Uber Rocket Problem

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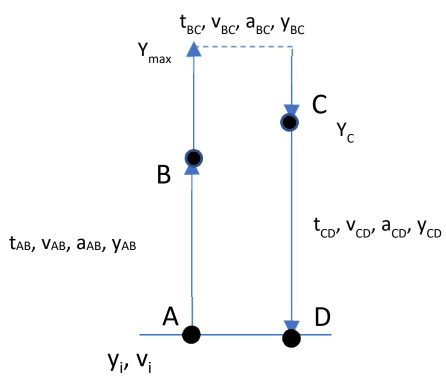
Description

One calm afternoon Calculus Cam decides to launch Hamster Huey into the air using a model rocket. The rocket is launched straight up off the ground, from rest. The rocket engine is designed to burn for specified time while producing non-constant net acceleration given by the equations below. After the engine stops the rocket continues upward in free-fall. A parachute opens after the rocket falls a specified vertical distance from its maximum height. When the parachute opens, assume the rocket instantly stops, and then increases speed to a terminal velocity given by the equation below. Assume the air resistance affects the rocket only during the parachute stage.



Givens

$t_{AB} = 4.0s$

$a_{AB}[t] = -1.4t^2 + 19$

$y_i = 0$ m

$v_i = 0$ m/s

$y_{max} - y_c = 115m$

$v_{CD}[t] = -15(1 - e^{-\frac{t}{6}})$

$a_{BC} = -9.8m/s^2$

Strategy

The goal of the problem is to find the total time the rocket stays in the air. In order to do this, the problem is separated into the three stages: AB, BC, and CD. The time is then determined for each part and then summed to get the final answer.

Stage AB

The time of this stage is given at 4.0 seconds. The position and velocity at the end of the stage need to be determined in order to solve for the time of the next stage. Because the acceleration equation is given, the integration allows us to find the to find the velocity equation and integrated again to find the position equation. These equations can be used to determine the height and velocity of the rocket at B.

$$\Delta v_{AB}[t] = \int a[t]\, dt$$

$$v_{AB}[t] - v_i = \int -1.4t^2 + 19\, dt$$

$$\underline{v_{AB}[t] = -0.4667t^3 + 19t + 0}$$

$$v_{AB}[4] = -0.4667 * 4^3 + 19 * 4 + 0$$

$$v_{AB}[4] = -29.867 + 76.00$$

$$\underline{v_{AB}[4] = v_B = 46.13m/s}$$

$$\Delta y_{AB}[t] = \int v[t]\, dt$$

$$y_{AB}[t] - y_i = \int -0.4667t^3 + 19t + 0\, dt$$

$$\underline{y_{AB}[t] = -0.1167t^4 + 9.50t^2 + 0}$$

$$y_{AB}[4] = -0.1167 * 4^4 + 9.50 * 4^2 + 0$$

$$y_{AB}[4] = y_B = -29.867 + 152$$

$$\underline{y_{AB}[4] = 122.13m}$$

Stage BC

Now that height at B and the velocity at B have been determined, it is possible to determine the t_{BC}. The rocket no longer has boosters, so its acceleration becomes -9.8m/s^2 constantly. The time at stage BC is found by using a constant acceleration equation that writes position in terms of time. The vertex is found to determine the maximum height of the rocket and the time it took to get there. It is given that the rocket's parachute deploys 115m below the max height, and then the time is found using the same equation.

$$y_{BC}[t] = \frac{1}{2}a_{BC}t^2 + v_B t + y_B$$

$$y_{BC}[t] = \frac{1}{2} * -9.8t^2 + 46.13t + 122.13$$

$$\underline{y_{BC}[t] = -4.9t^2 + 46.13t + 122.13}$$

$$t_{max} = \frac{-b}{2a}$$

$$t_{max} = \frac{-46.13}{-9.8}$$

$$t_{max} = 4.707s$$

$$y_{max} = y_{BC}[4.707]$$

$$y_{max} = -4.9 * 4.707^2 + 46.13 * 4.707 + 122.13$$

$$y_{max} = -108.56 + 217.13 + 122.13$$

$$y_{max} = 230.7m$$

$$y_{max} - y_c = 115m$$

$$230.7m - y_c = 115m$$

$$y_c = 115.7m$$

$$y_{BC}[t] = -4.9t^2 + 46.13t + 122.13$$

$$115.7 = -4.9t^2 + 46.13t + 122.13$$

$$0 = -4.9t^2 + 46.13t + 6.43$$

$$\underline{\cancel{t_{BC} = 0.1347s}, t_{BC} = 9.552s}$$

Stage CD

At point C a parachute deploys, and causes a new velocity equation to be introduced, but not before the rocket stops for a miniscule second meaning that the velocity at the end of stage BC is insignificant. From here, the time of this stage needs to be found, and the integration of the given velocity equation is used to find the position equation, which is then used to calculate t_{CD} based on the previously determined height at point C.

$$v_{CD}[t] = -15(1 - e^{-\frac{t}{6}})$$

$$\Delta y_{CD}[t] = \int v[t]\, dt$$

$$y_{CD}[t] = \int -15 + 15e^{-\frac{t}{6}}\, dt$$

$$\underline{y_{CD}[t] = -15t - 90e^{\frac{-t}{6}} + C}$$

$$y_c = y_{CD}[0] = -15 * 0 - 90e^0 + C$$

$$115.7 = -90 + C$$

$$C = 205.7$$

$$\underline{y_{CD}[t] = -15t - 90e^{\frac{-t}{6}} + 205.7}$$

$$0 = -15t - 90e^{\frac{-t}{6}} + 205.7$$

$$\underline{\cancel{t_{CD} = -7.607s}, t_{CD} = 13.03s}$$

Total

Once the time of all three stages are found, they are summed to produce the total time that the rocket flew in the air.

$$t_{total} = t_{AB} + t_{BC} + t_{CD}$$

$$t_{total} = 4.00s + 9.552s + 13.03s$$

$$\boxed{t_{total} = 26.58s}$$

Time	Points	Height	Velocity	Acceleration
(s)		(m)	(m/s)	(m/s^2)
0.00	A	0.00	0.00	19.00
1.00		9.38	18.53	17.60
2.00		36.13	34.27	13.40
3.00		76.05	44.40	6.40
4.00	B	122.12	46.13	-3.40
5.00		163.36	36.33	-9.80
6.00		194.79	26.53	-9.80
7.00		216.42	16.73	-9.80
8.00		228.25	6.93	-9.80
9.00		230.28	-2.87	-9.80
10.00		222.51	-12.67	-9.80
11.00		204.94	-22.47	-9.80
12.00		177.57	-32.27	-9.80
13.00		140.40	-42.07	-9.80
13.55	C	115.68	-47.48	-9.80
14.00		115.46	-1.08	-2.32
15.00		113.28	-3.22	-1.96
16.00		109.13	-5.03	-1.66
17.00		103.32	-6.56	-1.41
18.00		96.10	-7.85	-1.19
19.00		87.68	-8.95	-1.01
20.00		78.25	-9.88	-0.85
21.00		67.97	-10.67	-0.72
22.00		56.96	-11.33	-0.61
23.00		45.34	-11.89	-0.52
24.00		33.20	-12.37	-0.44
25.00		20.63	-12.77	-0.37
26.00		7.68	-13.12	-0.31
26.58	D	0.02	-13.29	-0.29