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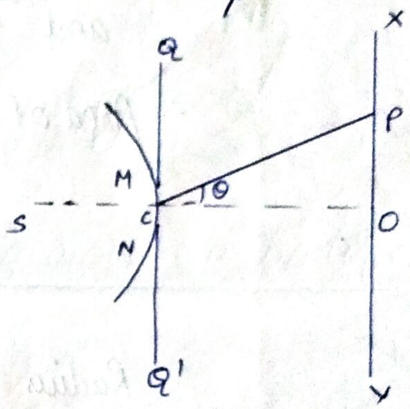
Diffraction

The phenomenon of bending of light wave around corners and their spreading into geometric shadow of object is called diffraction.

Fresnel's theory of rectilinear propagation of light

Fresnel considered that diffraction phenomenon is caused by the interference of the inumerable 2° wavelets produced by the unobstructed portions of the same wavefront.

According to Fresnel, the resultant effect at an external point due to a wavefront will depend on the factors given below.

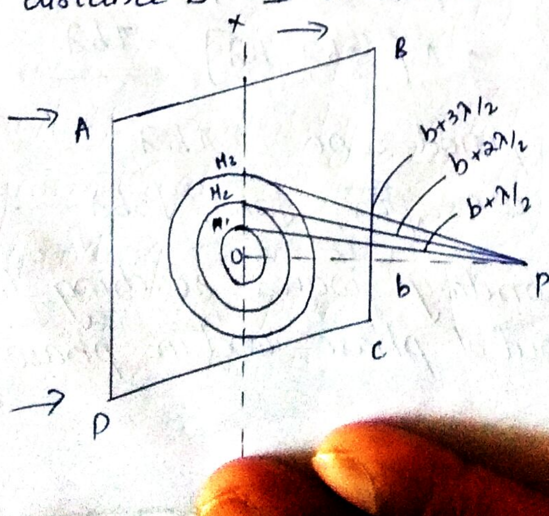


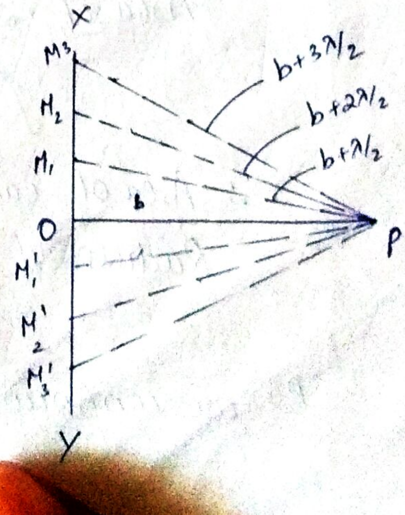
a) A wavefront can be divided into a large number of strips or zones called Fresnel's zones of small area

b) The effect at a point due to any particular zone will depend on the distance of the point from the zone.

c) The effect at P will depend on the obliquity of the point with reference to the zone under consideration.

Consider a plane wave front ABCD ⊥r to the plane of the paper and P is an external point at a distance 'b' ⊥r to ABCD.





To find the resultant intensity at P divide the wa
into Fresnel zones. If spheres are constructed with P
as centre and radii equal to $b+\lambda/2$, $b+2\lambda/2$, $b+3\lambda/2$
etc they will cut out circular areas of radii
OM_1, OM_2, OM_3 etc on the wave front. These circular
zones are called <u>half period zones</u>.

The area of the 1st circle is called 1st half period zone. The area b/w 1st & 2nd circle is called 2nd half period zone and so on. Each zone differs from its neighbour by a phase difference of π or path difference $\lambda/2$, hence called half period zones.

Here, $OP = b$, $OM_1 = r_1$, $OM_2 = r_2$ ....

and $M_1P = b+\lambda/2$, $M_2P = b+2\lambda/2$, ...

$\therefore$ Area of 1st H.P.Z $= \pi\, OM_1^2$

$$= \pi\left[M_1P^2 - OP^2\right] = \pi\left[\left(b+\frac{\lambda}{2}\right)^2 - b^2\right]$$

$$= \pi\left[b^2 + b\lambda + \lambda^2 - b^2\right] \simeq \pi b\lambda \qquad \lambda \ll 1$$

Radius of 1st HPZ $= \sqrt{b\lambda}$

~~Area of 2nd HPZ~~ $= \cancel{\pi\left(OM_2^2 - OM_1^2\right)}$

$= \pi[$

Radius of 2nd HPZ $= OM_2 = \left[M_2P^2 - OP^2\right]^{1/2}$

$$= \left[(b+\lambda)^2 - b^2\right]^{1/2}$$

$$= \sqrt{2b\lambda}$$

$\therefore$ Area of 2nd HPZ $= \pi\left[OM_2^2 - OM_1^2\right]$

$$= \pi\left[2b\lambda - b\lambda\right] = \pi b\lambda$$

$\therefore$ Area of each half period zone $= \pi b\lambda$

Radius of HPZ are $\sqrt{b\lambda}$, $\sqrt{2b\lambda}$, $\sqrt{3b\lambda}$ ...

The secondary waves reaching the point P are continuously out of phase and in phase with

with reference to the 1st half period zone. Let $m_1, m_2, m_3 \ldots$ be the amplitudes of the disturbance at P due to the 1st, 2nd, 3rd ... zones.

As the amplitudes are of gradually decreasing magnitude, the amplitude of vibration at P due to any zone can be approximately taken as the mean of the amplitudes due to the zone preceeding and succeeding it.

$$\therefore A = m_1 - m_2 + m_3 - m_4 + \ldots m_n$$

$$A = \frac{m_1}{2} + \left[\frac{m_1}{2} - m_2 + \frac{m_3}{2}\right] + \left[\frac{m_3}{2} - m_4 + \frac{m_5}{2}\right] + \ldots$$

but $m_2 = \frac{m_1 + m_3}{2}$, $m_4 = \frac{m_3 + m_5}{2} \ldots$

$$\therefore A = \frac{m_1}{2}$$

The intensity is proportional to the square of the amplitude,

$$I = \frac{m_1^2}{4}$$

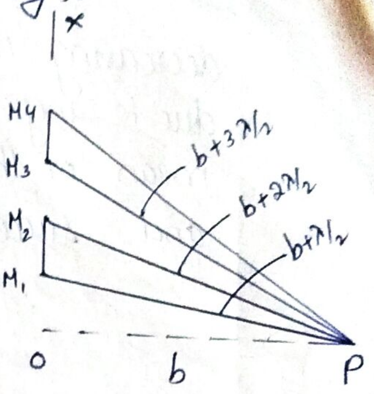
Thus the intensity at P is only 1/4th of that due to the 1st half period zone alone. A small obstacle of the size of half the area of the 1st half period zone placed at O will screen the effect of the whole wavefront and the intensity at P become zero; which shows the rectilinear propagation of light.

<u>Zone plate</u>

A zone plate is a specially constructed screen such that light is obstructed from every alternate zone. It can be designed so as to cut off light due to the even or odd numbered zone.

To construct a zone plate, concentric circles are drawn on white paper such that the radii are proportional to the square roots of the natural no:s.

odd numbered zones are covered with bla[ck] and a reduced photograph is taken. In the devel[oped] negative, odd zones are transparent to incident light and the even zones will cut off light.



If such a plate is held ⊥ to an incident beam of light and a screen is moved on the other side to get the image, it will be observed that maximum brightness is possible at some position of the screen from the zone plate.

$$r_n = \sqrt{nb\lambda} \Rightarrow b = \frac{r_n^2}{n\lambda}$$

If the source is at larger distance, even nos zones cutoff the light and the resultant amplitude at P is

$$A = m_1 + m_3 + m_5 + \ldots$$

∴ The focal length canbe written as

$$\boxed{f_n = b = \frac{r_n^2}{n\lambda}}$$

⇒ Zone plate has different foci for different wavelength.
∴ If a parallel beam of white light is incident on the zone plate, different colours come to focus at different points along the line OP.

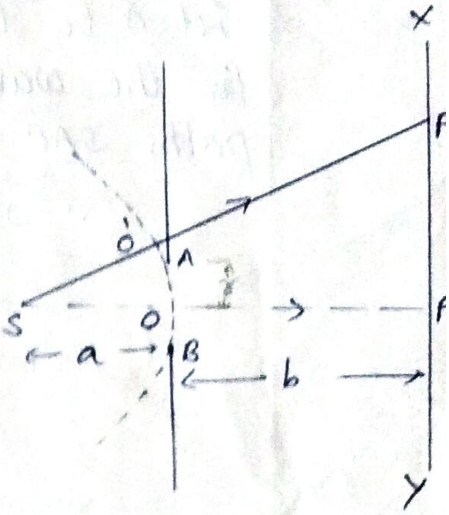
Types of diffraction

1) Fresnel diffraction - The source of light and the screen are effectively at finite distances from the obstacle. so this phenomenon doesnot require any lenses. The incident wave-front is either spherical or cylindrical.

2) Fraunhoffer diffraction :- The sourse of light and the screen are effectively at infinite distances from the

obstacle. The condition for this is achieved using two convex lenses, one to make the light from the source parallel and the other to focus the light after diffraction on to the screen.

Fresnel's diffraction at a circular aperture



Let AB be a small aperture and S is a point source of monochromatic light. XY is a screen. O is the centre of the aperture and 'r' is radius of the aperture. Let the distance of a source from the aperture be 'a' and the distance of the screen from the aperture be 'b'.

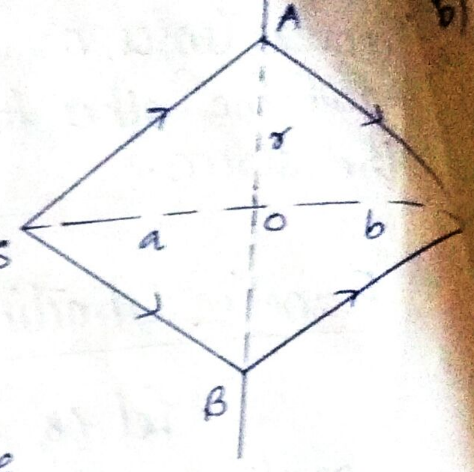
Results

1) Depending on the distance of P from the aperture, the number of HPZ that can be constructed may be odd or even. If even HPZs are exposed, the intensity at P will be minimum $(m_1 - m_2)$. If odd HPZs are exposed, the intensity at P will be maximum $(m_1 - m_2 + m_3)$

2) The no:- of HPZs can also be altered by changing the width of aperture, which in effect also causes maximum & minimum intensity at P.

3) To find intensity at P', we can construct HPZs by keeping O' as centre. There also if even zones are exposed intensity will be minimum and if odd zones are exposed intensity will be maximum. If the point P' happens to be of maximum intensity, then all the points lying on a circle of radius PP' on the screen also will be of maximum intensity. Thus with a circular aperture, the diffraction pattern will be concentric bright & dark rings with centre P.

Mathematical treatment:



a) Intensity at central point P :-

SO = a

OP = b , OA = r

Let δ be the path difference for the wave reaching P along the paths SAP and SOP.

$$\therefore \delta = SA + AP - SOP$$

$$= (a^2+r^2)^{1/2} + (b^2+r^2)^{1/2} - (a+b)$$

$$= a\left[1+\frac{r^2}{a^2}\right]^{1/2} + b\left[1+\frac{r^2}{b^2}\right]^{1/2} - a - b$$

$$= a\left[1+\frac{r^2}{2a^2}\right] + b\left[1+\frac{r^2}{2b^2}\right] - a - b$$

$$\delta = \frac{r^2}{2a} + \frac{r^2}{2b} = \frac{r^2}{2}\left[\frac{1}{a}+\frac{1}{b}\right]$$

$$\therefore \frac{1}{a}+\frac{1}{b} = \frac{2\delta}{r^2} \quad \text{———(1)}$$

If the position of the screen is such that n full no. of half period zones can be constructed on the aperture,

$\delta = \frac{n\lambda}{2} \Rightarrow 2\delta = n\lambda$

$$\therefore (1) \Rightarrow \boxed{\frac{1}{a}+\frac{1}{b} = \frac{n\lambda}{r^2}}$$

The point P will be of maximum or minimum intensity depending on whether n is odd or even.

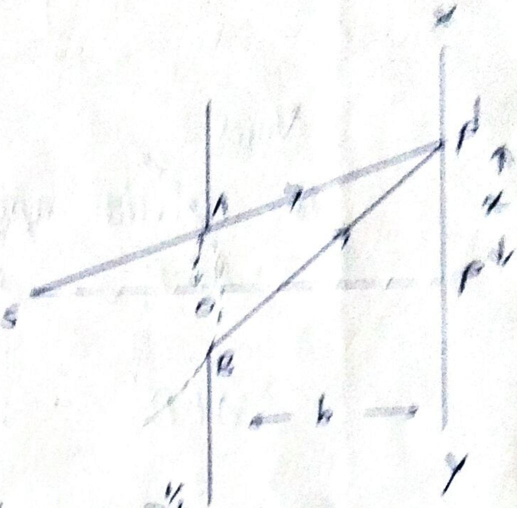
If the source is at infinite distance,

$a = \infty$ $\quad \frac{1}{\infty}+\frac{1}{b} = \frac{n\lambda}{r^2}$

$$\frac{1}{b} = \frac{1}{f} = \frac{n\lambda}{r^2}$$

b) Intensity at a point away from the centre :-

Let $PP' = x$, $OP = b$, $OA = r$



The path difference b/w the secondary waves from A & B and reaching P' is

$$\delta = BP' - AP'$$

$$= \sqrt{b^2 + (x+r)^2} - \sqrt{b^2 + (x-r)^2}$$

$$= b\left[1 + \frac{(x+r)^2}{b^2}\right]^{1/2} - b\left[1 + \frac{(x-r)^2}{b^2}\right]^{1/2}$$

$$= b\left[1 + \frac{(x+r)^2}{2b^2}\right] - b\left[1 + \frac{(x-r)^2}{2b^2}\right]$$

$$= \frac{(x+r)^2}{2b} - \frac{(x-r)^2}{2b} = \frac{1}{2b}\left[(x+r)^2 - (x-r)^2\right]$$

$$= \frac{1}{2b}\left[x^2 + 2xr + r^2 - (x^2 - 2xr + r^2)\right]$$

$$\delta = \frac{1}{2b}\left[4xr\right] \quad \text{—(1)}$$

The point P' will be dark if the path diff.

$$\delta = 2n\,\lambda/2 \quad (\text{even } \cancel{zo} \text{ no. of zones})$$

$$\frac{2n\lambda}{2} = \frac{2r x_n}{b}$$

or $$\boxed{x_n = \frac{nb\lambda}{2r}}$$ $x_n \rightarrow$ radius of n^{th} dark ring

The point P' will be bright if,

$$\delta = (2n+1)\frac{\lambda}{2}$$

$$\therefore (2n+1)\frac{\lambda}{2} = \frac{2r x_n}{b}$$

$$\boxed{x_n = \frac{(2n+1)b\lambda}{4r}}$$ $x_n \to$ radius of bright

Let ... wave f... strip

Notes

For circular aperture:

- At P ; $\frac{1}{a}+\frac{1}{b} = \frac{n\lambda}{r^2}$ } $n \to$ even $\to$ dark; $n \to$ odd $\to$ bright

- At P' ; $x_n = \frac{nb\lambda}{2r}$ for P' to be dark

$x_n = \frac{(2n+1)b\lambda}{4r}$ for P' to be bright

Q) A parallel beam of monochromatic light of $\lambda = 5000 A°$ is incident normally on a plate having a circular hole of diameter 1mm. The screen is at the farthest position for which the axial point is almost black. The screen is moved towards the plate so that the axial point is again seen black. How far is the screen moved from the first position to the second.

$\lambda = 5000 A°$

$D = 1mm$

$n_1 = 2$, $n_2 = 4$

$\frac{1}{b_1} = \frac{n_1\lambda}{r^2} \Rightarrow b_1 = \frac{(0.5\times10^{-3})^2}{2\times5000\times10^{-10}} = \underline{\underline{0.25 m}}$

$\frac{1}{b_2} = \frac{n_2\lambda}{r^2} \Rightarrow b_2 = \frac{(0.5\times10^{-3})^2}{4\times5000\times10^{-10}} = \underline{\underline{0.125m}}$

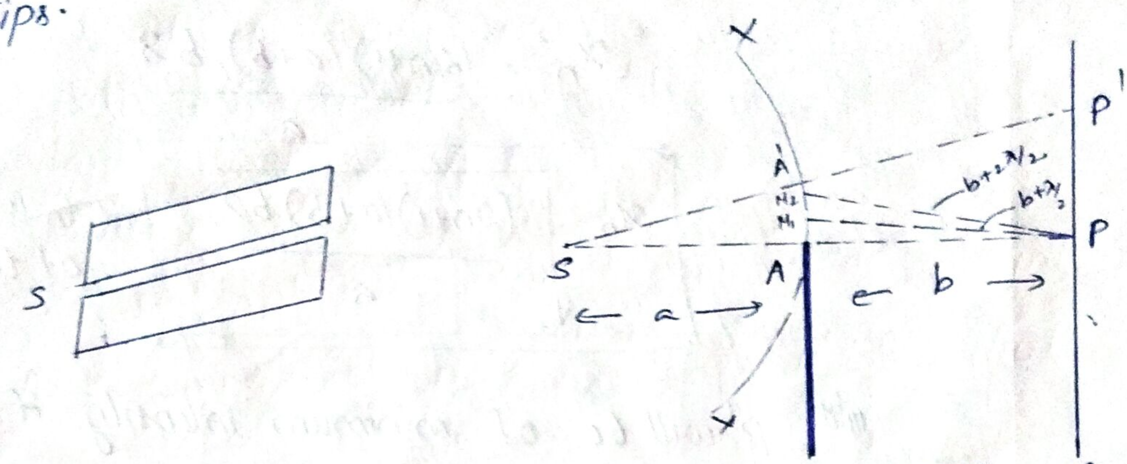
$b_1 - b_2 = \underline{\underline{0.125m}}$

Diffraction pattern due to a straight edge

Let 's' be narrow slit. AD is the straight edge and XY is the incident cylindrical wavefront. P is a point on screen. Below the point P is the geometrical shadow and above P is the illuminated portion.

Let the distance AP be b. Wrt to point P, the wave front can be divided into a nos of half period strips.

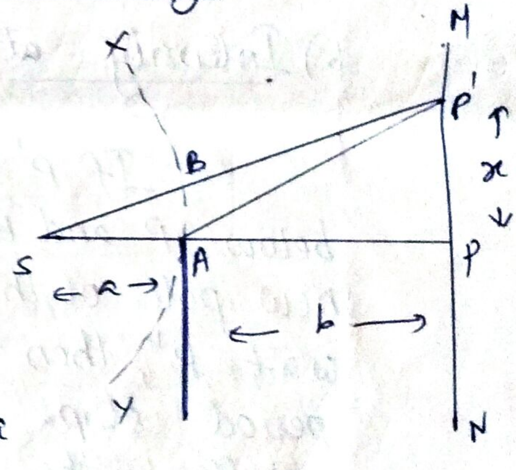
(5)



AM_1, M_1M_2, M_2M_3 etc measure the thickness of the 1st, 2nd, 3rd half period strips.

Let P' be a point on the screen in the illuminated portion. To calculate the resultant effect at P' due to the wave front XX, join S to P'.

a) <u>Position of maximum & minimum intensity :-</u>



$SA = a$, $AP = b$ $P'P = x$.

Path difference $\delta = AP' - BP'$

$$= (b^2+x^2)^{1/2} - (SP' - SB)$$

$$= (b^2+x^2)^{1/2} - \left[(a+b)^2+x^2\right]^{1/2} + a$$

$$= b\left[1+\frac{x^2}{b^2}\right]^{1/2} - (a+b)\left[1+\frac{x^2}{(a+b)^2}\right]^{1/2} + a$$

$$= b\left[1+\frac{x^2}{2b^2}\right] - (a+b)\left[1+\frac{x^2}{2(a+b)^2}\right] + a$$

$$= \frac{x^2}{2b} - \frac{x^2}{2(a+b)} = \frac{x^2}{2}\left[\frac{1}{b} - \frac{1}{(a+b)}\right]$$

$$\delta = \frac{x^2}{2}\left[\frac{a+b-b}{b(a+b)}\right] = \frac{x^2}{2}\frac{a}{b(a+b)} \quad \text{—(1)}$$

The point P' will be of maximum intensity if $\delta = (2n+1)\frac{\lambda}{2}$

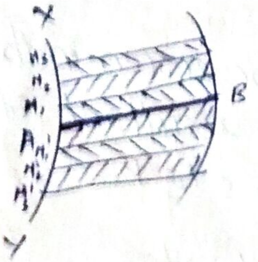
$$(2n+1)\frac{\lambda}{2} = \frac{x_n^2 a}{2b(a+b)}$$

$$x_n^2 = \frac{(2n+1)(a+b)b\lambda}{a}$$

$$\boxed{x_n = \sqrt{\frac{(2n+1)(a+b)b\lambda}{a}}}$$

x_n is the dis[tance] of the n^{th} bri[ght band]

M^{ly} P' will be of minimum intensity if $\delta = 2n\frac{\lambda}{2}$



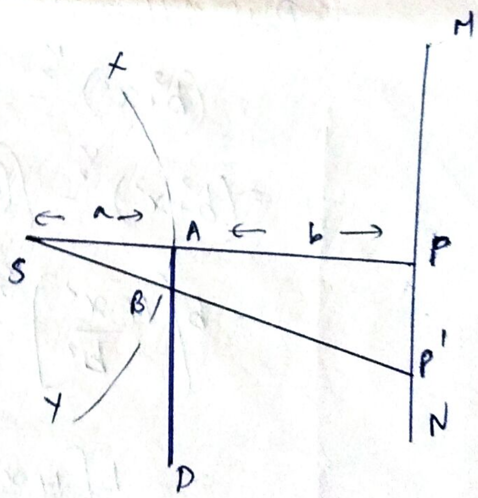
$$2n\frac{\lambda}{2} = \frac{a x_n^2}{2b(a+b)}$$

$$\boxed{x_n = \sqrt{\frac{2n(a+b)b\lambda}{a}}}$$

x_n is the distance of n^{th} dark band from P.

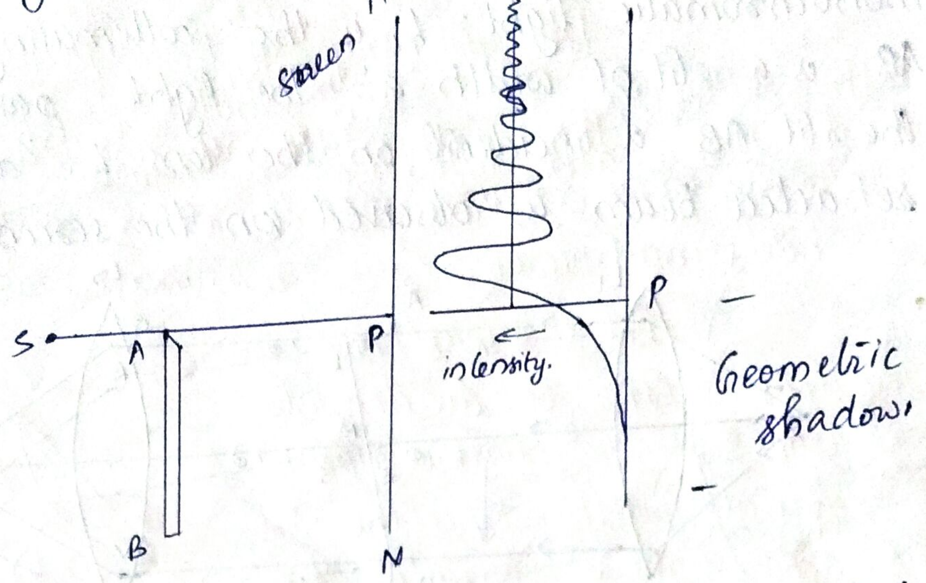
b) Intensity at a point inside the geometrical shadow.

If P' is a point below P and B is the new pole of the wave front w.r.t P', then the half period strips below B are cutoff by the obstacle and only the uncovered half period strips above B will be effective in producing the illumination at P'.



As P' moves farther from P, more no. of half period strips above B is also cutoff and the intensity gradually falls. Thus within the geometrical shadow, the intensity gradually falls off depending on the position of P' w.r.t P.

The intensity distribution on the screen due to a straight edge is shown below.



Q) In a particular experiment $\lambda = 600nm$, the distance b/w the slit source and the straight edge is 6m & b/w the edge & eyepiece is 4m. Calculate the position of the 1st 3 maxima & their seperation.

$$x_n = \sqrt{\frac{b(a+b)(2n+1)\lambda}{a}}$$

$a = 6m$, $b = 4m$, $\lambda = 600 \times 10^{-9} m$

$$\therefore x_n = \sqrt{\frac{4 \times 10 \times (2n+1) 600 \times 10^{-9}}{6}}$$

$$= \sqrt{4 \times 10^{-6} (2n+1)}$$

$$x_n = 2 \times 10^{-3} \sqrt{(2n+1)}$$

$\therefore$ Position of 1st maxima $=$, $n=0$; $\underline{2 \times 10^{-3} m}$

" " 2nd " $n=1$ $\underline{3.46 \times 10^{-3} m}$

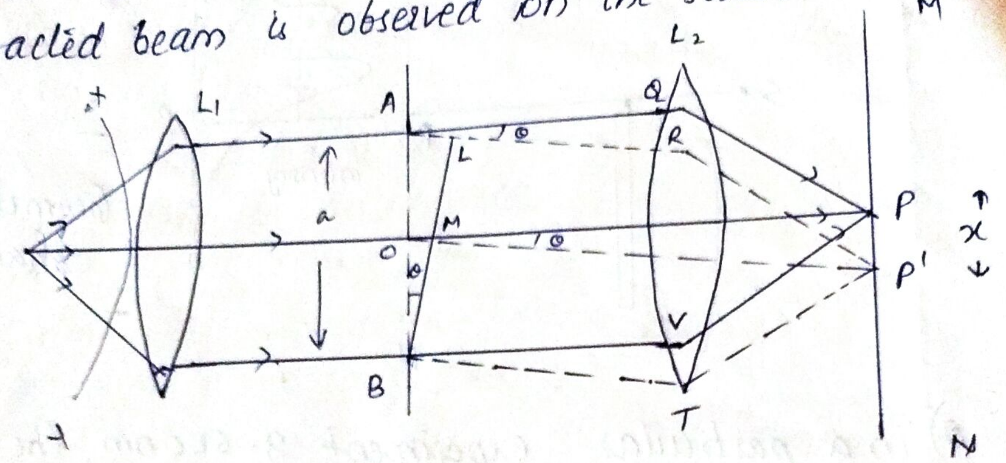
" " 3rd " $n=2$ $= \underline{4.47 \times 10^{-3} m}$

$x_2 - x_1 = 1.46 \times 10^{-3} m$

$x_3 - x_2 = 1.01 \times 10^{-3} m.$

Fraunhoffer diffraction at a single slit.

S is a narrow slit illuminated by monochromatic light. L_1 is the collimating lens. AB is a slit of width 'a'. The light passing through the slit AB is incident on the lens L_2 and the final refracted beam is observed on the screen MN.



A plane wave front is incident on the slit AB and each point on this wave front is a source of 2° disturbance. The 2° waves travelling in the direction ||el to OP come to focus at P and a bright central image is observed. The 2° waves from points equidistant from O and situated in the upper and lower ~~point~~ halves OA & OB of the wave front travel the same distance in reaching P and hence the path difference is zero. The 2° waves reinforce one another and P will be a point of maximum intensity.

Now consider 2° waves traveling in the direction AR, inclined at an angle θ to OP. All the 2° waves traveling in this direction reach the point P' on the screen. The point P' will be of maximum or minimum intensity depending on the path diff. b/w the 2° waves originating from the corresponding points of the wavefront. Draw a $\perp^r$ BL to AR.

In ΔABL, $\sin\theta = \frac{AL}{AB} = \frac{AL}{a}$ (7)

$$AL = \sin\theta \quad \text{——} (1)$$

$AL \rightarrow$ path difference.

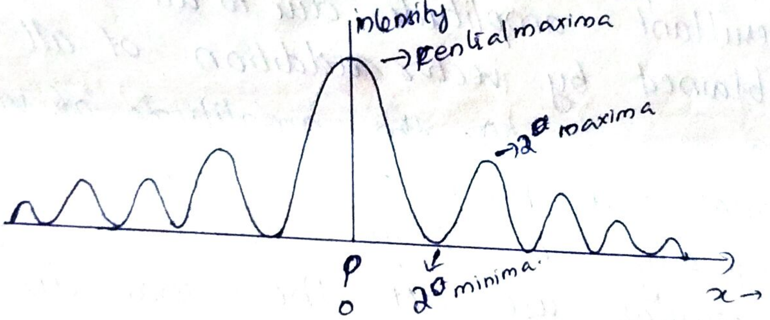
If the path difference is $n\lambda$, destructive interference takes place, on the other hand, if AL is an odd multiple of $\lambda/2$, 2° maxima is obtained.

$$a\sin\theta = n\lambda \quad \text{——} (2) \rightarrow \text{Condition for minima}$$

$$a\sin\theta = \frac{(2n+1)\lambda}{2} \quad \text{——} (3) \rightarrow \text{Condition for maxima.}$$

$n = 1, 2, 3, \ldots$

Thus the diffraction pattern due to a single slit consists of a central bright maximum at P followed by 2° maxima and minima on both sides.



If the lens L_2 is very near the slit or the screen is far away from the lens L_2 then,

$$\sin\theta = x/f \qquad f \rightarrow \text{focal length of } L_2$$

$$\text{but } \sin\theta = \frac{\lambda}{a} \qquad n=1$$

$$\therefore \frac{\lambda}{a} = \frac{x}{f}$$

$$x = \frac{f\lambda}{a} \qquad x \rightarrow \text{distance of secondary minimum from P}$$

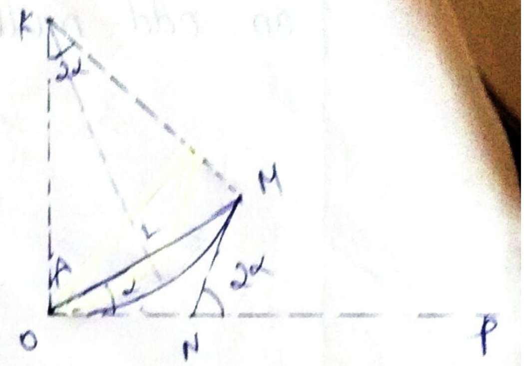
$\therefore$ Width of central maximum,

$$\boxed{W = 2x = \frac{2f\lambda}{a}}$$

Intensity distribution in diffraction pattern due to a single slit:

$\lambda \to 2\pi$
$1 \to \frac{2\pi}{\lambda}$
$a\sin\theta \to \frac{2\pi a\sin\theta}{\lambda}$

The intensity at P depends on the path difference 'δ' b/w waves originating from A & B. Here path difference is $a\sin\theta$. ∴ Corresponding phase difference is $\frac{2\pi}{\lambda} a\sin\theta = 2\alpha$



Let the wavefront incident of AB be divided into a large no. of small strips of equal width. Since the incident light is parallel, the amplitude of the wave from each strip can be taken to be the same, but the initial phase will differ by a const amount. The resultant amplitude due to all the strips can be obtained by vector addition of all amplitudes.

As the amplitude of vibration are equal and small and the phase diff. increases regularly from strip to strip, the polygon tends to a circular arc OM. The final direction of the vector is MN and initial direction is OP. 2α is the phase diff. b/w 2° waves from the extreme ends of the slits. Chord OM represents the resultant amplitude

In ΔOKL $\quad \sin\alpha = \frac{OL}{r}$

$\therefore OL = r\sin\alpha$ $\quad$ $r \to$ radius of circular arc.

Chord $OM = 2\,OL = 2r\sin\alpha$ —— (1)

$2\alpha = \frac{\text{Arc } OM}{\text{radius}} = \frac{A_0}{r}$ $\quad$ $A_0 \to$ ~~resultant amplitude~~

$\therefore 2r = \frac{A_0}{\alpha}$

Resultant amplitude $= \frac{A_0 \sin\alpha}{\alpha}$

∴

$\because$ Resultant intensity $= \dfrac{A_0^2 \sin^2\alpha}{\alpha^2}$

or $\boxed{I = I_0 \dfrac{\sin^2\alpha}{\alpha^2}}$

(a) Central maximum :-

For the point P on the screen, $\theta = 0,\ \alpha = 0$

$\therefore \lim_{\alpha \to 0} \dfrac{\sin\alpha}{\alpha} = 1$

$\therefore I = I_0 \rightarrow$ maximum

(b) Secondary Maxima :-

$\sin\theta_n = \dfrac{(2n+1)\lambda}{2a}$ $\quad\left(2\alpha = \dfrac{2\pi a \sin\theta}{\lambda}\right)$

but we have $\alpha = \dfrac{\pi a \sin\theta}{\lambda}$

$\therefore \alpha = \dfrac{\pi a}{\lambda} \dfrac{(2n+1)\lambda}{2a} = \dfrac{(2n+1)\pi}{2}$

Sub. $n = 1, 2, 3, \ldots$ $\alpha = \dfrac{3\pi}{2}, \dfrac{5\pi}{2}, \dfrac{7\pi}{2}$ e.t.c

(i) For the 1^{st} 2° maximum, $\alpha = \dfrac{3\pi}{2}$

$I = I_0 \left(\dfrac{\sin\alpha}{\alpha}\right)^2 = I_0 \left(\dfrac{\sin 3\pi/2}{3\pi/2}\right)^2 = \dfrac{(-)^2 I_0 (2)^2}{(3\pi)^2} = \dfrac{I_0}{22}$

(ii) For the 2^{nd} 2° maximum $\alpha = 5\pi/2$

$I = I_0 \left(\dfrac{\sin\alpha}{\alpha}\right)^2 = I_0 \left(\dfrac{\sin 5\pi/2}{5\pi/2}\right)^2 = I_0 \left(\dfrac{2}{5\pi}\right)^2 = \dfrac{I_0}{61}$

Thus the 2° maxima are of decreasing intensity and their direction is given by,

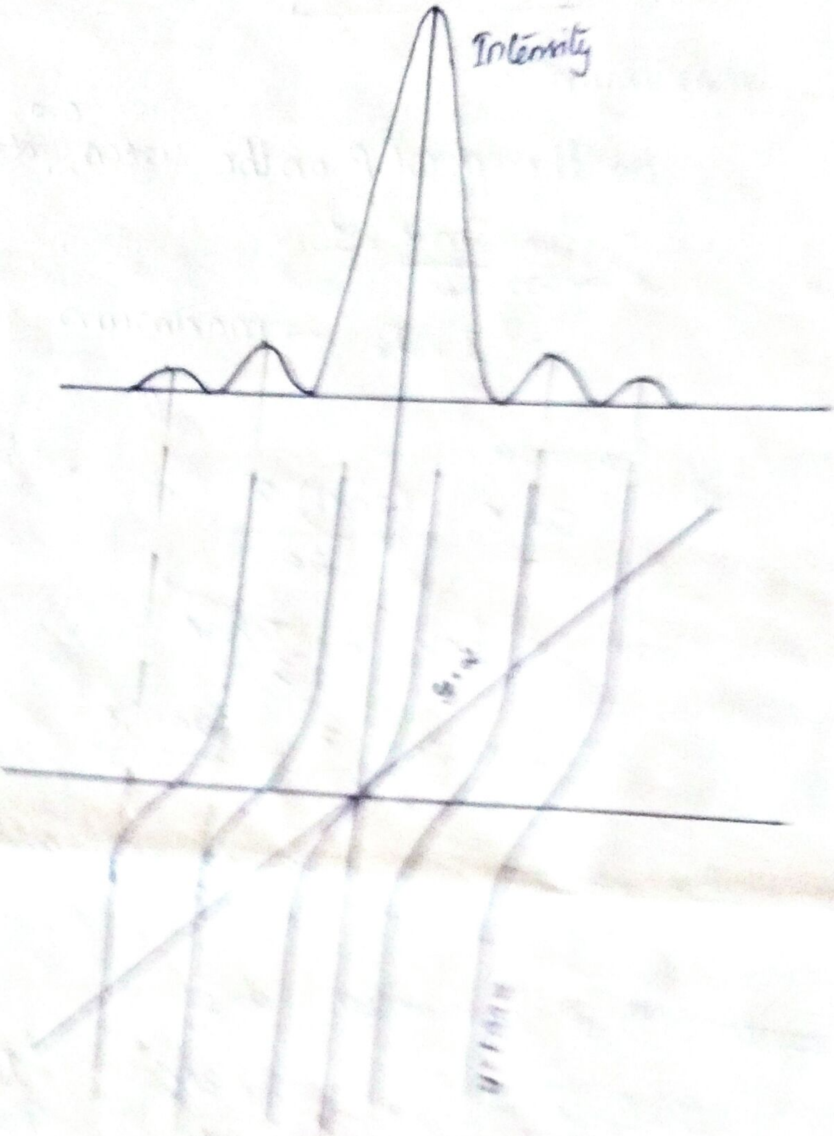
$I = I_0 \left(\dfrac{\sin\alpha}{\alpha}\right)^2$

$dI = I_0 \left[\dfrac{\alpha^2 2\sin\alpha \cos\alpha - \sin^2\alpha (2\alpha)}{\alpha^4}\right] d\alpha$

For I to be maximum, $\dfrac{dI}{d\alpha} = 0$

$$\alpha \cos\alpha = \sin\alpha$$

$$\Rightarrow \underline{\underline{\tan\alpha = \alpha}}$$



(c) Secondary minima

[illegible]

2) Red light of wavelength [illegible] falls on a slit [illegible] width [illegible] the [illegible] of diffraction pattern [illegible] from the slit.

$\lambda = 6500 A^{\circ}$ $a = 0.5 mm$

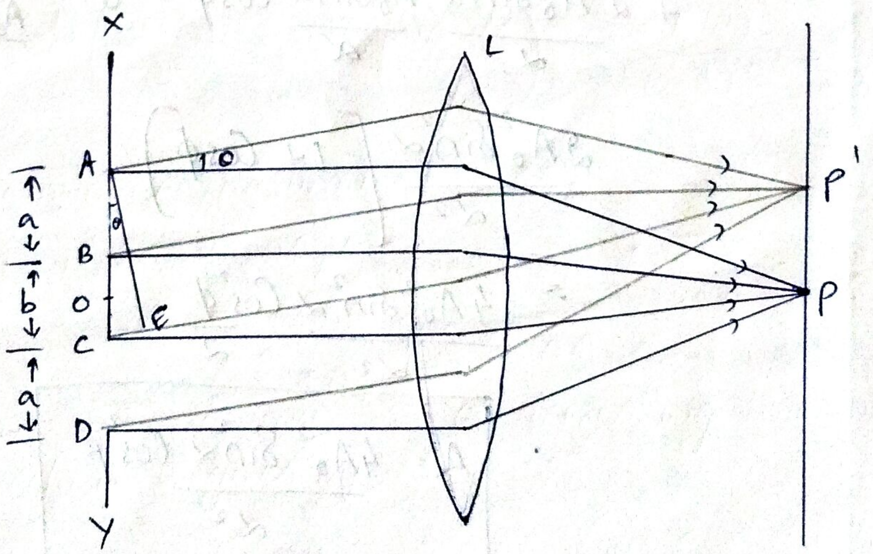
$D = 1.8 m$

$W = \frac{2D\lambda}{a} = \underline{4.68 mm}$

Fraunhoffer diffraction at double slit

Let 'a' be the width of the slit and 'b' the distance b/w slits so that the distance b/w corresponding points of the two slits is (a+b). Let a parallel beam of monochromatic light of wavelength λ be incident normally on the slits. Let the diffracted light be focussed on the screen by a convex lens.

The pattern obtained on the screen consists of equally spaced interference fringes in the region normally occupied by the central maximum in the single slit diffraction. The central interference maximum has maximum intensity while the maxima on either side of it are of gradually decreasing intensity. In the region of 2° maxima of single slit diffraction, the interference fringes of low intensity are observed. The diffraction at two parallel slits is a case of diffraction as well as interference.



According to Huygen's principle every point in the slits AB & CD sends out 2° wavelets in all directions. From the theory of diffraction at single slit the resultant amplitude due to wavelets diffracted from each slit in the direction P'_t is given

$$A' = \frac{A_0 \sin\alpha}{\alpha} \quad ; \quad \alpha = \frac{\pi a \sin\theta}{\lambda}$$

We can consider 2 slits as equivalent to two coherent sources placed at the middle points S_1 & S_2 of the slits each sending wavelets of amplitude $A_0 \sin\alpha/\alpha$ in the direction θ. The resultant amplitude at P'_t on the screen will be the result of interference b/w 2 waves of the same amplitude $A_0 \sin\alpha/\alpha$ and having phase difference ϕ.

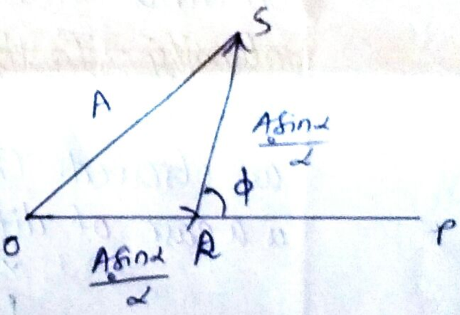
Draw a ⊥ʳ AE to CP'. The path diff is

$$CE = (a+b)\sin\theta$$

Corresponding phase difference $= \frac{2\pi}{\lambda}(a+b)\sin\theta$.

The resultant amplitude at P' can be determined by vector addition.

$$OS^2 = OR^2 + RS^2 + 2\, OR \times RS \cos\phi$$



$$= \frac{A_0^2 \sin^2\alpha}{\alpha^2} + \frac{A_0^2 \sin^2\alpha}{\alpha^2}$$

$$+ 2 \frac{A_0 \sin\alpha}{\alpha} \cdot \frac{A_0 \sin\alpha}{\alpha} \cos\phi$$

$$= \frac{2A_0^2 \sin^2\alpha}{\alpha^2}\left[1 + \cos\phi\right]$$

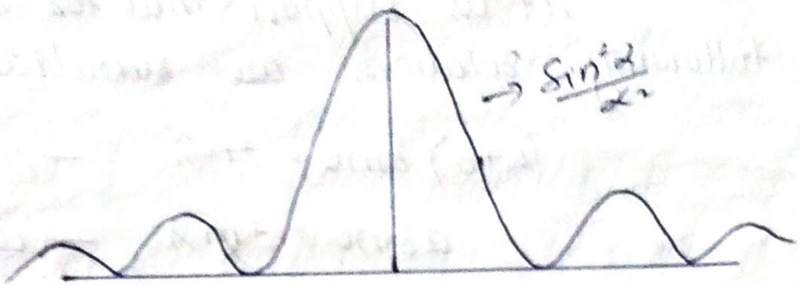
$$= \frac{4A_0^2 \sin^2\alpha}{\alpha^2} \cos^2\frac{\phi}{2} \qquad \frac{\phi}{2} = \beta$$

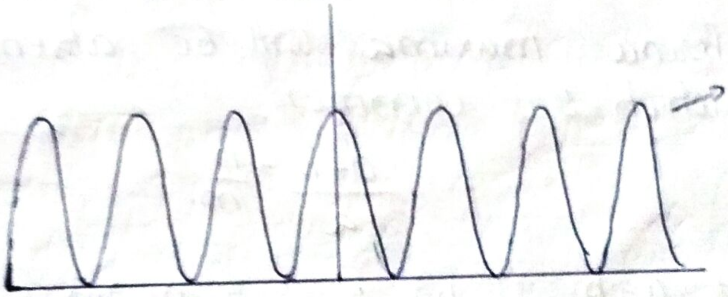
$$\therefore \boxed{A = \frac{4A_0^2 \sin^2\alpha \cos^2\beta}{\alpha^2}} \qquad \text{where } \beta = \frac{\phi}{2} = \frac{\pi}{\lambda}(a+b)\sin\theta$$

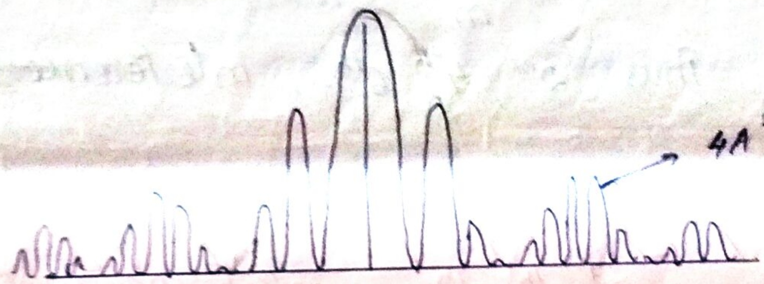
Thus the intensity of the resultant pattern depends on two factors $\frac{\sin^2\alpha}{\alpha^2}$ which gives the intensity of diffraction pattern due to each slit and

$\cos^2\beta$ which gives the intensity of interference pattern due to diffracted light waves from two slits.

The resultant intensity at any point on the screen is given by the product of these two factors and will be zero when either of these factors is zero.







- The diffraction minima is obtained when

$$\sin\alpha = 0 \Rightarrow \alpha = \pm m\pi \qquad m = 1, 2, 3, \ldots$$

or $$\frac{\pi a \sin\theta}{\lambda} = \pm m\pi$$

$$a\sin\theta = \pm m\lambda$$

- The interference maxima is obtained when

$$\cos^2\beta = 1 \Rightarrow \beta = \pm n\pi$$

$$\frac{\pi}{\lambda}(a+b)\sin\theta = \pm n\pi$$

$$(a+b)\sin\theta = \pm n\lambda \qquad n = 0, 1, 2, 3\ldots$$

Missing orders :-

If the width 'a' of the slit is kept const and the opaque distance b b/w the slits varied, it is observed that certain interference maxima are missing.

Let us suppose that for some value of θ the following relations are simultaneously satisfied

$$(a+b)\sin\theta = \pm n\lambda \quad \rightarrow \text{interference maxima}$$

$$a\sin\theta = \pm m\lambda \quad \rightarrow \text{diffraction minima}$$

When both these relations hold, the interference maxima will be absent in the direction for which θ is common.

$$\therefore \frac{a+b}{a} = \frac{n}{m}.$$

- if $a=b$; $\frac{n}{m} = 2$ or $n = 2m = 2, 4, 6, \ldots$

Thus $2^{nd}, 4^{th}, 6^{th} \ldots$ interference maxima will be missing.

- If $2a=b$ $\frac{n}{m} = 3$ or $n = 3m = 3, 6, 9. \ldots$

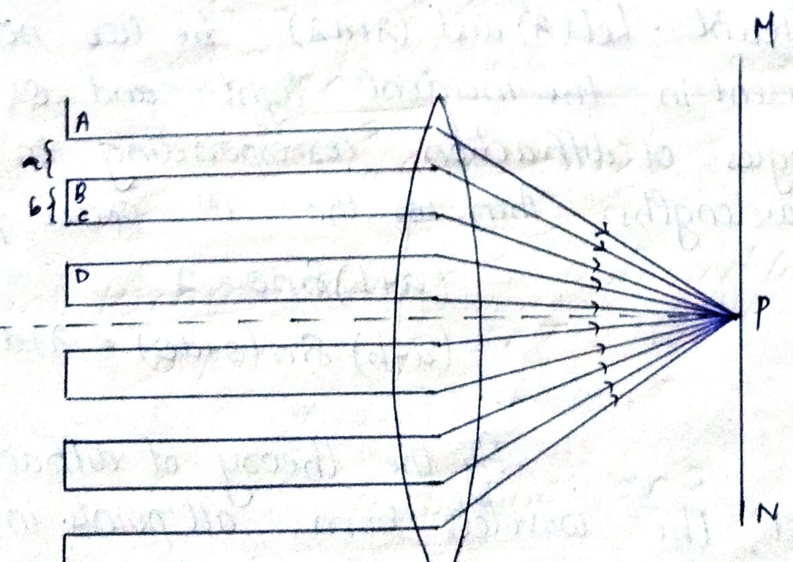
$3^{rd}, 6^{th}, 9^{th} \ldots$ interference maxima will be missing

Plane transmission grating.

Grating is an optical device which consists of a very large number of narrow slits. The slits are seperated by opaque spaces. When a wave front is incident on a grating surface, light is transmitted through the slits and obstructed by the opaque portions. Such a grating is called transmission grating. Gratings are prepared by ruling equidistant parallel lines on a glass surface. The lines are drawn with a fine diamond point.

If the spacing b/w the lines is of the order of the wavelength of light, then an appreciable deviation of the light is produced. Gratings used for the study of the visible region of the spectrum contain 10,000 lines per cm. Practically replicas of original grating are prepared. For that on the original grating surface a thin layer of collodion solution is poured and the solution is allowed to harden. Then the film of collodion is removed from the grating surface and then fixed b/w 2 glass plates.

Theory



In figure XY is the grating surface and M the screen. The slits are all parallel. Here AB is a slit and BC is an opaque portion. The width of each slit is a and the opaque spacing b/w any two consecutive slits is b. When a plane wave front incident, the secondary ~~slits~~ waves travelling in the same direction as that of the incident light will come to focus at the point P on the screen. Thus the point P will be a central bright maximum.

If the secondary waves travelling in a direction inclined at an angle θ, the intensity at P, will depend on the path difference b/w the 2° waves originating from the corresponding points A & C of the two neighbouring slits.

$\therefore$ Path diff $= (a+b)\sin\theta$

$(a+b)\sin\theta_n = n\lambda$ $\rightarrow$ condition for maximum intensity

$\theta_n \rightarrow$ direction of the n^{th} principal maximum.

$n = 1, 2, 3, \ldots$

If the incident light consists of more than one wavelength, the beam gets dispersed and the angles of diffraction for different wavelength will be different. Let (λ) and $(\lambda+d\lambda)$ be two nearby wavelengths present in the incident light and θ, $\theta+d\theta$ be the angles of diffraction corresponding to these two wavelengths. Then for the 1st order principal maxima

$$(a+b)\sin\theta = \lambda$$

$$(a+b)\sin(\theta+d\theta) = \lambda+d\lambda.$$

By the theory of diffraction at a single slit, the wavelets from all points in a slit diffracted at an angle θ are equivalent to a single wave of amplitude $\frac{A\sin\alpha}{\alpha}$ where $\alpha = \frac{\pi}{\lambda} a\sin\theta$

It can be shown that the resultant amplitude in the direction of θ is

$$A = \frac{A_0 \sin\alpha}{\alpha} \cdot \frac{\sin N\beta}{\sin\beta}$$

Hence resultant intensity,

$$\boxed{I = \frac{A_0^2 \sin^2\alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2\beta}}$$

The 1st factor $\frac{A_0^2 \sin^2\alpha}{\alpha^2}$ gives diffraction pattern due to a single slit and the second factor $\frac{\sin^2 N\beta}{\sin^2\beta}$ gives the interference pattern due to N slits.

Principal maximum :-

When $\sin\beta = 0$ ie, $\beta = n\pi$, then $\sin N\beta = 0$

$\therefore \frac{\sin N\beta}{\sin\beta} = \frac{0}{0}$ and is indeterminate

$$\therefore \lim_{\beta \to \pm n\pi} \frac{\sin N\beta}{\sin\beta} = \lim_{\beta \to \pm n\pi} \frac{\frac{d}{d\beta}(\sin N\beta)}{\frac{d}{d\beta}(\sin\beta)}$$

$$= \lim_{\beta \to \pm n\pi} \frac{N \cos N\beta}{\cos\beta} = \pm N$$

$$\therefore I = \frac{A_0^2 \sin^2\alpha}{\alpha^2} \cdot N^2 \rightarrow \text{which is maximum}$$

These maxima are most intense and hence they are called principal maxima

$\therefore$ for that $\beta = n\pi$

$$\frac{\pi}{\lambda}(a+b)\sin\theta = n\pi$$

$$(a+b)\sin\theta = n\lambda$$

$n = 0 \rightarrow$ central maximum

$n = 1, 2, 3 \ldots \Rightarrow$ principal maxima

Minima :-

$$\sin N\beta = 0 \quad ; \sin\beta \neq 0.$$

$$\therefore N\beta = m\pi$$

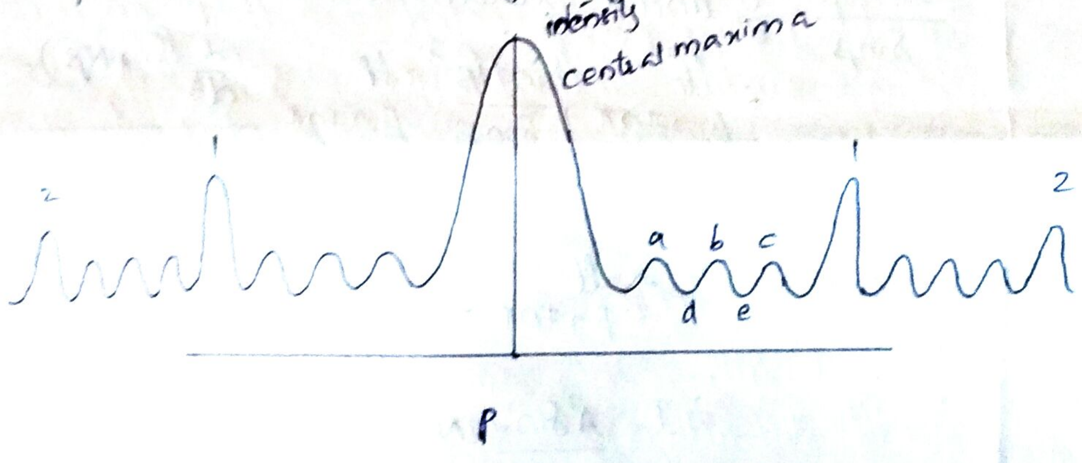
$$N\frac{\pi}{\lambda}(a+b)\sin\theta = m\pi$$

$$N(a+b)\sin\theta = m\lambda$$

$m =$ all integral values except $0, N, 2N, \ldots$

Secondary maxima :-

$m=0$ gives principal maxima of zero order (central maxima), $m=1,2,3,\ldots(N-1)$ give minima. ∴ there are $(N-1)$ equally spaced minima b/w two adjacent maxima. As there are $(N-1)$ minima, there must be $(N-2)$ other maxima known as secondary maxima b/w 2 principal maxima. The intensity of the 2° maxima decreases as N increases and when N is very large as in grating the secondary maxima are not visible in the spectrum. In such cases there is uniform darkness between any two principal maxima. This principle is used in gratings.



P → central maxima

1, 2 → 1st, 2nd principal maxima

a, b, c → secondary maxima

d, e → secondary minima

Diffraction

1) Fresnel diffraction :-

a) Circular aperture :-

(i) At P :-

$$\frac{1}{a}+\frac{1}{b}=\frac{n\lambda}{r^2}$$

$a \rightarrow$ source to slit distance
$b \rightarrow$ slit to screen distance
$r \rightarrow$ radius of aperture
$\lambda \rightarrow$ wavelength of light

$n \rightarrow$ odd (bright)
$n \rightarrow$ even (dark)

(ii) At P'

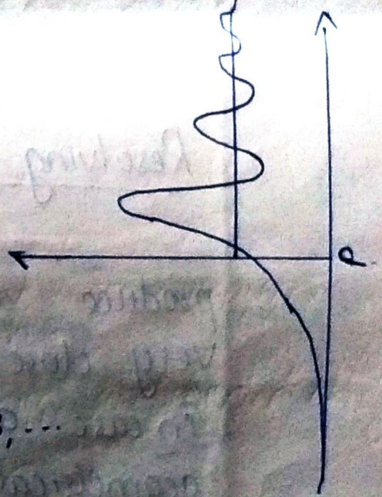
$$x_n = \frac{nb\lambda}{2r}$$

$x_n \rightarrow$ radius of n^{th} dark ring.
$n = 1, 2, 3, \ldots$

$$x_n = \frac{(2n+1)b\lambda}{4r}$$

$x_n \rightarrow$ radius of n^{th} bright ring
$n = 0, 1, 2, 3 \ldots$

b) Straight edge :-



At P' :

$$x_n = \sqrt{\frac{(2n+1)(a+b)b\lambda}{a}}$$

$n = 0, 1, 2, 3, \ldots$

$x_n \rightarrow$ distance of n^{th} bright band.

$$x_n = \sqrt{\frac{2n(a+b)b\lambda}{a}}$$

$n = 1, 2, 3, \ldots$

$x_n \rightarrow$ distance of n^{th} dark band.

2) Fraunhoffer diffraction :-

a) Single slit

Intensity, $I = I_0 \frac{\sin^2\alpha}{\alpha^2}$

$a\sin\theta = n\lambda \rightarrow$ Minima $n = 1, 2, 3, \ldots$

$n = 0 \rightarrow$ central

$a\sin\theta = (2n+1)\frac{\lambda}{2} \rightarrow$ Maxima $n = 1, 2, 3, \ldots$

Width of central maxima

$$W = 2x = \frac{2f\lambda}{a}$$

b) Two slits

$$I = \frac{4A_0^2 \sin^2\alpha \cos^2\beta}{\alpha^2}$$

$a\sin\theta = m\lambda$, $m = 1, 2, 3 \ldots \rightarrow$ Minima

$(a+b)\sin\theta = n\lambda$, $n = 0, 1, 2, 3 \ldots \rightarrow$ Maxima.

c) N slits

$$I = \frac{A_0^2 \sin^2\alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2\beta}$$

For principal maxima; $(a+b)\sin\theta = n\lambda$ $n = 1, 2, 3 \ldots$

For minima, $N(a+b)\sin\theta = m\lambda$, $m =$ integers except $0, N, 2N \ldots$

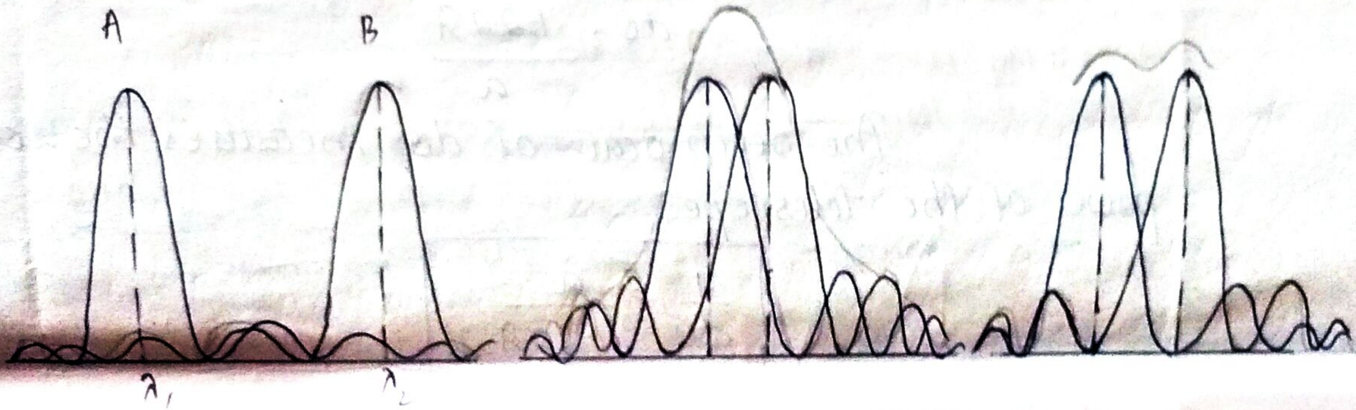
Resolving power

The ability of an optical instrument to produce distinctly seperate images of two objects located very close to each other is called its resolving power. In case of microscopes and telescopes, we talk of geometrical resolution where the geometrical positions between two nearby objects are to be resolved and in case of spectroscopes we refer to spectral resolution where differences of wavelengths of light in a given source are to be resolved. Resolving power is normally defined as the reciprocal of the smallest angle subtended at the objective of optical instrument by two dark point objects, which can just be distinguished as seperate.

Rayleigh Criterion

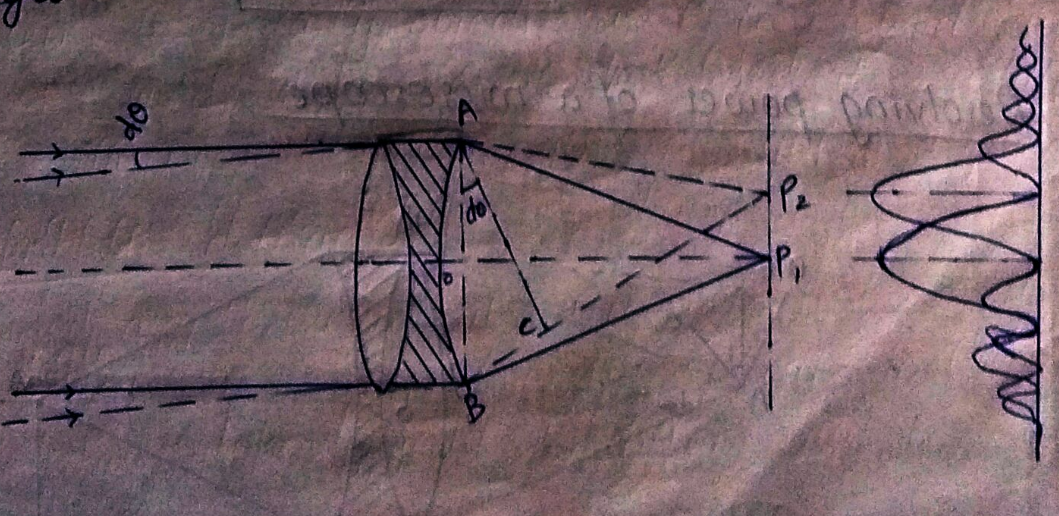
When a beam of light from a point object passes through the objective of a telescope, the lens act like a circular aperture and produces a diffraction pattern instead of a point image. This diffraction pattern is a bright disc surrounded by alternate dark and bright rings. It is known as Airy's disc.

According to Rayleigh, the two images of such point objects lying close to each other may be regarded as seperated if the central maximum of one falls on the first minimum of the other.



Resolving power of a telescope

Let 'a' be the diameter of the objective of the telescope. Consider the rays of light from two neighbouring points of a distant object. Let P_1 & P_2 be the positions of the central maxima of the two images.



Let the angle P_2AP_1 be $d\theta$. The path difference b/w the 2° waves travelling in the directions BP_2 AP_2 is equal to BC.

$\therefore$ $BC = AB\sin d\theta$ if $d\theta << 1$

$\sin d\theta \simeq d\theta$

$= a\,d\theta$

If this BC is equal to λ, the position of P_2 corresponds to the 1st minimum of the 1st image.

$\therefore a\,d\theta = \lambda$

$d\theta = \lambda/a$

but P_2 is the also the central maximum of 2nd image.

The above eqn holds good for rectangular apertures. For circular apertures this can be written as,

$$d\theta = \frac{1.22\lambda}{a}$$

The reciprocal of $d\theta$ measures the resolving power of the telescope

$$\boxed{\frac{1}{d\theta} = \frac{a}{1.22\lambda}}$$

$d\theta \rightarrow$ angle subtended by the two distant object points at the aperture.

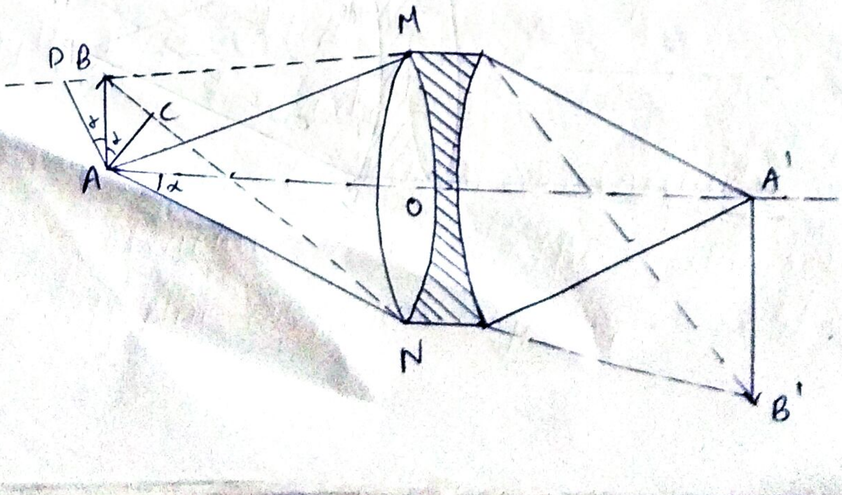
If 'f' is the focal length of the telescope objective, then

$$d\theta = \frac{r}{f} = \frac{1.22\lambda}{a}$$

$$\boxed{r = \frac{1.22f\lambda}{a}}$$

$r \rightarrow$ radius of central bright disc.

Resolving power of a microscope



The path difference b/w the extreme rays from the point B and reaching A' is

$$(BN+NA') - (BM+MA')$$

but $NA' = MA'$

$$\therefore P.d = BN - BM$$

In fig, AD is $\perp^r$ to DM and AC is $\perp^r$ to BN

$$\therefore BN - BM = (BC+CN) - (DM-DB)$$

but $CN = AN = AM = DM$

$$\therefore P.d = BC + DB.$$

$$BC = AB \sin\alpha = d\sin\alpha$$

$$DB = AB\sin\alpha = d\sin\alpha$$

$d \rightarrow$ distance b/w the two object point.

$$\therefore pd = 2d\sin\alpha$$

If this path difference is equal to 1.22λ, then A' corresponds to the 1st minimum of the image B' and the two images appear just resolved.

$$\therefore 2d\sin\alpha = 1.22\lambda$$

$$\therefore \boxed{d = \frac{1.22\lambda}{2\sin\alpha}}$$

This eqn is based on the assumption that the object points A and B are self luminous. But actually, the objects viewed with a microscope are not self luminous but are illuminated with light from a condenser. It is found that the resolving power depends upon the mode of illumination. According to Abbe, the least distance b/w two just resolvable object points is,

$$d = \frac{\lambda_0}{2\mu\sin\alpha}$$

$\lambda_0 \rightarrow$ wavelength of light in vacuum

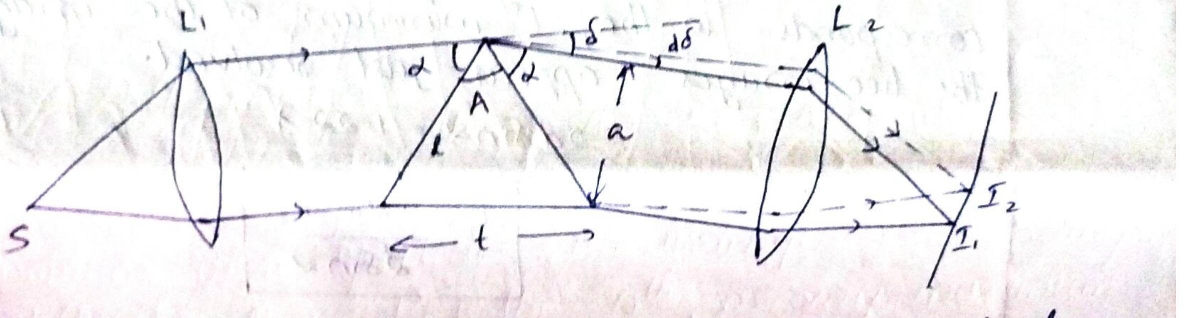
$\mu \rightarrow$ refractive index of the medium b/w the object and the objective.

The space b/w the objective and the object is filled with oil. This has two advantages. 1st ly the loss of light by reflection at first lens surface is decreased and 2nd ly the resolving power of the microscope is increased.

$\mu \sin\alpha \rightarrow$ numerical aperture of the objective.

Resolving power of a prism.

The resolving power of a spectrographic devices signifies the ability of the instrument to form two seperate spectral images of 2 neighbouring wavelengths (λ) & $(\lambda + d\lambda)$.



The image I_1 corresponds to the principal maximum for the wavelength λ and I_2 corresponds to the principal maximum for the wavelength $\lambda + d\lambda$. The face of the prism limits the incident beam to a rectangular section of width 'a'.

In the case of diffraction at a rectangular aperture, the position of I_2 will correspond to the 1st minimum of the image I_1 for wavelength λ provided

$$a\,d\delta = \lambda$$

$$d\delta = \frac{\lambda}{a} \quad \text{——— (I)}$$

Here δ is the angle of minimum deviation

Also $\alpha + A + \alpha + \delta = \pi$

$$\therefore 2\alpha = \pi - (A+\delta)$$

$$\alpha = \frac{\pi}{2} - \left(\frac{A+\delta}{2}\right)$$

$$\therefore \sin\alpha = \sin\left[\frac{\pi}{2} - \left(\frac{A+\delta}{2}\right)\right] = \cos\left(\frac{A+\delta}{2}\right) \quad \text{—(1)}$$

But $\sin\alpha = \dfrac{a}{l}$

$$\therefore \cos\left(\frac{A+\delta}{2}\right) = \frac{a}{l} \quad \text{—(2)}$$

$$\text{Also } \sin\frac{A}{2} = \frac{t}{2l} \quad \text{—(3)}$$

In the case of a prism, $\mu = \dfrac{\sin\left(\frac{A+\delta}{2}\right)}{\sin A/2}$

or $\mu \sin\dfrac{A}{2} = \sin\left(\dfrac{A+\delta}{2}\right)$

Diff w.r.t λ, $\dfrac{d\mu}{d\lambda}\sin\dfrac{A}{2} = \dfrac{1}{2}\cos\left(\dfrac{A+\delta}{2}\right)\dfrac{d\delta}{d\lambda}$ —(4)

Sub (2) & (3) in (4)

$$\frac{d\mu}{d\lambda}\frac{t}{2l} = \frac{1}{2}\frac{a}{l}\frac{d\delta}{d\lambda}$$

$$\boxed{a\frac{d\delta}{d\lambda} = t\frac{d\mu}{d\lambda}} \quad \text{—(5)}$$

Sub (I) in (5);

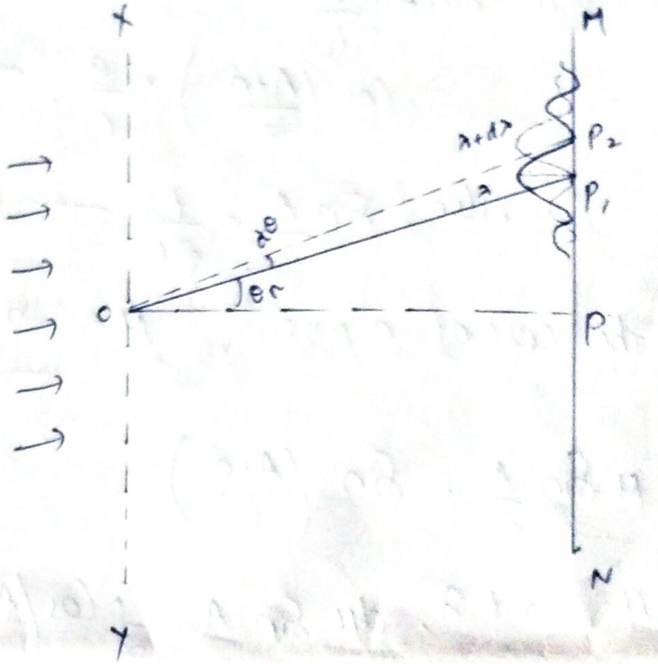
$$\frac{a\lambda}{d\lambda\, a} = t\frac{d\mu}{d\lambda}$$

$$\boxed{\frac{\lambda}{d\lambda} = t\frac{d\mu}{d\lambda}}$$

The expression $\lambda/d\lambda$ measures the resolving power of the prism.

∴ The resolving power of prism is directly proportional to the length of the base of the prism and rate of change of refractive index w.r.t. wavelength for that particular material.

Resolving power of a plane transmission grating.



XY is the grating surface. P_1 is the n^{th} primary maximum of a spectral of wavelength λ at an angle of diffraction θ_n. P_2 is the n^{th} primary maximum of a second spectral line of wavelength $\lambda+d\lambda$ at a diffracting angle $\theta_n+d\theta$. The direction of the n^{th} primary maximum for a wavelength λ is

$$(a+b)\sin\theta_n = n\lambda \quad \text{——(1)}$$

The direction of the n^{th} primary maximum for a wavelength $\lambda+d\lambda$ is

$$(a+b)\sin(\theta_n+d\theta) = n(\lambda+d\lambda) \quad \text{——(2)}$$

These two lines will appear just resolved if the angle of diffraction $(\theta_n+d\theta)$ also corresponds to the diffraction of the 1^{st} secondary minimum after the n^{th} primary maximum at P_1

This is possible if the extra path difference introduced is λ/N, where N is the total no. of lines on the grating surface

$$(a+b)\sin(\theta_n + d\theta) = n\lambda + \frac{\lambda}{N} \quad \text{--- (3)}$$

Equating (3) & (2)

$$n(\lambda + d\lambda) = n\lambda + \frac{\lambda}{N}$$

$$n\,d\lambda = \frac{\lambda}{N}$$

$$\boxed{\frac{\lambda}{d\lambda} = nN}$$

This measures the resolving power of grating. Here 'n' is the order of the spectrum and 'N' is the no. of lines per cm on the grating surface.