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B.E. IV Semester Examination

BE-IV/6(A)

26932

CIVIL/MECH. ENGINEERING

Course No. MTH-412

(Engineering Maths-III)

Time Allowed- 3 Hours *Maximum Marks-100*

Note :- Attempt any **five** questions, selecting at least **two** questions from **each** section. All carry **equal** marks. Use of calculator is allowed.

Section - I

1. a) Find the Laplace transform of :

i) $\cos^2(at) + \sinh at - e^{-t}\left(\sqrt{t} + 7\right)$

ii) $f(t) = \begin{cases} t & 0 \le t < a \\ 2a - t, & a \le t < 2a \end{cases}$

and $f(t+2a) = f(t), \forall t \ge 0$

b) State and prove sufficient conditions for existence of Laplace transform of a function f(t). Explain whether the conditions you consider and necessary also. (7,7,6)

2. a) Find the inverse Laplace transforms of the following functions :

i) $\dfrac{4}{(s-1)(s-2)(s-3)}$

ii) $\dfrac{s}{\left(s^2+a^2\right)^2}$

b) State initial value Theorem and verify it for :

$f(t)=t+\sin t$ (7,7,6)

3. a) Find the Fourier transform of $f(t)=\begin{cases} t^3, & |t|<a \\ 0, & \text{otherwise} \end{cases}$

b) Find the Fourier Sine transform of $\dfrac{e^{-ax}}{x}$

c) Find the inverse Fourier Cosine transform of the following function : $\underset{c}{f}(s)=\begin{cases} s^2 & \text{for } 0<s<a \\ 0, & \text{for } s\geq a \end{cases}$ (7,7,6)

4. a) Solve the differential equations :

i) $\left(D^2+D\right)y=3t^2$ with $y(0)=0,\ y'(0)=1$

ii) $y''+4y=f(t)$ with $y(0)=0,\ y'(0)=1$

b) Solve the following integral equation

$$\int_0^{\infty} f(x) \operatorname{Cos} ax \, dx = e^{-a}$$ (7,7,6)

Section - II

5. a) State and prove the Rodrigue's formula for Legendre polynomials. Also write $P_4(x)$ and $P_5(x)$.

b) Show that $J_{-n}(x) = (-1)^n J_n(x)$ for all integers n. Is it true if n is not integer. (12,8)

6. a) Show that $e^{\frac{x}{2}\left(t-\frac{1}{t}\right)}$ is the generating function for $J_n(x)$

b) Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre's polynomials. (10,10)

7. a) Define a lattice with example and show that a lattice L is distributive if and only if

$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c) \; \forall \; a, b, c, \in L$

b) Define a Boolean algebra with example and show that in a Boolean algebra L :

(i) $(x')' = x \ \forall \ x \in L$

(ii) $x \leq y$ if and only if $y' \leq x'$ (10,10)

8. Express the following Boolean functions in DNF :

(i) $a \wedge (b \vee c)$

(ii) $\left[(a \wedge b')' \vee c'\right] \wedge (a' \vee c)'$

(b) Draw the circuits of the following functions :

(i) $[x \vee (x \wedge y)] \wedge [y \vee (x' \wedge y)]$

(ii) $(x' \wedge z) \vee (x' \wedge y) \vee (x \wedge y' \wedge z) \vee (y \wedge z)$.

(10,10)