B.E. I Semester Examination

BE-I/12(A)

227922

MATHEMATICS - I

Course No. MTH-101

Time Allowed- 3Hours

Maximum Marks-100

Note: Attempt five questions in all, selecting at least two from each section. All carry equal marks. Use of calculator is allowed.

Section - A

- 1. a) Given $y = (\sin^{-1} x)^2$, find the value of its n^{th} derivative at x=0.
 - b) Find extreme value of $x^2+y^2+z^2$ when ax+by+cz=p.
 - c) Find the radius of curvature at origin of the curve:

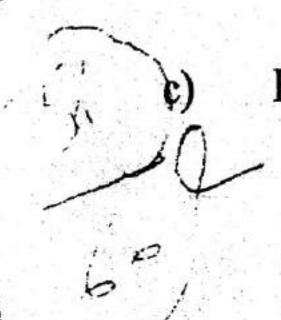
$$2x^3 + 4x^2y + xy^2 + 5y^3 + x^2 - 2xy + y^2 - 4x = 0.$$
(7,7,6)

2. a) Find all asymptotes of the curve given by

$$(2x-3y+1)^2(x+y)=8x-2y+9.$$

b) If $x^2 + y^2 + z^2 = e^{2u}$, then show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = e^{-2u}$$



Evaluate $\int_0^{\pi/2} \log(\sin \theta) d\theta$.

(7,7,6)

[Turn Over

3. a) Show that

$$\int_{0}^{t} erf_{c}(ax)dx = t \ erf_{c}(at) - \frac{1}{a\sqrt{\pi}} \left(e^{-a^{2}t^{2}} - 1\right).$$

- Find the length of the arc of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ in the first quadrant.
- Find the area of loop of the curve $a^2y^2 = x^3(2a x)$ above x-axis. (7,7,6)
- 4. a) Find the surface area of the solid generated by the revolution of the loops of the curve $r^2 = a^2 \cos 2\theta$ about the initial line.
 - b) Evaluate $\iint r \sin \theta dr d\theta$ over the cardioid $r = a(1 \cos \theta)$ above the initial line.
 - Evaluate $\iiint (x+y+z) dx dy dz$ over the tetrahedron bounded by the planes x = 0, y = 0, z = 0 and x+y+z=1.

(7,7,6)

Section - B

5. a) If $tan(x+iy) = cos \alpha + i sin \alpha$, prove that

$$x = \frac{\pi}{4}(2n+1) \text{ and } y = \frac{1}{4}\log\tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right).$$

b) Sum the series to infinity.

 $1 + x \cosh \alpha + x^2 \cosh 2\alpha + x^3 \cosh 3\alpha + \dots$ Show that

Show that $\sinh(x-y) = \sinh x \cosh y - \cosh x \sinh y \text{ and}$ $\cosh(x-y) = \cosh x \cosh y - \sinh x \sinh y$

Find the Eduation

(7,7,6)

and whose avic

a)
$$(1+e^{x^2})^{\frac{1}{2}}$$

b) $(1+x^2)^{\frac{1}{2}}(\frac{dy}{dx}-4x^2\cos^2y)+x\sin 2y=0$

b)
$$(D^2 + 5)y = x \sin x$$
.
c) $(D^2 + 5)y = x \sin x$.
Solve the following differential equations:

a)
$$y'' - 2y' + y = \frac{e^x}{x}$$

a)
$$x$$

b) $x^{y'''} + 2x^2y'' + 2y = 20(x + x^{-1})$

b)
$$(D^2y^2 + 2x^2y^2 + 2x^2y^2 + 2y^2 + 2y$$

- Find the equations of the lines in which the plane a) 8. 2x+y-z=0 cuts the cone $4x^2-y^2+3z^2=0$. Also find the angle between the lines.
 - Find the equation of the right circular cylinder whose axis b) is $\frac{x-1}{2} = \frac{y}{3} = \frac{z-3}{1}$ and radius 2.
 - c) Find the angle of intersection of the spheres $x^2 + y^2 + z^2 + 6y + 2z + 8 = 0$ and $x^{2} + y^{2} + z^{2} + 6x + 8y + 4z + 20 = 0.$

