



**BITS Pilani**  
Pilani | Dubai | Goa | Hyderabad

**Work Integrated Learning  
Programmes Division  
M.Tech (Data Science and  
Engineering)**

## **DSECFZC416 - Mathematical Foundations for Data Science**

### **INSTRUCTIONS**

1. Assignments have to be handwritten, images to be captured in a doc file. Convert the doc file to a pdf file and name it as your BITSID.pdf and upload the file
2. Assignments sent via email would not be accepted
3. Submissions beyond 31<sup>st</sup> of Aug, 20.00 hrs would not be graded.

### **Assignment 2 - Total Marks – 10**

(Each question carries 2 marks weightage)

Q.1

Let  $Q$  be the relation on the set  $\mathbb{R}$  of real numbers, where real numbers  $x$  and  $y$  satisfy  $xQy$  if and only if  $e^{x-y}$  is an integer. Determine

- (i) whether or not the relation  $Q$  is *reflexive*,
- (ii) whether or not the relation  $Q$  is *symmetric*,
- (iii) whether or not the relation  $Q$  is *anti-symmetric*,
- (iv) whether or not the relation  $Q$  is *transitive*,
- (v) whether or not the relation  $Q$  is a *equivalence relation*,
- (vi) whether or not the relation  $Q$  is a *partial order*.

[Justify your answers with short proofs and/or counterexamples.]

Q.2. Suppose that  $f$  is a function from  $A$  to  $B$ , where  $A$  and  $B$  are finite sets with  $|A| = |B|$ . Show that  $f$  is one – to – one if and only if it is onto

Q.3.

Let  $f: [0, 4] \rightarrow [0, 10]$  be the function defined so that

$$f(x) = \begin{cases} x^3 & \text{if } 0 \leq x \leq 2; \\ x + 6 & \text{if } 2 < x \leq 4. \end{cases}$$

Determine whether or not this function is injective, and whether or not it is surjective, giving brief reasons for your answers.

Q.4.

Find  $\bigcup_{i=1}^{\infty} A_i$  and  $\bigcap_{i=1}^{\infty} A_i$  if for every positive integer  $i$ ,

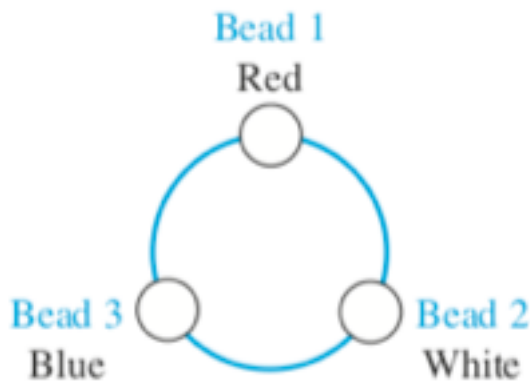
a)  $A_i = \{i, i + 1, i + 2, \dots\}$ .

b)  $A_i = \{0, i\}$ .

c)  $A_i = (0, i)$ , that is, the set of real numbers  $x$  with  $0 < x < i$ .

d)  $A_i = (i, \infty)$ , that is, the set of real numbers  $x$  with  $x > i$ .

Q.5. A bracelet is made of three beads and the choice of the color of the beads are Red, Blue and White. All possible combinations of the beads are possible.



Define the relation  $R$  between bracelets as:  $(B_1, B_2)$ , where  $B_1$  and  $B_2$  are bracelets, belongs to  $R$  if and only if  $B_2$  can be obtained from  $B_1$  by rotating it or rotating it and then reflecting it.

- Write the Cartesian product of combination
- Show that  $R$  is an equivalence relation
- What are the equivalence classes of  $R$ ?

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