Total No. of Questions - 8] [Total No. of Printed Pages-4

# B.E. IV Semester Examination

## BE-IV/6(A)

## 28797

## CIVIL/MECH. ENGG.

## Course No. : MTH - 412

## (Engg. Maths - III)

LATEST

10/7/15

*Time Allowed- 3 Hours* *Maximum Marks-100*

***Note :*** Attempt **five** questions in all selecting at least two questions from each section. All carry equal marks. Use of calculator is allowed.

### SECTION - I

1. a) Express the function f(t) in terms of unit step function and then find its L.T., where $f(t)=\begin{cases} 1 & , \quad 0\le t<\frac{1}{2} \\ t-1, & \frac{1}{2}\le t<1 \\ 0 & , \quad t\ge 1 \end{cases}$

b) Find L.T. of $\int_0^t \frac{e^{2x}\sin 3x}{x}dx$

c) State initial value and final value theorems. Also verify these for the function: $f(t)=2t^2e^{-3t}+4.$ **(7, 7, 6)**

2. a) Using L.T. techniques, solve the differential equations:

$$y''' - 2y'' + 5y' = 0 \text{ with } y(0) = 0, y'(0) = 1 \text{ and } y\left(\frac{\pi}{8}\right) = 1.$$

b) State the convolution theorem for $L^{-1}$ and apply it to evaluate the inverse L.T. of $\frac{1}{(s-2)(s+1)^4}$.

c) Evaluate L.T. of cosh 3t, sin 2t.

(7, 7, 6)

3. a) Find the Fourier integral representation of the function:

$$f(x) = \begin{cases} e^{-3x} & , x \geq 0 \\ 0 & , x < 0 \end{cases}$$

b) State Parseval's identities. Hence or otherwise evaluate

$$\int_0^\infty \frac{dt}{(t^2+4)(t^2+9)}$$

c) Find L $[f(t)]$, where $f(t)$ is a periodic function of $t$ with period 2 :

$f(t) = t^2$ for $0 \leq t < 2$ and

$f(t+2) = f(t)$ for all $t \geq 0$

(7, 7, 6)

4. a) Find the Fourier sine transform of $f(x) = \frac{1}{x(x^2+a^2)}$.

b) Find the inverse cosine transform of the function:

$$\bar{f}(s)=\begin{cases} s^2-a^2, & 0\le s<a \\ 0 & , s\ge a \end{cases}$$

c) State and prove convolution property for Fourier transforms.

**(7, 7, 6)**

## SECTION - II

5. a) Define Bessel's function of order $\alpha$ and of I[st] kind. Also prove the following relations:

i) $J_n^1(x)+\frac{n}{x}J_n(x)=J_{n-1}(x)\ \forall_n$

ii) $\frac{2n}{x}J_n(x)=J_{n-1}(x)+J_{n+1}(x)\ \forall_n$

b) Evaluate:

$$\int J_5(x)\,dx=-J_4(x)-\frac{4}{x}J_3(x)-\frac{8}{x^2}J_2(x)+c$$

c) Evaluate: $\int_{-1}^{1}x^5p_3(x)\,dx$ and $\int_{-1}^{1}x^3p_4(x)\,dx$

**(7, 7, 6)**

6. a) Define lattice with an example. Also prove the following in equality: $a\wedge(b\vee c)\ge(a\wedge b)\vee(a\wedge c)$

b) Define DNF and CNF of a Boolean function with examples. Also express the function:

[Turn Over

$f(x,y,z)=\left[x^1 \vee \left(y^1 \wedge z\right)\right]^1$ in DNF and CNF.

b) State and prove cancellation property in a distributive lattice.

(7, 7, 6)

7. a) Show that $\int_{-1}^{1} x\, p_n(x)\, p_{n-1}(x)\, dx = \frac{2n}{4n^2-1}$.

b) Using Ja Cobi series, prove that

$J_0^2(x)+2\left[J_1^2(x)+J_2^2(x)+J_3^2(x)+\ldots\ldots\right]=1.$

c) State and prove associative law for meet is a lattice

(7, 7, 6)

8. a) Prove the following:

i) $p_n(-x)=(-1)^n p_n(x)$.

ii) $p_{2n}(0)=(-1)^n \frac{(2n)!}{2^{2n}(n!)^2}$.

iii) $J_{-n}(x)=(-1)^n J_n(x)\ \forall_n$.

iv) $2J_0''(x)=J_2(x)-J_0(x)$.

b) Using axioms of Boolean algebra, express the function:

$f(x,y,z)=x \vee \left[\left(y \wedge z'\right)' \wedge (x \vee y)\right]$ in DNF and CNF. Also then draw their circuits.

(10, 10)

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