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Project Report on,

“DATA TRANSMISSION ANALYSIS AND SIMULATION APPLYING AMPLITUDE/FREQUENCY MODULATION AND EFFECTS OF ATTENUATION ON THE TRANSMITTED SIGNAL.”

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ABSTRACT

In this project we account for and study the area of data communication, the technical aspects, theory governing the transmission of signal, simulation of different modulations, transference of the signal across the transmitting channel, effects of attenuation on the transmitting signal and finally aspects of demodulation.

Rigorous mathematical analysis of the different signals are done using powerful mathematical tools such as Laplace and Fourier transforms for ease of analysis of different signals as well as in application. Simulations of transmission of different kinds of signals in MATLAB are shown in this project with exquisite detail.

We have covered different types of modulation in theory additionally, Amplitude and Frequency in particular for this project. The process of modulation with carrier signal and input/message signal is examined thoroughly.

We try to cater to the power of computer software such as MATLAB, open source software such as LaTeXto develop this project.

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2 Modulation

2.1 Need of Modulation

We must have some strong reasons for why to modulate our message signal as it makes the complete process of communication a bit complex.Here are a few points we need to have a look on :

1. No interference: Suppose there is a busy channel , where we need to transmit multiple signals , and if the range of frequency is common between any two signal there is a chance of interference among them. During Modulation, the message signal is processed using carrier signals having unique frequencies (carrier frequency) due to which there is no two signals in the channel having same range of frequency and hence the problem of interference is removed.For example , there are 3 signals viz. voice , audio and video whose frequency ranges from 300Hz - 35KHz, 20Hz-20KHz and 0Hz-4.5MHz respectively; being transmitted from the same channel so they are chances of interfering as the range is common.Modulating these signals with 3 different frequencies f1,f2 and f3 woulld help us deal with this.

2. Multiplexing: The second point is a byproduct of the above advantage. Since, we can have an interference-absent signal transmission for multiple signals through a single channel that is the very idea of multiplexing, which has numerous applications in the field of data communication.Muxing brings along one more advantage of cost reduction.

3. Height Of Antenna: Signals are actually transmitted and recieved using antennas, which is the most vital part of any bidirectional/unidirectional communication.We can see in the forthcoming sub-topic that height of the antenna could be reduced as a result of modulation.

4. Improves Signal To Noise Ratio: As the frequency of the modulated signal is very high the chances of noise entry decreases abruptly, due to which the signal to noise improves substantially.

5. Long distance Transmission: As the frequency increases, the power of signal increases and it does not attenuates easily while a low frequency signal fades out easily and cannot undergo long distance transmissions.

2.2 Antenna Theory

As we have already talked that,height of the transmitting/receiving antenna depends upon the the signal we are trying to work on.The theory talks about what should be the height of the antenna for a proper transmission; it says that

$$\text{height of antenna(h)} \propto \frac{wavelength(\lambda)}{4}$$

we know that,

$$\lambda = \frac{c}{f} \text{ ,c is the speed of light}$$

From the above famous relation,it is clear that wavelength(λ) varies inversely with frequency(f) of the signal.So the greater the frequency of the transmitting signal the smaller will be the value of λ and hence the feasible will be the height of the antenna(h). Suppose, we have a signal propagating with a frequency of 15kHz, then wavelength(λ) is given by

$$\lambda = \frac{3\times10^8}{15\times10^3}$$

$$= 20000 \text{ m}$$

and the height of the antenna is,

$$h = \frac{20000}{4}$$

$$= 5000 \text{ m}$$

which is ofcourse not feasible to build.Hence, the frequency of 10KHz is also falling short for purpose that is why the above mentioned signals have frequencies in the range of MHz.The carrier frequencies in case of modulation have frequencies in this range , which makes it possible to build antennas of practical length.

Suppose the frequency of the transmitting signal is 6MHz after modulating ,the wavelength of the signal is :

$$\lambda = \frac{3\times10^8}{6\times10^6}$$

$$= 50\text{m}$$

and now the height of the antenna is,

$$h = \frac{50}{4}$$

$$= 12.5 \text{ m}$$

This time the antenna seems practical to build and hence antenna theory plays a great role in deciding the carrier frequency or indirectly the height of the antennas.

2.3 Modulation Process

1. Continuous Wave Modulation(CWM): The modulated signal in this case is continuous in time domain.It is further divided into :

    - Amplitude Modulation(AM): The amplitude of the carrier signal varies in accordance with the input baseband signal.Different types of AM are :

        (a) **Double sideband Full Carrier(DSB-FC)** :The transmission of a modulated signal which contains a carrier along with sidebands is termed as DSB-FC.

        (b) **Double sideband Suppressed carrier(DSB-SC)** : The above discussed transmission is known to be inefficient as two-third of the total power in the carrier wave is being wasted having no information.If we suppress the above type of signals and the power that is saved is distributed among the two sidebands we get DSB-SC.

        (c) **Single sideband Suppressed carrier(SSB-SC)** : As both the sidebands are carrying same information two times, its better if we suppress one of the two. The suppression of one of them along with the carrier wave and transmitting a single sideband is what we call Single sideband Suppressed carrier system or SSB-SC.It has high power as compared to DSB-FC/DSB-SC.

        (d) **Vestige sideband Suppressed carrier(VSB-SC)** : As transmission of two sidebands is unnecassary , but transmitting single sideband leads to loss of information.So in vestigial modulation a part of signal(vestige) is modulated along with each sideband.In other words, a part of upper side band is also transmitted with lower sideband.To avoid interferences, a guard band having very small width is also laid along with each sidebands. VSB modulations are highly efficient and filter design is quite easy as high accuracy is not demanded.The only point where it lacks is demodulation becomes a difficult task in this kind of modulations.

        (e) **Independent sideband Suppressed carrier(ISB-SC)** :Input signal is modulated by two different modulating signals.Here the transmitter comprises of two independent SSB-SC modulators. one of the modulator generates the upper sideband and the other generates the lower sideband.The output signals from the two modulators are combined to form a DSB ,where the sidebands are completely independent but are symmetrical about a common carrier freqeuncy.

- Angle Modulation : It has been further divided into two parts :
  (a) *Frequency Modulation* : In this case,the carrier wave frequency is varied according to the modulating signal whereas the amplitude remains constant.Two types of FM are explained below :
    i. **Narrow Band FM :** As the name suggests,it is the FM wave with comparitively small bandwith.The modulation index is as small as 1 radian.

       Its uses are in the fields of FM mobile communications like ambulances,taxies,etc.
    ii. **Wide Band FM :** FM waves having infinitely large frequency comes under this category.It ideally contains the carrier wave and an infinite number of sidebands located symmetrically around it.

       Its widely applied in th fields of Television,FM radio,etc.
  (b) *Phase Modulation* : The variation of the phase of carrier wave linearly in accordance with the input signal is phase modulation.It is of two types :
    i. **Narrow Band Phase Modulation**
    ii. **Wide Band Phase Modulation**

2. Pulse Modulation(PM): In this kind of modulation,some parameter of a pulse train is varied in accordance with the input message signal.It is mainly divided in two parts :
   - Analog Pulse Modulation(APM):
     (a) **Amplitude PM**
     (b) **Position PM**
     (c) **Phase PM**
   - Digital Pulse Modulation(DPM):
     (a) **Baseband :**
       - Pulse Code Modulation(PCM)
       - Differential Pulse Code Modulation(D-PCM)
       - Delta Modulation
       - Adaptive Delta Modulation
     (b) **Bandpass :**
       - Binary
       - M-ary

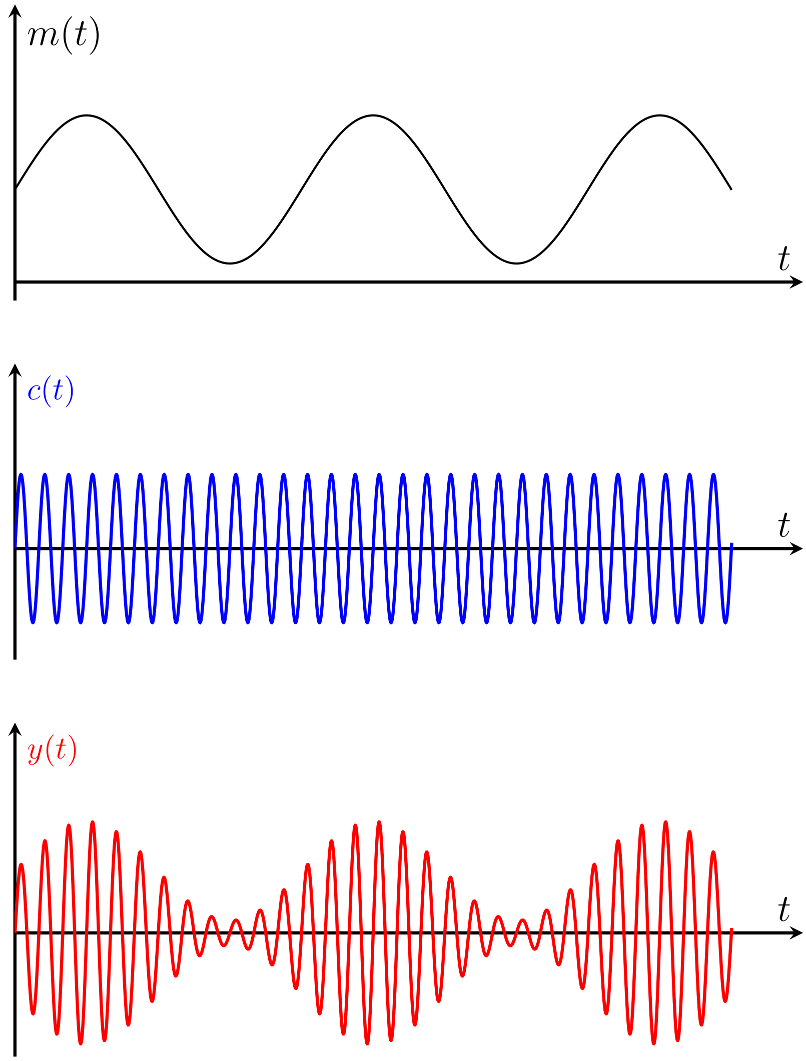
3 Amplitude Modulation(AM)

3.1 Introduction

Amplitude Modulation is a modulation technique in which the amplitude of another signal called *carrier signal* $c(t)$ is varied in accordance with the *message/input/modulating signal* $m(t)$. The modulating signal or $m(t)$ contains the intended message or information that is to be transmitted across the channel. The modulating signal contains the intended message or information sometimes consisting of audio data, as for an example in AM radio broadcasting, or two-way radio communications.

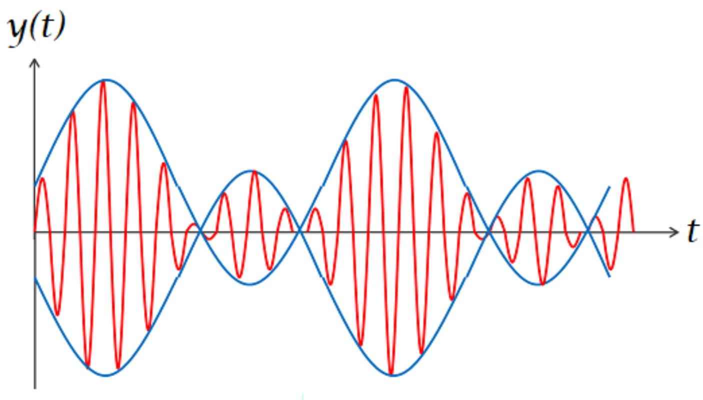
3.2 Modulation Process

The high-frequency sinusoidal waveform(i,e., *carrier signal*) is modulated with respect to the input signal by combining it with the message signal using a *multiplier* or *mixer*. It is worth mentioning that mixing is a non-linear operation because it generates new frequencies. An example of visual realization of Amplitude modulation is shown as follows:



The signal in red *i.e.,* $y(t)$ is the *amplitude modulated* wave.

Generally we have the modulated signal as $y(t) = m(t).c(t)$, but in case of amplitude modulation if we use this form, there is a serious problem of phase reversal *i.e.,* the signal crosses the $x - axis$, which in turn when multiplication causes the problem of phase reversals.



The mechanism of retrieval of original message signal from such a phase reversed signal is complicated and impractical for implementation. So, we improvise and solve this problem.

To do so, we shift our message signal $m(t)$, by some DC value say A and then multiply it with its carrier signal, for modulation purposes.

The general form of such signal is given by the following equation,

$$y(t) = A_c[1 + k_a m(t)]cos(2\pi f_c t) \tag{1}$$

From the above equation if we observe then, the message signal is measured in $volts$. Therefore, by dimensional analysis the unit of k_a is $volts^{-1}$.So,

k_a, is the amplitude sensitivity.
$m(t)$ is the message/modulating signal.
$y(t)$ is the modulated wave.

The *envelope* of $y(t)$ in particular has same shape as the baseband signal $m(t)$ provided that the following two requirements are satisfied,

- The amplitude of $k_a m(t)$ represented as $|k_a m(t)|$ is always less than unity,*i.e.,*

$$|k_a m(t)| \leq 1, \ \ \forall \ t \tag{2}$$

  - It ensures that the function $1 + k_a m(t) > 0$, and since an envelope is positive. We can represent it as $A_c[1 + k_a m(t)]$.
  - When, $|k_a m(t)| > 1$, $y(t)$ becomes overmodulated, resulting in carrier phase reversals whenever the factor as mentioned $1 + k_a m(t)$ crosses zero.
  - The absolute maximum value of $k_a m(t)$ multiplied by 100 is referred to as *percentage modulation*

- The carrier frequency f_c is much greater than the highest frequency component W of the message signal $m(t)$*i.e.,*

$$f_c \gg W \tag{3}$$

– If the above condition is not satisfied, an envelope cannot be realized successfully. The component W, is called the message bandwidth.

If we take the *fourier* transform of *equation* (), then we have,

$$F(y(t)) = Y(f) = \int_{-\infty}^{\infty} A_c[1 + k_a m(t)]cos(2\pi f_c t)dt$$

Upon, using the following results we simplify the above equation,

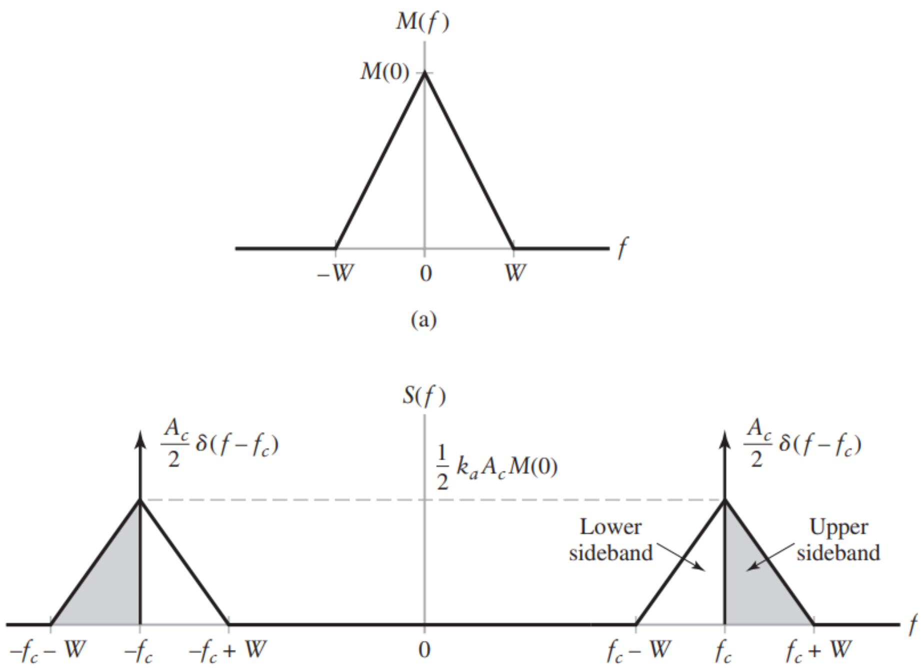
- $cos(x) = \dfrac{e^x - e^{-x}}{2}$
- $\int_{-\infty}^{\infty} e^{j2\pi f(t-a)}df = \delta(t-a) = \delta(a-t)$

We find that,

$$F(cos(2\pi f_c t)) = \frac{\delta(f-f_c) + \delta(f+f_c)}{2} \tag{4}$$

equation $Y(f)$ implies,

$$Y(f) = \frac{A_c}{2}[\delta(f+f_c) + \delta(f-f_c)] + \frac{k_a A_c}{2}[M(f+f_c) + M(f-f_c)] \tag{5}$$



The above figure represents all the signals given by *equation* 5. Figure (a) in above gives the frequency spectrum of signal $m(t)$

We now analyze the spectrum given by the above figure.

- The frequency of the carrier signal should be sufficiently higher than that of the modulating signal *i.e.,* the condition given by the *equation* (3) should hold. Otherwise, both the side bands will interfere each other.
- The frequency above f_c is known as upper sideband, below f_c is referred to as the lower sideband. For negative frequencies: The upper sideband is below $-f_c$ as shaded region shows clearly and the lower sideband is above $-f_c$. The condition $f_c > W$ ensures that the sidebands do not overlap, in accordance with point above.
- For positive frequencies, the highest frequency component of the AM wave equals $f_c + W$, and the lowest frequency component equals $f_c - W$. The difference between these two frequencies defines the *transmission bandwidth* B_T for an AM wave.

$$B_T = 2W \tag{6}$$

3.3 Types of Amplitude Modulation

- **Double Side Band - with Carrier(DSB-WC)** : This form of Amplitude Modulation is most widely used. All radio channels in the AM band use this type of modulation.
- **Double Side Band - Suppressed Carrier(DSB-SC)** : This form of Amplitude Modulation is same as the above mentioned except for the fact that it is without carrier(*suppressed*).
- **Single Side Band (SSB)** : In this form of AM the only half of the signal of DSB-SC is used.
- **Vestigial Side Band (VSB)** : This is the modified version of SSB, to ease the generation and reception of the signal.

3.4 Advantages

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4 Frequency Modulation

The angle modulation in which the instantaneous frequency $f_i(t)$ of the carrier signal is varied linearly with the message signal is called frequency modulation.Mathematically, if $m(t)$ be the message signal(voltage waveform) and f_c be the frequency of the unmodulated carrier signal then

$$f_i(t) = f_c + k_f m(t) \tag{7}$$

where k_f is the frequency-sensitivity factor of the modulator(in hertz per volt)

4.1 Time Equation for Frequency Modulated Wave

Let $\theta_i(t)$ denote the angle of a modulated sinusoidal carrier at time t and the resulting angle-modulated wave is

$$s(t) = A_c \cos\left[\theta_i(t)\right] \tag{8}$$

where A_c is the carrier amplitude.
If $f_i(t)$ be the instantaneous frequency of resulting FM wave then

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt} \tag{9}$$

Integrating Eq (3) with respect to time, we get

$$\theta_i(t) = 2\pi \int_0^t f_i(\tau)\, d\tau = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau$$

where the second term accounts for the increase or decrease in the instantaneous phase due to the message signal $m(t)$.
Therefore, the frequency modulated wave is given by

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right]$$

4.2 Narrow Band Frequency Modulation

Consider a sinusoidal modulating wave defined by

$$m(t) = A_m \cos 2\pi f_m t$$

Then instantaneous frequency of the resulting FM wave is

$$\begin{aligned} f_i(t) &= f_c + k_f A_m \cos 2\pi f_m t \\ &= f_c + \Delta f \cos 2\pi f_m t \end{aligned}$$

where $\Delta f = k_f A_m$ is called the frequency deviation which represents the maximum departure of the instantaneous frequency of the FM wave from the carrier frequency.
The angle $\theta_i(t)$ of the FM wave is given by

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t A_m \cos 2\pi f_m t\, d\tau$$

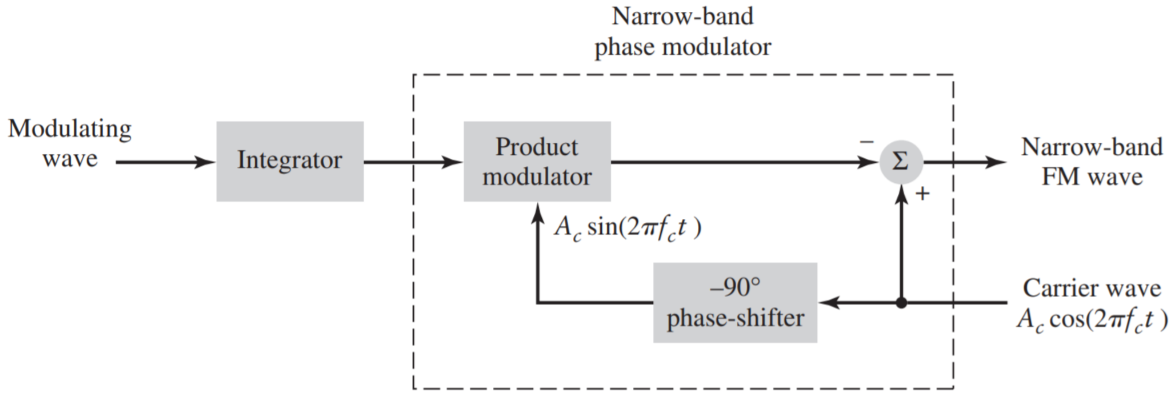

Figure 1: Block diagram of an indirect method for generating a narrow-band FM wave.

Or

$$\theta_i(t) = 2\pi f_c t + \frac{\Delta f}{f_m} \sin 2\pi f_m t$$

Or

$$\theta_i(t) = 2\pi f_c t + \beta \sin 2\pi f_m t$$

where $\beta = \dfrac{\Delta f}{f_m}$ is called the modulation index of the FM wave(measured in radians). The resulting FM wave is

$$s(t) = A_c \cos\left[2\pi f_c t + \beta \sin 2\pi f_m t\right] \tag{10}$$

Or

$$s(t) = A_c \cos\left[2\pi f_c t\right] \cos\left[\beta \sin 2\pi f_m t\right] - A_c \sin\left[2\pi f_c t\right] \sin\left[\beta \sin 2\pi f_m t\right]$$

For NBFM the modulation index is small compared to one radian.So we can assume

$$\cos\left[\beta \sin 2\pi f_m t\right] = 1$$

and

$$\sin\left[\beta \sin 2\pi f_m t\right] = \beta \sin 2\pi f_m t$$

Then the narrow band frequency modulated signal is

$$s(t) = A_c \cos\left[2\pi f_c t\right] - \beta A_c \sin\left[2\pi f_c t\right] \sin\left[2\pi f_m t\right] \tag{11}$$

A block diagram is shown above of generating NBFM signal. This modulator involves splitting the carrier wave $A_c \cos 2\pi f_c t$ into two paths. One path is direct and the other path contains a degree phase-shifting network and a product modulator, the combination of which generates a DSB-SC modulated wave. The difference between these two signals produces a narrow-band FM wave.

4.3 Wide Band Frequency Modulation

The frequency modulated signal for which modulation index β is greater than 1 is called wide band frequency modulated signal.

The time equation for WBFM signal can be written with the help of Bessel function. The wide band frequency modulated signal $s(t)$ can be written as

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos\left[2\pi(f_c + nf_m)t\right] \tag{12}$$

The discrete spectrum of $s(t)$ is obtained by taking the Fourier transforms of both sides of Eq (7) as below

$$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta)[\delta(f - f_c - nf_m) + \delta(f + f_c + nf_m)] \tag{13}$$

where $S(f)$ is Fourier Transform of $s(t)$. The above equation shows that the spectrum of $S(f)$ consists of an infinite number of delta functions spaced at $f = f_c \pm nf_m$ for n=0,1,2,$\cdots$

4.3.1 Power of a WBFM signal

The average power of an WBFM wave can be determined from

$$P_t = \sum_{n=-\infty}^{\infty} \frac{[A_c J_n(\beta)]^2}{2} \tag{14}$$

which can be simplified using the property

$$\sum_{n=-\infty}^{\infty} J_n(\beta)^2 = 1$$

The simplified average power of WBFM signal is

$$P_t = \frac{A_c^2}{2} \tag{15}$$

4.4 Carson's Rule

In theory, an FM wave contains an infinite number of side-frequencies so that the bandwidth required to transmit such a modulated wave is similarly infinite in extent.But in practice, the FM wave is effectively limited to a finite number of significant side-frequencies compatible with a specified amount of distortion. Let us consider first the case of an FM wave generated by a single-tone modulating wave of frequency f_m. In such an FM wave, the side-frequencies that are separated from the carrier frequency f_c by an amount greater than the frequency deviation Δf decrease rapidly toward zero, so that the bandwidth always exceeds the total frequency excursion, but nevertheless is limited. Specifically, we may identify two limiting cases:
1. For large values of the modulation index $\beta$the bandwidth approaches, and is only slightly greater than the total frequency excursion $2\Delta f$
2. For small values of the modulation index β the spectrum of the FM wave is effectively limited to the carrier frequency f_c and one pair of side-frequencies at$f_c \pm f_m$ so that the bandwidth approaches$2f_m$

Thus, the transmission bandwidth of an FM wave generated by a single-tone modulating wave of frequency f_m is defined by

$$B_T = 2\Delta f + 2f_m = 2\Delta f(1 + \frac{1}{\beta})$$

This is called Carson's rule.

4.5 Generation of FM Waves

There are two methods of generating frequency modulated waves, the first one is direct and the other indirect method.

4.5.1 Direct Method

The direct method uses a sinusoidal oscillator, with one of the reactive elements (like capacitive element) in the tank circuit of the oscillator being directly controllable by the message signal. It is capable of providing large frequency deviations. But a serious limitation of the direct method is the tendency for the carrier frequency to drift, which is usually unacceptable for commercial radio applications.
To overcome this limitation, frequency stabilization of the FM generator is required, which is realized through the use of feedback around the oscillator. Although the oscillator may be simple to build, the use of frequency stabilization adds system complexity to the design of the frequency modulator.

4.5.2 Indirect Method:Armstrong Modulator

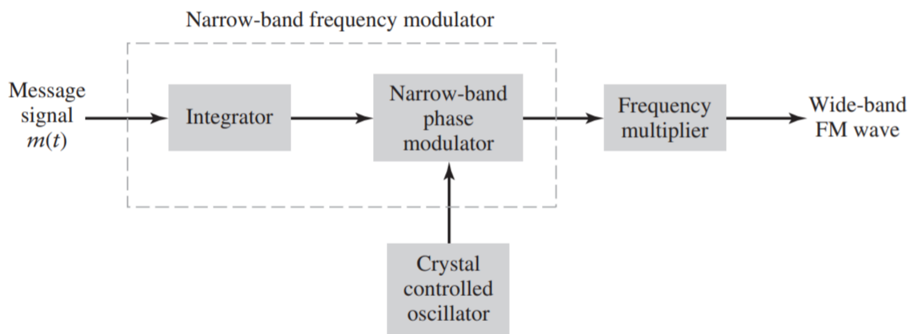

Figure 2: Block diagram of an indirect method for generating a narrow-band FM wave.

A simplified block diagram of this indirect FM system is shown in below fig. The message signal $m(t)$ is first integrated and then used to phase-modulate a crystal-controlled oscillator which provides frequency stability. In order to minimize the distortion inherent in the phase modulator, the maximum phase deviation or modulation index is purposely kept small, thereby resulting in a narrow-band FM wave. The narrow band FM wave is next multiplied in frequency by means of a frequency multiplier so as to produce the desired wide-band FM wave.

5 Experimentation

6 Simulation

7 Code

8 Attenuation

9 Demodulation