6th sem

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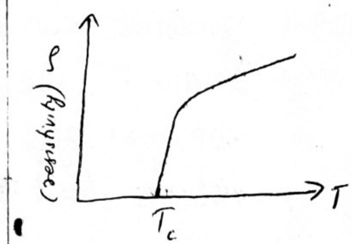
Superconductivity

When we cool certain metals and alloys to sufficiently low temperatures they exhibit almost zero electrical resistivity (ie, infinite conductivity). This zero resistivity is known as superconductivity. This phenomenon was first observed by H.k Onnes in 1911. He observed that as purified mercury is cooled, its resistivity vanishes abruptly at 4.2 k. The temp. at which the transition takes place is called the critical temperature (T_c)

Above T_c, a given specimen is in the usual normal state, while below T_c, it is in the superconducting state.



- Superconducting transition is reversible.
- For pure specimen, the transition will be sharp.
- From free electron model, the resistivity of a metal is,

$$\rho = \frac{m}{ne^2\tau}$$

$\tau \rightarrow$ collision time.

As T decreases, the lattice vibrations begin to freeze and hence the scattering of $\bar{e}$ diminishes. This results in longer τ and hence smaller ρ.

Sources of superconductivity

1) It is found to occur in metallic elements in which the no:- of valence $\bar{e}$ lies b/w 2 and 8.

2) Transition metals having odd valence $\bar{e}$s have higher T_c and are favourable to exhibit superconductivity while metals with even no:- of valence $\bar{e}$s are particularly unfavourable.

3) A small atomic volume, accompanied by a small atomic mass favours SC.

4) Materials having high normal resistivities exhibit SC.

Meissner effect / Perfect diagmagnetism

[illegible] field is applied [illegible] temp. it expels the magnetic flux totally held and acts as an ideal diamagnet. This phenomenon is called Meissner effect.

It was observed by Meissner & Ochsenfeld in 1933. By measuring the $\bar{B}$ in the neighbourhood of the specimen in a no. of cases, they established that as the temp is lowered to T_c, the specimen becomes SC and the flux is pushed out of it for all temps. $T < T_c$. The effect is reversible also ie, when the temp is raised from below T_c, the flux suddenly starts penetrating the specimen at $T = T_c$ as a result of which the specimen returns back to the normal state.

The magnetic induction inside the substance is,

$$B = \mu_0 (H + M) = \mu_0 (1 + \chi) H$$

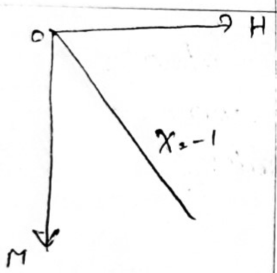
$H \rightarrow$ external applied field

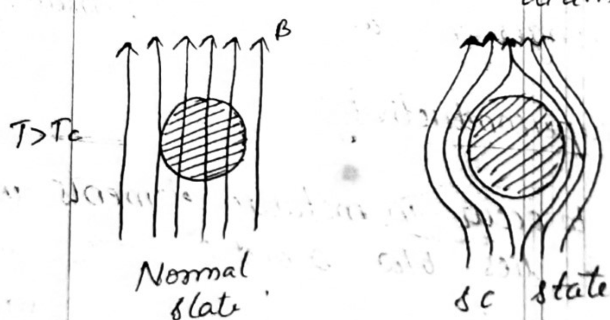
$M \rightarrow$ Magnetisation inside the material.

According to Meissner effect, $B = 0$

$$M = -H$$

or $\chi = -1 \Rightarrow$ The material is perfect diamagnetic





Critical field (H_c)

If a strong magnetic field, called the critical field, is applied to a superconducting specimen, it becomes normal and recovers its normal resistivity even at $T < T_c$.

H_c depends on the temp. For a given substance, the field decreases as the temp rises from $T = 0$ K to $T = T_c$.

$$H_c(T) = H_c(0)\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

The field has its maximum value $H_c(0)$ at $T=0k$ and vanishes at $T=T_c$.

The $H_c(T)$ neednot necessarily be an external field; It may also arise as a result of an electric current flowing through the sc specimen itself.

The critical current I_c flowing through a sc ring of radius r is $I_c = 2\pi r H_c$.

Q) A superconducting tin has a critical temp of 3.7k at zero magnetic field and a critical field of 0.0306 T at 0 k. Find the critical field at 2k.

$T_c = 3.7k$

$H_c(0) = 0.0306T$

$H_c(2) = ?$

$$H_c(2) = H_c(0)\left[1-\left(\frac{2}{3.7}\right)^2\right]$$

$= \underline{\underline{0.0216\,T}}$

Q) The magnetic field intensity in the tin material is zero at 3.69k and $(3\times10^5)/4\pi$ at 0 k. Calculate the temp of the sc if the field intensity is measured as $(2\times10^5)/4\pi$.

$T_c = 3.69\,k$

$B_c(0) = (3\times10^5)/4\pi$

$B_c(T) = (2\times10^5)/4\pi$

$$B_c(T) = B_c(0)\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

$$\frac{(2\times10^5)}{4\pi} = \frac{3\times10^5}{4\pi}\left[1-\left(\frac{T}{3.69}\right)^2\right]$$

$$1-\frac{2}{3} = \left(\frac{T}{3.69}\right)^2$$

$$\sqrt{\frac{1}{3}} = \frac{T}{3.69} \Rightarrow T = \frac{3.69}{\sqrt{3}} = \underline{\underline{2.13\,k}}$$

9) Calculate the critical current for a wire of lead having diameter of 1mm at 4.2k. The critical temp. for lead is 7.18 k and $H_c(0) = 6.5\times10^4$ A/m.

$H_c(0) = 6.5\times10^4$ A/m

$T_c = 7.18$ k

$T = 4.2$ k

$$H_c(T) = H_c(0)\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

$$= 6.5\times10^4\left[1-\left(\frac{4.2}{7.18}\right)^2\right]$$

$$= 4.28\times10^4 \text{ A/m}$$

$$I_c = 2\pi r H_c = 2\pi\times0.5\times10^{-3}\times4.28\times10^4$$

$$= 134.46 \text{ A}$$

Electrodynamics of SCs. (Phenomenological theory of sc)

London equations :-

On the Maxwell's electrodynamic eqns alone, the Meissner effect and field penetration cannot be explained. Thus based on two fluid model, London brothers put forward the electrodynamics of SCs.

According to two fluid model, a superconductor is supposed to be composed of two distinct types of e. normal ēs and super ēs. At $T=T_c$ all the ēs are normal and as the temp decreases no. of super ēs increases until at $T=0k$ all are super ēs.

∴ At any temp. the conduction ēs n becomes

$$n = n_n + n_s \qquad n_s = n\left[1-\left(\frac{T}{T_c}\right)^4\right]$$

The normal current and the supercurrent are assumed to flow parallel. ~~Since~~ the supercurrent flows with no resistance, on the contrary the normal current cannot flow without resistance and gets dissipated. ∴ We consider only super ēs.

The eqn of motion for a super ē in the presence of an electric field is,

$$m\frac{dv_s}{dt} = -eE$$

$V_s \rightarrow$ average velocity. The collision force is absent b'coz this type of ēs undergoes no collision.

Current density of super ēs is $J_s = -en_sV_s$

$$\frac{dJ_s}{dt} = -en_s\frac{dV_s}{dt}$$

$$\boxed{\frac{dJ_s}{dt} = \frac{e^2n_s}{m}E}$$

This is called First-London eqn. Thus in steady state, the electric field inside a SC vanishes. $(E=0)$ $\frac{dJ_s}{dt}=0 \Rightarrow E=0$

$\therefore$ From Maxwell's equation

$$\frac{dB}{dt} = -(\nabla \times E) = 0 \Rightarrow B = \text{const.}$$

The above eqn tells us that in steady state B is const inside a SC irrespective of its temperature. This result is not in agreement with the Meissner effect. In order to remove this discrepancy, London suggested some modifications.

Take curl of 1st London eqn & sub. for $\nabla \times E$

$$\nabla \times \frac{dJ_s}{dt} = \frac{e^2n_s}{m}\nabla \times E$$

$$\frac{d}{dt}(\nabla \times J_s) = -\frac{e^2n_s}{m}\frac{dB}{dt}$$

On integrating; $\therefore \boxed{\nabla \times J_s = -\frac{n_se^2B}{m}}$

This is the 2nd London eqn

We have the Maxwell's eqns

$$\nabla \times B = \mu_0 J_s$$

Taking curl; $\nabla \times \nabla \times B = \mu_0(\nabla \times J_s)$

but $\nabla \times \nabla \times B = \nabla(\nabla \cdot B) - \nabla^2 B$ $(\because \nabla \cdot B = 0)$

$= -\nabla^2 B$

$$\therefore -\nabla^2 B = \mu_0 (-)\frac{n_se^2B}{m}$$

$$\nabla^2 B = \frac{\mu_0 n_s e^2 B}{m}$$

or $\nabla^2 B = \frac{1}{\lambda^2} B$

where $\lambda = \left(\frac{m}{\mu_0 n_s e^2}\right)^{1/2}$ → London penetration depth.

1 D form of eqn is

$$\frac{\partial^2 B_z}{\partial x^2} = \frac{1}{\lambda^2} B_z$$ (Let the $\bar{B}$ be in z direction)

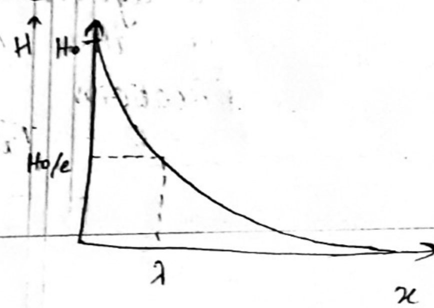
The soln to the above diff. eqn is

$$B_z(x) = B_z(0)\, e^{-x/\lambda}$$

Thus the field decreases exponentially as one proceeds from the surface into the SC. Thus the flux is not expelled entirely from the SC, as was once thought, but there is a small region near the surface in which there is an appreciable field.

The penetration depth is also ~~temp~~ dependent

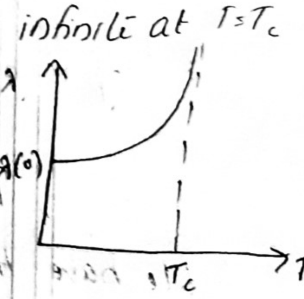
$$\lambda(T) = \lambda(0)\left(1-\left(\frac{T}{T_c}\right)^4\right)^{-1/2}$$



$\lambda(0) = \lambda$ at $T = 0$ K.

λ increases with T and become infinite at $T = T_c$

At $T = T_c$, the sub. become normal and the field penetrates the whole specimen.



Also $n_s = n_0\left[1-\left(\frac{T}{T_c}\right)^4\right]$

Q) The penetration depth of mercury at 3.5 K is about 750 Å. Estimate the penetration depth at 0 K.

$\lambda(3.5k) = 750$ Å

$T = 3.5$ K

$\lambda(0) = ?$

$$\lambda(T) = \lambda(0)\left(1-\left(\frac{T}{T_c}\right)^4\right)^{-1/2}$$

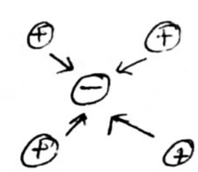
$$\lambda(0) = \lambda(T)\left[1-\left(\frac{T}{T_c}\right)^4\right]$$

Theory of superconductivity

The theory of sc was put forward by Bardeen, Cooper, Shrieffer and named as BCS theory. They theorized that the ēs in scs operate in pairs called Cooper pairs, which have opposite spin and opposite wave ~~kr~~ vectors. As a result, the pair of particles operate as though they have zero spin and have no ~~yet~~ net wave vector. Particles that have integer spins belongs to the class of particles called bosons. In the case of sc, ēs which are normally fermions, pair up and behave like bosons. In view of the zero wave vector of the Cooper pairs, they do not suffer from the typical scattering effects that normal ēs experience.

The theory of sc requires a net attractive interaction b/w a pair of ēs in the neighbourhood of Fermi surface. This is possible only if the interaction b/w the pair of ēs is taking place via +ve ions of the lattice, a direct Coulomb interaction b/w them always produces a repulsion.

Consider an ē passing through the packing of the +ve ions. Because it is -vely charged, it is attracted by the neighbouring +ve ions and gets screened by them. The screening greatly reduces the effective charge of this ē in fact the ion core may produce a net +ve charge on this assembly. At the same time, due to the attraction b/w the ēs and the ion core, the lattice gets deformed on local scale.



Now suppose another ē passes by the side of the assembly of the said ē and the ion core

The 2nd ē doesn't see simply the base ē but a deformed lattice and gets attracted towards the assembly. Thus it can be said that the 2nd ē interacts with the 1st via lattice deformation.

Thus the two ēs form a bound state and their motions are correlated. Cooper pairs can have a significant distance b/w them, of the order of several nm and still maintain the interaction b/w them. This is accomplished using lattice waves or by the exchange of phonons.

As a result of formation of cooper pairs, an energy gap occurs. This energy gap is highest at low temps. but vanishes at the transition temp. BCS theory gives the relation b/w energy gap at zero temp & T_c as.

$$\Delta(T=0) = 3.52\, kT_c.$$

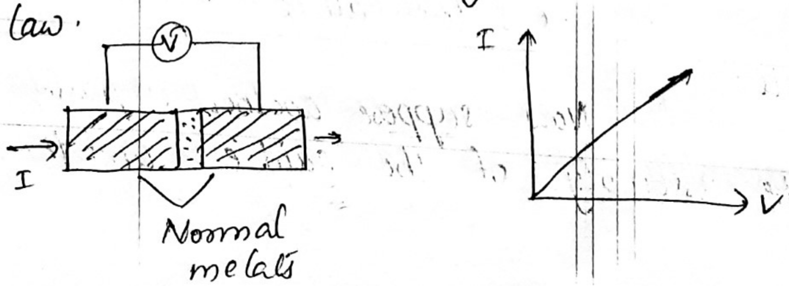
Isotope effect

Exptl study of sc'ing meterials shows that the transition temp varies with the average isotopic mass, M, of their constituents as,

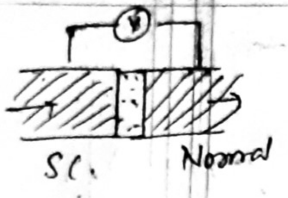
$$T_c \propto M^{-1/2}$$

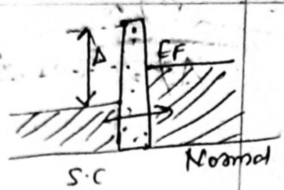
(1) Tunneling and the Josephson effect

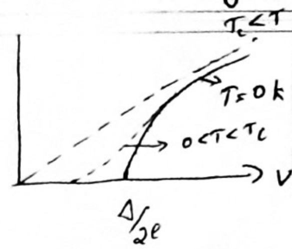
When a thin insulating layer is sandwitched b/w two metals, it acts as a potential barrier as far as the flow of conduction ēs is concerned. But by quantum mech. electrons can tunnel across the barrier, when a potential is applied. This current - voltage relation across the tunneling jn is observed to obey Ohm's law.



If one metal is SC; no current is observed to flow across the junction until the potential reaches a threshold value $eV = \Delta/2$. It is because the energy states lying horizontally below E_F in the normal metal are already occupied. As the temp. is increased towards T_c, the threshold voltage decreases. This consequently indicates a decrease in the energy gap itself. The above discussed tunnelling is called normal tunneling or single ē tunneling, where ēs tunnel in singles through the insulated layer.



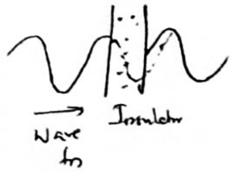




If both are SC, Josephson predicted that in addition to the normal tunneling of single p. ēs, the Cooper pairs not only can tunnel through the insulating layer from one SC, to another without dissociation, even at zero potential difference across the jn, but also their wave fns on both sides would be highly correlated. This is known as Josephson effect.

He showed that the effect of insulating layer is just to introduce a phase difference $\Delta\phi$ b/w the two parts of the wave fn on opposite sides of the junction.

He showed that the tunneling current is $I = I_0 \sin\phi_0$ $\left(\phi_0 = \frac{hc}{2e} \rightarrow \text{fluxoid}\right)$

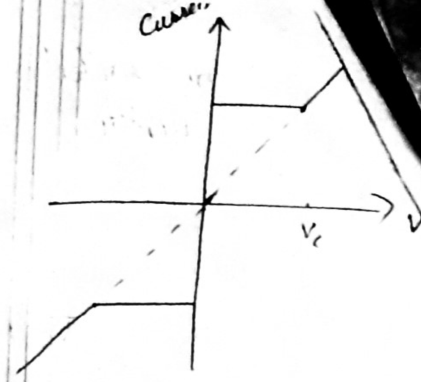


$I_0 \rightarrow$ Max. current, that the jn can carry without a pd.

Thus with no applied voltage, a dc current will flow across the jn with a value b/w I_0 & $-I_0$ according to the value of phase diff. $\phi_0 = \phi_2 - \phi_1$. This is called DC Josephson effect.

(3)

If a static potential V_0 is applied across the junction due to which an additional phase will be introduced by the Cooper pair during Tunneling across the jn.



$$\therefore \Delta\phi = \frac{Et}{\hbar}$$

$E \rightarrow$ total energy of the system

$$E = (2e)V_0$$

$$\therefore \Delta\phi = \frac{2eV_0 t}{\hbar}$$

$$\therefore I = I_0(\sin(\phi_0 + \Delta\phi)) = I_0 \sin\left(\phi_0 + \frac{2eV_0 t}{\hbar}\right)$$

$\rightarrow$ represents an alternating current, with an angular frequency

$$\omega = \frac{2eV_0}{\hbar}$$

This is ac Josephson effect. Thus a photon of energy $\hbar\omega = 2eV_0$ is emitted or absorbed when an e^- pair crosses the jn. By measuring the voltage & frequency, it is possible to obtain precise value of e/h.

Type I and type II ~~semiconductors~~ superconductors

In type I superconductors, superconducting is abruptly destroyed when the strength of the applied field rises above a critical value H_c. Thus they show perfect Meissner effect. The critical magnetic field of such superconductors will be low. They are also called as soft superconductors. This type of superconductivity is normally exhibited by pure metals eg:- aluminium, Lead, Mercury etc where A, B, C & D are consts. Their values can be obtained by applying the following boundary conditions

$$\psi_1(x)\big|_{x=0} = \psi_2(x)\big|_{x=0}$$

A type II super conductor keep the magnetic field out until a first critical field (H_{c1}) and then the vortices appear in the system (vortex is a magnetic flux that penetrates the superconductor). Although the vortices are formed in the system, the rest of the material stays superconducting until the field is increased to the 2nd critical field H_{c2} in which superconductor becomes normal. The critical field H_{c2} of type II superconductor will be very high. They donot exhibit a complete Meissner effect. They are also called as hard superconductors.