BE-I/6(B)

227293

MATHEMATICS-I

Course No. MTH - 101

Time Allowed-3Hours

Maximum Marks-100

Note: Attempt five questions in all, selecting atleast two questions from each section. All carry equal marks. Use of calculator is allowed.

Section - A

- 1. a) If $y = x^{n-1} \log x$, prove that $xy_n = (n-1)!$.
 - b) Find the value of the expression $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy}$,

where
$$u = \sec^{-1}\left(\frac{x^2 - y^2}{x^5 + y^5}\right)$$
.

- Find the radius of curvature at any point of the curve $y^2 = 4ax$. (7,7,6)
- 2. a) Find all the asymptotes of the curve $(x+9y+2)(x+y)^2 = x+3y-1$.
 - b) Trace the curve given by $x = a(t + \sin t), y = a(1 + \cos t).$
 - Examine the function $x^2 + y^2 + z^2$ for extreme values given that $xyz = k^3$. (7,7,6)

[Turn Over

- 3. a) Show that $\beta(m,n) = \frac{1000}{(m+n)}$, where $\frac{1}{(m+n)}$
 - Find the surface of the solid generated by the revolution of the ellipse $x^2 + 4y^2 = 16$ about its minor axis.
 - c) Find the area of a loop of the curve $r^2 = a^2 \cos 2\theta$ (7,7,6)
- 4. a) Find the total length of the curve $x^2(a^2 x^2) = 8a^2y^2$.
 - b) Change the order of integration in $\int_{0}^{2} \int_{v}^{2} \frac{x \, dx \, dy}{x^2 + y^2}$ and hence evaluate it.

c)
$$\int_{0}^{1} \int_{0}^{\sqrt{1-x^2}} \int_{0}^{\sqrt{1-x^2-y^2}} xyz \, dz \, dy \, dx \, . \tag{7,7,6}$$

Section - B

- 5. a) Prove that $\log \left(\frac{a+ib}{a-ib} \right) = 2i \tan^{-1} \left(\frac{b}{a} \right)$.
 - b) Sum of the following series to infinity: $1 + x \cos \theta + \frac{x^2}{2!} \cos 2\theta + \frac{x^3}{3!} \cos 3\theta + \dots$
 - c) Express the function $\tan z$ into real and imaginary parts.
- 6. Solve the following differential equations: (7,7,6)

a)
$$(y^2 e^{xy^2} + 4x^3) dx + (2xy e^{xy^2} - 3y^2) dy = 0$$

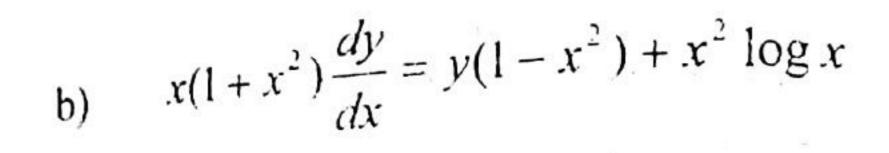
- 3. a) Show that $\beta(m,n) = \frac{\lceil (m), \lceil (n) \rceil}{\lceil (m+n) \rceil}$, where m,n>0.
 - Find the surface of the solid generated by the revolution of the ellipse $x^2 + 4y^2 = 16$ about its minor axis.
 - Find the area of a loop of the curve $r^2 = a^2 \cos 2\theta$ (7,7,6)
- 4. a) Find the total length of the curve $x^2(a^2 x^2) = 8a^2y^2$.
 - b) Change the order of integration in $\int_{0}^{2} \int_{y}^{2} \frac{x \, dx \, dy}{x^2 + y^2}$ and hence evaluate it.

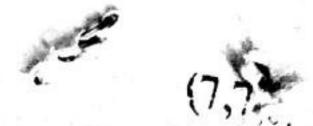
c)
$$\int_{0}^{1} \int_{0}^{\sqrt{1-x^2}} \int_{0}^{\sqrt{1-x^2-y^2}} xyz \, dz \, dy \, dx.$$
 (7,7,6)

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 $(1+e^{x})\cos x\,dx + e^{y}\sin x\,dy = 0$ c) Solve: $x^2y'' + 2xy' - 20y = (x+1)^2$.

a)

7.

- Using method of variation of parameters solve b) $y'' - 2y' + y = e^x \log x.$
- Solve: $(D^2 2D + 1)y = x \sin x$. (7,7,6)
- Find the equation of the tangent plane to the sphere 8. a) $3(x^2 + y^2 + z^2) - 2x - 3y - 4z - 22 = 0$ at the point (1,2,3).
 - b) Prove that the equation $4x^2 - y^2 + 2z^2 - 3yz + 2xy + 12x - 11y + 6z + 4 = 0$ represents a cone whose vertex is (-1,-2,-3).
 - Find the equation of the right circular cylinder whose guiding c) circle is $x^2 + y^2 + z^2 = 9$, x - y + z = 3. (7,7,6)

