On Max-min Fair Allocation for Multi-source Transmission

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ABSTRACT

Max-min fair is widely used in network traffic engineering to allocate available resources among different traffic transfers. Recently, as data replication technique developed, increasing systems enforce multi-source transmission to maximize network utilization. However, existing TE approaches fail to deal with multi-source transfers because the optimization becomes a joint problem of bandwidth allocation as well as flow assignment among different sources. In this paper, we present a novel allocation approach for multi-source transfers to achieve global max-min fairness. The joint bandwidth allocation and flow assignment optimization problem poses a major challenge with respect to nonlinearity and multiple objectives. We cope with this by deriving a novel transformation with simple equivalent canonical linear programming to achieve global optimality efficiently. Simulation results demonstrate that our approach is more max-min fair than other single-source and multi-source allocation approaches, meanwhile it outperforms others with substantial gains in terms of network throughput and transfer completion time.

1 INTRODUCTION

Max-min fair is a simple, classical and well-recognized sharing principle to define fairness in the field of data networks [13]. In particular, it deals with the flow control problem in network traffic engineering (TE), where available resources (such as link bandwidth) are allocated among different traffic transfers [12]. Aiming at allocating rates to available links as evenly as possible, the max-min fair allocation is such that the result for the transfer with smallest sharing has been maximized over all feasible resource allocation solutions. As the development of software-defined networking (SDN), max-min fair has become the most widely used principle in modern datacenter networks or WANs, such as B4 [9], BwE [11], SWAN [8], OWAN [10]. Leveraging a centralized controller, max-min fairness results in more stable service quality than other principles, *e.g.,* proportional fairness.

Data replication, a technique that amends both data availability and access efficiency, is emerging increasingly in recent datacenter networks and distributed filesystems [7, 14, 16]. To maximize network utilization and improve transmission performance, it is increasingly allowed to convey data in parallel from multiple sources for a transfer with multiple data replicas, *a.k.a* a multi-source transfer [1, 15]. However, a multi-source transfer will result in multiple flows, so multiple potential bottlenecks might be considered simultaneously to achieve transfer-level fairness. Existing TE approaches are no longer applicable because the allocation becomes a joint problem to optimally allocate each flow's bandwidth as well as to assign flows among the sources [8, 9, 11].

In this paper, we present a novel max-min fair allocation approach that jointly optimizes bandwidth allocation and flow assignment. The major challenge stems from the direct formulation which is nonlinear and multi-objective. We cope with this by deriving a novel transformation with simple equivalent canonical linear programming (LP) to achieve global optimality. We perform extensive simulations, which show that our approach leads to better performance on network throughput with a gain of up to 52% and reduces transfer completion time by up to 44%, compared to other single-source and multi-source allocation approaches.

2 NETWORK MODEL AND PROBLEM FORMULATION

Consider a telecommunications network composed of a set of nodes and a set of links $\mathcal{L}$. The capacity of link L_j ($j \in [1, M]$) is defined as C_j. Suppose there are a number of data transfers, each of which may come from multiple sources. Thus each transfer i ($i \in [1, N]$) is assigned with a set of flow paths $\{P_{i1}, ..., P_{ik}\}$, and each such path is identified with the set of links that it traverses, *i.e.,* $P_{ik} \subseteq \mathcal{L}$. Now let r_i denote the data rate of transfer i, which is the sum bandwidth of the constituent flows from all its sources. We use a variable set $\mathcal{X}_i = \{x_{i1}, ..., x_{ik}\}$ to express the flow assignment proportions from different sources for transfer i, and $\sum_{k=1}^{K_i} x_{ik} = 1$, where K_i is the total source number of transfer i.

We are interested in solving the joint bandwidth allocation and flow assignment problem, *i.e.,* finding the transmission rate r_i of each transfer, together with the assignment proportions $\mathcal{X}_i$ from all its sources. The solution is required to provide a fair and efficient allocation result, and its precise objective and constraints are described as below.

Objective. When computing allocated bandwidth, our goal is to maximize network utilization while in a max-min fair manner. A vector of transfer rate allocations $\{r_i\}$ is said to be max-min fair if and only if, for any other feasible allocation $\{r'_i\}$, the following has to be true: $\forall r'_p > r_p$ for the data transfer p, there exists another transfer q such that $p, q \in [1, N]$, $r'_q < r_q$, $r_q \le r_p$. In other words, increasing some components must be at the expense of decreasing some other existing smaller or equal components.

Constraints. The constraints of this problem are given as below. Constraint (1) is called the capacity constraint, which assures that for any link L_j, its load does not exceed its capacity C_j. Constraints (2) and (3) promise the sum fractions of a transfer from all available sources equal to 1.

$$s.t. \quad \sum_{L_j \subseteq P_{ik}} r_i \cdot x_{ik} \le C_j \qquad \forall j, \tag{1}$$

$$\sum_{k=1}^{K_i} x_{ik} = 1 \qquad \forall i, \tag{2}$$

$$0 \le x_{ik} \le 1 \qquad \forall i, k. \tag{3}$$

Figure 1 shows an example of a network consisting of six links $\{L_1, ..., L_6\}$, with capacities $\{8, 5, 4, 5, 7, 6\}$ respectively. Assume all

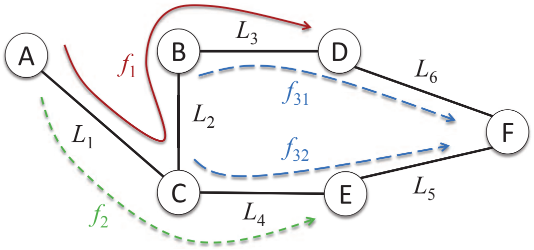


Figure 1: An example with 3 transfer requests. Transfer 3 comes from two feasible sources B and C, corresponding to two parallel flows f_{31} and f_{32}.

bandwidth numbers are in *Gbps* here. Suppose there are three data transfers, among which transfer 1 and 2 have a single data source while transfer 3 has two available sources. In total, there are four potential flows in the network, including $f_1 : A \to C \to B \to D$, $f_2 : A \to C \to E$, $f_{31} : B \to D \to F$ and $f_{32} : C \to E \to F$. Given the topology and values of link capacities, we aim at finding the max-min fair solution of the rate vector $\{r_1, r_2, r_3\}$ and the flow assignment $\{x_{31}, x_{32}\}$ for transfer 3.

3 APPROACH SPECIFICATION

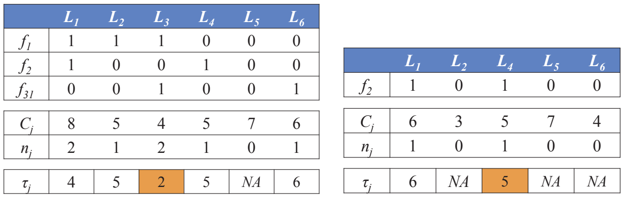
One of the biggest challenges for multi-source transmission stems from changing the optimized object from an individual 5-tuple flow into a group of flows that belong to the same transfer. Hence, multiple potential bottlenecks might be considered simultaneously. To this end, we design a MultiSource Water-Filling (MS-WF) algorithm which can jointly compute bandwidth allocation and flow assignment while providing global max-min fairness. The key to MS-WF algorithm is a novel transformation with simple equivalent canonical LP. In this section, we first briefly describe the traditional water-filling algorithm and its limitation. Then we present the MS-WF with one multi-source transfer for clear illustration. A generic solution that optimizes arbitrary flow transfers is provided afterward. At last we discuss some practical issues of the multi-source framework.

3.1 Flow-link Mapping Matrix and Traditional Water-Filling Algorithm

The proposed MS-WF algorithm is fundamentally based on a flow-link mapping matrix (FL matrix) with solvable variables. Here we define the FL matrix, and illustrate the traditional water-filling algorithm with this matrix.

Definition 1 (Flow-link Mapping Matrix)**.** The flow-link mapping matrix *(FL matrix)* $\{fl_{ij}\}$ expresses the flow paths and the traffic shares of each transfer in matrix form. The matrix element $fl_{ij} \in [0, 1]$ is defined as the proportion of flow i in its belonging transfer that traverses link L_j.

Water-Filling (WF) Algorithm. The traditional WF algorithm can compute the bandwidth allocations for single-source transmission. Using the FL matrix as an input, we first denote the saturated average bandwidth allocation as $\tau_j = C_j / n_j$, where n_j is the total number of flows that use link L_j. The WF algorithm iteratively finds the minimum τ^* and the corresponding bottleneck link L_{j^*}. Set the bandwidth of the flows that use link L_{j^*} to τ^*. Then the FL matrix is updated by subtracting those flows and the bottleneck link to calculate a new set of $\{\tau_j\}$. Such process iterates until all transfers obtain their allocated rates, *i.e.,* all flows have bottleneck links.



	L_1	L_2	L_3	L_4	L_5	L_6
f_1	1	1	1	0	0	0
f_2	1	0	0	1	0	0
f_{31}	0	0	1	0	0	1
C_j	8	5	4	5	7	6
n_j	2	1	2	1	0	1
τ_j	4	5	2	5	*NA*	6

(a)

	L_1	L_2	L_4	L_5	L_6
f_2	1	0	1	0	0
C_j	6	3	5	7	4
n_j	1	0	1	0	0
τ_j	6	*NA*	5	*NA*	*NA*

(b)

Figure 2: An example of the FL matrix for single-source transmission, where transfer 3 only uses one source. C_j is the bandwidth capacity, $n_j = \sum_i fl_{ij}$ is the total number of flows that use link L_j, and $\tau_j = C_j / n_j$ is the saturated average bandwidth share. (a) Illustration of the first iteration, where L_3 is found as the bottleneck link with minimum τ_j. (b) Illustration of the second iteration with the updated FL matrix, where L_4 is then found as the bottleneck link.

Example. Consider a single-source version of Figure 1, where we assume transfer 3 only uses source B, so there are 3 flows over the network. Figure 2 illustrates the allocation algorithm for this single-source example. In the first iteration, L_3 is first saturated because τ_3 is of the minimum value $\tau^* = 2$. We allocate the bandwidth of f_1 and f_{31} that traverse L_3 to 2, and then remove the rows of f_1, f_{31} and column L_3. The FL matrix is updated accordingly for the next iteration, where link L_4 is then found as the bottleneck and bandwidth of f_2 is set to 5. By this point, the ultimate solution to the rate allocation for the 3 transfers is (2, 5, 2).

The traditional WF algorithm succeeds in obtaining the max-min fair allocation for single-source transmission, yet it is incapable of dealing with multi-source transfers. This principally stems from the fact that the WF algorithm is based on flows, rather than on transfers. For the example in Figure 1, transfer 3 will have double weights by using the traditional WF algorithm, which is unfair to the others. A naïve solution is to normalize the weight of each transfer, and to assign an equal share to the flows from different sources. With regard to the example, the elements for f_{31} and f_{32} become 1/2 and 1/2 in the FL matrix, such that each transfer has the same sum weight of 1. The allocation result turns into (8/3, 10/3, 3), which is slightly better than the single-source cases, which are (2, 5, 2) for using only source B and (2.5, 4, 2.5) for using only source C. However, as we will soon learn, equally sharing the flow weight is still not the optimal solution. *The transfer-level max-min fair allocation is conditioned by the optimal flow assignment.*

3.2 MS-WF Algorithm with One Multi-source Transfer

Given the FL matrix and its application, now let's consider a more complex case by adding one multi-source transfer into the network. We assume there are K_m available sources for the particular transfer

Algorithm 1 MS-WF Algorithm with one multi-source transfer

```
Input:
    Flow-link mapping matrix: $FL = \{fl_{ij}\}$;
    Capacity of each link: $\{C_j\}$, $1 \leqslant j \leqslant M$;
Output:
    Transmission rate of each transfer: $\{r_i\}$, $1 \leqslant i \leqslant N$;
    Flow assignment of transfer $m$: $\mathcal{X}_m = \{x_{m1}, \ldots, x_{mk}\}$;
• Step 1:                                              ▷ Initiation
$r_i \leftarrow 0, \quad \forall i = 1, \ldots, N$;
$\mathcal{L} \leftarrow \{L_1, L_2, \ldots, L_M\}$;
• Step 2:                   ▷ Calculate the saturated average bandwidth
$n_j(\mathcal{X}_m) \leftarrow \sum_i fl_{ij}, \quad \forall j \in \mathcal{L}$;
$\tau_j(\mathcal{X}_m) \leftarrow C_j / n_j(\mathcal{X}_m), \quad \forall j \in \mathcal{L}$;
• Step 3:                            ▷ Find the bottleneck fair share $\tau^*$
if $min\{\tau_j(\mathcal{X}_m)\}$ is a constant then
    $\mathcal{X}^* \leftarrow \varnothing$;
else
    $\mathcal{X}^* \leftarrow \mathcal{X}_m | max\ min\{\tau_j(\mathcal{X}_m)\}$;
end if
$\tau^* \leftarrow min\{\tau_j(\mathcal{X}_m)\}$;
• Step 4:                      ▷ Set data rate and update the FL matrix
$\mathcal{L}_{j^*} \leftarrow \{L_j | \tau_j(\mathcal{X}_m) = \tau^*\}$;
$\mathcal{L} \leftarrow \complement_{\mathcal{L}_{j^*}} \mathcal{L}$;
for $i | P_i \cap \mathcal{L}_{j^*} \neq \varnothing$ do
    $r_i \leftarrow r_i + \tau^*$;
    Remove $f_i$ from $FL$;
end for
Update $FL$;
• Step 5:                                               ▷ Iteration
if No transfers left then
    return $\{r_i\}$ and $\mathcal{X}_m$;
else
    goto Step 2.
end if
```

m, though the flow assignment $\mathcal{X}_m$ is unknown and needs to be solved. The MS-WF algorithm continues to use the FL matrix as an input, but the elements are no longer 0 and 1 as in the single-source case. Instead, the matrix elements related to transfer m are replaced by the unknown variables in $\mathcal{X}_m$. The MS-WF algorithm with one multi-source transfer (**Algorithm 1**) can be described in the following high-level steps:

(1) Start from zero allocation with the whole link set, and build the FL matrix with variables $\mathcal{X}_m$.
(2) Calculate the saturated average bandwidth $\tau_j(\mathcal{X}_m)$ on each link L_j.
(3) Find the bottleneck fair share $\tau^* = min\{\tau_j(\mathcal{X}_m)\}$ by solving $\mathcal{X}^* = \mathcal{X}_m | max\, min\{\tau_j(\mathcal{X}_m)\}$.
(4) Set the data rate to τ^* for the flows that traverse the bottleneck links, and update the FL matrix by removing those flows and links.
(5) If there are no transfers left then stop, otherwise return to Step 2.

In Step 2, similar to the traditional WF algorithm, we first calculate $n_j(\mathcal{X}_m)$ by summarizing the elements of column j in the FL matrix. Here $n_j(\mathcal{X}_m)$ denotes the number of transfers that use link L_j. Due to the fact that transfer m comes from multiple parallel flows, there might be only a fraction of the transfer using link L_j. As a result, n_j becomes a function of $\mathcal{X}_m$ instead of an integer value as in the single-source case. Next, we compute the average bandwidth $\tau_j(\mathcal{X}_m) = C_j / n_j(\mathcal{X}_m)$, which is also a function of $\mathcal{X}_m$.

	L_1	L_2	L_3	L_4	L_5	L_6
f_1	1	1	1	0	0	0
f_2	1	0	0	1	0	0
f_{31}	0	0	x	0	0	x
f_{32}	0	0	0	$1-x$	$1-x$	0
C_j	8	5	4	5	7	6
n_j	2	1	$1+x$	$2-x$	$1-x$	x
τ_j	4	5	$\frac{4}{1+x}$	$\frac{5}{2-x}$	$\frac{7}{1-x}$	$\frac{6}{x}$

Figure 3: An example of MS-WF, where transfer 3 comes as two flows. x is the flow assignment variable to be calculated. τ_j in orange cell is found as the minimum bottleneck share.

In Step 3, given $\{\tau_j(\mathcal{X}_m)\}$ on the current link set, one or several bottleneck links are found. Specifically, since all the variables $\mathcal{X}_m$ are within a certain range $[0, 1]$, $min\{\tau_j(\mathcal{X}_m)\}$ is sometimes a constant value. In that case, no flow assignment variable is calculated, *i.e.*, $\mathcal{X}^* = \varnothing$. Otherwise, the flow assignment is determined by $\mathcal{X}^* = \mathcal{X}_m | max\, min\{\tau_j(\mathcal{X}_m)\}$. We use τ^* to denote the minimum bandwidth share, and the set of $\{L_{j^*}\}$ is found as the bottleneck links.

Back to the example in Figure 1, transfer 3 has two available sources to access, resulting in two parallel flows f_{31} and f_{32}, respectively. For simplicity, we use only one variable x to denote the proportion of f_{31}, and that of f_{32} will thus be $1 - x$. Figure 3 illustrates the FL matrix, followed by the saturated average bandwidths $\{\tau_j(\mathcal{X}_m)\}$.

From the example, we can tell that *Step 3, finding $\mathcal{X}^*$ that maximizes $min\{\tau_j(\mathcal{X}_m)\}$, is the major challenge in MS-WF.* Accordingly, it is to find $x^* = x | max\, min(4, 5, 4/(1+x), 5/(2-x), 7/(1-x), 6/x)$ in Figure 3. First, as formulated in **Problem 1**, it is a nonlinear programming problem, which can not be solved directly. Second, even if we can find a linear expression, we still need long sequences of LPs for the max-min objective (multi-objective), which is computationally intense in real implementation.

Problem 1 (The optimization problem in Step 3)**.**

$$max \quad min\{\tau_j(\mathcal{X}_m)\}, \tag{4}$$

$$s.t. \quad \sum_{k=1}^{K_m} x_{ik} = 1 \qquad x_{ik} \in \mathcal{X}_m, \tag{5}$$

$$0 \le x_{ik} \le 1 \qquad \forall i, k. \tag{6}$$

To cope with this, we transform this nonlinear optimization problem into a canonical form of LP problem based on **Theorem 1**. As a result, the equivalent canonical LP problem is expressed as **Problem 2**, which can be solved efficiently with limited computational complexity.

Problem 2 (The equivalent LP problem in Step 3)**.**

$$min \quad t \tag{7}$$

$$s.t. \quad t \geq \tau'_j(\mathcal{X}_m) \qquad \forall j, \tag{8}$$

$$\sum_{k=1}^{K_m} x_{ik} = 1 \qquad x_{ik} \in \mathcal{X}_m, \tag{9}$$

$$0 \leq x_{ik} \leq 1 \qquad \forall i, k. \tag{10}$$

Theorem 1. *Problem 1 is equivalent to Problem 2 as a canonical LP problem, where* $\tau'_j(\mathcal{X}_m) = 1/\tau_j(\mathcal{X}_m)$.

Proof. Given an arbitrary instance of the FL matrix, $\tau_j(\mathcal{X}_m)$ satisfies two conditions: i) $\tau_j(\mathcal{X}_m) \geq 0$, and ii) the inverse of $\tau_j(\mathcal{X}_m)$ is a linear function of $\mathcal{X}_m$. So we let $\tau'_j(\mathcal{X}_m) = 1/\tau_j(\mathcal{X}_m)$, and the objective of $max\, min\{\tau_j(\mathcal{X}_m)\}$ is then equivalent to $min\, max\{\tau'_j(\mathcal{X}_m)\}$, which becomes linear accordingly. Next, we introduce a temporary variable $t = max\{\tau'_j(\mathcal{X}_m)\}$, and use a sequence of inequality constraints $t \geq \tau'_j(\mathcal{X}_m)$ for all j to express t. Since $\tau'_j(\mathcal{X}_m)$ is a linear function of $\mathcal{X}_m$, the constraint (8) as a set of inequalities is also linear. In the end, the optimization problem in Step 3 (**Problem 1**) turns into an equivalent canonical LP problem (**Problem 2**), where the decision variables to be solved are the flow assignment set $\mathcal{X}_m$ and t. □

As a result, the optimization problem of the example in Figure 3 is well transformed into a simple equivalent LP problem, which is expressed as below.

$$min \quad t \tag{11}$$

$$s.t. \quad t \geq \frac{1}{4}, \tag{12}$$

$$t \geq \frac{1}{5}, \tag{13}$$

$$4t - x \geq 1, \tag{14}$$

$$5t + x \geq 2, \tag{15}$$

$$7t + x \geq 1, \tag{16}$$

$$6t - x \geq 0, \tag{17}$$

$$0 \leq x \leq 1. \tag{18}$$

The results of the above LP come out as $x = 1/3$ and $t = 1/3$. Then $\tau^* = 1/t = 3$ is the minimum fair share, and L_3 and L_4 are the bottleneck links that are saturated in this iteration. We set $r_1 = r_2 = 3$ as the bandwidth of f_1 and f_2, and $r_3 = 3$ as the sum bandwidth of f_{31} and f_{32}. Meanwhile, the data volume assignment of transfer 3 concludes with 1/3 from source B and 2/3 from source C. The final rate allocation to the 3 transfers are (3, 3, 3), which is more max-min fair than the allocation (2, 5, 2) where transfer 3 uses only source B (as in Figure 2), as well as the allocation (2.5, 4, 2.5) where transfer 3 uses only source C. In addition, if we don't consider the impact of date volume and assume each transfer has the equal volume of $3Gbits$, then MS-WF outperforms the single source approaches in terms of the average completion time (MS-WF: (3/3+3/3+3/3)/3=1, source B: (3/2+3/5+3/2)/3=1.2) and source C: (3/2.5+3/4+3/2.5)/3=1.05), as well as total completion time (MS-WF: 3/3=1, source B: 3/2=1.5 and source C: 3/2.5=1.2).

3.3 Generic MS-WF

Having solved the preliminary instance with one multi-source transfer, now we consider the generic MS-WF with arbitrary transfer combinations. Here the variables become the flow assignments $\{\mathcal{X}_i\}$ ($i \in [1, N]$) for all transfers. Except for the transfers with single source, whose $\mathcal{X}_i = \{1\}$ for all the time, the rest of $\{\mathcal{X}_i\}$ are to be computed by MS-WF. The main challenge is that the flow assignments of different transfers correlate to each other and can not be calculated independently. One transfer's assignment plan may affect another's optimal decision. *Therefore, the max-min fair allocation requires joint calculation for all flow assignments.*

The generic MS-WF mainly follows the procedures in **Algorithm 1**. Exceptionally, we put all sets of the variables $\{\mathcal{X}_i\}$ into the FL matrix, such that τ_j turns into a function of $\{\mathcal{X}_1, ..., \mathcal{X}_N\}$. By the same token, we transform the nonlinear optimization problem in Step 3 into an equivalent canonical LP problem with the help of one additional decision variable t. In each iteration, parts of the flow assignment sets are solved by LP based on **Theorem 2**. Then we plug the values into the FL matrix and remove them from $\{\mathcal{X}\}$. Continue iterating until all flow assignment variables $\mathcal{X}_i$ are determined. Finally, we sum up the constituent flow rates as the multi-source transfer rate, *i.e.,* $r_i = \sum_{k=1}^{K_i} r_{ik}$, where K_i is the total source number of transfer i. The detailed proof of **Theorem 2** can be found in **Appendix A**.

Theorem 2. *Multiple sets of variables* $\mathcal{X}_i$ *can be jointly calculated by MS-WF.*

3.4 Discussion

Extension. The MS-WF algorithm is scalable and extensible for more complex use cases. For instance, differentiated qualities of service lead to transfers with variations in priority or other requirements, such that MS-WF is capable of supporting weighted fair allocation by taking priority factors into account. The max-min fair principle can also be applied to different optimization objectives (*e.g.,* transfer completion time), and the adaption of MS-WF with consideration of data size can likewise yield the optimal results for multi-source transfers.

Applicability. The driving application scenario for the MS-WF algorithm is the Large Hadron Collider (LHC) network, which requires deadline scheduling of large-scale datasets (*e.g.,* petabytes) to be transferred around over 180 member sites all over the world [4]. Fairness among large-scale science dataflows is one of the most important metrics for the LHC network, and the current scheduling system performs poorly in terms of fairness due to missing a fairness-aware scheduling framework for the multi-source transfers. Our multi-source transmission framework is part of the pre-production deployment of Unicorn [17], an SDN geo-distributed data analytic system in the CMS (one of the largest scientific experiments in the LHC network).

The experimental SDN system relies on a logically centralized controller to orchestrate bulk transfers. Since the system is for large-scale datasets, the average flow duration may vary from hours to several days. The computation complexity is significantly reduced in MS-WF by transforming a nonlinear multi-objective problem into a single LP. Experimental results show that the computation time for 1000 concurrent flows in MS-WF is at most 40 seconds, and it can be further reduced by removing the redundant constraints in implementation. Therefore, our approach is scalable to practically handle large-scale datasets and networks.

4 PERFORMANCE EVALUATION

To evaluate how MS-WF algorithm works on a large-scale network, we implement a flow-level datacenter network simulator.

4.1 Simulation Methodology

Topologies: We conduct our experiments by emulating a 3-tier datacenter network topology with 8:1 oversubscription. The topology contains 64 servers, whereby each edge link is of $1Gbps$ capacity, and aggregated link is of $10Gbps$ capacity.

Workloads: We synthesize a stream of transfer requests with a total number of 1000. The request arrival is modeled as a Poisson process, where the arrival rate λ is defined as the average number of new transfers per time slot. We set the slot length as one second for fast simulation. A transfer has multiple sources with probability ρ. A multi-source transfer is assumed to have a random number of replicas between [2,5], and the replicas are randomly placed in servers. In simulations, we do not consider the fluctuation of transfer size, and assume all transfers have an uniform size of V.

Performance metrics: We use *network throughput* and *average transfer completion time* to show the improvements of MS-WF over other approaches.

Alternative approaches: We compare the following bandwidth allocation approaches, each of which adopts the traditional WF algorithm.

- **Best-source:** This approach selects a best replica source based on the algorithm in [15] for multi-source transfers.
- **Equal-share:** This approach equally splits the transfer across different sources. For example, if a transfer has 3 replicas, each replica will send 1/3 of the data.
- **Random-source:** This approach randomly selects an available source to transmit data.

4.2 Simulation Results

Figure 4 shows the simulation results of network throughput for various approaches. Here we set the arrival rate $\lambda = 2$ and the data size $V = 10Gbits$ for all of the 1000 transfers. As the results show, Random-source approach disregards the source dissimilarity, therefore performs the worst with a constant throughput value. Equal-share approach takes advantage of source diversity simplistically. When the diversity is limited to a small number of multi-source transfers (at low multi-source probabilities), equal flow sharing approximates the optimal assignment, and thus obtains the similar performance as MS-WF. But as the multi-source proportion increases, there are more flows entering the network. The effect of "bad flows" enlarges, and hence drags down the overall throughput improvement. Accordingly, Best-source approach begins to outperform Equal-share. By jointly optimizing the bandwidth allocation and flow assignment, MS-WF achieves a much higher throughput than the others, resulting in higher utilization of the network. When all the transfers have multiple sources ($\rho = 1$), MS-WF obtains a substantial throughput gain of up to 52% compared to the single-source transmission.

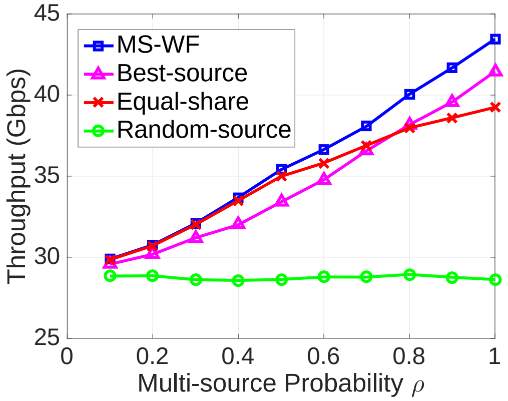


Figure 4: Network throughput vs. multi-source probability ρ, where the arrival rate $\lambda = 2$ and the data size $V = 10Gbits$.

Figure 5 compares the transfer completion time versus three factors respectively: the multi-source probability ρ, the transfer size V and the transfer arrival rate λ. It is shown that, MS-WF achieves the smallest transfer completion time across all parameter configurations. This improvement is mainly derived from a more max-min fair allocation by MS-WF.

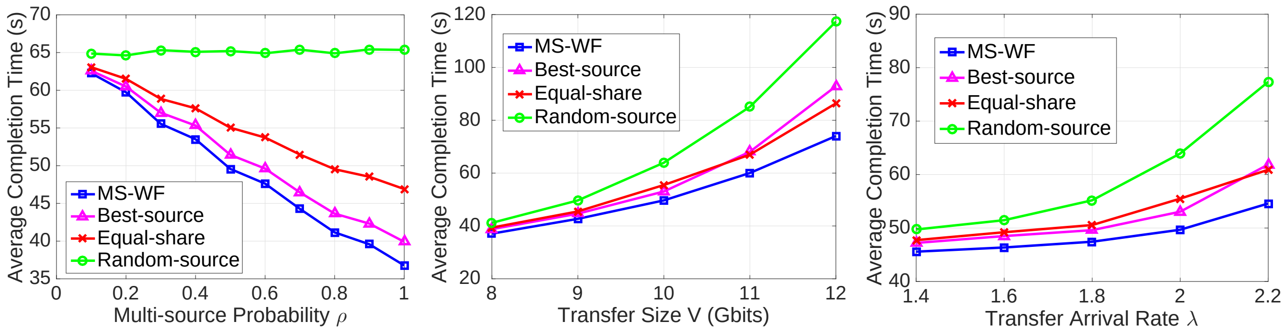
Figure 5(a) focuses on the impact of multi-source probability ρ. Approximately, ρ equals to the proportion of multi-source transfers. The gap between Random-source and the other approaches verifies that, leveraging multiple sources is more efficient for data transmission. Moreover, the sustained decline in completion time with the multi-source probability implies that, the multi-source transmission obtains more performance gains by placing more replicas in the network. Compared to single-source transmission, MS-WF reduces the average completion time by up to 44%.

Figure 5(b) shows the relationship between completion time and the transfer size V. As the transfer size increases, the transfer backlog starts to cause more bottleneck links, which leads to the degradation of transmission rates. Therefore the completion time increases superlinearly along with the transfer size for all the allocation approaches. But the almost linear completion time growth of MS-WF suggests that, by completing transfers as quick as possible, MS-WF is capable of optimizing data transfers for small files as well as large files.

Figure 5(c) illustrates the impact of transfer arrival rate λ. As expected, at higher rates, the number of transfers over the network potentially increases, and links are more likely to become congested. Accordingly, the performance degrades quickly for all the approaches except MS-WF. The smaller growth in completion time demonstrates that, by effectively avoiding the congestion point, MS-WF manages to handle relatively a larger amount of traffic without degrading the performance.

5 RELATED WORK

Distributed filesystems. Several high-performance distributed filesystems with sufficient data replicas have been developed, including GFS [7], HDFS [16] and Quantacast File System [14]. Leveraging SDN, Mayflower [15] performs global optimizations to make



(a)

(b)

(c)

Figure 5: Impact of (a) the multi-source probability ρ, (b) the data size V and (c) the transfer arrival rate λ on average transfer completion time. In each subfigure, we adjust one factor and fixed the other two. The three default values are $\rho = 0.5$, $V = 10$ and $\lambda = 2$.

intelligent replica selection and flow scheduling decisions based on both filesystem and network information. Nevertheless, current solutions focus heavily on best replica selection and data replication placement, instead of multi-source transmission as in our work. As shown in the proceeding sections, single-source transmission fails to achieve transfer-level max-min fairness, therefore provides sub-optimal performance.

Coflow scheduling. The works that schedule parallel flows have been developed to optimize transfers at the level of coflow rather than individual ones. Coflow [2], Varys [3] and Barrat [6] improve application-level performance by minimizing coflow completion times and guaranteeing predictable completions. However, their basic assumption is that the flows are streamed for different data and the volume of each flow is designated in advance, so they can easily predict the completion time and allocate the rate to meet their deadlines. The improvement of our approach over them is that the flow volume assignment is jointly optimized with bandwidth allocation to achieve global optimality.

6 CONCLUSION

We present a novel max-min fair allocation approach for multi-source transmission which conveys data in parallel from multiple sources and dynamically adjusts the flow volumes to maximize network utilization. The allocation relies on a MultiSource Water-Filling algorithm that jointly computes the bandwidth allocation and flow assignment with simple equivalent canonical LP to achieve global optimality. Extensive simulations validate that, compared to other single-source and multi-source allocation approaches, our approach achieves a better throughput gain of up to 52% and decreases transfer completion time by up to 44% for large-scale transfers. We believe this approach is applicable to various traffic management systems that orchestrate arbitrary bulk transfers. Developing such systems will be the next step of this research.

A PROOF OF THEOREM 2

We give the full proof of **Theorem 2** as below.

Proof. Consider the linear programming (LP) problem with multiple sets of variables in each iteration.

First, we can prove that an optimal solution for this problem exists: Given the transformation of the LP, the objective variable t is in constraints of τ', which is a linear and continuous function of $\{x_{ik}\}$. The variables $\{x_{ik}\}$ are bounded by an interval $[0, 1]$. So based on the intermediate value theorem (**Theorem 3**) and its corollary (**Corollary 1**), the continuous function τ' is also bounded. Therefore the minimum of t must exist. The proofs of **Theorem 3** and **Corollary 1** can be found in book [5].

Theorem 3 (Intermediate Value Theorem)**.** *If a continuous function, f, with an interval, $[a, b]$ as its domain, takes values $f(a)$ and $f(b)$ at each end of the interval, then it also takes any value between $f(a)$ and $f(b)$ at some point within the interval.*

Corollary 1. *The image of a continuous function over an interval is itself an interval.*

According to the property of LP, its constraints include a convex set (*a.k.a* feasible solution set), and the optimal solution of the problem must be on the vertexes of its feasible solution set. If it obtains the optimal solution on more than one vertex, we can arbitrarily choose one, because they will return the same result. We denote the optimal solution as $\{x^*_{ik}|\forall i, k\}$ and t^*. Since it is on the vertex of the feasible solution set, at least one of the equation can be satisfied in the LP's constraints using x^*_{ik} and t^*. Let $\tilde{\mathcal{X}}$ denote all the variables of $\{x_{ik}\}$ contained in the satisfied equations. Then we can substitute all the variables $\tilde{x}_{ik} \in \tilde{\mathcal{X}}$ by $\tilde{x}^*_{ik}$ and plug the values into the FL matrix. The corresponding variables and columns and are then removed from $\mathcal{X}$ as well as from the matrix. When the set $\tilde{\mathcal{X}}$ is empty, which means the minimum value of t is a constant, we can simply remove it from the FL matrix and begin the next iteration.

As a result, at least one column can be removed from the FL matrix in each iteration, and all the variables (if exist) in that column can be calculated by an LP. Therefore given an FL matrix with finite columns and rows, all sets of variables $\{\mathcal{X}_i\}$ can be calculated in finite iteration steps by MS-WF. □

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