ISM - Assignment

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Q2) A random variable has foll. distribution

X	0	1	2	3	4	5	6	7	8
$P(x)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$	$15k$	$17k$

i) Find the value of k

ii) Find $P(X<4)$, $P(0<X<4)$

iii) Find the smallest value of x for which $P(X$ less then or equal to $k) > 0.5$?

Ans 1) Value of k

$$\sum_{i=0}^{8} P(x_i) = 1$$

$$k + 3k + 5k + 7k + 9k + 11k + 13k + 15k + 17k = 1$$

$$81k = 1 \quad \boxed{k = \frac{1}{81}}$$

(ii) $P(x<4)$

$$P(x<4) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$$

$$= k + 3k + 5k + 7k$$

$$= 16k$$

$$= 16\left(\frac{1}{81}\right) \quad \left[\because k = \frac{1}{81}\right]$$

$$P(x<4) = \frac{16}{81}$$

$$P(0<x<4) = P(x=1) + P(x=2) + P(x=3)$$

$$= 3k + 5k + 7k$$

$$= 15k$$

$$= \frac{15}{81} \quad \left(\because k = \frac{1}{81}\right)$$

$$= \frac{5}{27}$$

(iii) Smallest value of x for which $P(x \leq k) > 0.5$

$$P(x \leq 0) = P(x=0) = \frac{1}{81} = 0.012$$

$$P(x \leq 1) = P(x=0) + P(x=1) = \frac{1}{81} + \frac{3}{81}$$

$$= \frac{4}{81} = \underline{0.49}$$

$$P(x \leq 2) = P(x=0) + P(x=1) + P(x=2)$$

$$= \frac{1}{81} + \frac{3}{81} + \frac{5}{81}$$

$$= \frac{9}{81} = 0.111$$

$$P(x \leq 3) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$$

$$= \frac{1}{81} + \frac{3}{81} + \frac{5}{81} + \frac{7}{81}$$

$$= \frac{16}{81} = 0.197$$

$$P(x \leq 4) = P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4)$$

$$= \frac{1}{81} + \frac{3}{81} + \frac{5}{81} + \frac{7}{81} + \frac{9}{81} = \cancel{0.308}$$

$$= \frac{25}{81}$$

$$= 0.308$$

$P(x \leq 5) = P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4)$
$+ P(x=5)$

$= \frac{1}{81} + \frac{3}{81} + \frac{5}{81} + \frac{7}{81} + \frac{9}{81} + \frac{11}{81}$

$= \frac{36}{81} = \underline{0.444}$

$P(x \leq 6) = P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4)$
$+ P(x=5) + P(x=6)$

$= \frac{1}{81} + \frac{3}{81} + \frac{5}{81} + \frac{7}{81} + \frac{9}{81} + \frac{11}{81} + \frac{13}{81}$

$= 0.\underline{604}$

$\therefore$ The min. value of x for which $P(x \leq k) > 0.5$ is $x = 6$.

Q5) In a city A, out of 600 men, 325 men were found to be smokers. Does this information support the statement "Majority of men in city are smokers"?

Ans) Let us check or develop our Null and Alternative hypotheses:

Null hypothesis is that portion of smokers in the whole city is 50%, ie $50/100 = 0.5$

$H_0 : P = 0.5$

we are interested to see if the proportion of smoker is more than 50%. i.e alternative hypothesis is

$H_1 : P > 0.05$

The portion of smokers in sample is 325 out n=60

Using the sample portion,

Observed value ~~(P_0) = 0.5~~ (p) = $325/600 = 0.542$

If the null hypothesis H_0 is true,

Expected value $(P_0) = 0.5$

and S.E of $p = \sqrt{\{0.5 * (1-0.5)/600\}} = 0.0204$

The test statistic is:

$z =$ (Observed Value − Expected Value)/SE

$= (0.542 - 0.5)/0.0204$

$= \underline{2.1}$

Since, the alternative hypothesis $H_1 : P > 0.5$ is one-sided, the critical region of test is one-tailed.

At 5% level of significance

Critical Region is $Z \geq 1.645$

The value of test statistic z i.e. 2.1 lies in the critical region, and hence is significant.

$\therefore$ We reject the null hypothesis that majority of men in the city are smokers.

Q6) Foll. are the details of Sales (in lakhs) done by A, B & C. Test for significant ~~of~~ diff. in their performance.

	Sales Man	Monthly Sales				$\bar{x}$
m	A	48	49	50	49	49
	B	47	49	48	48	48
	C	49	51	50	50	50
		n				

$\bar{x} \rightarrow$ Mean of each Sales man

$m \times n = 12$

$$\text{grand mean} = \frac{48+49+50+49+47+49+48+48+49+51+50+50}{12}$$

$= 49$

$$SST = (48-49)^2 + (49-49)^2 + (50-49)^2 + (49-49)^2 + (47-49)^2 + (49-49)^2 + (45-49)^2 + (48-49)^2 + (49-49)^2 + (51-48)^2 + (50-49)^2 + (50-49)^2$$

$= 1+0+1+0+4+0+1+1+0+4+1+1$

$= 14$ — 1

Degrees of freedom $= m.n-1$

$= 12-1$

$= 11$ — 2

$$SSW = (48-49)^2 + (49-49)^2 + (50-49)^2 + (49-49)^2 + (47-48)^2 + (49-48)^2 + (48-48)^2 + (48-48)^2 + (49-50)^2 + (51-50)^2 + (50-50)^2 + (50-50)^2$$

$= 1+0+1+0+1+1+0+0+1+1+0+0$
$= 6 \quad — \quad (3)$

Degrees of freedom $= m(n-1)$
$= 3(3)$
$= 9 \quad — \quad (4)$

$SSB = (49-49)^2 + (49-49)^2 + (49-49)^2 + (49-49)^2 +$
$(48-49)^2 + (48-49)^2 + (48-49)^2 + (48-49)^2 +$
$(50-49)^2 + (50-49)^2 + (50-49)^2 + (50-49)^2$

$= 0 + 4 + 4 = 8 \quad — \quad (5)$

Degrees of freedom $= m-1$
$= 3-1$
$= 2 \quad — \quad (6)$

- As per relation,
$SSW + SSB = SST$
Substituting (1), (3), (5) in above eqn

$6 + 8 = 14$

$df\,SSW + df\,SSB = df\,SST$
sub. (2), (4), (6)
$9 + 2 = 11$

i.e $m-1+m(n-1)$ [df . SSW + df SSB]
$= m-1+mn-m$
$= mn-1 \Rightarrow$ df . SST

Null hypothesis:
H_0 : There is no significant diff. in performance
~~$\mu_1 = \mu_2 = \mu_3$~~ i.e. $pr_1 = pr_2 = pr_3$

Alternate hypothesis:
H_1 : There is significant diff. in performance

Assume H_0

$$F\text{-Statistic} = \frac{\frac{SSB}{m-1}}{\frac{SSW}{m(n-1)}}$$

$$= \frac{8/2}{6/9} = 4 \times 3/2 = \underline{\underline{6}}$$

Let's take significant level of 10%.
$\alpha = 0.10$
$F_c = 3.00645$ (F from distribution table)

We observe that $F_c < F$ - statistic
Hence we can reject Null hypothesis.
There is a significant diff. in performance.