

## Conduction in metals & free electron model

- When free atoms form a metal, all the valence electrons become conduction electrons and their states are profoundly modified, while the core electrons remain localised and their character remains essentially unchanged. Just as valence electrons are responsible for chemical properties, so conduction electrons are responsible for most of the properties of metals.

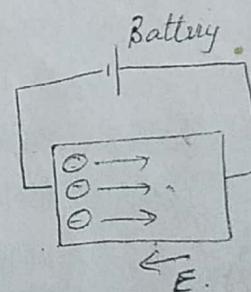
### The free-electron gas

- A metal crystal consists of positive metal ions whose valence electrons are free to move between the ions as if they constitute an electron gas.
- The crystal is held together by electrostatic force of attraction between the positively charged ions and the negatively charged electron gas.
- The mutual repulsion between the electrons is ignored.
- The potential field due to positive ions is completely uniform, so that electrons can move from place to place in the crystal without any change in their energy.

### Electrical conductivity

The Ohm's law is given by

$$I = \frac{V}{R}$$



where  $I$  is the current,  $V$  the potential difference and  $R$  the resistance of the wire.

$$\text{But we have } J = \frac{I}{A}, E = \frac{V}{L}, R = \frac{\rho L}{A}$$

where  $J$  is the current density,  $E$  the electric field and  $\rho$  the electrical resistivity.

$\sigma'$  is the conductivity where  $\sigma = \frac{1}{P}$

$\therefore$  On substituting, we will get

$$\underline{\underline{J = \sigma E}}$$

According to the free electron theory, electrons move freely in a conductor. In the absence of an electric field, the electron gas is in an equilibrium state described by equilibrium distribution functions. Because of the fact that in a conductor the number of electrons moving in opposite directions is always the same, their average velocity in any direction is zero. This explains the fact that in the absence of an external electric field there is no electric current in a conductor, no matter how many free electrons it contains.

When an electric field  $E$  is applied to a conductor, the random motion of the  $e$ 's gets modified in such a way that they drift slowly, in a direction opposite to that of the field, with an average drift velocity  $V_d$ . To calculate  $V_d$ , consider a free electron in an electric field  $E$ . It will experience a force  $eE$ , which accelerates the electron according to Newton's 2<sup>nd</sup> Law of motion.

$$a = \frac{eE}{m}$$

But due to the collision with impurities, lattice imperfections, velocity will not increase indefinitely and it will experience an opposing force given by

$$F = -\frac{mv_d}{\tau}, \quad \tau \rightarrow \text{relaxation time.}$$

$$\therefore m \frac{dv_d}{dt} = eE - \frac{mv_d}{\tau} \quad (1)$$

When  $e$  attain drift velocity, the acceleration will become zero and

$$(2) \quad \boxed{V_d = \frac{eE\tau}{m}}$$

since an  $e$  has a negative charge, it drifts in a direction opposite to that of the field.

Relaxation time and mean free path

②

Suppose that as soon as the velocity of the directional motion of the electrons attains a const value  $v_d$ , the field is turned off. This velocity starts diminishing as a result of collisions, and the electron gas ultimately return to an equilibrium state. Such a process leading to the establishment of equilibrium in a system is termed as relaxation process. Thus for  $E=0$ ,

$$\frac{dV_d(t)}{dt} = \frac{v_d}{\tau}$$

$$V_d(t) = V_d e^{-t/\tau}$$

$V_d(t)$  → velocity of the electrons  
 $t$  → time after the field is turned off.

b/w two successive collisions is taken as the mean free path of the electron. At room temp., since the velocity imparted to the electrons by an electric field is much smaller than thermal velocity, the time  $\tau$  taken by the electrons in travelling the distance  $\lambda$  will thus be decided not by the drift velocity due to the field but by the average velocity  $\bar{v}$ , due to the random thermal motion.

$$\tau = \frac{n \lambda}{\bar{v}}$$

$n \rightarrow$  no. of collisions  
                   that are required to  
                   nullify the directional  
                   velocity completely.

Electrical conductivity and Ohm's law.

knowing the drift velocity of the electrons, it is easy to calculate the current density and hence the conductivity of metal. Consider a cylindrical conductor of length  $v_d$  and area of cross section  $a$ .

Suppose it contains  $N$  electrons / unit volume. Imagine section of the conductor and count the no. of charges passing through this section / second.

$$\frac{Q}{t} = \frac{N(1 \times V_d) e}{t}$$

$$I = N(1 \times V_d) e \quad (t=1s)$$

$$\therefore J = \frac{I}{A} = NV_d e \quad (A = \text{unity})$$

From (2)

$$J = Ne \left( \frac{eE\tau}{m} \right)$$

$$J = \sigma \left( \frac{Ne^2 \tau}{m} \right) E$$

Conductivity  $\boxed{\sigma = \frac{Ne^2 \tau}{m} = Ne\mu}$

$\mu = \frac{e\tau}{m}$  is called carrier mobility and is defined as the average drift velocity per unit electric field

$$\text{i.e., } \mu = \frac{V_d}{E} = \frac{e\tau E}{mE} = \frac{e\tau}{m}$$

Resistivity,  $\boxed{\rho = \frac{m}{Ne^2 \tau}}$

- (Q) Na metal with a bcc structure has two atoms per unit cell. The radius of the Na atom is  $1.85 \text{ \AA}$ . Calculate its electrical resistivity at  $0^\circ\text{C}$  if the classical value of the mean free time at this temperature is  $3 \times 10^{-14} \text{ s}$ .

$$\tau = 3 \times 10^{-14} \text{ s}$$

$$D = 2$$

$$\text{for BCC, } a = \frac{4\sigma}{\sqrt{3}} = \frac{4}{\sqrt{3}} \times 1.85 \times 10^{-10} = \underline{\underline{4.27 \times 10^{-10} \text{ m}}}$$

The no. of es / unit volume in Na atom is

$$N \sigma' = \frac{n}{a^3} = \frac{2}{(4.27 \times 10^{-10})^3} = \underline{\underline{2.57 \times 10^{28} / \text{m}^3}}$$

(3)

We have the resistivity  $\rho = \frac{m}{Ne^2 \tau}$

$$= \frac{9.1 \times 10^{-31}}{2.57 \times 10^{28} \times (1.6 \times 10^{-19})^2 \times 3.1 \times 10^{-14}}$$

$$= 4.46 \times 10^8 \Omega m$$

### Electrical resistivity versus temperature

Electrical conductivity of a metal varies with the metal's temperature. At room temp., the electrical resistivity of most metals is dominated by the collision of conduction electrons with the phonons of the lattice (due to thermal vibrations). On the other hand, at very low temp. it is due to the collisions of electrons with the impurity atoms or other imperfections that are present in a real crystal. Thus in general, the resistivity of a metal containing imperfections is given by

$$\rho = \rho_i + \rho_{ph}(T) \quad (1)$$

$\rho(T) \rightarrow$  resistivity caused by the thermal vibrations of the lattice which is temp. dependent.

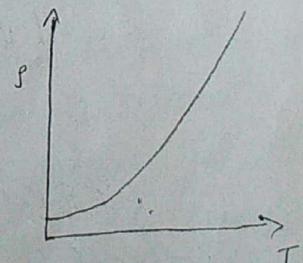
$\rho_i \rightarrow$  [Also called residual resistivity].

$\rho_i \rightarrow$  resistivity caused by the scattering of es by impurity atoms. [also called residual resistivity]

The statement that  $\rho$  can be split into two parts, one of which is independent of  $T$  is known as the Maltiessen rule (eqn 1). This becomes less accurate at high temp. or at high impurity content.

At low temperatures the scattering by phonon is negligible because of negligibly small amplitude of vibration.

Therefore as  $T \rightarrow 0$ ,  $\rho = \rho_i$



2) As the temp increases, the scattering by phonons becomes more effective and  $S_{ph}(T)$  increases linearly with temperature.

Matthiessen's rule is not always valid. Calculation of resistivity due to lattice vibrations  $S_{ph}$  have been quite successful in some metals. Empirically, one finds that  $S_{ph}$  is rather well represented by a universal function,

$$S_{ph} \propto \frac{T}{M\Omega_R^2} \cdot f\left(\frac{T}{\Theta_R}\right)$$

where  $f$  goes to unity at high temp. At low temp,  $f \propto (T/\Theta_R)^4$ .  $\Theta_R \rightarrow$  resistive characteristics temp

### Heat capacity of conduction electrons.

According to free electron model (Drude-Lorentz model), the conduction electrons are treated as free particles as if they are electron gas molecules which obey the classical laws of mechanics and statistical mechanics. Further, from kinetic theory of gases, the average energy of a free electron is  $\frac{3}{2}kT$ . Thus if a metal contains  $N_A$  free es per mole, then the average energy of the electrons per mole should be

$$k \langle \bar{E} \rangle = \frac{3}{2} N_A k T = \frac{3}{2} R T$$

$N_A \rightarrow$  Avagadro number

$R = N_A k =$  universal gas const

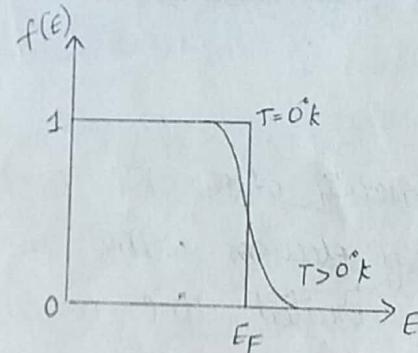
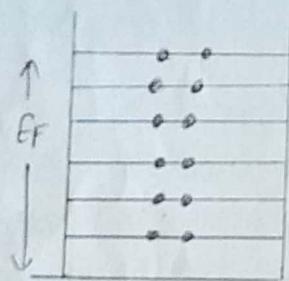
∴ Electronic specific heat is given by

$$C_e = \frac{\partial \langle \bar{E} \rangle}{\partial T} = \frac{3}{2} R \approx 3 \text{ cal/mol}^\circ\text{K}$$

∴ Total heat capacity in metals, including phonons should be,  
(At high temp)  $C = C_{ph} + C_e = \frac{3R}{2} + 3R = 9 \text{ cal/mol}^\circ\text{K}$

However, the exp'l measurements show that the electronic specific heat is smaller than the classical value by a factor of about  $10^3$ . This discrepancy was removed by the introduction of quantum statistics developed by E. Fermi and P.A.M. Dirac and is known as Fermi-Dirac statistics. (4)

The energy of the es in a metal is quantized according to QM. The es in the metal occupy these levels according to Pauli exclusion principle, which states that an energy level can accommodate at most 2 es, one with spin up and the other with spin down. Thus in filling the energy levels, 2 es occupy the lowest level, 2 more in the next level and so forth. The energy of the highest occupied level is called the Fermi energy level. A typical value for the Fermi energy in metals is about 5eV.



Thus contrary to classical mechanics, the conduction es in metals possess some energy even at 0°K.

The distribution of es among the levels is usually described by the distribution function  $f(E)$  which is defined as the probability that the level  $E$  is occupied by an e. Thus if the level is certainly empty, then  $f(E)=0$ , while if it is certainly full, then  $f(E)=1$ . In general  $f(E)$  has a value b/w zero & unity.

Thus at absolute zero ( $T=0^{\circ}\text{K}$ )

$$f(E) = \begin{cases} 1 & E < E_F \\ 0 & E > E_F \end{cases}$$

When the temp. is increased, thermal energy excites the electrons and they absorb thermal energy of the order of  $kT$ . In order to conserve pauli-exclusion principle, only those  $\bar{e}$ s close to the Fermi level can be excited. Thus only these  $\bar{e}$ s, which are a small fraction of the total no: are capable of being thermally excited and thus explains the low electronic specific heat.

The distribution function  $f(E)$  at  $\text{temp} \neq 0^{\circ}\text{K}$  is given by,

$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

This is known as Fermi-Dirac distribution.

We can find the thermal energy and heat capacity of the  $\bar{e}$ s using F-D distribution. Since only electrons within the range  $kT$  of the fermi level are excited, we conclude that only a fraction  $kT/E_F$  of the electrons is affected. Therefore the no: of  $\bar{e}$ s excited per mole is about  $N_A(kT/E_F)$  and since each  $\bar{e}$  absorbs an energy  $kT$ , on average, it follows that the thermal energy per mole is approximately,

$$\bar{E} = \frac{N_A k T \cdot k T}{E_F} = \frac{N_A (k T)^2}{E_F}$$

$$\therefore C_e = \frac{\partial(\bar{E})}{\partial T} = 2R \frac{k T}{E_F}$$

Thus the specific heat of the es is reduced from its classical value. For  $E_F = 5 \text{ eV}$ , at  $T = 300^\circ\text{K}$ , this factor is equal to  $1/200$ . This great reduction is in agreement with expt. (5)

The Fermi Temperature is defined as,

$$E_F = kT_F$$

$$\therefore C_e = \frac{2R \cdot T}{T_F}$$

An exact evaluation of the electronic heat capacity yields the value

$$C_e = \frac{\pi^2 R k T}{3} \frac{1}{E_F}$$

### The Fermi Surface

The electrons in a metal are in a continuous state of random motion. Thus they possess some velocity. We can consider a velocity space, whose axes are  $v_x$ ,  $v_y$  and  $v_z$ . Each point in this space represents a unique velocity, both in magnitude and direction.

Consider the conduction es in this velocity space. These es have many different velocities and since these velocities are random, the points representing them fill the space uniformly. Thus we can construct a sphere outside which all points are empty. The radius of this sphere is the Fermi speed  $v_F$  which is given by

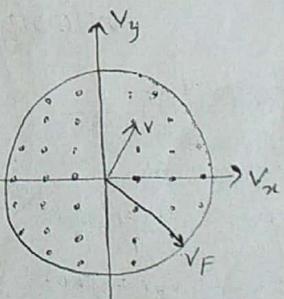
$$E_F = \frac{1}{2} m v_F^2$$

This sphere is called Fermi sphere and its surface as Fermi surface (Fs).

If we substitute  $E_F = 5 \text{ eV}$ , we get  $v_F = 10^6 \text{ m/s}$ . Thus electrons at the Fs are moving very fast. Also Fermi speed and Fermi surface are independent of temp.

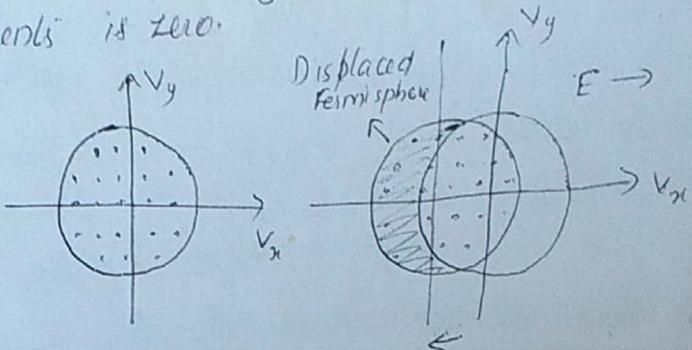
$$E_F = \frac{\hbar^2}{2m} (3\pi^2 N)^{2/3}$$

As  $N \uparrow$ ,  $E_F \uparrow$



## Electrical conductivity - Effects of the Fermi surface..

In the absence of an electric field, the Fermi sphere is centred at the origin. The various  $\vec{e}$ s are all moving, some at very high speeds and they carry individual currents. But the total current of the system is zero, b'coz for every  $\vec{e}$  at velocity ' $v$ ' there exists another  $\vec{e}$  with velocity ' $-v$ ' and the sum of their two currents is zero.



The situation changes when a field is applied. If the field is in the +ve  $x$ -direction, each  $\vec{e}$  acquires a drift velocity  $v_d = eE/m$ . Thus the whole Fermi sphere is displaced to the left. Even though displacement is very small and  $\vec{e}$ s still cancel each other, some  $\vec{e}$ s in the shaded crescent remain uncompensated. It is these  $\vec{e}$ s which produce the observed current.

The fraction of  $\vec{e}$ s which remain uncompensated is approximately  $v_d/v_F$ . The concentration of these  $\vec{e}$ s is therefore  $N(v_d/v_F)$  and since each electron has a velocity of approximately ' $-v_F$ ', the current density is

$$\begin{aligned} \text{Conc} &= NVd/V_F \\ \text{distance travelled} &= -V_F \Delta t \\ \therefore \text{Volume} &= A V_F \Delta t \\ \therefore \text{No. of } \vec{e} &= \frac{NVd}{V_F} \cdot (-AV_F \Delta t) \end{aligned}$$

$$\therefore Q = -\frac{eNVd(-V_F)A\Delta t}{V_F}$$

$$\frac{\partial}{\partial t} Q = eNVdA$$

$$I = eNVdA$$

$$J = \frac{I}{A} = eNVd$$

$$J = -eN \left( \frac{Vd}{V_F} \right) (-V_F)$$

$$= NeV_d$$

$$\text{but } V_d = eE/m$$

$$\therefore J = \frac{Ne^2 Z_F E}{m}$$

$Z_F \rightarrow$  collision time of an  $\vec{e}$  at the FS

$$\tau = \frac{Ne^2 Z_F}{m}$$

$$\begin{aligned} J &= \frac{I}{A} \\ I &= Q \\ \therefore J &= \frac{Q}{A} \\ J &= \frac{Q}{Z_A} \\ &= eN \left( \frac{Vd}{V_F} \right) \frac{Z_A}{A} \\ &= \frac{eN(Vd)}{V_F} \frac{Z_A}{A} \\ \text{for unit area} \\ J &= \frac{eNVd}{V_F} Z_A \\ Z_A &= \frac{V_F}{V_F} = 1 \end{aligned}$$

(6)

## Thermal conductivity in metals

The quantity of heat flowing through a metal is given by  $Q = -kA\frac{dT}{dx}$   $k \rightarrow$  Thermal conductivity

In insulators, heat is carried entirely by phonons, but in metals heat may be transported by both  $\bar{e}$ s and phonons.

$$\therefore k = k_e + k_{ph}$$

In most cases, the contribution of the electrons greatly exceeds that of the phonons, because of the great concentration of electrons.  $\therefore$  Conductivity of phonons can be neglected.

If a metal is heated,  $\bar{e}$ s at the hot end travel in all directions, but a certain fraction travel to the right and carry energy to the cold end.  $\text{H}^{\text{H}}$ , a certain fraction of the  $\bar{e}$ s at the cold end travel to the left and carry energy to the hot end. These oppositely travelling  $\bar{e}$  currents are equal, but because those at the hot end are more energetic on the average than those on the right, a net energy is transported to the right, resulting in a current of heat.

To find thermal conductivity, we can use the equation  $k_e = \frac{1}{3} C_v V_F l_F$ . Here the heat carriers are  $\bar{e}$ s.

$$\therefore k_e = \frac{1}{3} C_e V_F l_F \\ = \frac{1}{3} \frac{\pi^2 N k^2 T}{2 E_F} V_F l_F$$

$$\text{but } E_F = \frac{1}{2} m v_F^2 \quad \text{and } \frac{l_F}{V_F} = \tau_F$$

$$\therefore k_e = \frac{\pi^2 N k^2 T \tau_F}{3m} \quad \text{but } \sigma = \frac{N e^2 \tau_F}{m}$$

$$k_e = \frac{\pi^2 k^2 T \sigma}{3 e^2}$$

$$\therefore \frac{k_e}{\sigma T} = \frac{1}{3} \left( \frac{\pi k^2}{e} \right)^2$$

where  $\frac{k_e}{\sigma T} \cdot L$

$$\therefore \boxed{L = \frac{1}{3} \left( \frac{\pi k^2}{e} \right)^2} \rightarrow \text{Lorenz number.}$$

$$= 2.45 \times 10^{-8} \text{ W m K}^{-2}$$

### Klemm - Franz Law

The law states that the ratio of thermal conductivity ( $k$ ) to the electrical conductivity ( $\sigma$ ) of a metal is proportional to the temperature ( $T$ ).

$$\frac{k_e}{\sigma} \propto LT$$

The proportionality const is called Lorenz number.  $L$

$$L = \frac{1}{3} \left( \frac{\pi k^2}{e} \right)^2$$

- a) A uniform copper wire of length 0.5m and diameter 0.3mm has a resistance of 0.12Ω at 293K. If the thermal conductivity of the specimen at the same temp. is 390 W m⁻¹ K⁻¹. Calculate the Lorenz number. Compare this value with the theoretical value.

$$l = 0.5 \text{ m} \quad d = 0.3 \text{ mm}, r = 0.15 \text{ mm} \\ R = 0.12 \Omega \quad T = 293 \text{ K} \quad (\text{radius } r)$$

$$k_e = 390 \text{ W m}^{-1} \text{ K}^{-1} \quad L = ?$$

$$L = \frac{k_e}{\sigma T}$$

$$R = \frac{\rho l}{A}$$

$$\frac{1}{\rho} =$$

$$\pi r^2 = A$$

$$\sigma = \frac{l}{RA} = \frac{0.5}{(0.15 \times 10^{-3}) \pi \times 0.12}$$

$$\sigma = 5.89 \times 10^7 \Omega^{-1} \text{ m}^{-1}$$

$$\therefore L = \frac{k_e}{\sigma T} = \frac{390}{5.89 \times 10^7 \times 293} = 2.26 \times 10^{-8} \text{ W m K}^{-2}$$

$$\text{The theoretic value of } L = \frac{1}{3} \left( \frac{\pi k}{e} \right)^2$$

$$= \frac{1}{3} \frac{\pi^2 (1.38 \times 10^{-23})^2}{(1.6 \times 10^{-19})^2} = \underline{\underline{2.84 \times 10^{-8} \text{ Ws} \text{ A}^{-2}}}$$

Comparing the two values, we observe that the theoretical value is about 1.26 times higher than the exptl value.

### Hall effect

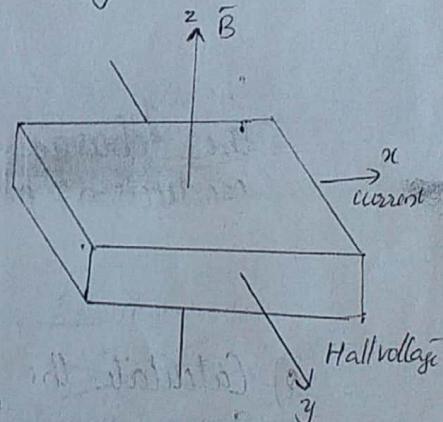
Consider a rectangular metal slab carrying a current density  $J_x$  in the positive  $x$ -direction and placed in a uniform magnetic field of induction  $B$  acting  $\perp$  to both the conductor & the current. Then a Hall voltage will develop at right angles to both the current & the magnetic field.

In the absence of the magnetic field, the conduction  $e^-$  drift with a velocity  $v_x$  in the  $-ve\ x$ -direction. However when the field is applied, a force (Lorentz force) causes the  $e^-$  path to deflect towards the front face of the rectangular block. As a result, an excess of  $e^-$  accumulates on the front face of the slab. Simultaneously equal no. of  $+ve$  charge appears on the opposite face of the slab. The appearance of the opposite charges on the opposite faces creates an electric field (Hall field) directed towards the  $y$ -axis.

The Lorentz force  $F_L$  acting on an  $e^-$  which is moving from right to left with a velocity  $v$  is

$$F_L = -ev_x B_z$$

$$F_L = -ev_x B_z$$



Further, the accumulated charges on the opposite faces produce a force opposite to the Lorentz force. Hence, the accumulation process continues until the Hall force completely cancels the Lorentz force. In equilibrium

$$F_H = F_L$$

$$eE_H = -eV_x B_z$$

$$E_H = -V_x B_z$$

But the current density is  $J_x$

$$J_x = NeV_x \Rightarrow V_x = \frac{J_x}{Ne}$$

$$\boxed{E_H = -\left(\frac{1}{Ne}\right) B_z J_x}$$

$N \rightarrow$  no. per unit volume

Hall field is proportional to  $\bar{B}$  &  $\bar{J}$ . The constant of proportionality is called Hall constant.

$$\boxed{R_H = -\frac{1}{Ne}}$$

The sign of the Hall constant indicates the nature of the charge carrier that predominate in the conduction process.

$$\therefore \boxed{E_H = R_H B_z J_x}$$

a) Calculate the Hall coefficient of Na based on free electron model. Sodium has bcc structure and the side of the cube is

$$4.28 \text{ \AA}^\circ$$

$$a = 4.28 \text{ \AA}^\circ$$

bcc  $\rightarrow$  no. of atoms / unit cell = 2

$$\therefore N = \frac{2}{a^3} = \frac{2}{(4.28)^3 \times 10^{-30}} = \underline{\underline{2.55 \times 10^{28} / m^3}}$$

$$R_H = -\frac{1}{Ne} = -\frac{1}{2.55 \times 10^{28} \times 1.6 \times 10^{-19}}$$

$$= -2.45 \times 10^{-10} \text{ m}^3 \text{ C}^{-1}$$

$\underline{\underline{\quad}}$

## Failure of the free electron model

- 1) It fails to explain the electric specific heat and the specific heat capacity of metals.
- 2) It fails to explain superconducting properties of metals.
- 3) It fails to explain new phenomena like photo-electric effect, Compton effect, Black body radiation.
- 4) It fails to explain electrical conductivity of semiconductors or insulators.
- 5) Ferromagnetism could not be explained by this theory.

## Important equations

$$1) V_d = \frac{eE\tau}{m}$$

$V_d \rightarrow$  drift velocity  
 $\tau \rightarrow$  relaxation time

$$2) V_d(t) = V_d e^{-t/\tau}$$

$V_d(t) \rightarrow$  velocity of free  $e^-$  at time  $t$   
when  $E=0$

$$3) \sigma = \frac{Ne^2\tau}{m}$$

$\sigma \rightarrow$  conductivity  
 $N \rightarrow$  no. of  $e^-$ s / unit volume.

$$4) \mu = \frac{V_d}{E} = \frac{e\tau}{m}$$

$\mu \rightarrow$  carrier mobility

$$5) \sigma = N e \mu$$

$\rho \rightarrow$  resistivity

$$6) \rho = \frac{m}{N e^2 \tau}$$

$$7) N = V \cdot \frac{n}{a^3}$$

$V \rightarrow$  valency  
 $n \rightarrow$  no. of atoms / unit cell  
 $a \rightarrow$  unit cell dimension

$$8) \rho = \rho_i + \rho_{ph}(\tau)$$

$\rightarrow$  Matthiessen rule  
 $\rho_i \rightarrow$  residual resistivity  
 $\rho_{ph} \rightarrow$  ideal resistivity

$$9) f(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

$f(E) \rightarrow$  Fermi-Diatomic distribution  
 $f_n$

$E_F \rightarrow$  Fermi energy

$k \rightarrow$  Boltzmann const

$$10) C_e = \frac{\pi^2}{2} \frac{RKT}{E_F}$$

$C_e \rightarrow$  electronic specific heat  
 $R \rightarrow$  universal gas const

$$11) E_F = kT_F$$

$T_F \rightarrow$  Fermi temperature

$$12) \frac{k_e}{\sigma T} = L \quad \rightarrow$$

Wiedemann-Franz law

$k_e \rightarrow$  thermal conductivity

$\sigma \rightarrow$  electrical conductivity

$L \rightarrow$  Lorenz no:-

$$13) L = \frac{1}{3} \left( \frac{\pi k}{e} \right)^2 = 2.45 \times 10^{-8} \text{ W} \Omega \text{K}^{-1}$$

$$14) E_H = R_H B_z J_x$$

$E_H \rightarrow$  Hall voltage

$R_H \rightarrow$  Hall constant

$J_x \rightarrow$  surface current density

$$15) R_H = -\frac{1}{Ne}$$