(2)

Total No. of Questions-8] [Total No. of Printed Pages-4

B.E. IV Semester Examination

BE - IV/6 (B)

214545

CIVIL/MECH. ENGG

Course No. : MTH - 412

(Engg. Maths - III)

Time Allowed- 3Hours *Maximum Marks-100*

Note:- Attempt five questions in all, selecting at least two questions from each section. All questions carry equal marks . use of calculator is allowed.

SECTION - I

1. a) Find the Laplace transform of the following functions.

i) $t^2 e^{2t} \text{Cos}2t$.

ii) Cost Cos2t Cos3t

b) Find the inverse Laplace transform of the following functions.

i) $\dfrac{s}{(s^4+s^2+1)}$

ii) $\dfrac{s}{(s^2+a^2)^2}$ (10,10)

[Turn Over

2\. a) State convolution theorem for inverse Laplace transform and use it to find the inverse Laplace transform of

$$\frac{1}{(s-1)\sqrt{s}}$$

b) Evaluate

i) $\int_0^{\infty} \frac{e^{2t} - e^{3t}}{t} dt.$

ii) $L\left[e^{t} f_c \sqrt{t}\right]$

c) Using L.T techniques, solve the equation $y'' - 3y' + 2y = 4e^{2t}$, *with* $y(o) = -3,\ y'(o) = 5$ (7,6,7)

3\. a) Find L[f(t)], where f(t) is a periodic function given by

$$f(t) = \begin{cases} 1, & for\ o < t < 1 \\ -1, & for\ 1 < t < 2 \end{cases}$$

b) State final value theorem and verify it for $f(t) = t^2 e^{-at}$

c) Find the Fourier sine and cosine transform of

$$f(x) = \begin{cases} 1, & for\ o \le x < 1 \\ 0, & for\ x \ge 1 \end{cases}$$

Also apply parseval's identity to evaluate $\int_0^{\infty} \frac{\sin^2 t}{t^2} dt.$

(7,6,7)

4. **a)** Find the inverse fourier sine [illegible]

$$\bar{f}(s) = \begin{cases} e^{s}, & for\ o < s < a \\ 0, & for\ \ s \geq a \end{cases}$$

b) Find the fourier [illegible]

$$f(x) = \begin{cases} 1 - x^2 & , for\ |x| \leq 1 \\ 0 & , for\ |x| > 1 \end{cases}$$

c) Find the fourier integral [illegible]

$$f(x) = \begin{cases} 0, & for\ x < 0 \\ \frac{1}{2}, & for\ x = 0 \\ e^{-x}, & for\ x > 0 \end{cases}$$

(7, 5, 7)

Section - II

5. **a)** Prove that

$$4J_n''(x) = J_{n-2}(x) - 2J_n(x) + J_{n+2}(x) \quad for\ all\ n.$$

b) Prove that $\int_0^{\pi/2} \sqrt{\pi x}\ J_{\frac{1}{2}}(2x)\,dx = 1$

c) Prove that

$$\int_{-1}^{1} x^2 P_{n+1}(x) P_{n-1}(x)\,dx = \frac{2n(n+ \ldots)}{(2n-1)(2n+1) \ldots}$$

6. a) State and Prove Rodrigue's formula.

b) Express the function $x^3 - 5x^2 + 6x + 1$ in terms of legendre polynomial.

c) Prove that

$$\int J_5(x)\,dx = -J_4(x) - \frac{4}{x}J_3(x) - \frac{8}{x^2}J_2(x) + C$$ **(7,6,7)**

7. a) State two De - Morgan's laws in a Boolean algebra and prove any one of them.

b) Prove that every Boolean function can be expressed in DNF.

c) Draw the Circuit represented by the function $y' \vee [x \vee \{(y \vee z) \wedge x'\}] \vee z \vee [x \wedge (y' \vee z)]$.

(7,6,7)

8. a) Define a lattice. Prove that a lattice L is a distributive if and only if

$$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c) \quad \forall\, a, b, c \in L$$

b) In a Boolean algebra B, prove that $a \vee (a \wedge b) = a$ and $a \wedge (a \vee b) = a$ for a, b $\in B$

c) Write the function $f = (xy' + xz') + x'$ in DNF as well as in CNF. **(7,6,7)**