

21 REINFORCEMENT LEARNING

In which we examine how an agent can learn from success and failure, from reward and punishment.

21.1 INTRODUCTION

Chapters 18, 19, and 20 covered methods that learn functions, logical theories, and probability models from examples. In this chapter, we will study how agents can learn *what to do* in the absence of labeled examples of what to do.



REINFORCEMENT

Consider, for example, the problem of learning to play chess. A supervised learning agent needs to be told the correct move for each position it encounters, but such feedback is seldom available. In the absence of feedback from a teacher, an agent can learn a transition model for its own moves and can perhaps learn to predict the opponent's moves, but *without some feedback about what is good and what is bad, the agent will have no grounds for deciding which move to make*. The agent needs to know that something good has happened when it (accidentally) checkmates the opponent, and that something bad has happened when it is checkmated—or vice versa, if the game is suicide chess. This kind of feedback is called a **reward**, or **reinforcement**. In games like chess, the reinforcement is received only at the end of the game. In other environments, the rewards come more frequently. In ping-pong, each point scored can be considered a reward; when learning to crawl, any forward motion is an achievement. Our framework for agents regards the reward as *part* of the input percept, but the agent must be “hardwired” to recognize that part as a reward rather than as just another sensory input. Thus, animals seem to be hardwired to recognize pain and hunger as negative rewards and pleasure and food intake as positive rewards. Reinforcement has been carefully studied by animal psychologists for over 60 years.

Rewards were introduced in Chapter 17, where they served to define optimal policies in **Markov decision processes** (MDPs). An optimal policy is a policy that maximizes the expected total reward. The task of **reinforcement learning** is to use observed rewards to learn an optimal (or nearly optimal) policy for the environment. Whereas in Chapter 17 the agent has a complete model of the environment and knows the reward function, here we assume no

prior knowledge of either. Imagine playing a new game whose rules you don't know; after a hundred or so moves, your opponent announces, "You lose." This is reinforcement learning in a nutshell.

In many complex domains, reinforcement learning is the only feasible way to train a program to perform at high levels. For example, in game playing, it is very hard for a human to provide accurate and consistent evaluations of large numbers of positions, which would be needed to train an evaluation function directly from examples. Instead, the program can be told when it has won or lost, and it can use this information to learn an evaluation function that gives reasonably accurate estimates of the probability of winning from any given position. Similarly, it is extremely difficult to program an agent to fly a helicopter; yet given appropriate negative rewards for crashing, wobbling, or deviating from a set course, an agent can learn to fly by itself.

Reinforcement learning might be considered to encompass all of AI: an agent is placed in an environment and must learn to behave successfully therein. To keep the chapter manageable, we will concentrate on simple environments and simple agent designs. For the most part, we will assume a fully observable environment, so that the current state is supplied by each percept. On the other hand, we will assume that the agent does not know how the environment works or what its actions do, and we will allow for probabilistic action outcomes. Thus, the agent faces an unknown Markov decision process. We will consider three of the agent designs first introduced in Chapter 2:

- A **utility-based agent** learns a utility function on states and uses it to select actions that maximize the expected outcome utility.
- A **Q-learning** agent learns an **action-utility function**, or **Q-function**, giving the expected utility of taking a given action in a given state.
- A **reflex agent** learns a policy that maps directly from states to actions.

A utility-based agent must also have a model of the environment in order to make decisions, because it must know the states to which its actions will lead. For example, in order to make use of a backgammon evaluation function, a backgammon program must know what its legal moves are *and how they affect the board position*. Only in this way can it apply the utility function to the outcome states. A Q-learning agent, on the other hand, can compare the expected utilities for its available choices without needing to know their outcomes, so it does not need a model of the environment. On the other hand, because they do not know where their actions lead, Q-learning agents cannot look ahead; this can seriously restrict their ability to learn, as we shall see.

Q-LEARNING
Q-FUNCTION

PASSIVE LEARNING

ACTIVE LEARNING
EXPLORATION

We begin in Section 21.2 with **passive learning**, where the agent's policy is fixed and the task is to learn the utilities of states (or state-action pairs); this could also involve learning a model of the environment. Section 21.3 covers **active learning**, where the agent must also learn what to do. The principal issue is **exploration**: an agent must experience as much as possible of its environment in order to learn how to behave in it. Section 21.4 discusses how an agent can use inductive learning to learn much faster from its experiences. Section 21.5 covers methods for learning direct policy representations in reflex agents. An understanding of Markov decision processes (Chapter 17) is essential for this chapter.

21.2 PASSIVE REINFORCEMENT LEARNING

To keep things simple, we start with the case of a passive learning agent using a state-based representation in a fully observable environment. In passive learning, the agent's policy π is fixed: in state s , it always executes the action $\pi(s)$. Its goal is simply to learn how good the policy is—that is, to learn the utility function $U^\pi(s)$. We will use as our example the 4×3 world introduced in Chapter 17. Figure 21.1 shows a policy for that world and the corresponding utilities. Clearly, the passive learning task is similar to the **policy evaluation** task, part of the **policy iteration** algorithm described in Section 17.3. The main difference is that the passive learning agent does not know the **transition model** $P(s' | s, a)$, which specifies the probability of reaching state s' from state s after doing action a ; nor does it know the **reward function** $R(s)$, which specifies the reward for each state.

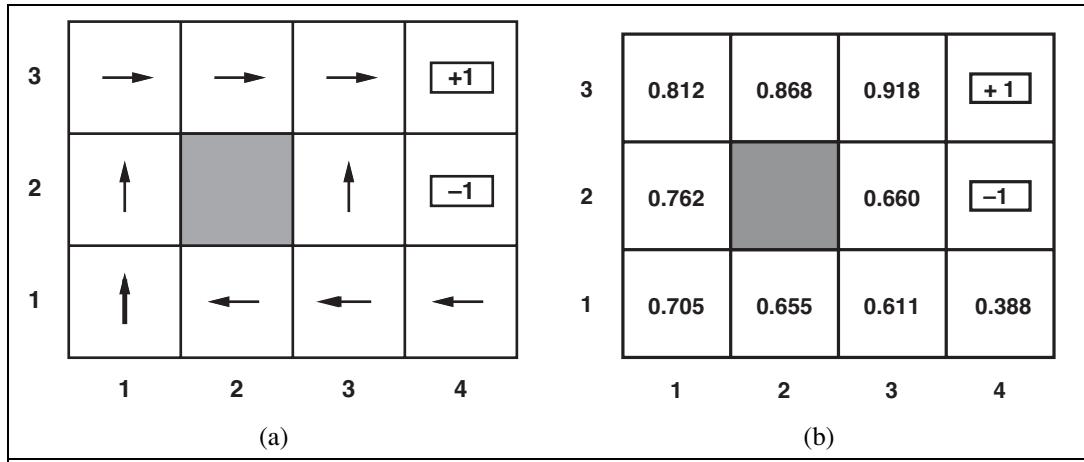


Figure 21.1 (a) A policy π for the 4×3 world; this policy happens to be optimal with rewards of $R(s) = -0.04$ in the nonterminal states and no discounting. (b) The utilities of the states in the 4×3 world, given policy π .

TRIAL

The agent executes a set of **trials** in the environment using its policy π . In each trial, the agent starts in state $(1,1)$ and experiences a sequence of state transitions until it reaches one of the terminal states, $(4,2)$ or $(4,3)$. Its percepts supply both the current state and the reward received in that state. Typical trials might look like this:

$$\begin{aligned}
 &(1, 1).-04 \rightsquigarrow (1, 2).-04 \rightsquigarrow (1, 3).-04 \rightsquigarrow (1, 2).-04 \rightsquigarrow (1, 3).-04 \rightsquigarrow (2, 3).-04 \rightsquigarrow (3, 3).-04 \rightsquigarrow (4, 3)+1 \\
 &(1, 1).-04 \rightsquigarrow (1, 2).-04 \rightsquigarrow (1, 3).-04 \rightsquigarrow (2, 3).-04 \rightsquigarrow (3, 3).-04 \rightsquigarrow (3, 2).-04 \rightsquigarrow (3, 3).-04 \rightsquigarrow (4, 3)+1 \\
 &(1, 1).-04 \rightsquigarrow (2, 1).-04 \rightsquigarrow (3, 1).-04 \rightsquigarrow (3, 2).-04 \rightsquigarrow (4, 2).-1 .
 \end{aligned}$$

Note that each state percept is subscripted with the reward received. The object is to use the information about rewards to learn the expected utility $U^\pi(s)$ associated with each nonterminal state s . The utility is defined to be the expected sum of (discounted) rewards obtained if

policy π is followed. As in Equation (17.2) on page 650, we write

$$U^\pi(s) = E \left[\sum_{t=0}^{\infty} \gamma^t R(S_t) \right] \quad (21.1)$$

where $R(s)$ is the reward for a state, S_t (a random variable) is the state reached at time t when executing policy π , and $S_0 = s$. We will include a **discount factor** γ in all of our equations, but for the 4×3 world we will set $\gamma = 1$.

21.2.1 Direct utility estimation

DIRECT UTILITY
ESTIMATION
ADAPTIVE CONTROL
THEORY
REWARD-TO-GO

A simple method for **direct utility estimation** was invented in the late 1950s in the area of **adaptive control theory** by Widrow and Hoff (1960). The idea is that the utility of a state is the expected total reward from that state onward (called the expected **reward-to-go**), and each trial provides a *sample* of this quantity for each state visited. For example, the first trial in the set of three given earlier provides a sample total reward of 0.72 for state (1,1), two samples of 0.76 and 0.84 for (1,2), two samples of 0.80 and 0.88 for (1,3), and so on. Thus, at the end of each sequence, the algorithm calculates the observed reward-to-go for each state and updates the estimated utility for that state accordingly, just by keeping a running average for each state in a table. In the limit of infinitely many trials, the sample average will converge to the true expectation in Equation (21.1).

It is clear that direct utility estimation is just an instance of supervised learning where each example has the state as input and the observed reward-to-go as output. This means that we have reduced reinforcement learning to a standard inductive learning problem, as discussed in Chapter 18. Section 21.4 discusses the use of more powerful kinds of representations for the utility function. Learning techniques for those representations can be applied directly to the observed data.



Direct utility estimation succeeds in reducing the reinforcement learning problem to an inductive learning problem, about which much is known. Unfortunately, it misses a very important source of information, namely, the fact that the utilities of states are not independent! *The utility of each state equals its own reward plus the expected utility of its successor states.* That is, the utility values obey the Bellman equations for a fixed policy (see also Equation (17.10)):

$$U^\pi(s) = R(s) + \gamma \sum_{s'} P(s' | s, \pi(s)) U^\pi(s') . \quad (21.2)$$

By ignoring the connections between states, direct utility estimation misses opportunities for learning. For example, the second of the three trials given earlier reaches the state (3,2), which has not previously been visited. The next transition reaches (3,3), which is known from the first trial to have a high utility. The Bellman equation suggests immediately that (3,2) is also likely to have a high utility, because it leads to (3,3), but direct utility estimation learns nothing until the end of the trial. More broadly, we can view direct utility estimation as searching for U in a hypothesis space that is much larger than it needs to be, in that it includes many functions that violate the Bellman equations. For this reason, the algorithm often converges very slowly.

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function PASSIVE-ADP-AGENT(percept) returns an action
  inputs: percept, a percept indicating the current state  $s'$  and reward signal  $r'$ 
  persistent:  $\pi$ , a fixed policy
     $mdp$ , an MDP with model  $P$ , rewards  $R$ , discount  $\gamma$ 
     $U$ , a table of utilities, initially empty
     $N_{sa}$ , a table of frequencies for state-action pairs, initially zero
     $N_{s'|sa}$ , a table of outcome frequencies given state-action pairs, initially zero
     $s, a$ , the previous state and action, initially null

  if  $s'$  is new then  $U[s'] \leftarrow r'; R[s'] \leftarrow r'$ 
  if  $s$  is not null then
    increment  $N_{sa}[s, a]$  and  $N_{s'|sa}[s', s, a]$ 
    for each  $t$  such that  $N_{s'|sa}[t, s, a]$  is nonzero do
       $P(t | s, a) \leftarrow N_{s'|sa}[t, s, a] / N_{sa}[s, a]$ 
     $U \leftarrow \text{POLICY-EVALUATION}(\pi, U, mdp)$ 
  if  $s'.\text{TERMINAL?}$  then  $s, a \leftarrow \text{null}$  else  $s, a \leftarrow s', \pi[s']$ 
  return  $a$ 

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Figure 21.2 A passive reinforcement learning agent based on adaptive dynamic programming. The POLICY-EVALUATION function solves the fixed-policy Bellman equations, as described on page 657.

21.2.2 Adaptive dynamic programming

ADAPTIVE DYNAMIC
PROGRAMMING

An **adaptive dynamic programming** (or ADP) agent takes advantage of the constraints among the utilities of states by learning the transition model that connects them and solving the corresponding Markov decision process using a dynamic programming method. For a passive learning agent, this means plugging the learned transition model $P(s' | s, \pi(s))$ and the observed rewards $R(s)$ into the Bellman equations (21.2) to calculate the utilities of the states. As we remarked in our discussion of policy iteration in Chapter 17, these equations are linear (no maximization involved) so they can be solved using any linear algebra package. Alternatively, we can adopt the approach of **modified policy iteration** (see page 657), using a simplified value iteration process to update the utility estimates after each change to the learned model. Because the model usually changes only slightly with each observation, the value iteration process can use the previous utility estimates as initial values and should converge quite quickly.

The process of learning the model itself is easy, because the environment is fully observable. This means that we have a supervised learning task where the input is a state-action pair and the output is the resulting state. In the simplest case, we can represent the transition model as a table of probabilities. We keep track of how often each action outcome occurs and estimate the transition probability $P(s' | s, a)$ from the frequency with which s' is reached when executing a in s . For example, in the three trials given on page 832, *Right* is executed three times in (1,3) and two out of three times the resulting state is (2,3), so $P((2, 3) | (1, 3), Right)$ is estimated to be 2/3.

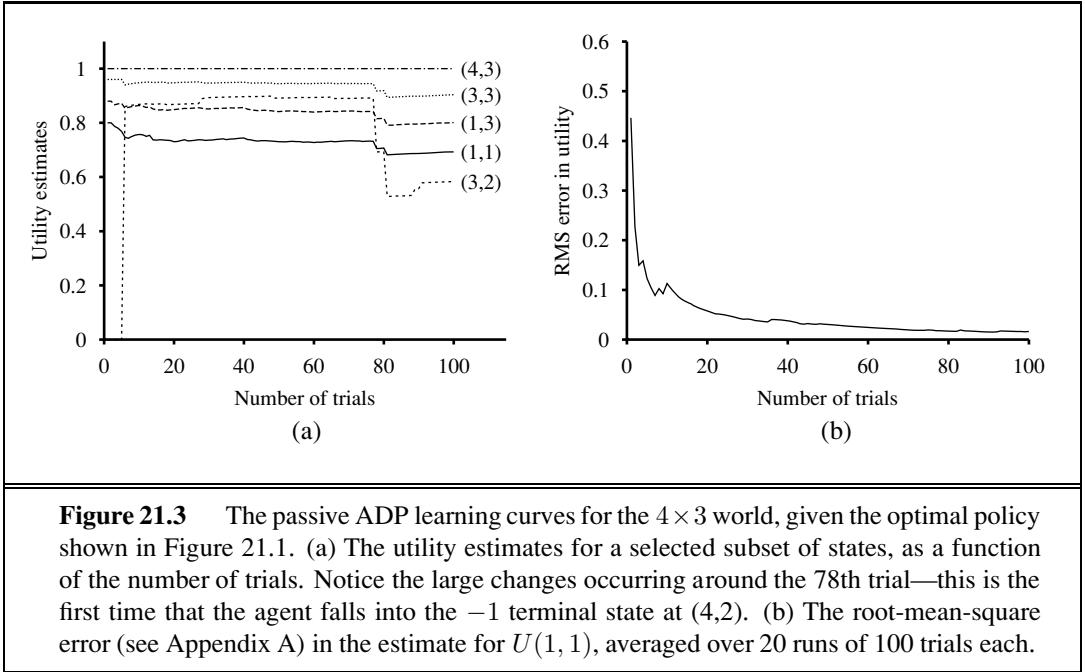


Figure 21.3 The passive ADP learning curves for the 4×3 world, given the optimal policy shown in Figure 21.1. (a) The utility estimates for a selected subset of states, as a function of the number of trials. Notice the large changes occurring around the 78th trial—this is the first time that the agent falls into the -1 terminal state at $(4,2)$. (b) The root-mean-square error (see Appendix A) in the estimate for $U(1, 1)$, averaged over 20 runs of 100 trials each.

The full agent program for a passive ADP agent is shown in Figure 21.2. Its performance on the 4×3 world is shown in Figure 21.3. In terms of how quickly its value estimates improve, the ADP agent is limited only by its ability to learn the transition model. In this sense, it provides a standard against which to measure other reinforcement learning algorithms. It is, however, intractable for large state spaces. In backgammon, for example, it would involve solving roughly 10^{50} equations in 10^{50} unknowns.

A reader familiar with the Bayesian learning ideas of Chapter 20 will have noticed that the algorithm in Figure 21.2 is using maximum-likelihood estimation to learn the transition model; moreover, by choosing a policy based solely on the *estimated* model it is acting *as if* the model were correct. This is not necessarily a good idea! For example, a taxi agent that didn't know about how traffic lights might ignore a red light once or twice without no ill effects and then formulate a policy to ignore red lights from then on. Instead, it might be a good idea to choose a policy that, while not optimal for the model estimated by maximum likelihood, works reasonably well for the whole range of models that have a reasonable chance of being the true model. There are two mathematical approaches that have this flavor.

The first approach, **Bayesian reinforcement learning**, assumes a prior probability $P(h)$ for each hypothesis h about what the true model is; the posterior probability $P(h | \mathbf{e})$ is obtained in the usual way by Bayes' rule given the observations to date. Then, if the agent has decided to stop learning, the optimal policy is the one that gives the highest expected utility. Let u_h^π be the expected utility, averaged over all possible start states, obtained by executing policy π in model h . Then we have

$$\pi^* = \operatorname{argmax}_\pi \sum_h P(h | \mathbf{e}) u_h^\pi.$$

In some special cases, this policy can even be computed! If the agent will continue learning in the future, however, then finding an optimal policy becomes considerably more difficult, because the agent must consider the effects of future observations on its beliefs about the transition model. The problem becomes a POMDP whose belief states are distributions over models. This concept provides an analytical foundation for understanding the exploration problem described in Section 21.3.

ROBUST CONTROL THEORY

The second approach, derived from **robust control theory**, allows for a *set* of possible models \mathcal{H} and defines an optimal robust policy as one that gives the best outcome in the *worst case* over \mathcal{H} :

$$\pi^* = \operatorname{argmax}_{\pi} \min_h u_h^\pi .$$

Often, the set \mathcal{H} will be the set of models that exceed some likelihood threshold on $P(h | \mathbf{e})$, so the robust and Bayesian approaches are related. Sometimes, the robust solution can be computed efficiently. There are, moreover, reinforcement learning algorithms that tend to produce robust solutions, although we do not cover them here.

21.2.3 Temporal-difference learning

Solving the underlying MDP as in the preceding section is not the only way to bring the Bellman equations to bear on the learning problem. Another way is to use the observed transitions to adjust the utilities of the observed states so that they agree with the constraint equations. Consider, for example, the transition from (1,3) to (2,3) in the second trial on page 832. Suppose that, as a result of the first trial, the utility estimates are $U^\pi(1, 3) = 0.84$ and $U^\pi(2, 3) = 0.92$. Now, if this transition occurred all the time, we would expect the utilities to obey the equation

$$U^\pi(1, 3) = -0.04 + U^\pi(2, 3) ,$$

so $U^\pi(1, 3)$ would be 0.88. Thus, its current estimate of 0.84 might be a little low and should be increased. More generally, when a transition occurs from state s to state s' , we apply the following update to $U^\pi(s)$:

$$U^\pi(s) \leftarrow U^\pi(s) + \alpha(R(s) + \gamma U^\pi(s') - U^\pi(s)) . \quad (21.3)$$

TEMPORAL-DIFFERENCE

Here, α is the **learning rate** parameter. Because this update rule uses the difference in utilities between successive states, it is often called the **temporal-difference**, or TD, equation.

All temporal-difference methods work by adjusting the utility estimates towards the ideal equilibrium that holds locally when the utility estimates are correct. In the case of passive learning, the equilibrium is given by Equation (21.2). Now Equation (21.3) does in fact cause the agent to reach the equilibrium given by Equation (21.2), but there is some subtlety involved. First, notice that the update involves only the observed successor s' , whereas the actual equilibrium conditions involve all possible next states. One might think that this causes an improperly large change in $U^\pi(s)$ when a very rare transition occurs; but, in fact, because rare transitions occur only rarely, the *average value* of $U^\pi(s)$ will converge to the correct value. Furthermore, if we change α from a fixed parameter to a function that decreases as the number of times a state has been visited increases, then $U^\pi(s)$ itself will converge to the

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function PASSIVE-TD-AGENT(percept) returns an action
  inputs: percept, a percept indicating the current state  $s'$  and reward signal  $r'$ 
  persistent:  $\pi$ , a fixed policy
     $U$ , a table of utilities, initially empty
     $N_s$ , a table of frequencies for states, initially zero
     $s, a, r$ , the previous state, action, and reward, initially null

  if  $s'$  is new then  $U[s'] \leftarrow r'$ 
  if  $s$  is not null then
    increment  $N_s[s]$ 
     $U[s] \leftarrow U[s] + \alpha(N_s[s])(r + \gamma U[s'] - U[s])$ 
  if  $s'.\text{TERMINAL?}$  then  $s, a, r \leftarrow \text{null}$  else  $s, a, r \leftarrow s', \pi[s'], r'$ 
  return  $a$ 

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Figure 21.4 A passive reinforcement learning agent that learns utility estimates using temporal differences. The step-size function $\alpha(n)$ is chosen to ensure convergence, as described in the text.

correct value.¹ This gives us the agent program shown in Figure 21.4. Figure 21.5 illustrates the performance of the passive TD agent on the 4×3 world. It does not learn quite as fast as the ADP agent and shows much higher variability, but it is much simpler and requires much less computation per observation. Notice that *TD does not need a transition model to perform its updates*. The environment supplies the connection between neighboring states in the form of observed transitions.



The ADP approach and the TD approach are actually closely related. Both try to make local adjustments to the utility estimates in order to make each state “agree” with its successors. One difference is that TD adjusts a state to agree with its *observed* successor (Equation (21.3)), whereas ADP adjusts the state to agree with *all* of the successors that might occur, weighted by their probabilities (Equation (21.2)). This difference disappears when the effects of TD adjustments are averaged over a large number of transitions, because the frequency of each successor in the set of transitions is approximately proportional to its probability. A more important difference is that whereas TD makes a single adjustment per observed transition, ADP makes as many as it needs to restore consistency between the utility estimates U and the environment model P . Although the observed transition makes only a local change in P , its effects might need to be propagated throughout U . Thus, TD can be viewed as a crude but efficient first approximation to ADP.

Each adjustment made by ADP could be seen, from the TD point of view, as a result of a “pseudoexperience” generated by simulating the current environment model. It is possible to extend the TD approach to use an environment model to generate several pseudoexperiences—transitions that the TD agent can imagine *might* happen, given its current model. For each observed transition, the TD agent can generate a large number of imaginary

¹ The technical conditions are given on page 725. In Figure 21.5 we have used $\alpha(n) = 60/(59 + n)$, which satisfies the conditions.

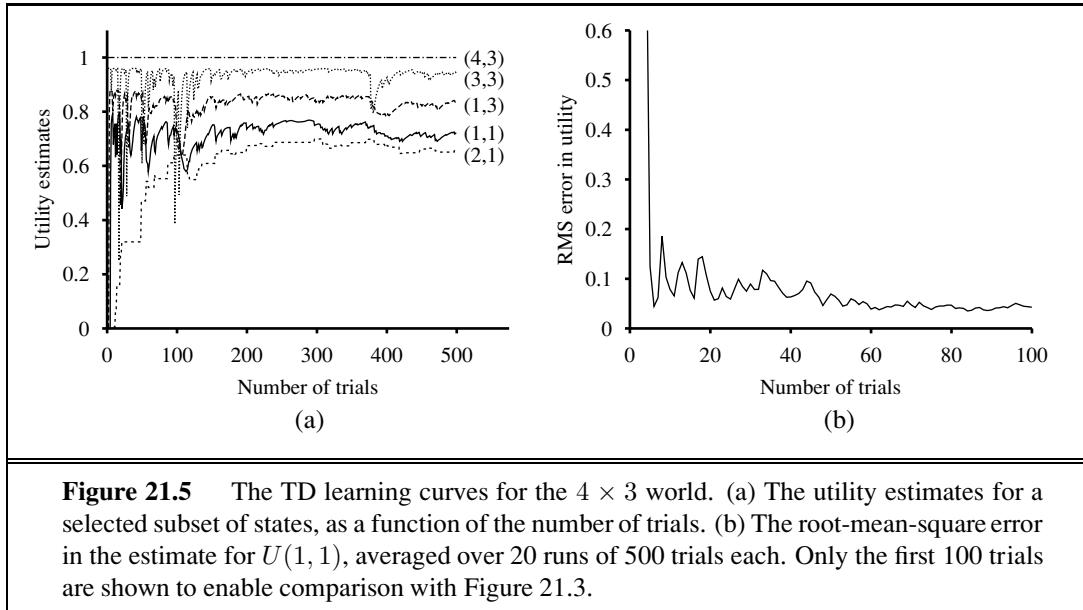


Figure 21.5 The TD learning curves for the 4×3 world. (a) The utility estimates for a selected subset of states, as a function of the number of trials. (b) The root-mean-square error in the estimate for $U(1, 1)$, averaged over 20 runs of 500 trials each. Only the first 100 trials are shown to enable comparison with Figure 21.3.

transitions. In this way, the resulting utility estimates will approximate more and more closely those of ADP—of course, at the expense of increased computation time.

In a similar vein, we can generate more efficient versions of ADP by directly approximating the algorithms for value iteration or policy iteration. Even though the value iteration algorithm is efficient, it is intractable if we have, say, 10^{100} states. However, many of the necessary adjustments to the state values on each iteration will be extremely tiny. One possible approach to generating reasonably good answers quickly is to bound the number of adjustments made after each observed transition. One can also use a heuristic to rank the possible adjustments so as to carry out only the most significant ones. The **prioritized sweeping** heuristic prefers to make adjustments to states whose *likely* successors have just undergone a *large* adjustment in their own utility estimates. Using heuristics like this, approximate ADP algorithms usually can learn roughly as fast as full ADP, in terms of the number of training sequences, but can be several orders of magnitude more efficient in terms of computation. (See Exercise 21.3.) This enables them to handle state spaces that are far too large for full ADP. Approximate ADP algorithms have an additional advantage: in the early stages of learning a new environment, the environment model P often will be far from correct, so there is little point in calculating an exact utility function to match it. An approximation algorithm can use a minimum adjustment size that decreases as the environment model becomes more accurate. This eliminates the very long value iterations that can occur early in learning due to large changes in the model.

21.3 ACTIVE REINFORCEMENT LEARNING

A passive learning agent has a fixed policy that determines its behavior. An active agent must decide what actions to take. Let us begin with the adaptive dynamic programming agent and consider how it must be modified to handle this new freedom.

First, the agent will need to learn a complete model with outcome probabilities for all actions, rather than just the model for the fixed policy. The simple learning mechanism used by PASSIVE-ADP-AGENT will do just fine for this. Next, we need to take into account the fact that the agent has a choice of actions. The utilities it needs to learn are those defined by the *optimal* policy; they obey the Bellman equations given on page 652, which we repeat here for convenience:

$$U(s) = R(s) + \gamma \max_a \sum_{s'} P(s' | s, a) U(s') . \quad (21.4)$$

These equations can be solved to obtain the utility function U using the value iteration or policy iteration algorithms from Chapter 17. The final issue is what to do at each step. Having obtained a utility function U that is optimal for the learned model, the agent can extract an optimal action by one-step look-ahead to maximize the expected utility; alternatively, if it uses policy iteration, the optimal policy is already available, so it should simply execute the action the optimal policy recommends. Or should it?

21.3.1 Exploration

Figure 21.6 shows the results of one sequence of trials for an ADP agent that follows the recommendation of the optimal policy for the learned model at each step. The agent *does not* learn the true utilities or the true optimal policy! What happens instead is that, in the 39th trial, it finds a policy that reaches the +1 reward along the lower route via (2,1), (3,1), (3,2), and (3,3). (See Figure 21.6(b).) After experimenting with minor variations, from the 276th trial onward it sticks to that policy, never learning the utilities of the other states and never finding the optimal route via (1,2), (1,3), and (2,3). We call this agent the **greedy agent**. Repeated experiments show that the greedy agent *very seldom* converges to the optimal policy for this environment and sometimes converges to really horrendous policies.

GREEDY AGENT

How can it be that choosing the optimal action leads to suboptimal results? The answer is that the learned model is not the same as the true environment; what is optimal in the learned model can therefore be suboptimal in the true environment. Unfortunately, the agent does not know what the true environment is, so it cannot compute the optimal action for the true environment. What, then, is to be done?

EXPLOITATION
EXPLORATION

What the greedy agent has overlooked is that actions do more than provide rewards according to the current learned model; they also contribute to learning the true model by affecting the percepts that are received. By improving the model, the agent will receive greater rewards in the future.² An agent therefore must make a tradeoff between **exploitation** to maximize its reward—as reflected in its current utility estimates—and **exploration** to maxi-

² Notice the direct analogy to the theory of information value in Chapter 16.

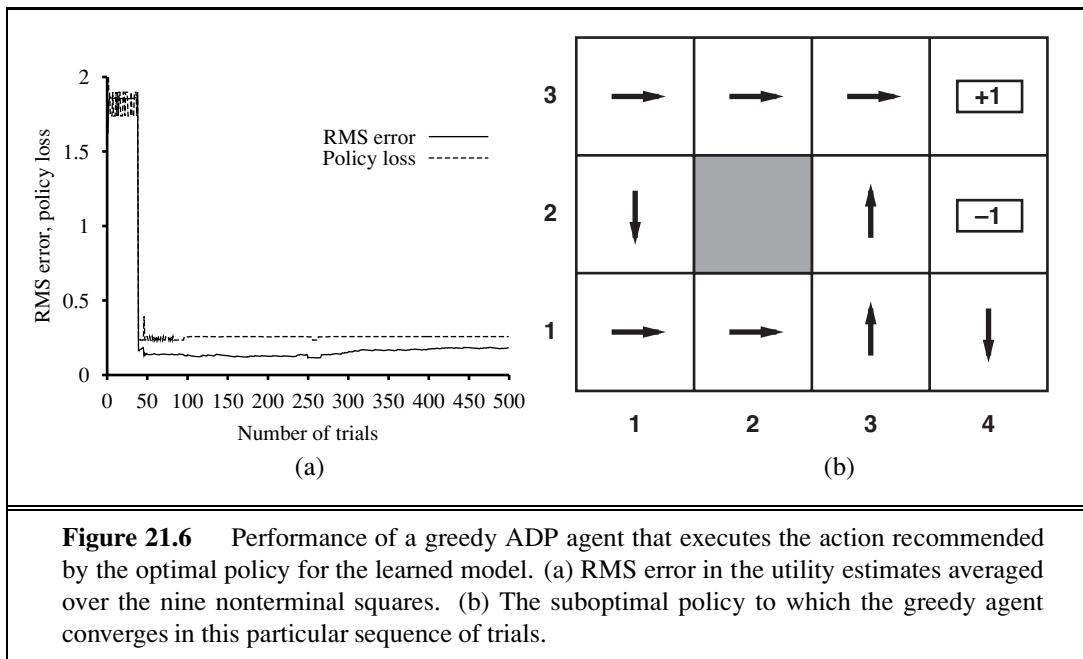


Figure 21.6 Performance of a greedy ADP agent that executes the action recommended by the optimal policy for the learned model. (a) RMS error in the utility estimates averaged over the nine nonterminal squares. (b) The suboptimal policy to which the greedy agent converges in this particular sequence of trials.

mize its long-term well-being. Pure exploitation risks getting stuck in a rut. Pure exploration to improve one's knowledge is of no use if one never puts that knowledge into practice. In the real world, one constantly has to decide between continuing in a comfortable existence and striking out into the unknown in the hopes of discovering a new and better life. With greater understanding, less exploration is necessary.

Can we be a little more precise than this? Is there an *optimal* exploration policy? This question has been studied in depth in the subfield of statistical decision theory that deals with so-called **bandit problems**. (See sidebar.)

BANDIT PROBLEM

GLIE

Although bandit problems are extremely difficult to solve exactly to obtain an *optimal* exploration method, it is nonetheless possible to come up with a *reasonable* scheme that will eventually lead to optimal behavior by the agent. Technically, any such scheme needs to be greedy in the limit of infinite exploration, or **GLIE**. A GLIE scheme must try each action in each state an unbounded number of times to avoid having a finite probability that an optimal action is missed because of an unusually bad series of outcomes. An ADP agent using such a scheme will eventually learn the true environment model. A GLIE scheme must also eventually become greedy, so that the agent's actions become optimal with respect to the learned (and hence the true) model.

There are several GLIE schemes; one of the simplest is to have the agent choose a random action a fraction $1/t$ of the time and to follow the greedy policy otherwise. While this does eventually converge to an optimal policy, it can be extremely slow. A more sensible approach would give some weight to actions that the agent has not tried very often, while tending to avoid actions that are believed to be of low utility. This can be implemented by altering the constraint equation (21.4) so that it assigns a higher utility estimate to relatively

EXPLORATION AND BANDITS

In Las Vegas, a *one-armed bandit* is a slot machine. A gambler can insert a coin, pull the lever, and collect the winnings (if any). An *n-armed bandit* has n levers. The gambler must choose which lever to play on each successive coin—the one that has paid off best, or maybe one that has not been tried?

The *n*-armed bandit problem is a formal model for real problems in many vitally important areas, such as deciding on the annual budget for AI research and development. Each arm corresponds to an action (such as allocating \$20 million for the development of new AI textbooks), and the payoff from pulling the arm corresponds to the benefits obtained from taking the action (immense). Exploration, whether it is exploration of a new research field or exploration of a new shopping mall, is risky, is expensive, and has uncertain payoffs; on the other hand, failure to explore at all means that one never discovers *any* actions that are worthwhile.

To formulate a bandit problem properly, one must define exactly what is meant by optimal behavior. Most definitions in the literature assume that the aim is to maximize the expected total reward obtained over the agent’s lifetime. These definitions require that the expectation be taken over the possible worlds that the agent could be in, as well as over the possible results of each action sequence in any given world. Here, a “world” is defined by the transition model $P(s' | s, a)$. Thus, in order to act optimally, the agent needs a prior distribution over the possible models. The resulting optimization problems are usually wildly intractable.

In some cases—for example, when the payoff of each machine is independent and discounted rewards are used—it is possible to calculate a **Gittins index** for each slot machine (Gittins, 1989). The index is a function only of the number of times the slot machine has been played and how much it has paid off. The index for each machine indicates how worthwhile it is to invest more; generally speaking, the higher the expected return and the higher the uncertainty in the utility of a given choice, the better. Choosing the machine with the highest index value gives an optimal exploration policy. Unfortunately, no way has been found to extend Gittins indices to sequential decision problems.

One can use the theory of *n*-armed bandits to argue for the reasonableness of the selection strategy in genetic algorithms. (See Chapter 4.) If you consider each arm in an *n*-armed bandit problem to be a possible string of genes, and the investment of a coin in one arm to be the reproduction of those genes, then it can be proven that genetic algorithms allocate coins optimally, given an appropriate set of independence assumptions.

unexplored state–action pairs. Essentially, this amounts to an optimistic prior over the possible environments and causes the agent to behave initially as if there were wonderful rewards scattered all over the place. Let us use $U^+(s)$ to denote the optimistic estimate of the utility (i.e., the expected reward-to-go) of the state s , and let $N(s, a)$ be the number of times action a has been tried in state s . Suppose we are using value iteration in an ADP learning agent; then we need to rewrite the update equation (Equation (17.6) on page 652) to incorporate the optimistic estimate. The following equation does this:

$$U^+(s) \leftarrow R(s) + \gamma \max_a f \left(\sum_{s'} P(s' | s, a) U^+(s'), N(s, a) \right). \quad (21.5)$$

EXPLORATION
FUNCTION

Here, $f(u, n)$ is called the **exploration function**. It determines how greed (preference for high values of u) is traded off against curiosity (preference for actions that have not been tried often and have low n). The function $f(u, n)$ should be increasing in u and decreasing in n . Obviously, there are many possible functions that fit these conditions. One particularly simple definition is

$$f(u, n) = \begin{cases} R^+ & \text{if } n < N_e \\ u & \text{otherwise} \end{cases}$$

where R^+ is an optimistic estimate of the best possible reward obtainable in any state and N_e is a fixed parameter. This will have the effect of making the agent try each action–state pair at least N_e times.

The fact that U^+ rather than U appears on the right-hand side of Equation (21.5) is very important. As exploration proceeds, the states and actions near the start state might well be tried a large number of times. If we used U , the more pessimistic utility estimate, then the agent would soon become disinclined to explore further afield. The use of U^+ means that the benefits of exploration are propagated back from the edges of unexplored regions, so that actions that lead *toward* unexplored regions are weighted more highly, rather than just actions that are themselves unfamiliar. The effect of this exploration policy can be seen clearly in Figure 21.7, which shows a rapid convergence toward optimal performance, unlike that of the greedy approach. A very nearly optimal policy is found after just 18 trials. Notice that the utility estimates themselves do not converge as quickly. This is because the agent stops exploring the unrewarding parts of the state space fairly soon, visiting them only “by accident” thereafter. However, it makes perfect sense for the agent not to care about the exact utilities of states that it knows are undesirable and can be avoided.

21.3.2 Learning an action-utility function

Now that we have an active ADP agent, let us consider how to construct an active temporal-difference learning agent. The most obvious change from the passive case is that the agent is no longer equipped with a fixed policy, so, if it learns a utility function U , it will need to learn a model in order to be able to choose an action based on U via one-step look-ahead. The model acquisition problem for the TD agent is identical to that for the ADP agent. What of the TD update rule itself? Perhaps surprisingly, the update rule (21.3) remains unchanged. This might seem odd, for the following reason: Suppose the agent takes a step that normally

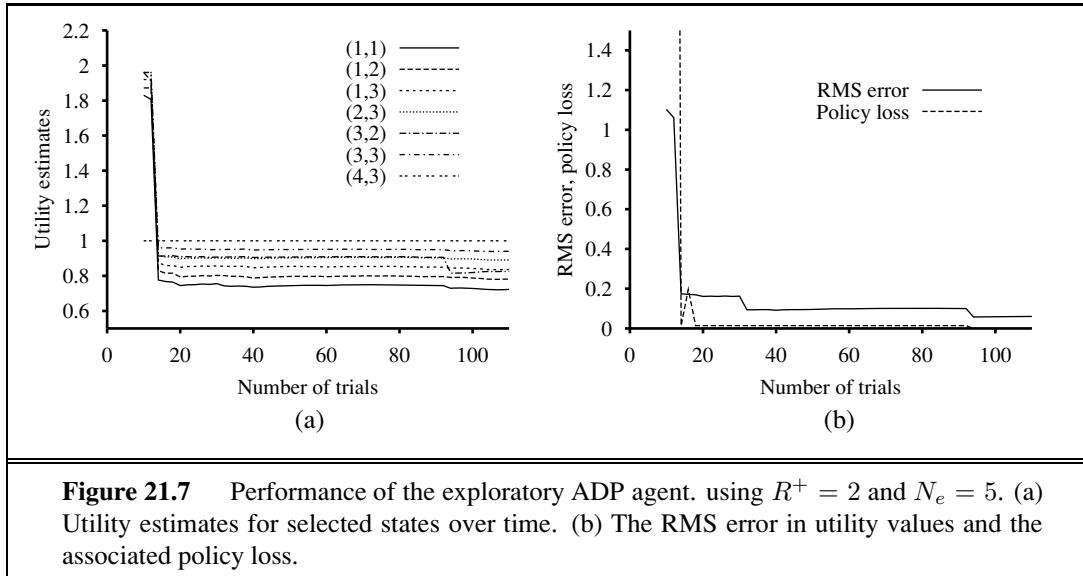


Figure 21.7 Performance of the exploratory ADP agent, using $R^+ = 2$ and $N_e = 5$. (a) Utility estimates for selected states over time. (b) The RMS error in utility values and the associated policy loss.

leads to a good destination, but because of nondeterminism in the environment the agent ends up in a catastrophic state. The TD update rule will take this as seriously as if the outcome had been the normal result of the action, whereas one might suppose that, because the outcome was a fluke, the agent should not worry about it too much. In fact, of course, the unlikely outcome will occur only infrequently in a large set of training sequences; hence in the long run its effects will be weighted proportionally to its probability, as we would hope. Once again, it can be shown that the TD algorithm will converge to the same values as ADP as the number of training sequences tends to infinity.

There is an alternative TD method, called **Q-learning**, which learns an action-utility representation instead of learning utilities. We will use the notation $Q(s, a)$ to denote the value of doing action a in state s . Q-values are directly related to utility values as follows:

$$U(s) = \max_a Q(s, a). \quad (21.6)$$



Q-functions may seem like just another way of storing utility information, but they have a very important property: *a TD agent that learns a Q-function does not need a model of the form $P(s' | s, a)$, either for learning or for action selection*. For this reason, Q-learning is called a **model-free** method. As with utilities, we can write a constraint equation that must hold at equilibrium when the Q-values are correct:

$$Q(s, a) = R(s) + \gamma \sum_{s'} P(s' | s, a) \max_{a'} Q(s', a'). \quad (21.7)$$

As in the ADP learning agent, we can use this equation directly as an update equation for an iteration process that calculates exact Q-values, given an estimated model. This does, however, require that a model also be learned, because the equation uses $P(s' | s, a)$. The temporal-difference approach, on the other hand, requires no model of state transitions—all

```

function Q-LEARNING-AGENT(percept) returns an action
  inputs: percept, a percept indicating the current state  $s'$  and reward signal  $r'$ 
  persistent:  $Q$ , a table of action values indexed by state and action, initially zero
     $N_{sa}$ , a table of frequencies for state-action pairs, initially zero
     $s, a, r$ , the previous state, action, and reward, initially null

  if TERMINAL?( $s$ ) then  $Q[s, \text{None}] \leftarrow r'$ 
  if  $s$  is not null then
    increment  $N_{sa}[s, a]$ 
     $Q[s, a] \leftarrow Q[s, a] + \alpha(N_{sa}[s, a])(r + \gamma \max_{a'} Q[s', a'] - Q[s, a])$ 
     $s, a, r \leftarrow s', \text{argmax}_{a'} f(Q[s', a'], N_{sa}[s', a']), r'$ 
  return  $a$ 

```

Figure 21.8 An exploratory Q-learning agent. It is an active learner that learns the value $Q(s, a)$ of each action in each situation. It uses the same exploration function f as the exploratory ADP agent, but avoids having to learn the transition model because the Q-value of a state can be related directly to those of its neighbors.

it needs are the Q values. The update equation for TD Q-learning is

$$Q(s, a) \leftarrow Q(s, a) + \alpha(R(s) + \gamma \max_{a'} Q(s', a') - Q(s, a)), \quad (21.8)$$

which is calculated whenever action a is executed in state s leading to state s' .

The complete agent design for an exploratory Q-learning agent using TD is shown in Figure 21.8. Notice that it uses exactly the same exploration function f as that used by the exploratory ADP agent—hence the need to keep statistics on actions taken (the table N). If a simpler exploration policy is used—say, acting randomly on some fraction of steps, where the fraction decreases over time—then we can dispense with the statistics.

SARSA

Q-learning has a close relative called **SARSA** (for State-Action-Reward-State-Action). The update rule for SARSA is very similar to Equation (21.8):

$$Q(s, a) \leftarrow Q(s, a) + \alpha(R(s) + \gamma Q(s', a') - Q(s, a)), \quad (21.9)$$

OFF-POLICY
ON-POLICY

where a' is the action *actually taken* in state s' . The rule is applied at the end of each s, a, r, s', a' quintuplet—hence the name. The difference from Q-learning is quite subtle: whereas Q-learning backs up the *best* Q-value from the state reached in the observed transition, SARSA waits until an action is actually taken and backs up the Q-value for that action. Now, for a greedy agent that always takes the action with best Q-value, the two algorithms are identical. When exploration is happening, however, they differ significantly. Because Q-learning uses the best Q-value, it pays no attention to the actual policy being followed—it is an **off-policy** learning algorithm, whereas SARSA is an **on-policy** algorithm. Q-learning is more flexible than SARSA, in the sense that a Q-learning agent can learn how to behave well even when guided by a random or adversarial exploration policy. On the other hand, SARSA is more realistic: for example, if the overall policy is even partly controlled by other agents, it is better to learn a Q-function for what will actually happen rather than what the agent would like to happen.

Both Q-learning and SARSA learn the optimal policy for the 4×3 world, but do so at a much slower rate than the ADP agent. This is because the local updates do not enforce consistency among all the Q-values via the model. The comparison raises a general question: is it better to learn a model and a utility function or to learn an action-utility function with no model? In other words, what is the best way to represent the agent function? This is an issue at the foundations of artificial intelligence. As we stated in Chapter 1, one of the key historical characteristics of much of AI research is its (often unstated) adherence to the **knowledge-based** approach. This amounts to an assumption that the best way to represent the agent function is to build a representation of some aspects of the environment in which the agent is situated.

Some researchers, both inside and outside AI, have claimed that the availability of model-free methods such as Q-learning means that the knowledge-based approach is unnecessary. There is, however, little to go on but intuition. Our intuition, for what it's worth, is that as the environment becomes more complex, the advantages of a knowledge-based approach become more apparent. This is borne out even in games such as chess, checkers (draughts), and backgammon (see next section), where efforts to learn an evaluation function by means of a model have met with more success than Q-learning methods.

21.4 GENERALIZATION IN REINFORCEMENT LEARNING

So far, we have assumed that the utility functions and Q-functions learned by the agents are represented in tabular form with one output value for each input tuple. Such an approach works reasonably well for small state spaces, but the time to convergence and (for ADP) the time per iteration increase rapidly as the space gets larger. With carefully controlled, approximate ADP methods, it might be possible to handle 10,000 states or more. This suffices for two-dimensional maze-like environments, but more realistic worlds are out of the question. Backgammon and chess are tiny subsets of the real world, yet their state spaces contain on the order of 10^{20} and 10^{40} states, respectively. It would be absurd to suppose that one must visit all these states many times in order to learn how to play the game!

FUNCTION APPROXIMATION
BASIS FUNCTION

One way to handle such problems is to use **function approximation**, which simply means using any sort of representation for the Q-function other than a lookup table. The representation is viewed as approximate because it might not be the case that the *true* utility function or Q-function can be represented in the chosen form. For example, in Chapter 5 we described an **evaluation function** for chess that is represented as a weighted linear function of a set of **features** (or **basis functions**) f_1, \dots, f_n :

$$\hat{U}_\theta(s) = \theta_1 f_1(s) + \theta_2 f_2(s) + \dots + \theta_n f_n(s).$$

A reinforcement learning algorithm can learn values for the parameters $\theta = \theta_1, \dots, \theta_n$ such that the evaluation function \hat{U}_θ approximates the true utility function. Instead of, say, 10^{40} values in a table, this function approximator is characterized by, say, $n = 20$ parameters—an *enormous* compression. Although no one knows the true utility function for chess, no one believes that it can be represented exactly in 20 numbers. If the approximation is good



enough, however, the agent might still play excellent chess.³ Function approximation makes it practical to represent utility functions for very large state spaces, but that is not its principal benefit. *The compression achieved by a function approximator allows the learning agent to generalize from states it has visited to states it has not visited.* That is, the most important aspect of function approximation is not that it requires less space, but that it allows for inductive generalization over input states. To give you some idea of the power of this effect: by examining only one in every 10^{12} of the possible backgammon states, it is possible to learn a utility function that allows a program to play as well as any human (Tesauro, 1992).

On the flip side, of course, there is the problem that there could fail to be any function in the chosen hypothesis space that approximates the true utility function sufficiently well. As in all inductive learning, there is a tradeoff between the size of the hypothesis space and the time it takes to learn the function. A larger hypothesis space increases the likelihood that a good approximation can be found, but also means that convergence is likely to be delayed.

Let us begin with the simplest case, which is direct utility estimation. (See Section 21.2.) With function approximation, this is an instance of **supervised learning**. For example, suppose we represent the utilities for the 4×3 world using a simple linear function. The features of the squares are just their x and y coordinates, so we have

$$\hat{U}_\theta(x, y) = \theta_0 + \theta_1 x + \theta_2 y. \quad (21.10)$$

Thus, if $(\theta_0, \theta_1, \theta_2) = (0.5, 0.2, 0.1)$, then $\hat{U}_\theta(1, 1) = 0.8$. Given a collection of trials, we obtain a set of sample values of $\hat{U}_\theta(x, y)$, and we can find the best fit, in the sense of minimizing the squared error, using standard linear regression. (See Chapter 18.)

For reinforcement learning, it makes more sense to use an *online* learning algorithm that updates the parameters after each trial. Suppose we run a trial and the total reward obtained starting at $(1, 1)$ is 0.4. This suggests that $\hat{U}_\theta(1, 1)$, currently 0.8, is too large and must be reduced. How should the parameters be adjusted to achieve this? As with neural-network learning, we write an error function and compute its gradient with respect to the parameters. If $u_j(s)$ is the observed total reward from state s onward in the j th trial, then the error is defined as (half) the squared difference of the predicted total and the actual total: $E_j(s) = (\hat{U}_\theta(s) - u_j(s))^2/2$. The rate of change of the error with respect to each parameter θ_i is $\partial E_j / \partial \theta_i$, so to move the parameter in the direction of decreasing the error, we want

$$\theta_i \leftarrow \theta_i - \alpha \frac{\partial E_j(s)}{\partial \theta_i} = \theta_i + \alpha (u_j(s) - \hat{U}_\theta(s)) \frac{\partial \hat{U}_\theta(s)}{\partial \theta_i}. \quad (21.11)$$

WIDROW-HOFF RULE
DELTA RULE

This is called the **Widrow-Hoff rule**, or the **delta rule**, for online least-squares. For the linear function approximator $\hat{U}_\theta(s)$ in Equation (21.10), we get three simple update rules:

$$\begin{aligned}\theta_0 &\leftarrow \theta_0 + \alpha (u_j(s) - \hat{U}_\theta(s)), \\ \theta_1 &\leftarrow \theta_1 + \alpha (u_j(s) - \hat{U}_\theta(s))x, \\ \theta_2 &\leftarrow \theta_2 + \alpha (u_j(s) - \hat{U}_\theta(s))y.\end{aligned}$$

³ We do know that the exact utility function can be represented in a page or two of Lisp, Java, or C++. That is, it can be represented by a program that solves the game exactly every time it is called. We are interested only in function approximators that use a *reasonable* amount of computation. It might in fact be better to learn a very simple function approximator and combine it with a certain amount of look-ahead search. The tradeoffs involved are currently not well understood.



We can apply these rules to the example where $\hat{U}_\theta(1, 1)$ is 0.8 and $u_j(1, 1)$ is 0.4. θ_0 , θ_1 , and θ_2 are all decreased by 0.4α , which reduces the error for (1,1). Notice that *changing the parameters θ in response to an observed transition between two states also changes the values of \hat{U}_θ for every other state!* This is what we mean by saying that function approximation allows a reinforcement learner to generalize from its experiences.

We expect that the agent will learn faster if it uses a function approximator, provided that the hypothesis space is not too large, but includes some functions that are a reasonably good fit to the true utility function. Exercise 21.5 asks you to evaluate the performance of direct utility estimation, both with and without function approximation. The improvement in the 4×3 world is noticeable but not dramatic, because this is a very small state space to begin with. The improvement is much greater in a 10×10 world with a +1 reward at (10,10). This world is well suited for a linear utility function because the true utility function is smooth and nearly linear. (See Exercise 21.8.) If we put the +1 reward at (5,5), the true utility is more like a pyramid and the function approximator in Equation (21.10) will fail miserably. All is not lost, however! Remember that what matters for linear function approximation is that the function be linear in the *parameters*—the features themselves can be arbitrary nonlinear functions of the state variables. Hence, we can include a term such as $\theta_3 f_3(x, y) = \theta_3 \sqrt{(x - x_g)^2 + (y - y_g)^2}$ that measures the distance to the goal.

We can apply these ideas equally well to temporal-difference learners. All we need do is adjust the parameters to try to reduce the temporal difference between successive states. The new versions of the TD and Q-learning equations (21.3 on page 836 and 21.8 on page 844) are given by

$$\theta_i \leftarrow \theta_i + \alpha [R(s) + \gamma \hat{U}_\theta(s') - \hat{U}_\theta(s)] \frac{\partial \hat{U}_\theta(s)}{\partial \theta_i} \quad (21.12)$$

for utilities and

$$\theta_i \leftarrow \theta_i + \alpha [R(s) + \gamma \max_{a'} \hat{Q}_\theta(s', a') - \hat{Q}_\theta(s, a)] \frac{\partial \hat{Q}_\theta(s, a)}{\partial \theta_i} \quad (21.13)$$

for Q-values. For passive TD learning, the update rule can be shown to converge to the closest possible⁴ approximation to the true function when the function approximator is *linear* in the parameters. With active learning and *nonlinear* functions such as neural networks, all bets are off: There are some very simple cases in which the parameters can go off to infinity even though there are good solutions in the hypothesis space. There are more sophisticated algorithms that can avoid these problems, but at present reinforcement learning with general function approximators remains a delicate art.

Function approximation can also be very helpful for learning a model of the environment. Remember that learning a model for an *observable* environment is a *supervised* learning problem, because the next percept gives the outcome state. Any of the supervised learning methods in Chapter 18 can be used, with suitable adjustments for the fact that we need to predict a complete state description rather than just a Boolean classification or a single real value. For a *partially observable* environment, the learning problem is much more difficult. If we know what the hidden variables are and how they are causally related to each other and to the

⁴ The definition of distance between utility functions is rather technical; see Tsitsiklis and Van Roy (1997).

observable variables, then we can fix the structure of a dynamic Bayesian network and use the EM algorithm to learn the parameters, as was described in Chapter 20. Inventing the hidden variables and learning the model structure are still open problems. Some practical examples are described in Section 21.6.

21.5 POLICY SEARCH

POLICY SEARCH

The final approach we will consider for reinforcement learning problems is called **policy search**. In some ways, policy search is the simplest of all the methods in this chapter: the idea is to keep twiddling the policy as long as its performance improves, then stop.

Let us begin with the policies themselves. Remember that a policy π is a function that maps states to actions. We are interested primarily in *parameterized* representations of π that have far fewer parameters than there are states in the state space (just as in the preceding section). For example, we could represent π by a collection of parameterized Q-functions, one for each action, and take the action with the highest predicted value:

$$\pi(s) = \max_a \hat{Q}_\theta(s, a). \quad (21.14)$$



Each Q-function could be a linear function of the parameters θ , as in Equation (21.10), or it could be a nonlinear function such as a neural network. Policy search will then adjust the parameters θ to improve the policy. Notice that if the policy is represented by Q-functions, then policy search results in a process that learns Q-functions. *This process is not the same as Q-learning!* In Q-learning with function approximation, the algorithm finds a value of θ such that \hat{Q}_θ is “close” to Q^* , the optimal Q-function. Policy search, on the other hand, finds a value of θ that results in good performance; the values found by the two methods may differ very substantially. (For example, the approximate Q-function defined by $\hat{Q}_\theta(s, a) = Q^*(s, a)/10$ gives optimal performance, even though it is not at all close to Q^* .) Another clear instance of the difference is the case where $\pi(s)$ is calculated using, say, depth-10 look-ahead search with an approximate utility function \hat{U}_θ . A value of θ that gives good results may be a long way from making \hat{U}_θ resemble the true utility function.

STOCHASTIC POLICY
SOFTMAX FUNCTION

One problem with policy representations of the kind given in Equation (21.14) is that the policy is a *discontinuous* function of the parameters when the actions are discrete. (For a continuous action space, the policy can be a smooth function of the parameters.) That is, there will be values of θ such that an infinitesimal change in θ causes the policy to switch from one action to another. This means that the value of the policy may also change discontinuously, which makes gradient-based search difficult. For this reason, policy search methods often use a **stochastic policy** representation $\pi_\theta(s, a)$, which specifies the *probability* of selecting action a in state s . One popular representation is the **softmax function**:

$$\pi_\theta(s, a) = e^{\hat{Q}_\theta(s, a)} / \sum_{a'} e^{\hat{Q}_\theta(s, a')}.$$

Softmax becomes nearly deterministic if one action is much better than the others, but it always gives a differentiable function of θ ; hence, the value of the policy (which depends in

a continuous fashion on the action selection probabilities) is a differentiable function of θ . Softmax is a generalization of the logistic function (page 725) to multiple variables.

POLICY VALUE

POLICY GRADIENT

Now let us look at methods for improving the policy. We start with the simplest case: a deterministic policy and a deterministic environment. Let $\rho(\theta)$ be the **policy value**, i.e., the expected reward-to-go when π_θ is executed. If we can derive an expression for $\rho(\theta)$ in closed form, then we have a standard optimization problem, as described in Chapter 4. We can follow the **policy gradient** vector $\nabla_\theta \rho(\theta)$ provided $\rho(\theta)$ is differentiable. Alternatively, if $\rho(\theta)$ is not available in closed form, we can evaluate π_θ simply by executing it and observing the accumulated reward. We can follow the **empirical gradient** by hill climbing—i.e., evaluating the change in policy value for small increments in each parameter. With the usual caveats, this process will converge to a local optimum in policy space.

When the environment (or the policy) is stochastic, things get more difficult. Suppose we are trying to do hill climbing, which requires comparing $\rho(\theta)$ and $\rho(\theta + \Delta\theta)$ for some small $\Delta\theta$. The problem is that the total reward on each trial may vary widely, so estimates of the policy value from a small number of trials will be quite unreliable; trying to compare two such estimates will be even more unreliable. One solution is simply to run lots of trials, measuring the sample variance and using it to determine that enough trials have been run to get a reliable indication of the direction of improvement for $\rho(\theta)$. Unfortunately, this is impractical for many real problems where each trial may be expensive, time-consuming, and perhaps even dangerous.

For the case of a stochastic policy $\pi_\theta(s, a)$, it is possible to obtain an unbiased estimate of the gradient at θ , $\nabla_\theta \rho(\theta)$, directly from the results of trials executed at θ . For simplicity, we will derive this estimate for the simple case of a nonsequential environment in which the reward $R(a)$ is obtained immediately after doing action a in the start state s_0 . In this case, the policy value is just the expected value of the reward, and we have

$$\nabla_\theta \rho(\theta) = \nabla_\theta \sum_a \pi_\theta(s_0, a) R(a) = \sum_a (\nabla_\theta \pi_\theta(s_0, a)) R(a).$$

Now we perform a simple trick so that this summation can be approximated by samples generated from the probability distribution defined by $\pi_\theta(s_0, a)$. Suppose that we have N trials in all and the action taken on the j th trial is a_j . Then

$$\nabla_\theta \rho(\theta) = \sum_a \pi_\theta(s_0, a) \cdot \frac{(\nabla_\theta \pi_\theta(s_0, a)) R(a)}{\pi_\theta(s_0, a)} \approx \frac{1}{N} \sum_{j=1}^N \frac{(\nabla_\theta \pi_\theta(s_0, a_j)) R(a_j)}{\pi_\theta(s_0, a_j)}.$$

Thus, the true gradient of the policy value is approximated by a sum of terms involving the gradient of the action-selection probability in each trial. For the sequential case, this generalizes to

$$\nabla_\theta \rho(\theta) \approx \frac{1}{N} \sum_{j=1}^N \frac{(\nabla_\theta \pi_\theta(s, a_j)) R_j(s)}{\pi_\theta(s, a_j)}$$

for each state s visited, where a_j is executed in s on the j th trial and $R_j(s)$ is the total reward received from state s onwards in the j th trial. The resulting algorithm is called REINFORCE (Williams, 1992); it is usually much more effective than hill climbing using lots of trials at each value of θ . It is still much slower than necessary, however.



Consider the following task: given two blackjack⁵ programs, determine which is best. One way to do this is to have each play against a standard “dealer” for a certain number of hands and then to measure their respective winnings. The problem with this, as we have seen, is that the winnings of each program fluctuate widely depending on whether it receives good or bad cards. An obvious solution is to generate a certain number of hands in advance and *have each program play the same set of hands*. In this way, we eliminate the measurement error due to differences in the cards received. This idea, called **correlated sampling**, underlies a policy-search algorithm called PEGASUS (Ng and Jordan, 2000). The algorithm is applicable to domains for which a simulator is available so that the “random” outcomes of actions can be repeated. The algorithm works by generating in advance N sequences of random numbers, each of which can be used to run a trial of any policy. Policy search is carried out by evaluating each candidate policy using the *same* set of random sequences to determine the action outcomes. It can be shown that the number of random sequences required to ensure that the value of *every* policy is well estimated depends only on the complexity of the policy space, and not at all on the complexity of the underlying domain.

21.6 APPLICATIONS OF REINFORCEMENT LEARNING

We now turn to examples of large-scale applications of reinforcement learning. We consider applications in game playing, where the transition model is known and the goal is to learn the utility function, and in robotics, where the model is usually unknown.

21.6.1 Applications to game playing

The first significant application of reinforcement learning was also the first significant learning program of any kind—the checkers program written by Arthur Samuel (1959, 1967). Samuel first used a weighted linear function for the evaluation of positions, using up to 16 terms at any one time. He applied a version of Equation (21.12) to update the weights. There were some significant differences, however, between his program and current methods. First, he updated the weights using the difference between the current state and the backed-up value generated by full look-ahead in the search tree. This works fine, because it amounts to viewing the state space at a different granularity. A second difference was that the program did *not* use any observed rewards! That is, the values of terminal states reached in self-play were ignored. This means that it is theoretically possible for Samuel’s program not to converge, or to converge on a strategy designed to lose rather than to win. He managed to avoid this fate by insisting that the weight for material advantage should always be positive. Remarkably, this was sufficient to direct the program into areas of weight space corresponding to good checkers play.

Gerry Tesauro’s backgammon program TD-GAMMON (1992) forcefully illustrates the potential of reinforcement learning techniques. In earlier work (Tesauro and Sejnowski, 1989), Tesauro tried learning a neural network representation of $Q(s, a)$ directly from ex-

⁵ Also known as twenty-one or pontoon.

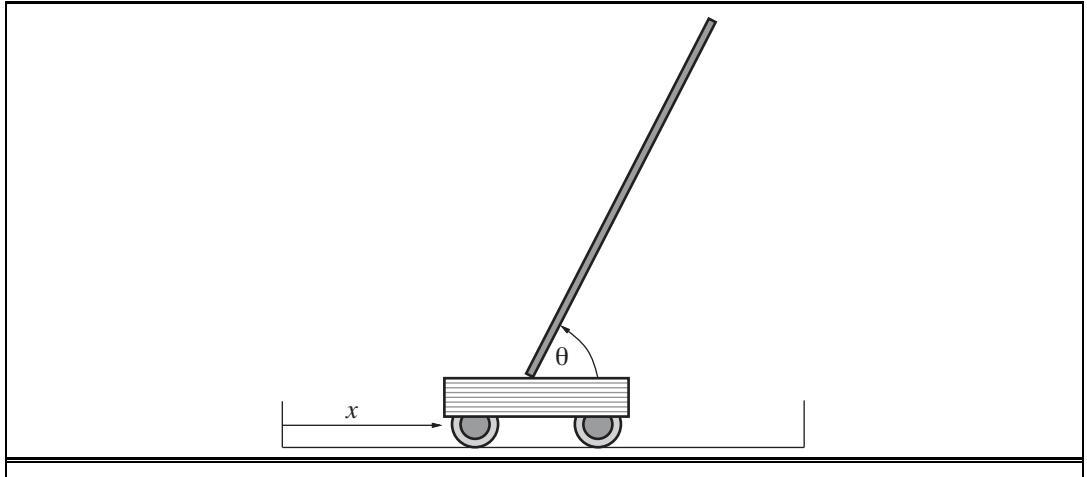


Figure 21.9 Setup for the problem of balancing a long pole on top of a moving cart. The cart can be jerked left or right by a controller that observes x , θ , \dot{x} , and $\dot{\theta}$.

amples of moves labeled with relative values by a human expert. This approach proved extremely tedious for the expert. It resulted in a program, called NEUROGAMMON, that was strong by computer standards, but not competitive with human experts. The TD-GAMMON project was an attempt to learn from self-play alone. The only reward signal was given at the end of each game. The evaluation function was represented by a fully connected neural network with a single hidden layer containing 40 nodes. Simply by repeated application of Equation (21.12), TD-GAMMON learned to play considerably better than NEUROGAMMON, even though the input representation contained just the raw board position with no computed features. This took about 200,000 training games and two weeks of computer time. Although that may seem like a lot of games, it is only a vanishingly small fraction of the state space. When precomputed features were added to the input representation, a network with 80 hidden nodes was able, after 300,000 training games, to reach a standard of play comparable to that of the top three human players worldwide. Kit Woolsey, a top player and analyst, said that “There is no question in my mind that its positional judgment is far better than mine.”

21.6.2 Application to robot control

CART-POLE
INVERTED
PENDULUM

BANG-BANG
CONTROL

The setup for the famous **cart–pole** balancing problem, also known as the **inverted pendulum**, is shown in Figure 21.9. The problem is to control the position x of the cart so that the pole stays roughly upright ($\theta \approx \pi/2$), while staying within the limits of the cart track as shown. Several thousand papers in reinforcement learning and control theory have been published on this seemingly simple problem. The cart–pole problem differs from the problems described earlier in that the state variables x , θ , \dot{x} , and $\dot{\theta}$ are continuous. The actions are usually discrete: jerk left or jerk right, the so-called **bang-bang control** regime.

The earliest work on learning for this problem was carried out by Michie and Chambers (1968). Their BOXES algorithm was able to balance the pole for over an hour after only about 30 trials. Moreover, unlike many subsequent systems, BOXES was implemented with a

real cart and pole, not a simulation. The algorithm first discretized the four-dimensional state space into boxes—hence the name. It then ran trials until the pole fell over or the cart hit the end of the track. Negative reinforcement was associated with the final action in the final box and then propagated back through the sequence. It was found that the discretization caused some problems when the apparatus was initialized in a position different from those used in training, suggesting that generalization was not perfect. Improved generalization and faster learning can be obtained using an algorithm that *adaptively* partitions the state space according to the observed variation in the reward, or by using a continuous-state, nonlinear function approximator such as a neural network. Nowadays, balancing a *triple* inverted pendulum is a common exercise—a feat far beyond the capabilities of most humans.

Still more impressive is the application of reinforcement learning to helicopter flight (Figure 21.10). This work has generally used policy search (Bagnell and Schneider, 2001) as well as the PEGASUS algorithm with simulation based on a learned transition model (Ng *et al.*, 2004). Further details are given in Chapter 25.



Figure 21.10 Superimposed time-lapse images of an autonomous helicopter performing a very difficult “nose-in circle” maneuver. The helicopter is under the control of a policy developed by the PEGASUS policy-search algorithm. A simulator model was developed by observing the effects of various control manipulations on the real helicopter; then the algorithm was run on the simulator model overnight. A variety of controllers were developed for different maneuvers. In all cases, performance far exceeded that of an expert human pilot using remote control. (Image courtesy of Andrew Ng.)

21.7 SUMMARY

This chapter has examined the reinforcement learning problem: how an agent can become proficient in an unknown environment, given only its percepts and occasional rewards. Reinforcement learning can be viewed as a microcosm for the entire AI problem, but it is studied in a number of simplified settings to facilitate progress. The major points are:

- The overall agent design dictates the kind of information that must be learned. The three main designs we covered were the model-based design, using a model P and a utility function U ; the model-free design, using an action-utility function Q ; and the reflex design, using a policy π .
- Utilities can be learned using three approaches:
 1. **Direct utility estimation** uses the total observed reward-to-go for a given state as direct evidence for learning its utility.
 2. **Adaptive dynamic programming** (ADP) learns a model and a reward function from observations and then uses value or policy iteration to obtain the utilities or an optimal policy. ADP makes optimal use of the local constraints on utilities of states imposed through the neighborhood structure of the environment.
 3. **Temporal-difference** (TD) methods update utility estimates to match those of successor states. They can be viewed as simple approximations to the ADP approach that can learn without requiring a transition model. Using a learned model to generate pseudoexperiences can, however, result in faster learning.
- Action-utility functions, or Q-functions, can be learned by an ADP approach or a TD approach. With TD, Q-learning requires no model in either the learning or action-selection phase. This simplifies the learning problem but potentially restricts the ability to learn in complex environments, because the agent cannot simulate the results of possible courses of action.
- When the learning agent is responsible for selecting actions while it learns, it must trade off the estimated value of those actions against the potential for learning useful new information. An exact solution of the exploration problem is infeasible, but some simple heuristics do a reasonable job.
- In large state spaces, reinforcement learning algorithms must use an approximate functional representation in order to generalize over states. The temporal-difference signal can be used directly to update parameters in representations such as neural networks.
- Policy-search methods operate directly on a representation of the policy, attempting to improve it based on observed performance. The variation in the performance in a stochastic domain is a serious problem; for simulated domains this can be overcome by fixing the randomness in advance.

Because of its potential for eliminating hand coding of control strategies, reinforcement learning continues to be one of the most active areas of machine learning research. Applications in robotics promise to be particularly valuable; these will require methods for handling con-

tinuous, high-dimensional, partially observable environments in which successful behaviors may consist of thousands or even millions of primitive actions.

BIBLIOGRAPHICAL AND HISTORICAL NOTES

Turing (1948, 1950) proposed the reinforcement-learning approach, although he was not convinced of its effectiveness, writing, “the use of punishments and rewards can at best be a part of the teaching process.” Arthur Samuel’s work (1959) was probably the earliest successful machine learning research. Although this work was informal and had a number of flaws, it contained most of the modern ideas in reinforcement learning, including temporal differencing and function approximation. Around the same time, researchers in adaptive control theory (Widrow and Hoff, 1960), building on work by Hebb (1949), were training simple networks using the delta rule. (This early connection between neural networks and reinforcement learning may have led to the persistent misperception that the latter is a subfield of the former.) The cart–pole work of Michie and Chambers (1968) can also be seen as a reinforcement learning method with a function approximator. The psychological literature on reinforcement learning is much older; Hilgard and Bower (1975) provide a good survey. Direct evidence for the operation of reinforcement learning in animals has been provided by investigations into the foraging behavior of bees; there is a clear neural correlate of the reward signal in the form of a large neuron mapping from the nectar intake sensors directly to the motor cortex (Montague *et al.*, 1995). Research using single-cell recording suggests that the dopamine system in primate brains implements something resembling value function learning (Schultz *et al.*, 1997). The neuroscience text by Dayan and Abbott (2001) describes possible neural implementations of temporal-difference learning, while Dayan and Niv (2008) survey the latest evidence from neuroscientific and behavioral experiments.

The connection between reinforcement learning and Markov decision processes was first made by Werbos (1977), but the development of reinforcement learning in AI stems from work at the University of Massachusetts in the early 1980s (Barto *et al.*, 1981). The paper by Sutton (1988) provides a good historical overview. Equation (21.3) in this chapter is a special case for $\lambda = 0$ of Sutton’s general $TD(\lambda)$ algorithm. $TD(\lambda)$ updates the utility values of all states in a sequence leading up to each transition by an amount that drops off as λ^t for states t steps in the past. $TD(1)$ is identical to the Widrow–Hoff or delta rule. Boyan (2002), building on work by Bradtko and Barto (1996), argues that $TD(\lambda)$ and related algorithms make inefficient use of experiences; essentially, they are online regression algorithms that converge much more slowly than offline regression. His LSTD (least-squares temporal differencing) algorithm is an online algorithm for passive reinforcement learning that gives the same results as offline regression. Least-squares policy iteration, or LSPI (Lagoudakis and Parr, 2003), combines this idea with the policy iteration algorithm, yielding a robust, statistically efficient, model-free algorithm for learning policies.

The combination of temporal-difference learning with the model-based generation of simulated experiences was proposed in Sutton’s DYNA architecture (Sutton, 1990). The idea of prioritized sweeping was introduced independently by Moore and Atkeson (1993) and

Peng and Williams (1993). Q-learning was developed in Watkins's Ph.D. thesis (1989), while SARSA appeared in a technical report by Rummery and Niranjan (1994).

Bandit problems, which model the problem of exploration for nonsequential decisions, are analyzed in depth by Berry and Fristedt (1985). Optimal exploration strategies for several settings are obtainable using the technique called **Gittins indices** (Gittins, 1989). A variety of exploration methods for sequential decision problems are discussed by Barto *et al.* (1995). Kearns and Singh (1998) and Brafman and Tennenholtz (2000) describe algorithms that explore unknown environments and are guaranteed to converge on near-optimal policies in polynomial time. Bayesian reinforcement learning (Dearden *et al.*, 1998, 1999) provides another angle on both model uncertainty and exploration.

CMAC

Function approximation in reinforcement learning goes back to the work of Samuel, who used both linear and nonlinear evaluation functions and also used feature-selection methods to reduce the feature space. Later methods include the **CMAC** (Cerebellar Model Articulation Controller) (Albus, 1975), which is essentially a sum of overlapping local kernel functions, and the associative neural networks of Barto *et al.* (1983). Neural networks are currently the most popular form of function approximator. The best-known application is TD-Gammon (Tesauro, 1992, 1995), which was discussed in the chapter. One significant problem exhibited by neural-network-based TD learners is that they tend to forget earlier experiences, especially those in parts of the state space that are avoided once competence is achieved. This can result in catastrophic failure if such circumstances reappear. Function approximation based on **instance-based learning** can avoid this problem (Ormoneit and Sen, 2002; Forbes, 2002).

The convergence of reinforcement learning algorithms using function approximation is an extremely technical subject. Results for TD learning have been progressively strengthened for the case of linear function approximators (Sutton, 1988; Dayan, 1992; Tsitsiklis and Van Roy, 1997), but several examples of divergence have been presented for nonlinear functions (see Tsitsiklis and Van Roy, 1997, for a discussion). Papavassiliou and Russell (1999) describe a new type of reinforcement learning that converges with any form of function approximator, provided that a best-fit approximation can be found for the observed data.

Policy search methods were brought to the fore by Williams (1992), who developed the **REINFORCE** family of algorithms. Later work by Marbach and Tsitsiklis (1998), Sutton *et al.* (2000), and Baxter and Bartlett (2000) strengthened and generalized the convergence results for policy search. The method of correlated sampling for comparing different configurations of a system was described formally by Kahn and Marshall (1953), but seems to have been known long before that. Its use in reinforcement learning is due to Van Roy (1998) and Ng and Jordan (2000); the latter paper also introduced the **PEGASUS** algorithm and proved its formal properties.

As we mentioned in the chapter, the performance of a *stochastic* policy is a continuous function of its parameters, which helps with gradient-based search methods. This is not the only benefit: Jaakkola *et al.* (1995) argue that stochastic policies actually work better than deterministic policies in partially observable environments, if both are limited to acting based on the current percept. (One reason is that the stochastic policy is less likely to get "stuck" because of some unseen hindrance.) Now, in Chapter 17 we pointed out that

optimal policies in partially observable MDPs are deterministic functions of the *belief state* rather than the current percept, so we would expect still better results by keeping track of the belief state using the **filtering** methods of Chapter 15. Unfortunately, belief-state space is high-dimensional and continuous, and effective algorithms have not yet been developed for reinforcement learning with belief states.

Real-world environments also exhibit enormous complexity in terms of the number of primitive actions required to achieve significant reward. For example, a robot playing soccer might make a hundred thousand individual leg motions before scoring a goal. One common method, used originally in animal training, is called **reward shaping**. This involves supplying the agent with additional rewards, called **pseudorewards**, for “making progress.” For example, in soccer the real reward is for scoring a goal, but pseudorewards might be given for making contact with the ball or for kicking it toward the goal. Such rewards can speed up learning enormously and are simple to provide, but there is a risk that the agent will learn to maximize the pseudorewards rather than the true rewards; for example, standing next to the ball and “vibrating” causes many contacts with the ball. Ng *et al.* (1999) show that the agent will still learn the optimal policy provided that the pseudoreward $F(s, a, s')$ satisfies $F(s, a, s') = \gamma\Phi(s') - \Phi(s)$, where Φ is an arbitrary function of the state. Φ can be constructed to reflect any desirable aspects of the state, such as achievement of subgoals or distance to a goal state.

The generation of complex behaviors can also be facilitated by **hierarchical reinforcement learning** methods, which attempt to solve problems at multiple levels of abstraction—much like the **HTN planning** methods of Chapter 11. For example, “scoring a goal” can be broken down into “obtain possession,” “dribble towards the goal,” and “shoot;” and each of these can be broken down further into lower-level motor behaviors. The fundamental result in this area is due to Forestier and Varaiya (1978), who proved that lower-level behaviors of arbitrary complexity can be treated just like primitive actions (albeit ones that can take varying amounts of time) from the point of view of the higher-level behavior that invokes them. Current approaches (Parr and Russell, 1998; Dietterich, 2000; Sutton *et al.*, 2000; Andre and Russell, 2002) build on this result to develop methods for supplying an agent with a **partial program** that constrains the agent’s behavior to have a particular hierarchical structure. The partial-programming language for agent programs extends an ordinary programming language by adding primitives for unspecified choices that must be filled in by learning. Reinforcement learning is then applied to learn the best behavior consistent with the partial program. The combination of function approximation, shaping, and hierarchical reinforcement learning has been shown to solve large-scale problems—for example, policies that execute for 10^4 steps in state spaces of 10^{100} states with branching factors of 10^{30} (Marthi *et al.*, 2005). One key result (Dietterich, 2000) is that the hierarchical structure provides a natural *additive decomposition* of the overall utility function into terms that depend on small subsets of the variables defining the state space. This is somewhat analogous to the representation theorems underlying the conciseness of Bayes nets (Chapter 14).

The topic of distributed and multiagent reinforcement learning was not touched upon in the chapter but is of great current interest. In distributed RL, the aim is to devise methods by which multiple, coordinated agents learn to optimize a common utility function. For example,

REWARD SHAPING

PSEUDOREWARD

HIERARCHICAL
REINFORCEMENT
LEARNING

PARTIAL PROGRAM

SUBAGENT

can we devise methods whereby separate **subagents** for robot navigation and robot obstacle avoidance could cooperatively achieve a combined control system that is globally optimal? Some basic results in this direction have been obtained (Guestrin *et al.*, 2002; Russell and Zimdars, 2003). The basic idea is that each subagent learns its own Q-function from its own stream of rewards. For example, a robot-navigation component can receive rewards for making progress towards the goal, while the obstacle-avoidance component receives negative rewards for every collision. Each global decision maximizes the sum of Q-functions and the whole process converges to globally optimal solutions.

Multiagent RL is distinguished from distributed RL by the presence of agents who cannot coordinate their actions (except by explicit communicative acts) and who may not share the same utility function. Thus, multiagent RL deals with sequential game-theoretic problems or **Markov games**, as defined in Chapter 17. The consequent requirement for randomized policies is not a significant complication, as we saw on page 848. What *does* cause problems is the fact that, while an agent is learning to defeat its opponent’s policy, the opponent is changing its policy to defeat the agent. Thus, the environment is **nonstationary** (see page 568). Littman (1994) noted this difficulty when introducing the first RL algorithms for zero-sum Markov games. Hu and Wellman (2003) present a Q-learning algorithm for general-sum games that converges when the Nash equilibrium is unique; when there are multiple equilibria, the notion of convergence is not so easy to define (Shoham *et al.*, 2004).

Sometimes the reward function is not easy to define. Consider the task of driving a car. There are extreme states (such as crashing the car) that clearly should have a large penalty. But beyond that, it is difficult to be precise about the reward function. However, it is easy enough for a human to drive for a while and then tell a robot “do it like that.” The robot then has the task of **apprenticeship learning**; learning from an example of the task done right, without explicit rewards. Ng *et al.* (2004) and Coates *et al.* (2009) show how this technique works for learning to fly a helicopter; see Figure 25.25 on page 1002 for an example of the acrobatics the resulting policy is capable of. Russell (1998) describes the task of **inverse reinforcement learning**—figuring out what the reward function must be from an example path through that state space. This is useful as a part of apprenticeship learning, or as a part of doing science—we can understand an animal or robot by working backwards from what it does to what its reward function must be.

This chapter has dealt only with atomic states—all the agent knows about a state is the set of available actions and the utilities of the resulting states (or of state-action pairs). But it is also possible to apply reinforcement learning to structured representations rather than atomic ones; this is called **relational reinforcement learning** (Tadepalli *et al.*, 2004).

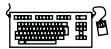
APPRENTICESHIP LEARNING

INVERSE REINFORCEMENT LEARNING

RELATIONAL REINFORCEMENT LEARNING

The survey by Kaelbling *et al.* (1996) provides a good entry point to the literature. The text by Sutton and Barto (1998), two of the field’s pioneers, focuses on architectures and algorithms, showing how reinforcement learning weaves together the ideas of learning, planning, and acting. The somewhat more technical work by Bertsekas and Tsitsiklis (1996) gives a rigorous grounding in the theory of dynamic programming and stochastic convergence. Reinforcement learning papers are published frequently in *Machine Learning*, in the *Journal of Machine Learning Research*, and in the International Conferences on Machine Learning and the Neural Information Processing Systems meetings.

EXERCISES



21.1 Implement a passive learning agent in a simple environment, such as the 4×3 world. For the case of an initially unknown environment model, compare the learning performance of the direct utility estimation, TD, and ADP algorithms. Do the comparison for the optimal policy and for several random policies. For which do the utility estimates converge faster? What happens when the size of the environment is increased? (Try environments with and without obstacles.)

21.2 Chapter 17 defined a **proper policy** for an MDP as one that is guaranteed to reach a terminal state. Show that it is possible for a passive ADP agent to learn a transition model for which its policy π is improper even if π is proper for the true MDP; with such models, the POLICY-EVALUATION step may fail if $\gamma = 1$. Show that this problem cannot arise if POLICY-EVALUATION is applied to the learned model only at the end of a trial.



21.3 Starting with the passive ADP agent, modify it to use an approximate ADP algorithm as discussed in the text. Do this in two steps:

- Implement a priority queue for adjustments to the utility estimates. Whenever a state is adjusted, all of its predecessors also become candidates for adjustment and should be added to the queue. The queue is initialized with the state from which the most recent transition took place. Allow only a fixed number of adjustments.
- Experiment with various heuristics for ordering the priority queue, examining their effect on learning rates and computation time.

21.4 Write out the parameter update equations for TD learning with

$$\hat{U}(x, y) = \theta_0 + \theta_1 x + \theta_2 y + \theta_3 \sqrt{(x - x_g)^2 + (y - y_g)^2}.$$



21.5 Implement an exploring reinforcement learning agent that uses direct utility estimation. Make two versions—one with a tabular representation and one using the function approximator in Equation (21.10). Compare their performance in three environments:

- The 4×3 world described in the chapter.
- A 10×10 world with no obstacles and a +1 reward at (10,10).
- A 10×10 world with no obstacles and a +1 reward at (5,5).

21.6 Devise suitable features for reinforcement learning in stochastic grid worlds (generalizations of the 4×3 world) that contain multiple obstacles and multiple terminal states with rewards of +1 or -1.



21.7 Extend the standard game-playing environment (Chapter 5) to incorporate a reward signal. Put two reinforcement learning agents into the environment (they may, of course, share the agent program) and have them play against each other. Apply the generalized TD update rule (Equation (21.12)) to update the evaluation function. You might wish to start with a simple linear weighted evaluation function and a simple game, such as tic-tac-toe.

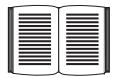
21.8 Compute the true utility function and the best linear approximation in x and y (as in Equation (21.10)) for the following environments:

- a. A 10×10 world with a single +1 terminal state at (10,10).
- b. As in (a), but add a -1 terminal state at (10,1).
- c. As in (b), but add obstacles in 10 randomly selected squares.
- d. As in (b), but place a wall stretching from (5,2) to (5,9).
- e. As in (a), but with the terminal state at (5,5).

The actions are deterministic moves in the four directions. In each case, compare the results using three-dimensional plots. For each environment, propose additional features (besides x and y) that would improve the approximation and show the results.



21.9 Implement the REINFORCE and PEGASUS algorithms and apply them to the 4×3 world, using a policy family of your own choosing. Comment on the results.



21.10 Is reinforcement learning an appropriate abstract model for evolution? What connection exists, if any, between hardwired reward signals and evolutionary fitness?