, solve the second equation for x and substitute:

$$\begin{aligned} x-5y &= 2 \\ x &= 2+5y \\ 4(2+5y)+3y &= 14-y \\ 8+20y+3y &= 14-y \\ 8+23y &= 14-y \\ 24y &= 6 \\ y &= \frac{6}{24} = \frac{1}{4} \end{aligned}$$

Next, substitute this value back into $x = 2 + 5y$ and simplify:

$$\begin{aligned} x &= 2+5\left(\frac{1}{4}\right) \\ x &= \frac{8}{4}+\frac{5}{4} \\ x &= \frac{13}{4} \end{aligned}$$

Finally, evaluate $x - y$ to find the difference:

$$\frac{13}{4}-\frac{1}{4}=\frac{12}{4}=3$$

(C) is correct.

4. B

Difficulty: Medium

Category: Algebra

Getting to the Answer: Since the second equation is easier to solve for one variable and the question asks for $\frac{b}{2}$, solve the second equation for a. Then substitute the result into the first equation to create an equation in only one variable:

$$\begin{aligned} a &= b+3 \\ 5(b+3) &= 6b+7 \\ 5b+15 &= 6b+7 \\ 8 &= b \end{aligned}$$

So $\frac{b}{2} = 4$, and **(B)** is correct.

5. A

Difficulty: Medium

Category: Algebra

Getting to the Answer: Set up two equations: one for the number of items sold and one for the money collected. Let N = the number of bags of nuts sold and G = the number of granola bars sold.

The equation for the total items is $N + G = 112$.

The equation for the money collected is $1.25N + 1.75G = 160$.

At this point, you could solve by either combination or substitution. If you use substitution, solve the first equation for N, the number of bags of nuts, because the question asks for G, the number of granola bars. Solving the first equation for N yields $N = 112 - G$. Substituting that equation into the second equation gives:

$$\begin{aligned} 1.25(112-G)+1.75G &= 160 \\ 140-1.25G+1.75G &= 160 \\ 0.5G &= 20 \\ G &= 40 \end{aligned}$$

Remember that the question asks for the number of cases of granola bars the owner purchased, so divide 40 by 20, the number of granola bars per case. **(A)** is correct.

As an alternative approach, if you read the question carefully and recognized you're solving for the number of cases, not the number of granola bars, the correct answer would have to be either (A) or (B), since (C) and (D) are way too big. You could then test one of those choices, for example, (A) 2, by multiplying by 20 granola bars per case: $2 \times 20 = 40$. Subtract that number from 112 to get the number of bags of nuts: $112 - 40 = 72$. Multiply each quantity by the price per package: $72 \times 1.25 = 90$ and $40 \times 1.75 = 70$. Then add the sales of the two items together: $90 + 70 = 160$. You've now confirmed the correct answer because \$160 is the total sale amount. If you had tested (B) instead, then you could have eliminated it and still arrived at the correct answer with no more work because you eliminated all choices except **(A)**.

If you chose any of the other options, you likely answered the wrong question. (B) is the number of cases of nuts purchased, (C) is the number of granola bars sold, and (D) is the number of bags of nuts sold.

6. D

Difficulty: Easy

Category: Algebra

Getting to the Answer: This system is already set up perfectly to solve using combination because the y terms ($-3y$ and $3y$) are opposites. Add the two equations to cancel $-3y$ and $3y$. Then solve the resulting equation for x:

$$\begin{array}{r} 2x - 3y = 14 \\ +(5x + 3y = 21) \\ \hline 7x \quad\quad = 35 \\ x \quad\quad = 5 \end{array}$$

Choice **(D)** is correct. The question asks only for the value of x, so you don't need to substitute x back into either equation to find the value of y.

7. D

Difficulty: Easy

Category: Algebra

Getting to the Answer: If you're not asked to find the value of an individual variable, the question may lend itself to combination. This question asks for $b + c$, so don't waste your time finding the variables individually if you can avoid it. After rearranging the equations so that variables and constants are aligned, you can add the equations together:

$$\begin{array}{r} -2b + 7c = 15 \\ +(3b - 6c = 2) \\ \hline b + c = 17 \end{array}$$

This matches **(D)**.

8. 59

Difficulty: Hard

Category: Algebra

Getting to the Answer: You're asked for the value of an expression rather than the value of one of the variables, so try combination. Start by rearranging the two equations so that variables and constants are aligned:

$$x + y = -15$$
$$\frac{x}{2} + \frac{5y}{2} = 37$$

Clear the fractions in the second equation and then add the equations:

$$2\left(\frac{x}{2} + \frac{5y}{2} = 37\right) \rightarrow x + 5y = 74$$

$$\begin{array}{r} x + y = -15 \\ +(x + 5y = 74) \\ \hline 2x + 6y = 59 \end{array}$$

This is precisely what the question asks for, so you're done. Enter **59**.

9. 1/2, .5, or 0.5

Difficulty: Medium

Category: Algebra

Getting to the Answer: None of the coefficients in either equation is 1, so using combination is a better strategy than substitution here. Examine the coefficients, looking for whether the x or the y terms will be easier to cancel. The y terms are both factors of 15, so multiply each equation to give y a coefficient of 15.

$$5(5x + 3y = 13) \rightarrow 25x + 15y = 65$$
$$3(8x + 5y = 21) \rightarrow 24x + 15y = 63$$

Subtract the second equation from the first:

$$\begin{array}{r} 25x + 15y = 65 \\ -(24x + 15y = 63) \\ \hline x \quad\quad = 2 \end{array}$$

Next, you need y so you can determine the value of $\frac{y}{x}$. Substitute 2 for x in one of the original equations:

$$\begin{aligned} 5(2) + 3y &= 13 \\ 10 + 3y &= 13 \\ 3y &= 3 \\ y &= 1 \end{aligned}$$

Plug your x- and y-values into $\frac{y}{x}$ to get $\frac{1}{2}$. Enter **1/2**, **.5**, or **0.5**.

10. 5

Difficulty: Hard

Category: Algebra

Getting to the Answer: Begin by choosing variables to represent tacos, t, and bags of chips, c. Then set up equations to represent the total purchase price at both restaurants. Restaurant A's total is $2.2t + 1.95c = 18.55$. Restaurant B's total is $3t + 1.5c = 19.5$. You're asked to find the number of bags of chips, so rewrite the equations to eliminate the t variable with combination. Multiply each equation by the t-coefficient of the other equation:

$$\begin{aligned} 3(2.2t + 1.95c = 18.55) &\rightarrow 6.6t + 5.85c = 55.65 \\ 2.2(3t + 1.5c = 19.5) &\rightarrow 6.6t + 3.3c = 42.9 \end{aligned}$$

Subtract the second equation from the first to cancel the t terms:

$$\begin{aligned} 6.6t + 5.85c &= 55.65 \\ -(6.6t + 3.3c &= 42.9) \\ \hline 2.55c &= 12.75 \\ c &= 5 \end{aligned}$$

You only need the number of bags of chips, so enter **5**.

An alternate strategy, which eliminates many of the calculations above, is to make use of the built-in graphing calculator. Again, determine the two costs equations, $2.2t + 1.95c = 18.55$ and $3t + 1.5c = 19.5$. Enter these into the graphing calculator using x for t and y for c, and note the point of intersection, (4, 5). Since the y variable represents the number of bags of chips, **5** is your final answer.

11. D

Difficulty: Medium

Category: Algebra

Strategic Advice: Note that (C) is impossible. There are only three possibilities: the lines intersect, in which case there is one solution; the lines are parallel, in which case there is no solution; or the equations describe the same line, in which case there are infinitely many solutions.

Getting to the Answer: Get the two equations into the same format so that you can distinguish among these possibilities:

$$\begin{aligned} 21x - 6y &= 54 \\ 3.5x - y &= 9 \end{aligned}$$

Now it's easier to see that the first equation is equivalent to multiplying every term in the second equation by 6. Both equations describe the same line, so there are infinitely many solutions; **(D)** is correct.

12. B

Difficulty: Hard

Category: Algebra

Getting to the Answer: A system of equations that has infinitely many solutions describes a single line. Therefore, manipulation of one equation will yield the other. Look at the constant terms: to turn the 18 into a -2, divide the first equation by -9:

$$\begin{aligned} \frac{(6x + 3y = 18)}{-9} &\rightarrow -\frac{6}{9}x - \frac{3}{9}y = -2 \\ &\rightarrow -\frac{2}{3}x - \frac{1}{3}y = -2 \end{aligned}$$

The y terms and constants in the second equation now match those in the first; all that's left is to set the coefficients of x equal to each other: $q = -\frac{2}{3}$. Choice **(B)** is correct.

Note that you could also write each equation in slope-intercept form and set the slopes equal to each other to solve for q.

13. C

Difficulty: Medium

Category: Algebra

Getting to the Answer: If the graphs intersect at $(-3, 1)$, then the solution to the system is $x = -3$ and $y = 1$. Substitute these values into both equations and go from there:

$$\begin{aligned} hx - 4y &= -10 & kx + 3y &= -15 \\ h(-3) - 4(1) &= -10 & k(-3) + 3(1) &= -15 \\ -3h - 4 &= -10 & -3k + 3 &= -15 \\ -3h &= -6 & -3k &= -18 \\ h &= 2 & k &= 6 \end{aligned}$$

So, $\frac{k}{h} = \frac{6}{2} = 3$, making **(C)** correct.

14. B

Difficulty: Easy

Category: Algebra

Getting to the Answer: The solution to a system of linear equations represented graphically is the point of intersection. If the lines do not intersect, the system has no solution.

According to the graph, the lines intersect, or cross each other, at (6, 3). The question asks for the *y*-coordinate of the solution, which is 3, so **(B)** is correct.

15. D

Difficulty: Hard

Category: Algebra

Getting to the Answer: A system of linear equations has infinitely many solutions if both lines in the system have the same slope and the same *y*-intercept (in other words, they are the same line).

To have the same slope, the *x*- and *y*-coefficients of the two equations must be the same. Use the *x*-coefficients here: to turn $\frac{1}{2}$ into 3, multiply by 6. So c becomes $6c$, and $6c = -6$, or $c = -1$, which is **(D)**.

Note that you could also write each equation in slope-intercept form and set the *y*-intercepts equal to each other to solve for c.

Check Your Work – Chapter 6

1. C

Difficulty: Easy

Category: Algebra

Getting to the Answer: Begin by multiplying all parts of the inequality by 6 to clear the fractions: $-a - 6a > -\frac{24}{3}$, which, when simplified, is $-7a > -8$. Divide both sides by -7, remembering to switch the direction of the sign: $a < \frac{8}{7}$. Therefore, **(C)** is correct.

2. A

Difficulty: Medium

Category: Algebra

Getting to the Answer: Don't solve for c on autopilot. Instead, solve for $15c$, then add 7 to both sides. To do this, first multiply both sides of the inequality by -1 to get $5c + 7 \geq -8$. (Notice that the inequality sign had to be flipped due to multiplication by a negative number.) Then subtract 7 from both sides to yield $5c \geq -15$. Multiply both sides by 3: $15c \geq -45$. Finally, add 7 to both sides: $15c + 7 \geq -38$. Choice **(A)** is correct.

3. C

Difficulty: Medium

Category: Algebra

Getting to the Answer: First, to clear the fraction, multiply both sides of the inequality by -8, remembering to flip the direction of the inequality sign: $8 - 10x < -24x + 16$.

Next, subtract 8 from both sides, and then add $24x$ to both sides. Divide both sides by 14, and then simplify the fraction:

$$\begin{aligned} -10x &< -24x + 8 \\ 14x &< 8 \\ x &< \frac{8}{14} \\ x &< \frac{4}{7} \end{aligned}$$

Thus, **(C)** is correct.

4. B

Difficulty: Medium

Category: Algebra

Getting to the Answer: Begin by subtracting 3 from both sides of the inequality to get $\frac{1}{4}a - \frac{1}{16}b < 2$. Next, clear the fractions in that inequality by multiplying all terms by 16: $16 \times \left(\frac{1}{4}a - \frac{1}{16}b < 2\right) \rightarrow 4a - b < 32$. **(B)** is correct.

5. B

Difficulty: Medium

Category: Algebra

Strategic Advice: Because the given inequality contains $4c$ and the value you're solving for has $12c$ (a multiple of $4c$), solve for $4c$ and then multiply by 3 to find $12c$.

Getting to the Answer: Subtract 20 from both sides of the inequality to find that $4c \geq 11$. Multiply by 3: $12c \geq 33$. Finally, add 7 to both sides: $12c + 7 \geq 40$. So the least permissible value for $12c + 7$ is 40. **(B)** is correct.

6. A

Difficulty: Easy

Category: Algebra

Getting to the Answer: The question offers a value range of a in terms of $6b$ and a value range for $3b$. Therefore, use the known value range of $3b$ to find the value range of $6b$.

Since $6b$ is two times $3b$, multiply both sides of the second inequality by 2 to find the value range of $6b$: $2 \times 3b < 2 \times 8$, so $6b < 16$.

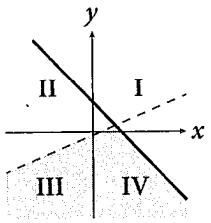
Because the signs are the same in the two inequalities (both are less-than signs), you can plug 16 in for $6b$ in the first inequality: $a < 16 + 4$, or $a < 20$. Hence, **(A)** is correct.

7. B

Difficulty: Medium

Category: Algebra

Getting to the Answer: Draw a sketch of the two lines (or use the graphing calculator) to visualize the system of inequalities.



Quadrants III and IV contain solutions to the system. Eliminate (C). Quadrant II contains no solutions, so you can also eliminate (A). Look closely at quadrant I. The line for $y \leq -x + 1$ intersects both the x and y axes at $+1$. The line for $y < \frac{1}{2}x$ passes through the origin and upward into quadrant I. Thus, there is a very small triangle of solutions to both inequalities that lies in quadrant I. Therefore, **(B)** is correct.

8. 400

Difficulty: Medium

Category: Algebra

Getting to the Answer: The first task is to express both inequalities in terms of y. Multiply $-y \leq 6x - 2{,}200$ by -1 to get $y \geq -6x + 2{,}200$; don't forget to flip the sign since you are multiplying by a negative number. Divide $3y \geq 9x - 1{,}500$ by 3 to get $y \geq 3x - 500$.

The solution set for these inequalities is the area of the coordinate plane on or above both lines. Thus, the minimum y-value will occur at the intersection of the two lines, whose equations are $y = -6x + 2{,}200$ and $y = 3x - 500$. To find the point of intersection, set these two equations equal to each other: $-6x + 2{,}200 = 3x - 500$. Isolate the x-values on one side to yield $-9x = -2{,}700$ and $x = 300$. Plug this value into one of the equations (it doesn't matter which one because $x = 300$ is where they intersect) to obtain the y-coordinate at the point of intersection: $3(300) - 500 = 400$. The y-coordinate at the point of intersection is the minimum possible value of b that the question was asking for, so enter **400**.

Alternatively, you could graph each inequality in the graphing calculator. Again, since the solution set will be above the lines, the minimum y-value will be at the intersection of the lines. Inspect the intersection to find that its y-value is **400**.

9. C

Difficulty: Medium

Category: Algebra

Getting to the Answer: You could plot the two lines on the graphing calculator and identify the solution set on a graph, checking to see if each ordered pair falls within it. However, testing the choices to see if they satisfy both inequalities may be a more efficient approach, particularly since one inequality is "less than" and the other is "less than or equal to." If you take this approach, you don't even need to rearrange the inequalities to isolate y on one side.

(A): Substituting these values for x and y in $x < 4 - 2y$ gives you $(-1) < 4 - 2(3)$, which is $-1 < -2$ (not a true statement). You don't need to evaluate the other inequality since this ordered pair is not in the solution set for $x < 4 - 2y$. Eliminate (A).

(B): Substituting these values for x and y in $x < 4 - 2y$ gives you $(1) < 4 - 2(1)$, or $1 < 2$. The ordered pair is in the solution set for this inequality, so plug the values into $y \leq -2x + 1$ to get $(1) \leq -2(1) + 1$, which simplifies to $1 \leq -1$. Thus, this ordered pair is not in the solution set for the second inequality. Eliminate (B).

(C): Substituting these values for x and y in $x < 4 - 2y$ gives you $(2) < 4 - 2(-3)$, or $2 < 10$. The ordered pair is in the solution set for this inequality, so plug the values into $y \leq -2x + 1$ to get $(-3) \leq -2(2) + 1$, which simplifies to $-3 \leq -3$. Since the sign for this inequality is "less than or equal to," this ordered pair is in the solution set. **(C)** is correct.

(D): Since you already identified the correct choice, you do not need to check this pair, but, for the record, substituting these values for x and y in $x < 4 - 2y$ gives you $(4) < 4 - 2(4)$, or $4 < -4$. Since this is incorrect, this ordered pair is not in the solution set for the first inequality.

10. C

Difficulty: Hard

Category: Algebra

Getting to the Answer: Since the question states that $x = y = 1$ is a solution to the system, plug those values in to get $1 > 1 + r$ and $1 < s - 1$. These inequalities further simplify to $r < 0$ and $s > 2$. Check each pair of values in the answer choices to see which complies with these limitations. Only **(C)** has both $r < 0$ and $s > 2$, and is correct. (B) is incorrect because s must be greater than 2, and (D) reverses r and s.

11. C

Difficulty: Medium

Category: Algebra

Getting to the Answer: Translate each part of the word problem into its mathematical equivalent. Because x, y, and z represent the numbers of each of the three ad sizes, the total number of ads will be $x + y + z$. "No fewer than 15" ads means $x + y + z \geq 15$, so eliminate (A) and (B).

The total cost of the ads sold will be represented by the number of each size ad sold times the respective cost of each size ad. Thus, $110x + 70y + 50z$ represents the total cost of all of the ads; "at least \$1,500" means that $110x + 70y + 50z \geq 1{,}500$. Therefore, **(C)** is correct.

12. A

Difficulty: Medium

Category: Algebra

Getting to the Answer: Translate each part of the word problem into its mathematical equivalent. The total weight of produce will be represented by the combined pounds of watermelons, w, cantaloupes, c, and tomatoes, t. This combined weight cannot exceed 250 pounds, so the inequality is $w + c + t \leq 250$. Eliminate (C) and (D).

The total money for the produce sold is represented by the price per pound of each type of produce times the number of pounds of that type. This must be "at least" (greater than or equal to) \$200, so the inequality is $0.5w + 1c + 2.5t \geq 200$. **(A)** is correct.

13. D

Difficulty: Medium

Category: Algebra

Getting to the Answer: Translate each part of the word problem into its mathematical equivalent, beginning with the easiest-to-translate components. There needs to be at least 5 apple trees and at least 3 pear trees, so the correct inequalities are $x \geq 5$ and $y \geq 3$. Eliminate (B) and (C). The total number of apple and pear trees must be at least 15, so the correct inequality is $x + y \geq 15$. Eliminate (A). Only **(D)** is left and is correct.

Note that, by being strategic, you never even have to determine the first, most complicated inequality in each of the answer choices. For the record: apple trees cost \$120 and pear trees cost \$145, so the total amount spent on trees is $120x + 145y$. This total amount cannot go above \$2,050, which means it must be less than or equal to \$2,050. Therefore, $120x + 145y \leq 2{,}050$.

14. B

Difficulty: Medium

Category: Algebra

Getting to the Answer: First, define the relationship between the weight of each kind of container and the weight the shelf can hold. Since x is the number of 50-pound containers of paint and y is the number of 35-pound containers of varnish, the combined weight of the containers will be represented by $50x + 35y$. This needs to be no more than 1,450 pounds, so the inequality that represents this is $50x + 35y \leq 1{,}450$. Eliminate (A) and (C).

The question also states that the total number of containers the shelf can hold is no more than 32, so the combined number of containers of paint and containers of varnish must be no more than 32. This is represented by the inequality $x + y \leq 32$. Thus, **(B)** is correct.

15. C

Difficulty: Hard

Category: Algebra

Getting to the Answer: Begin by translating the weight of the combined bags of flour and sugar. The weight of each bag of flour times the number of bags of flour—plus the weight of each bag of sugar times the number of bags of sugar—will yield the total. Thus, $50f + 20s$ will describe the weight of all of the bags combined. Since the weight that the supplier can deliver is no more than 750 pounds, the inequality that describes this situation is $50f + 20s \leq 750$. Eliminate (A) and (D).

The question also specifies that the bakery needs to buy at least three times as many bags of sugar as bags of flour. In other words, the number of bags of sugar must be equal to or greater than three times the number of bags of flour. This is represented by $s \geq 3f$, which can be rewritten as $3f \leq s$. **(C)** is correct.

Check Your Work – Chapter 7

1. A

Difficulty: Medium

Category: Algebra

Getting to the Answer: The notation $g(x-2)$ asks for the value of $g(x)$ when x is $x-2$, so substitute $x-2$ for x and simplify. Don't forget to use the correct order of operations as you work.

$$\begin{aligned} g(x-2) &= 7(x-2)-3 \\ &= 7x-14-3 \\ &= 7x-17 \end{aligned}$$

Choice **(A)** is correct.

2. 15

Difficulty: Easy

Category: Algebra

Getting to the Answer: The notation $k(4)$ is the output value of the function k when 4 is substituted for the input x. Similarly, $k(1)$ is the output value of the function k when 1 is substituted for the input x. Substitute 4 and 1 into the function, one at a time, and then subtract the results.

$$\begin{aligned} k(4) &= 5(4)+2 = 20+2 = 22 \\ k(1) &= 5(1)+2 = 5+2 = 7 \\ k(4)-k(1) &= 22-7 = 15 \end{aligned}$$

Enter **15**.

3. A

Difficulty: Medium

Category: Algebra

Getting to the Answer: The notation $g(h(x))$ is read "g of h of x." It means that the output of $h(x)$ becomes the input of $g(x)$. First, use the table on the right to find the value of $h(x)$ when x is 3. At $x=3$, $h(3)=0$. This is your new input. Now, use the table on the left to find $g(0)$, which is -1, making **(A)** the correct answer.

4. D

Difficulty: Medium

Category: Algebra

Getting to the Answer: Evaluate the numerator and denominator separately. Start with the innermost parentheses and work your way out.

$$\begin{aligned} p(5) &= 2(5)+8 = 18 \\ q(p(5)) &= q(18) = 18-3 = 15 \\ q(5) &= 5-3 = 2 \\ p(q(5)) &= p(2) = 2(2)+8 = 12 \end{aligned}$$

Combine to get $\frac{q(p(5))}{p(q(5))} = \frac{15}{12} = \frac{5}{4} = 1.25$. The correct answer is **(D)**.

5. D

Difficulty: Hard

Category: Algebra

Getting to the Answer: Determine the linear change of the functions relative to the change in n, and then extrapolate to get the values of $f(6)$ and $g(6)$. You don't need to determine the algebraic expressions for the functions. As a shortcut, find the changes per a 2-unit increase of n and apply that to the values of the functions when $n=4$. For $f(n)$, the increase from $n=2$ to $n=4$ is $16.2-11.6=4.6$. Thus, the value of $f(6)$ is $f(4)+4.6=16.2+4.6=20.8$. The change in $g(n)$ from $n=2$ to $n=4$ is $0.5-1.5=-1$. So the value of $g(6)$ is $g(4)+(-1)=0.5-1=-0.5$. Now, calculate $h(6)$: $h(6)=2\times f(6)-g(6)=2(20.8)-(-0.5)=41.6+0.5=42.1$. Choice **(D)** is correct.

6. A

Difficulty: Easy

Category: Algebra

Getting to the Answer: Rewrite $f(x)-g(x)=0$ as $f(x)=g(x)$. You're looking for the x-value that gives the same y-value in each function; that is, the point where the functions f and g intersect. According to the graph, the functions intersect at $(-2,-1)$, so the value of x for which $f(x)-g(x)=0$ is -2. **(A)** is correct.

7. B

Difficulty: Hard

Category: Algebra

Getting to the Answer: Parallel lines have the same slope. First, determine the slope of linear function f from the graph. Function f passes through the points $(-3, 0)$ and $(0, 6)$. Thus, the slope is $\frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 0}{0 - (-3)} = \frac{6}{3} = 2$. Now use the slope-intercept form, $y = mx + b$, to determine b, the y-intercept, which is the y-value when x is 0 or $g(0)$. Plugging in $(1, 1)$ for (x, y) and 2 for m gives $1 = 2(1) + b$, and $b = -1$. **(B)** is correct.

8. 0

Difficulty: Medium

Category: Algebra

Getting to the Answer: According to the graph, when $x = 6$, the y-value of the graph of f is 6, so $f(6) = 6$. When $x = -3$, the y-value of the graph of g is 6, so $g(-3) = 6$. Thus, $f(6) - g(-3) = 6 - 6 = 0$. Enter **0**.

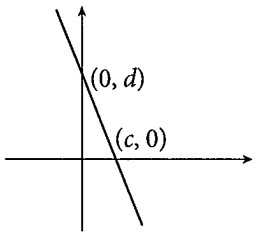
9. B

Difficulty: Medium

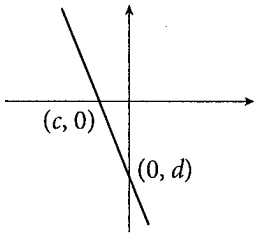
Category: Algebra

Strategic Advice: Quickly sketching the different possibilities can be helpful.

Getting to the Answer: Because $2c = d$, both the x-intercept, c, and the y-intercept, d, must have the same sign. If both are positive, then d would be greater than c, and the graph of f would look something like this:



This is all you need to do to solve the question. According to the choices, the slope is always the same regardless of the sign of c and d. In other words, if the slope is negative at one point, then it must be negative all the time. Therefore, **(B)** is correct. On test day, you would move on to the next question without needing to check what the line looks like when c and d are negative. For the record, if c and d are negative, then d will be less than c, and the graph would look like this:



If you're curious to see the algebra, plug $(0, d)$ and $(c, 0)$ into the slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{d - 0}{0 - c} = -\frac{d}{c}$$

The question states that $d = 2c$, so sub in $2c$ for d:

$$-\frac{d}{c} = -\frac{(2c)}{c} = -2$$

Therefore, the slope is -2, and the answer is indeed **(B)**.

10. B

Difficulty: Medium

Category: Algebra

Getting to the Answer: Since $g(x) = 2f(x) - 9$, when $x = 0$, $g(0) = 2f(0) - 9$. Begin by using the graph of f to find $f(0)$. At $x = 0$, $f(x) = 1$. Then substitute 1 for $f(0)$ into $g(0) = 2f(0) - 9$ to solve for $g(0)$. This gives $g(0) = 2(1) - 9 = -7$. **(B)** is correct.

11. C

Difficulty: Medium

Category: Algebra

Getting to the Answer: Thinking about the y-intercept (the starting amount) for the function will reduce the amount of work you need to do. The table indicates that Paulo had 21 votes on day 3, when $t = 3$, not at the start, when $t = 0$. This means that (B) and (D) are incorrect. To evaluate the remaining answer choices, pick a point and try it in a function. If it works, then you've found the correct answer. If it doesn't work, then you can confidently select the other answer choice. The result of checking point (4, 28) is as shown.

$$\text{(A)}: 28 \neq \frac{4}{7}$$

Choice (A) is incorrect, so **(C)** must be correct. On test day, you would stop here. For the record, here is the reason why **(C)** is correct.

$$\text{(C)}: 28 = 7(4)$$

12. C

Difficulty: Medium

Category: Algebra

Getting to the Answer: The words "rate of increase" (more generally, "rate of change") mean slope. Examine the graph and look for trends in the slope of the two lines. The line representing the truck has a significantly steeper slope than the line representing the car. Choice **(C)** is correct.

13. D

Difficulty: Medium

Category: Algebra

Getting to the Answer: Compare each answer choice to the graph, eliminating false statements as you go.

(A): Carmel went to the library first, so the library (not the grocery store) is 5 miles from her home. Eliminate (A).

(B): Carmel traveled 7 miles away from her home (between $t = 0$ minutes and $t = 30$ minutes), but then also traveled 7 miles back (between $t = 45$ minutes and $t = 60$ minutes), so she traveled a total of 14 miles. Eliminate (B).

(C): When Carmel reached the library, she was 5 miles from home; when she reached the grocery store, she was 7 miles from home. This means the grocery store must be $7 - 5 = 2$ miles farther away. Eliminate (C).

(D) must be correct. Carmel is the same distance from home (5 miles) between $t = 15$ minutes and $t = 25$ minutes, so she spent 10 minutes at the library. She is stopped once again (at the grocery store) between $t = 30$ minutes and $t = 45$ minutes, so she spent 15 minutes at the grocery store.

14. 930

Difficulty: Hard

Category: Algebra

Strategic Advice: The fact that the number of visitors each 15 minutes is constant means that the cumulative number of visitors is a linear function.

Getting to the Answer: Because the time between the numbers of cumulative visitors in the table varies, pick an interval that is easy to work with to determine the number of visitors who enter every 15 minutes. Next, use that value to find how many entered by 4:45 p.m. There are six 15-minute periods between 12:30 p.m. and 2:00 p.m. The number of visitors admitted during that time was $600 - 420 = 180$. So, $\frac{180}{6} = 30$ visitors enter every 15 minutes.

In order to project the cumulative, or total, number of visitors for a specific time, set up a function, $f(v)$. Pick a time that is convenient, such as 2:00 p.m. Since you know that there were 600 visitors by 2:00 p.m. you can write $f(v) = 600 + 30v$, where v is the number of 15-minute periods after 2:00 p.m. The question asks for the cumulative visitors admitted by 4:45 p.m. Thus, v is the number of 15-minute periods between 2:00 and 4:45, which is 11. So, $f(11) = 600 + 30(11) = 930$. Enter **930**.

Check Your Work – How Much Have You Learned: Algebra

1. B

Difficulty: Easy

Category: Algebra

Getting to the Answer: To find the cost of carrots on Saturday, multiply their weight a by the sale price. If there is a 10% discount, then the customer actually pays $100\% - 10\% = 90\%$. Therefore, a pounds of carrots cost $0.50 \times 90\% \times a = 0.50 \times 0.90 \times a = 0.45a$. Now, add the cost of three bags of oranges, $7.25 \times 3 = 21.75$, to get the total cost: $c = 0.45a + 21.75$. Choice **(B)** is correct.

2. −5

Difficulty: Medium

Category: Algebra

Getting to the Answer: The first equation contains fractions, which are difficult to work with. Clear the fractions by multiplying both sides of the equation by 12, the least common denominator:

$$12\left(\frac{x}{4}+\frac{y}{3}=5\right) \rightarrow 3x+4y=60$$

Using similar logic, multiply both sides of the second equation by 5:

$$5\left(2x+\frac{4}{5}y=5\right) \rightarrow 10x+4y=25$$

The question asks for the value of x. You can eliminate the y terms by subtracting the second equation from the first:

$$\begin{array}{r} 3x+4y=60 \\ -(10x+4y=25) \\ \hline -7x=35 \\ x=-5 \end{array}$$

Enter **−5**.

3. 32

Difficulty: Hard

Category: Algebra

Getting to the Answer: Linear functions have a constant rate of change, or slope. Examining the bottom two rows of the table, you see that as x is increased by 2 units, $f(x)$ is increased by 4 units. Therefore, the slope is $\frac{4}{2}=2$. The table also supplies the point (0, 13), which means that the y-intercept is 13. Therefore, the equation of this linear function is $f(x) = 2x + 13$. The question asks for $f(9.5)$, so plug in 9.5 for x and simplify: $f(9.5) = 2(9.5) + 13 = 19 + 13 = 32$. Enter **32**.

4. C

Difficulty: Easy

Category: Algebra

Getting to the Answer: Distributing the $\frac{4}{9}$ would create more messy fractions. To avoid this, multiply both sides of the equation by 9:

$$\begin{aligned} 9\left[\frac{4}{9}(x-8)=2\right] \\ 4(x-8)&=18 \\ 4x-32&=18 \\ 4x&=50 \\ x&=12.5 \end{aligned}$$

Choice **(C)** is correct. Backsolving is another viable approach. When backsolving, start with either (B) or (C). Try plugging in $x = 11.5$:

$$\begin{aligned} \frac{4}{9}(x-8)&=2 \\ \frac{4}{9}(11.5-8)&=2 \\ \frac{4}{9}(3.5)&=2 \\ \frac{14}{9}&=2 \end{aligned}$$

This does not result in a true statement, so (B) is incorrect. Since the left-hand side is smaller than the right-hand side, move up to (C):

$$\begin{aligned} \frac{4}{9}(x-8)&=2 \\ \frac{4}{9}(12.5-8)&=2 \\ \frac{4}{9}(4.5)&=2 \\ 2&=2 \end{aligned}$$

This is a true statement, so **(C)** is correct.

5. D

Difficulty: Medium

Category: Algebra

Getting to the Answer: A linear relationship can be modeled by $y = mx + b$. Begin by choosing any two points from the table and substituting them into the slope formula. Using the points (0, 7) and (−6, −5), the slope is $\frac{y_2-y_1}{x_2-x_1}=\frac{-5-7}{-6-0}=\frac{-12}{-6}=2$. The point (0, 7) indicates that 7 is the y-intercept. Plugging these values in for m and b in the $y = mx + b$ equation yields $y = 2x + 7$. Finally, substitute in $x = 2$ to find the missing value of y. This yields $y = 2(2) + 7 = 4 + 7 = 11$. Choice **(D)** is correct.

6. B

Difficulty: Hard

Category: Algebra

Getting to the Answer: Use known points from the graph to determine the equations for the two lines. Then convert them into inequalities.

The solid line passes through the points (9, 7) and (0, 4). Its slope is therefore $\frac{y_2 - y_1}{x_2 - x_1} = \frac{4-7}{0-9} = \frac{-3}{-9} = \frac{1}{3}$. The graph shows that the y-intercept is 4, so the equation for the solid line is $y = \frac{1}{3}x + 4$. Multiply both sides by 3 to clear the fraction: $3y = x + 12$. This can be restated as $3y - x = 12$. Because the shaded region is below this line and the line is solid, the inequality defined by the line is $3y - x \leq 12$. Eliminate (A) and (C).

The dotted line passes through the points (4, 0) and (7, 6). Thus, the slope of this line is $\frac{6-0}{7-4} = \frac{6}{3} = 2$. The y-intercept is unclear from the graph, so plug the x-intercept (4, 0) into $y = 2x + b$ to get $0 = 2(4) + b$. Thus, $b = -8$. The equation for this line is therefore $y = 2x - 8$. Since the line is dotted and the values of the solution are below the line, the inequality is $y < 2x - 8$. This converts to $2x - y > 8$. These two inequalities match the ones in **(B)**.

7. A

Difficulty: Medium

Category: Algebra

Getting to the Answer: Solve the second inequality for x and substitute this result into the first inequality. If $x > 2x + 4$, then $-x > 4$. This simplifies to $x < -4$ (remember that you must flip the inequality if you multiply or divide by a negative number). Plugging into the first inequality gives $5y < 2(-4) - 22$, or $5y < -30$. This means that $y < -6$. Choice **(A)** is correct.

8. C

Difficulty: Hard

Category: Algebra

Getting to the Answer: Graphically, a "solution" to a system of equations is where the two lines touch each other. For a system of equations to have infinitely many solutions, they must touch each other in an infinite number of places. This means that the two lines must be on top of each other. In other words, the two lines are identical and have the same slope and y-intercept. Although you could rearrange each equation into $y = mx + b$ form, it is easier to get both equations into the same format and simply compare each part of the equations piece by piece.

Since the first equation contains fractions and the second does not, multiply the first equation by 7 to clear the fractions:

$$7\left(\frac{2}{7}x - 6 = 3ay\right) \rightarrow 2x - 42 = 21ay$$

Next, rearrange the equation so that the x and y variables are on the left and the constant is on the right. This will make it match the format of the second equation.

$$2x - 21ay = 42$$

Now compare this to the second equation. Both contain a $2x$ and a 42. For the first equation to match the second, $-21ay$ must equal $-14y$. This means that $a = \frac{-14y}{-21y} = \frac{2}{3}$. Choice **(C)** is correct.

9. C

Difficulty: Medium

Category: Algebra

Getting to the Answer: Rather than solving for each variable individually, see if you can solve for the desired expression all at once. Begin by getting the x and y variables together on the left-hand side of the equation and the constants on the right:

$$\begin{aligned} -3x - 2y &= 1 \\ 4x + 3y &= 8 \end{aligned}$$

There isn't a quick way to simply add or subtract these equations to get to $8x + 8y$, but since $8x + 8y = 8(x + y)$, finding $x + y$ and multiplying the results by 8 will be an efficient route to get to the answer. You can obtain $x + y$ by adding the two equations:

$$\begin{aligned} -3x - 2y &= 1 \\ +(4x + 3y &= 8) \\ \hline x + y &= 9 \end{aligned}$$

Since $x + y = 9$, $8(x + y) = 8(9) = 72$. Choice **(C)** is correct.

10. 6

Difficulty: Medium

Category: Algebra

Getting to the Answer: To evaluate $f(g(8))$, start with the innermost function. Plug 8 into g to obtain $g(8) = 2(8) + 5 = 16 + 5 = 21$. Now take this value and plug it into the f function: $f(21) = \frac{21}{3} - 1 = 7 - 1 = 6$. Enter **6**.