Total No. of Questions-8] [Total No. of Printed Pages-4

B.E IV Semester Examination

BE-IV/6(A) Faizan (10 56/ 2014)

215236

CIVIL/MECH. ENGG.

Course No. : MTH - 412

(Engg. Maths. - III)

Time Allowed-3 Hours *Maximum Marks-100*

Note : Attempt any five questions selecting at least two questions from each section. All carry equal marks. Use of calculator is allowed.

Section - I

1. a) Find the Laplace transform of the following functions :

i) $e^{-3t}[t^4 - 2t^{3/2} + 2\sin 3t - 3\cos 3t + 4]$.

ii) $\int_o^t x^2 . \sin(x-t)dx$

b) Find the inverse Laplace transform of the following functions.

i) $\log\left(\frac{S^2+9}{S^2+4}\right)$

ii) $\frac{10}{(S^2+9)^2}$ **(10,10)**

2. a) State and prove initial value theorem. Also verify it for the function $f(t) = t^5 e^{-4t}$

b) Describe Laplace transform of derivatives and use it to find L.T. of $\frac{\cos\sqrt{t}}{\sqrt{t}}$

c) Using L.T. techniques. Solve the equation: $(y'' - y' - 2)y = 20 \sin 2t$ with $y(0) = -1$ and $y'(0) = 2$.

(7,7,6)

3. a) Using L.T. Evaluate the integral $\int_0^\infty te^{-2t} \cos t \, dt$

b) Derive the Fourier integral representation of the function

$$f(x) = \begin{cases} 2, & 0 \le x \le 5 \\ 10, & 7 \le x \le 15 \\ 0, & \textit{Otherwise} \end{cases}$$

c) Determine the inverse Fourier sine and cosine transforms of e^{-as}, *where* $0 < s < \infty$

(7,7,6)

4. a) Find the Fourier sine transform of the function : $\frac{x}{x^2+1}$

b) Using Parseval's identity for F.T., evaluate $\int_0^\infty \frac{dx}{(x^2+1)(x^2+4)}$

c) Define convolution product of two functions in $(-\infty, \infty)$ Also state and prove convolution theorem for Fourier transforms. (7,7,6)

Section - II

5. a) Define Bessel's function of order n and prove that $J_{-n}(x) = (-1)^n J_n(x)$, When n is integer. Is this result true when n is not an integer.

b) State and Prove orthogonality of legendre polynomials.

c) Express $J_5(x)$ in terms of $J_0(x)$ and $J_1(x)$ (7,7,6)

6. a) Prove that $\int_{-1}^{1} x^k . P_n(x)\ dx = 0$ for k = 0,1,2,...,n-1 and also deduce that $\int_{-1}^{1} P_m(x) P_n(x) dx = 0$ *for* $m \neq n$.

b) Using Jacobi series, prove that

$$Sin\ x = 2[J_1(x) - J_3(x) + J_5(x) - .....]$$ and

$$\cos x = J_o(x) - 2J_2(x) + 2J_4(x) - .....]$$

c) Express $f(x) = 4x^4 + 4x^3 - 2x + 5$ in terms of legendre's polynomials. (7,7,6)

7. a) Define a distributive lattice with an example Also prove associative property for the meet '$\wedge$'

b) Let L be a distributive lattice then prove that

$(x \wedge y) \vee (y \wedge z) \vee (z \wedge x) = (x \vee y) \wedge (y \vee z) \wedge (z \vee x)$

for any $x, y, z \in L$.

c) Prove that $P_n'(1) = \frac{n(n+1)}{2}$ and

$$P_n'(-1) = (-1)^{n-1} \frac{n(n+1)}{2}.$$ (7,7,6)

8. a) Define a Boolean algebra with an example. Also prove a De - Morgan law in a Boolean algebra.

b) Using axioms of a Boolean algebra, express the function f in C N F, where $f = (a \vee b) \wedge [(b \wedge c') \vee (a' \wedge c)]$

c) Write tabular form of the function f and express it in DNF, where $f = [(a \vee b \vee c') \wedge (a' \wedge c')] \vee c'$. (7,7,6)