Total No. of Questions—13] [Total No. of Printed Pages—4+2

BE-I/11(A)

236169

(New Course)

ENGINEERING MATHEMATICS—COURSE NO. BSC-101

Time Allowed—3 *Hours* *Maximum Marks*—100

Note : (*i*) Attempt Q. No. **1**, which is compulsory and it consists of short answer type questions of total **10** marks.

(*ii*) From each unit, attempt *one* question of **15** marks each (Total marks : 15 × 6 = 90)

1. Verify whether the following statements are true *or* false : 10×1=10

(*i*) If :

$$z = \frac{x^2 + y^2}{x - y},$$

then :

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0.$$

(*ii*) The curve $x^2y^2 - x^2 + y^2 = 1$ has three asymptotes.

(*iii*) The area of a loop of the curve $r^2 = a^2 \cos 2\theta$ is $\frac{1}{2}a^2$.

(*iv*) $\mathrm{erf}_C(x) = 1 - \mathrm{erf}(x)$

[*Turn over*

(*v*) $\sinh^2 z + \cosh^2 z = 1$.

(*vi*) Two non-zero vectors are perpendicular if their dot product is zero.

(*vii*) The gradient of a constant function is zero.

(*viii*) The differential equation $(D^3 - D)y = 0$ has two only solutions e^x amd e^{-x}.

(*ix*) The P.I. of $(D - 3)^2 y = 2e^{3x}$ is x^2e^{3x}.

(*x*) $\int_0^2\int_0^3 (x + y)\, dx\, dy = 15$.

Unit I

2. (*a*) Trace the curve :

$$x(x^2 + y^2) = a(x^2 - y^2), \text{ where } a > 0.$$

(*b*) Find all the asymptotes of the curve :

$$x(y - 3)^3 = 4y(x - 1)^3.$$ 8,7

Or

3. (*a*) Verify Euler's theorem for the function :

$$z = 2x^2 + 3xy + 4y^2.$$

(*b*) Find the radius of curvature at any point of the curve :

$$xy = c^2.$$

(c) Find the nature of double points at the origin on the curve :

$$x^3 + y^3 - 4axy = 0.$$ 5,5,5

Unit II

4. (a) Verify Rolle's theorem for :

$$f(x) = e^x(\sin x - \cos x) \text{ in } \left[\frac{\pi}{4}, \frac{5\pi}{4}\right].$$

(b) Find the extreme values of the function

$\cos x \cdot \cos y \cdot \cos z$, where $x + y + z = \pi$. 8,7

Or

5. (a) State and prove mean value theorem.

(b) Evaluate :

$$\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{x \sin x}\right).$$

(c) Expand $f(x, y) = e^{x+y}$ as a series in powers of x and y upto third degree terms. 5,5,5

Unit III

6. (a) The area between the curves $y^2 = 4ax$ and $x^2 = 4ay$ revolves about the x-axis. Find the volume of the solid thus generated.

(b) Evaluate $\iint_R (x^2 + y^2)\,dx\,dy$ over the area R of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in the first quadrant by using the transformation $x = au$ and $y = bv$. 8,7

Or

7. (a) Evaluate :

$$\int_0^1 (x \log x)^3\, dx.$$

(b) Show that :

$$\operatorname{erf}_C(x) + \operatorname{erf}_C(-x) = 2\ \forall\ x.$$

(c) Find the perimeter of the curve :

$$r = a(1 + \sin\theta).$$ 5,5,5

Unit IV

8. (a) Verify Green's theorem for $\oint_C (3x^2 - 8y^2)\,dx + (4y - 6xy)\,dy$, where C is the boundary of the region defined by $x = 0$, $y = 0$ and $x + y = 1$.

(b) Verify divergence theorem for $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$ taken over the cube bounded by $x = 0$, $x = a$, $y = 0$, $y = a$, $z = 0$ and $z = a$. 8,7

Or

9. (*a*) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then show that :

$$\nabla^2(r^m) = m(m-1)r^{m-2}.$$

(*b*) Find the divergence and curl of the vector function $\vec{F} = x^2y\hat{i} + xy\hat{j} + 3yz\hat{k}$ at at the point (–1, 1, 1).

(*c*) Find the circulation of $\vec{F}$ around the curve C, where $\vec{F} = y\hat{i} + z\hat{j} + x\hat{k}$ and C is the circle $x^2 + y^2 = 1$, $z = 0$.

5,5,5

Unit V

10. (*a*) If $\sin(\alpha + i\beta) = re^{i\theta}$, then show that $2r^2 = \cosh 2\beta - \cos 2\alpha$ and $\tan\theta = \tanh\beta \cot\alpha$.

(*b*) If $\cosh x = \sec\theta$, then show that :

$$\theta = \frac{\pi}{2} - 2\tan^{-1}(e^{-x}).$$

8,7

Or

11. (*a*) Express $\cosh z$ into real and imaginary parts.

(*b*) Express $\log(\sqrt{3} + i)$ into real and imaginary parts.

(*c*) Sum the series to infinity :

$\sin\alpha \,.\, \sin\beta + \sin 2\alpha \,.\, \sin^2\beta + \sin 3\alpha \,.\, \sin^3\beta + ...$ 5,5,5

Unit VI

12. Solve :

(a) $(D^2 + 5D + 4)y = x^2 + 7x + 9$

(b) $(D^2 + 1)y = 4x \cos x$

(c) $(D^2 - 2D + 1)y = e^x \log x$ 5,5,5

Or

13. Solve :

(a) $(x^3 + xy^4)\, dx + 2y^3 dy = 0$

(b) $(x^3y^3 - xy)\, dx = dy$

(c) $dx = (x + y + 1)dy.$ 5,5,5