Uber Pulley Problem

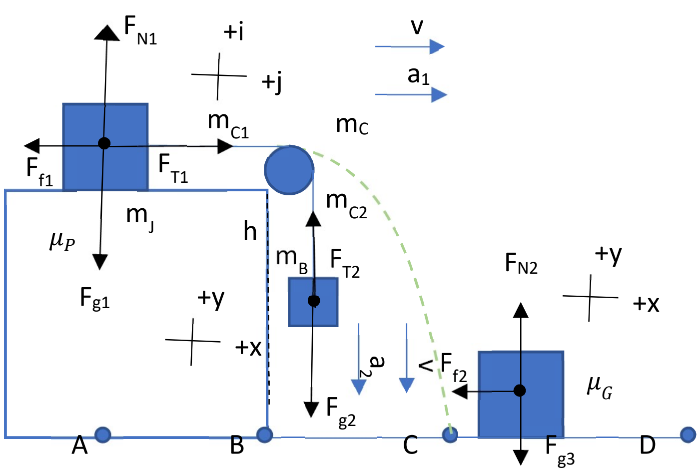
Alexander Sun October 31, 2019 Section L

Description

"Jerky" Jerry decided to make a jabberwocky jumper using a pulley system (see diagram). His method was to attach one end of a chain to a barrel of rocks, and the other end to the jumper. He placed the barrel and chain over a massless frictionless pulley, and then walked along a platform away from the pulley to point A (the full length of the chain). When he sat in the jumper he accelerated along the platform to point B and then launched off it while releasing the chain from the jumper and avoiding the pulley. He flew through the air as a projectile to point C, transitioning 75% of his speed into the horizontal direction, and eventually slid to a stop at point D. Note: Ignore any heights of the jumper, pulley, and barrel. Ignore any frictional and normal forces of the chain.

Diagram

μ



Givens

$$m_J = 69kg$$
$$m_B = 163\ kg$$
$$m_C = 61kg$$
$$L_C = 13m$$
$$h = 21m$$
$$\mu_p = 0.11$$

$$\Delta x_{BD} = 54m$$
$$x[t] = \frac{1}{2}at^2 + v_i t + x_i$$
$$v_f = v_i + a * t$$
$$v_f^2 = v_i^2 + 2a\Delta x$$

Assumptions

$$F_{T1} = F_{T2} \qquad a_1 = -a_2$$

Stage AB

First, the goal of this stage is to calculate the velocity by which Jerry flies off the platform. This can be done writing the sum of the forces acting on it, shown in the FBD.

$$\sum F_i : F_{N1} - F_{g1} = m_J *\ 0$$

$$F_{N1} = m_J * g$$

$$F_{N1} = 69 * 9.8$$

$$\underline{F_{N1} = 676.2N}$$

$$F_{f1} = \mu_P * F_{N1}$$

$$F_{f1} = 0.11 * 676.2$$

$$\underline{F_{f1} = 74.382N}$$

$$\sum F_j : F_{T1} - F_{f1} = m_1 *\ a_1$$

$$F_{T1} - 74.382 = m_1 *\ a_1$$

$$\underline{F_{T1} = m_1 *\ a_1 + 74.382}$$

The chain contains weight, so as the system moves, more mass gets added onto part pulling downward, and mass gets removed from the part being moved. The following equations write the mass in terms of position of the chain, where a position of 13 represents the entire chain off the platform and hanging off. This fact can then be used to write acceleration as a function of position of the chain, allowing the velocity at the end to be found.

$$m_1[x] = m_J + m_{C1}[x]$$

$$m_{C1}[x] = (m_C - \frac{x}{l_C} * m_C)$$

$$m_1[x] = 69 + (61 - \frac{x}{13} * 61)$$

$$m_2[x] = m_B + m_{C2}[x]$$

$$m_{C2}[x] = (\frac{x}{l_C} * m_C)$$

$$m_2[x] = 163 + (\frac{x}{13} * 61)$$

$$\sum F_y : F_{T2} - F_{g2} = m_2 *\ a_2$$

$$F_{T2} - m_2 * g = m_2 *\ a_2$$

$$\underline{F_{T2} = m_2 *\ a_2 + m_2 * g}$$

$$Substitute: F_{T1} = F_{T2}$$

$$m_1 * a_1 + 74.382 = m_2 * a_2 + g * m_2$$

$$m_1 * a_1 + 74.382 = m_2 * -a_1 + g * m_2$$

$$a_1(m_1 + m_2) = -74.382 + 9.8 * m_2$$

$$a_1 = \frac{-74.382 + 9.8 * m_2}{m_1 + m_2}$$

$$a_1[x] = \frac{-74.382 + 9.8 * (163 + (\frac{x}{13} * 61))}{69 + (61 - \frac{x}{13} * 61) + 163 + (\frac{x}{13} * 61)}$$

$$a_1[x] = \frac{-74.382 + 1597.4 + 45.9846x}{293}$$

$$\underline{a_1[x] = 5.19802 + 0.156944x}$$

$$a \equiv \frac{dv}{dt}$$

$$a = \frac{dx}{dx} * \frac{dv}{dt}$$

$$a = v * \frac{dv}{dx}$$

$$a[x]dx = v\,dv$$

$$\int_{x_0}^{x} a[x]dx = \int_{v_0}^{v} v\,dv$$

$$\int_{0}^{x} a[x]dx = \int_{0}^{v} v\,dv$$

$$\int_{0}^{x} 5.19802 + 0.156944x\,dx = \frac{1}{2}v^2, calculator$$

$$0.078472x^2 + 5.19802x = \frac{1}{2}v^2$$

$$v^2 = 0.156944x^2 + 10.396x$$

$$\underline{v[x] = \sqrt{0.156944x^2 + 10.396x}}$$

$$v[13] = \sqrt{0.156944 * 13^2 + 10.396 * 13}$$

$$v[13] = \sqrt{26.5235 + 135.149}$$

$$v[13] = \sqrt{161.6725}$$

$$\underline{v_{Bx} = 12.71505m/s}$$

Stage BC

This stage uses kinematics to determine the net velocity upon reaching the ground, and the distance traveled from BC. These values become helpful in completing the next stage of the journey.

$$y[t] = \frac{1}{2}a_y t^2 + v_{By}t + y_i$$

$$y[t] = \frac{1}{2} * -9.8 * t^2 + 21$$

$$\underline{y[t] = -4.9 * t^2 + 21}$$

$$0 = -4.9 * t^2 + 21$$

$$4.9 * t^2 = 21$$

$$t^2 = 4.28571$$

$$\underline{t = 2.07019s, \cancel{t = -2.0702}}$$

$$x[t] = \frac{1}{2}a_x t^2 + v_{Bx}t + x_i$$

$$\underline{x[t] = 12.71505 * t}$$

$$x[2.0782] = 12.71505 * 2.07019$$

$$\underline{x_{BC} = 26.3226m}$$

$$v_{Cy} = v_{By} + a * t$$

$$v_{Cy} = 0 - 9.8 * 2.07019$$

$$\underline{v_{Cy} = -20.2879m/s}$$

$$v_{net} = \sqrt{v_{Cx}^2 + v_{Cy}^2}$$

$$v_{net} = \sqrt{12.71505^2 + (-20.2879)^2}$$

$$\underline{v_{net} = 23.9431m/s}$$

Stage CD

This stage uses the information of the previous stage to determine the coefficient of friction. It loses a forth of its net velocity upon hitting the ground, and travels from C to D. The velocity and travel distance can be used to find the acceleration. Using the second FBD a sum of forces can be created to find the coefficient of friction of the surface.

$$v_C = 0.75 * 23.9431$$

$$\underline{v_C = 17.9573}$$

$$v_f^2 = v_C^2 + 2a\Delta x$$

$$v_f^2 = v_C^2 + 2 * a * (x_D - x_C)$$

$$0 = 17.9573^2 + 2 * a_x * (54 - 26.3226)$$

$$-55.3548a = 322.466$$

$$\underline{a_x = -5.8254m/s^2}$$

$$\sum F_x : -F_f = m_J * a_x$$

$$-F_f = 69 * -5.8254$$

$$\underline{F_f = +401.9526N}$$

$$\sum F_y : F_{N2} - F_{g3} = m_J * a_y$$

$$F_{N2} = 0 + m_J * g$$

$$F_{N2} = 0 + 69 * 9.8$$

$$\underline{F_{N2} = 676.2N}$$

$$F_{f2} = \mu_G * F_{N2}$$

$$401.9526 = \mu_G * 676.2$$

$$\mu_G = \frac{401.9526}{676.2}$$

$$\boxed{\mu_G = 0.5944}$$