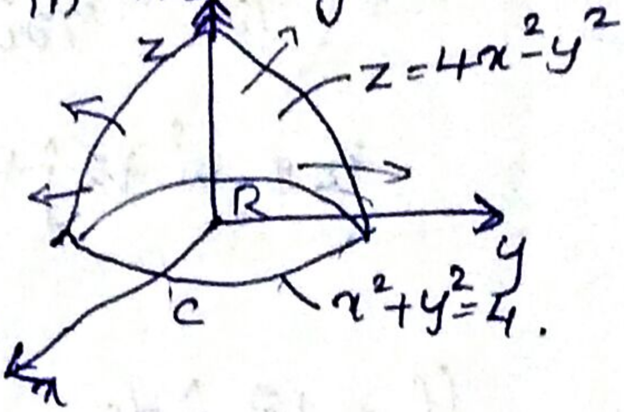
$\vec{F}(x,y,z) = \cancel{2x\hat{i} + 3z\hat{j} + 5y\hat{k}}\ 2z\hat{i} + 3x\hat{j} + 5y\hat{k}$

taking σ to be the portion of the parabola in $z = 4 - x^2 - y^2$ for which $z \geq 0$ with upward oriatation & C to be tuk oriented circle $x^2+y^2=4$ that ~~fall~~ forms the boundray of σ in the xy plane.



An:- Parametric eq^n of circle

$x = 2\cos\theta,\quad y = 2\sin\theta$

$0 \leq \theta \leq 2\pi$.

$\vec{F} = 2z\hat{i} + 3x\hat{j} + 5y\hat{k} \qquad dr = dx\hat{i} + dy\hat{j} + dz\hat{k}$

$\vec{F}.dr = 2zdx + 3xdy + 5ydz$

$\oint_C \vec{F}.dr = \int_0^{2\pi} 2zdx + 3xdy + 5ydz$

$= \int_0^{2\pi} 2\times 0 + 3\times 2\cos\theta \times 2\cos\theta\, d\theta + 5\times 0.$

$= \int_0^{2\pi} \cancel{6\cos\theta d\theta} = \cancel{6\sin\theta} \int_0^{2\pi} 12\cos^2\theta\, d\theta$

$= 12\int_0^{2\pi} \frac{1+\cos 2\theta}{2} d\theta = 6\int_0^{2\pi} 1+\cos 2\theta\, d\theta$

$= 6\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{2\pi} = 12\pi$

$$\operatorname{curl} \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2z & -3x & 5y \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial}{\partial y}5y - \frac{\partial}{\partial z}3x\right) - \hat{j}\left(\partial/\partial x\, 5y - \partial/\partial z\, 2z\right) + \hat{k}\left(\partial/\partial x\, 3x - \partial/\partial z\, 2z\right)$$

$$= \cancel{\text{tos}}\ 5\hat{i} + 2\hat{j} + 3k$$

$z = 4 - x^2 - y^2$

$\frac{\partial z}{\partial x} = -2x$

$\frac{\partial z}{\partial y} = -2y.$

$$\iint_S \operatorname{curl}\vec{F}\cdot\hat{n}\, ds$$

$$= \iint_R (5i+2j+3k)\cdot\left(-\frac{\partial z}{\partial x}\hat{i} - \frac{\partial z}{\partial y}\hat{j} + k\right) dA$$

$$= \iint_R (5\hat{i}+2\hat{j}+3k)\cdot(2x\hat{i} + 2y\hat{j} - \hat{k})\, dA$$

$$= \iint_R 10x + 4y + 3\ dA.$$

polar form

$x = r\cos\theta$

$y = r\sin\theta$

$0 \le \theta \le 2\pi$

$0 \le r \le 2$

$\therefore dA = r\,dr\,d\theta$

$$= \int_0^{2\pi}\int_0^2 (10r\cos\theta + 4r\sin\theta + 3)\, r\, dr\, d\theta$$

$$= \int_0^{2\pi} \left[10\frac{r^3}{3}\cos\theta + 4\frac{r^3}{3}\sin\theta + \frac{3r^2}{2}\right]_0^2 d\theta$$

$$= \int_0^{2\pi} \frac{80}{3}\cos\theta + \frac{32}{3}\sin\theta + 8\ d\theta.$$

$=$

$$\cancel{\frac{80}{3}\left[-\cos\theta\right]_0^{2\pi} + \frac{32}{3}\left[\sin\theta\right]_0^{2\pi}}$$

$$= \frac{80}{3}(\sin\theta)_0^{2\pi} + \frac{32}{3}(-\cos\theta)_0^{2\pi} + (6\theta)_0^{2\pi}$$

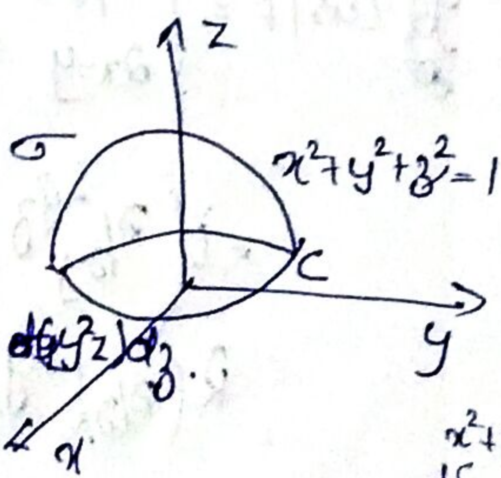
$\cos 0 = 1$
$\cos\pi = -1$
$\cos 2\pi = 1$
$\cos 3\pi = -1$

$$= \frac{80}{3}\times 0 - \frac{32}{3}(1-1) + 12\pi$$

$$= \underline{\underline{12\pi}}$$

$\iint_S \text{curl}\,\vec{F}\,\hat{n}\,ds = \oint_C \vec{F}\cdot d\vec{r}$. Hence its verified.

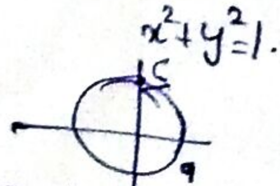
Q. Verify stokes theorem when $\vec{F} = (2x-y)\hat{i} - (yz^2)\hat{j} - (y^2 z)\hat{k}$ where S is the upper half surface of the unit sphere $x^2+y^2+z^2=1$ and C is it boundary.



$\oint_C \vec{F}\cdot d\vec{r}$

$$\vec{F}\cdot d\vec{r} = (2x-y)\,dx - (yz^2)\,dy - (y^2z)\,dz.$$

$$= \int_0^{2\pi} (2x-y)\,dx - (y^2z)\,dy - (y^2z)\,dz.$$



$x = \cos\theta$
$y = \sin\theta$
$0 \le \theta$

$$= \int_0^{2\pi} \cancel{2\cos\theta - \sin^2\theta\times 0 - 0}.$$

$$\cancel{\int_0^{2\pi} 2\cos\theta \, d\theta = \left[2\sin\theta\right]_0^{2\pi} =}$$

$$= \int_0^{2\pi} (2\cos\theta - \sin\theta) - \sin\theta \, d\theta \quad -\sin\theta \times 0 - y^2 \times 0.$$

$$= \int_0^{2\pi} -2\cos\theta\sin\theta + \sin^2\theta \, d\theta \qquad \sin^2\theta$$

$$= \int_0^{2\pi} -\sin 2\theta + \frac{1-\cos 2\theta}{2} \, d\theta = \left(\frac{\cos 2\theta}{2}\right)_0^{2\pi} + \frac{1}{2}\left(\theta - \frac{\sin 2\theta}{2}\right)_0^{2\pi}$$

$$= \frac{1}{2}\left[\cos 4\pi - \cos 0\right] + \frac{1}{2}\left[2\pi - 0\right]$$

$$= \frac{1}{2}(1-1) + \frac{2\pi}{2} = \pi //$$

$$\iint \text{curl}\,\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2x-y & -yz^2 & -y^2z \end{vmatrix}$$

$$= \hat{i}\left(\partial/\partial y\,(-y^2z) - \partial/\partial z\,(-yz^2)\right) - \hat{j}\left(\partial/\partial x\,(-y^2z) + \partial/\partial z\,(2x-y)\right)$$

$$\hat{k}\left(\partial/\partial x\,(-yz^2) - \partial/\partial y\,(2x-y)\right)$$

$$= -2yz + 2yz\,\hat{i} - \hat{j} \times 0 + \hat{k}(1)$$

$$= \hat{k} //$$

$$\sigma : z = \sqrt{1-x^2-y^2} \qquad \partial z/\partial x = \frac{1}{2\sqrt{1-x^2-y^2}} - 2x$$

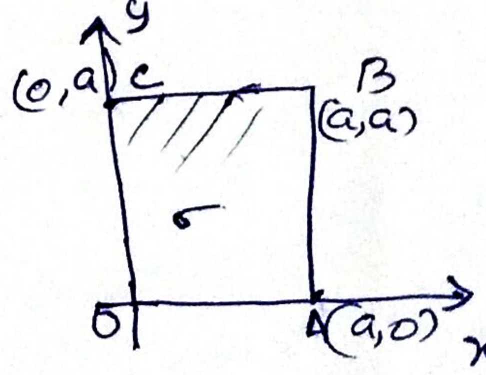
$$\frac{\partial z}{\partial y} = \frac{1}{2\sqrt{1-x^2-y^2}} \times -2y .$$

$$\iint (\text{curl}\,\vec{F}\cdot \hat{n}\, dS = \iint_R \hat{k}\cdot\left(\frac{-\partial z}{\partial x}\hat{i} - \frac{\partial z}{\partial y}\hat{j} + k\right) dA$$

$$= \iint_R dA = \text{Area of } \overset{R}{\cancel{\text{circle}}} = \pi r^2 = \pi \times 1 = \pi //$$

Hence verified. $\oint_C \vec{F}\cdot dr = \iint_G \text{curl}\,\vec{F}\cdot\hat{n}\, dS.$

Q Verify stoke theorem for the function $\vec{F} = x^2\hat{i} + xy\hat{j}$ intergated around the squre in the plane $z=0$ whoes sides are along the lines $x=0$, $y=0$, $x=a$, $y=a$.



$\hat{n} = \hat{k}$

$\text{curl}\,\vec{F} = yk$,

$$\iint_R \text{curl}\vec{F}\, n\, dA = \int_0^a\int_0^a y\,dy\,dx$$

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