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B.E. IV Semester Examination

BE-IV/11(B)

237726

CIVIL/MECH. ENGG.

Course No. MTH - 412

(Engg. Maths - III)

Time Allowed-3 Hours *Maximum Marks-100*

Note : Attempt **five** questions in all, selecting at least two questions from each section. Use of calculator is allowed.

Section - I

1. a) Find the Laplace transforms of the following functions

i. $\dfrac{e^{3t}\sin^2 t}{t}$

ii. $t^2\cosh 2t$

b) Find the Inverse L. T. of the following functions.

i. $\dfrac{5S^2-15S-11}{(S+1)(S-2)^3}$

ii. $\dfrac{S}{(S^2+4)^2}$ (5×4)

2. a) State and prove initial value theorem. Also verify it for the function $f(t)=t^3e^{-2t}$

b) Using L.T. Technique, solve

$y''+4y'+5y=e^{-2t}(\cos t-\sin t)$ with $y(0)=1$ and $y'(0)=-3$

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(7,7,6)

c. Evaluate : $\int_0^\infty e^{-t} erf \sqrt{t}\, dt$.

3. a. State convolution theorem for L^{-1} and use it to. Find the inverse L.T. of $\frac{1}{(S-1)^5(S+2)}$

b. Find the Fourier integral representation of the function

$$f(t)=\begin{cases}\sin t, & |t|\le\pi\\ 0, & otherwise\end{cases}$$

c. Find the Fourier cosine transform of $e^{-x^2/2}$ (7,7,6)

4. a. Solve the integral equation :

$$\int_0^\infty f(x)\sin ax\,dx=\begin{cases}a^2-1, & a\in[0,1]\\ 0, & otherwise\end{cases}$$

b. State Parseval's identities for F_s and F_c. Apply these to evaluate $\int_0^\infty \frac{(1-\cos t)^2}{t^2}dt$ and $\int_0^\infty \frac{\sin^2 t}{t^2}dt$, where

$$f(x)=\begin{cases}1, & 0\le x\le 1\\ 0, & x>1\end{cases}$$

c. Evaluate inverse Fourier sine and cosine transforms of the function: $\bar{f}(s)=\begin{cases}s, & 0\le s<1\\ s-3, & 1\le s<3\\ 0, & s\ge 3\end{cases}$ (7,7,6)

Section - II

5. a. Define Bessel's function of order n and of Ist kind. Also prove that $J_{-n}(x) = (-1)^n J_n(x)$, when n is an integer.

b. Prove that

$$\int J_5(x)dx = -J_4(x) - \frac{4}{x}J_3(x) - \frac{8}{x^2}J_2(x) + c$$

c. Using Jacobi series. prove that

i. $Cos(x\cos\theta) = J_0(x) - 2J_2(x)\cos 2\theta + 2J_4(x)\cos 4\theta - \ldots\ldots$

ii. $Sin(x\cos\theta) = 2[J_1(x)\cos\theta - J_3(x)\cos 3\theta + J_5(x)\cos 5\theta - \ldots\ldots]$ **(7,7,6)**

6. a. For distinct positive integers m and n. show that $\int_{-1}^{1}(1-x^2)P_m{}'(x)P_n{}'(x)dx = 0$

b. Find the Legendre - Fourier series of the function

$$f(x) = \begin{cases} 0, & -1 \le x < 0 \\ x, & 0 \le x \le 1 \end{cases}$$

c Evaluate : $\int_{-1}^{1} x^5 P_4(x)dx$ and $\int_{-1}^{1} x^6 P_4(x)dx$ **(7,7,6)**

7. a. Define distributive lattice with an example. Also prove that a lattice L is distributive if

$$(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) = (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$$

for any elements a, b. c, of L.

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b. If L is a modular Lattice, prove that $(a \vee b) \wedge c = b \wedge c$ implies that $(c \vee b) \wedge a = b \vee a$ for any elements a, b, c of L.

c. State and prove cancellation law in a modular lattice.

(7,7,6)

a. Define Boolean algebra with an example. Also prove that

i. $x'' = x$

ii. $x \leq y$ *iff* $x \wedge y' = 0$

b. Define DNF and CNF of a Boolean function. Also express the function $f(x, y, z) = \left[(x \vee y) \wedge z'\right]'$ in CNF and draw its circuit.

c. Write tabular form of the function

$f(a, b, c) = (a \wedge b' \wedge c) \vee (a' \vee b)' \vee c$ Also express it in DNF and CNF with circuit representation. **(7,7,6)**

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