

The interference of water waves. Constructive interference occurs along the line AB and destructive interference occurs along the line CD.

When two or more trains of light waves meet in a region, they **interfere** to produce a new wave there whose instantaneous amplitude is the sum of those of the original waves. **Constructive interference** refers to the reinforcement of waves with the same phase to produce a greater amplitude, and destructive interference refers to the partial or complete cancellation of waves whose phases differ (Fig. 2.3). If the original waves have different frequencies, the result will be a mixture of constructive and destructive interference, as in Fig. 3.4.

The interference of light waves was first demonstrated in 1801 by Thomas Young, who used a pair of slits illuminated by monochromatic light from a single source (Fig. 2.4). From each slit secondary waves spread out as though originating at the slit; this is an example of **diffraction**, which, like interference, is a characteristic wave phenomenon. Owing to interference, the screen is not evenly lit but shows a pattern of alternate bright and dark lines. At those places on the screen where the path lengths from the two slits differ by an odd number of half wavelengths ($\lambda/2, 3\lambda/2, 5\lambda/2, \dots$), destructive interference occurs and a dark line is the result. At those places where the path lengths are

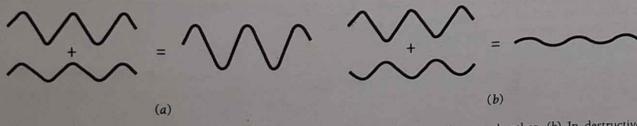


Figure 2.3 (a) In constructive interference, superposed waves in phase reinforce each other. (b) In destructive interference, waves out of phase partially or completely cancel each other.

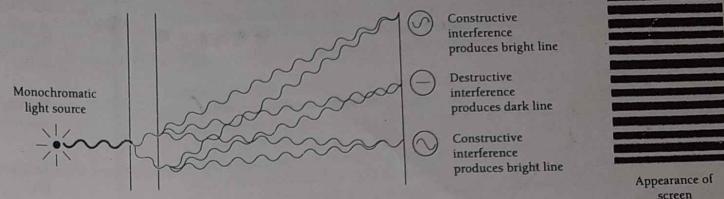


Figure 2.4 Origin of the interference pattern in Young's experiment. Constructive interference occurs where the difference in path lengths from the slits to the screen is $0, \lambda, 2\lambda, \dots$. Destructive interference occurs where the path difference is $\lambda/2, 3\lambda/2, 5\lambda/2, \dots$

equal or differ by a whole number of wavelengths ($\lambda, 2\lambda, 3\lambda, \dots$), constructive interference occurs and a bright line is the result. At intermediate places the interference is only partial, so the light intensity on the screen varies gradually between the bright and dark lines.

Interference and diffraction are found only in waves—the particles we are familiar with do not behave in those ways. If light consisted of a stream of classical particles, the entire screen would be dark. Thus Young's experiment is proof that light consists of waves. Maxwell's theory further tells us what kind of waves they are: electromagnetic. Until the end of the nineteenth century the nature of light seemed settled forever.

2.2 BLACKBODY RADIATION

Only the quantum theory of light can explain its origin

Following Hertz's experiments, the question of the fundamental nature of light seemed clear: light consisted of em waves that obeyed Maxwell's theory. This certainty lasted only a dozen years. The first sign that something was seriously amiss came from attempts to understand the origin of the radiation emitted by bodies of matter.

We are all familiar with the glow of a hot piece of metal, which gives off visible light whose color varies with the temperature of the metal, going from red to yellow to white as it becomes hotter and hotter. In fact, other frequencies to which our eyes do not respond are present as well. An object need not be so hot that it is luminous for it to be radiating em energy. All objects radiate such energy continuously whatever their temperatures, though which frequencies predominate depends on the temperature. At room temperature most of the radiation is in the infrared part of the spectrum and hence is invisible.

The ability of a body to radiate is closely related to its ability to absorb radiation. This is to be expected, since a body at a constant temperature is in thermal equilibrium with its surroundings and must absorb energy from them at the same rate as it emits energy. It is convenient to consider as an ideal body one that absorbs all radiation incident upon it, regardless of frequency. Such a body is called a **blackbody**.

The point of introducing the idealized blackbody in a discussion of thermal radiation is that we can now disregard the precise nature of whatever is radiating, since

Constructive interference
Destructive interference

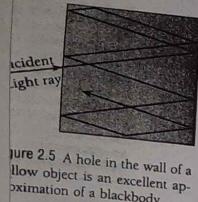


Figure 2.5 A hole in the wall of a hollow object is an excellent approximation of a blackbody.

all blackbodies behave identically. In the laboratory a blackbody can be approximated by a hollow object with a very small hole leading to its interior (Fig. 2.5). Any radiation striking the hole enters the cavity, where it is trapped by reflection back and forth until it is absorbed. The cavity walls are constantly emitting and absorbing radiation, and it is in the properties of this radiation (blackbody radiation) that we are interested.

Experimentally we can sample blackbody radiation simply by inspecting what emerges from the hole in the cavity. The results agree with everyday experience. A blackbody radiates more when it is hot than when it is cold, and the spectrum of a hot blackbody has its peak at a higher frequency than the peak in the spectrum of a cooler one. We recall the behavior of an iron bar as it is heated to progressively higher temperatures: at first it glows dull red, then bright orange-red, and eventually it becomes "white hot." The spectrum of blackbody radiation is shown in Fig. 2.6 for two temperatures.

The Ultraviolet Catastrophe

Why does the blackbody spectrum have the shape shown in Fig. 2.6? This problem was examined at the end of the nineteenth century by Lord Rayleigh and James Jeans. The details of their calculation are given in Chap. 9. They started by considering the radiation inside a cavity of absolute temperature T whose walls are perfect reflectors to be a series of standing em waves (Fig. 2.7). This is a three-dimensional generalization of standing waves in a stretched string. The condition



color and brightness of an heated until it glows, such filament of this light bulb, is upon its temperature, here is about 3000 K. An that glows white is hotter is when it glows red, and off more light as well.

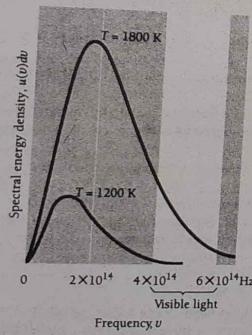


Figure 2.6 Blackbody spectra. The spectral distribution of energy in the radiation depends only on the temperature of the body. The higher the temperature, the greater the amount of radiation and the higher the frequency at which the maximum emission occurs. The dependence of the latter on temperature follows a formula called Wien's displacement law, which is discussed in Sec. 9.6.

for standing waves in such a cavity is that the path length from wall to wall, whatever the direction, must be a whole number of half-wavelengths, so that a node occurs at each reflecting surface. The number of independent standing waves $G(\nu)d\nu$ in the frequency interval between ν and $\nu + d\nu$ per unit volume in the cavity turned out to be

$$\text{Density of standing waves in cavity} \quad G(\nu)d\nu = \frac{8\pi\nu^2d\nu}{c^3} \quad (2.1)$$

This formula is independent of the shape of the cavity. As we would expect, the higher the frequency ν , the shorter the wavelength and the greater the number of possible standing waves.

The next step is to find the average energy per standing wave. According to the theorem of equipartition of energy, a mainstay of classical physics, the average energy per degree of freedom of an entity (such as a molecule of an ideal gas) that is a member of a system of such entities in thermal equilibrium at the temperature T is $\frac{1}{2}kT$. Here k is Boltzmann's constant:

$$\text{Boltzmann's constant} \quad k = 1.381 \times 10^{-23} \text{ J/K}$$

A degree of freedom is a mode of energy possession. Thus a monatomic ideal gas molecule has three degrees of freedom, corresponding to kinetic energy of motion in three independent directions, for an average total energy of $\frac{3}{2}kT$.

(A one-dimensional harmonic oscillator has two degrees of freedom, one that corresponds to its kinetic energy and one that corresponds to its potential energy. Because each standing wave in a cavity originates in an oscillating electric charge in the cavity wall, two degrees of freedom are associated with the wave and it should have an average energy of $2\frac{1}{2}kT$.)

$$\text{Classical average energy per standing wave} \quad \bar{\epsilon} = kT \quad (2.2)$$

The total energy $u(\nu)d\nu$ per unit volume in the cavity in the frequency interval from ν to $\nu + d\nu$ is therefore

$$\text{Rayleigh-Jeans formula} \quad u(\nu)d\nu = \bar{\epsilon}G(\nu)d\nu = \frac{8\pi kT}{c^3}\nu^2d\nu \quad (2.3)$$

This radiation rate is proportional to this energy density for frequencies between ν and $\nu + d\nu$. Equation (2.3), the Rayleigh-Jeans formula, contains everything that classical physics can say about the spectrum of blackbody radiation.

Even a glance at Eq. (2.3) shows that it cannot possibly be correct. As the frequency ν increases toward the ultraviolet end of the spectrum, this formula predicts that the energy density should increase as ν^2 . In the limit of infinitely high frequencies, $u(\nu)d\nu$ therefore should also go to infinity. In reality, of course, the energy density (and radiation rate) falls to 0 as $\nu \rightarrow \infty$ (Fig. 2.8). This discrepancy became known as the ultraviolet catastrophe of classical physics. Where did Rayleigh and Jeans go wrong?

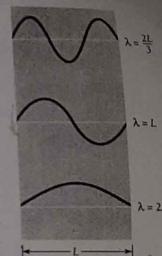


Figure 2.7 Em radiation in a cavity whose walls are perfect reflectors consists of standing waves that have nodes at the walls, which restricts their possible wavelengths. Shown are three possible wavelengths when the distance between opposite walls is L .

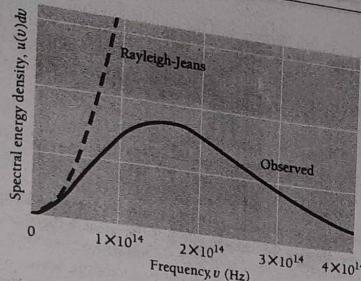


Figure 2.8 Comparison of the Rayleigh-Jeans formula for the spectrum of the radiation from a blackbody at 1500 K with the observed spectrum. The discrepancy is known as the ultraviolet catastrophe because it increases with increasing frequency. This failure of classical physics led Planck to the discovery that radiation is emitted in quanta whose energy is $h\nu$.

Planck Radiation Formula

In 1900 the German physicist Max Planck used "lucky guesswork" (as he later called it) to come up with a formula for the spectral energy density of blackbody radiation:

Planck radiation formula

$$u(\nu) d\nu = \frac{8\pi h}{c^3} \frac{\nu^3 d\nu}{e^{h\nu/kT} - 1} \quad (2.4)$$

Here h is a constant whose value is

Planck's constant

$$h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s}$$

Max Planck (1858–1947) was born in Kiel and educated in Munich and Berlin. At the University of Berlin he studied under Kirchhoff and Helmholtz, as Hertz had done earlier. Planck realized that blackbody radiation was important because it was a fundamental effect independent of atomic structure, which was still a mystery in the late nineteenth century, and worked at understanding it for six years before finding the formula the radiation obeyed. He "strived from the day of its discovery to give it a real physical interpretation." The result was the discovery that radiation is emitted in energy steps of $h\nu$. Although this discovery, for which he received the Nobel Prize in 1918, is now considered to mark the start of

modern physics, Planck himself remained skeptical for a long time of the physical reality of quanta. As he later wrote, "My vain attempts to somehow reconcile the elementary quantum with classical theory continued for many years and cost me great effort. . . Now I know for certain that the quantum of action has a much more fundamental significance than I originally suspected."

Like many physicists, Planck was a competent musician (he sometimes played with Einstein) and in addition enjoyed mountain climbing. Although Planck remained in Germany during the Hitler era, he protested the Nazi treatment of Jewish scientists and lost his presidency of the Kaiser Wilhelm Institute as a result. In 1945 one of his sons was implicated in a plot to kill Hitler and was executed. After World War II the Institute was renamed after Planck and he was again its head until his death.



At high frequencies, $h\nu \gg kT$ and $e^{h\nu/kT} \rightarrow \infty$, which means that $u(\nu) d\nu \rightarrow 0$ as observed. No more ultraviolet catastrophe. At low frequencies, where the Rayleigh-Jeans formula is a good approximation to the data (see Fig. 2.8), $h\nu \ll kT$ and $h\nu/kT \ll 1$. In general,

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

If x is small, $e^x \approx 1 + x$, and so for $h\nu/kT \ll 1$ we have

$$\frac{1}{e^{h\nu/kT} - 1} \approx \frac{1}{1 + \frac{h\nu}{kT} - 1} \approx \frac{kT}{h\nu} \quad h\nu \ll kT$$

Thus at low frequencies Planck's formula becomes

$$u(\nu) d\nu \approx \frac{8\pi h}{c^3} \nu^3 \left(\frac{kT}{h\nu} \right) d\nu = \frac{8\pi kT}{c^3} \nu^2 d\nu$$

which is the Rayleigh-Jeans formula. Planck's formula is clearly at least on the right track; in fact, it has turned out to be completely correct.

Next Planck had the problem of justifying Eq. (2.4) in terms of physical principles. A new principle seemed needed to explain his formula, but what was it? After several weeks of "the most strenuous work of my life," Planck found the answer: The oscillators in the cavity walls could not have a continuous distribution of possible energies ϵ but must have only the specific energies

$$\text{Oscillator energies} \quad \epsilon_n = nh\nu \quad n = 0, 1, 2, \dots \quad (2.5)$$

An oscillator emits radiation of frequency ν when it drops from one energy state to the next lower one, and it jumps to the next higher state when it absorbs radiation of frequency ν . Each discrete bundle of energy $h\nu$ is called a quantum (plural quanta) from the Latin for "how much."

With oscillator energies limited to $nh\nu$, the average energy per oscillator in the cavity walls—and so per standing wave—turned out to be not $\bar{\epsilon} = kT$ as for a continuous distribution of oscillator energies, but instead

$$\text{Actual average energy per standing wave} \quad \bar{\epsilon} = \frac{h\nu}{e^{h\nu/kT} - 1} \quad (2.6)$$

This average energy leads to Eq. (2.4). Blackbody radiation is further discussed in Chap. 9.

Example 2.1

Assume that a certain 660-Hz tuning fork can be considered as a harmonic oscillator whose vibrational energy is 0.04 J. Compare the energy quanta of this tuning fork with those of an atomic oscillator that emits and absorbs orange light whose frequency is 5.00×10^{14} Hz.

Solution

(a) For the tuning fork,

$$h\nu_1 = (6.63 \times 10^{-34} \text{ J} \cdot \text{s}) (660 \text{ s}^{-1}) = 4.38 \times 10^{-31} \text{ J}$$

The total energy of the vibrating tines of the fork is therefore about 10^{29} times the quantum energy $h\nu$. The quantization of energy in the tuning fork is obviously far too small to be observed, and we are justified in regarding the fork as obeying classical physics.

(b) For the atomic oscillator,

$$h\nu_2 = (6.63 \times 10^{-34} \text{ J} \cdot \text{s}) (5.00 \times 10^{14} \text{ s}^{-1}) = 3.32 \times 10^{-19} \text{ J}$$

In electronvolts, the usual energy unit in atomic physics,

$$h\nu_2 = \frac{3.32 \times 10^{-19} \text{ J}}{1.60 \times 10^{-19} \text{ eV}} = 2.08 \text{ eV}$$

This is a significant amount of energy on an atomic scale, and it is not surprising that classical physics fails to account for phenomena on this scale.

The concept that the oscillators in the cavity walls can interchange energy with standing waves in the cavity only in quanta of $h\nu$ is, from the point of view of classical physics, impossible to understand. Planck regarded his quantum hypothesis as an "act of desperation" and, along with other physicists of his time, was unsure of how seriously to regard it as an element of physical reality. For many years he held that, although the energy transfers between electric oscillators and em waves apparently are quantized, em waves themselves behave in an entirely classical way with a continuous range of possible energies.

2.3 PHOTOELECTRIC EFFECT

The energies of electrons liberated by light depend on the frequency of the light

During his experiments on em waves, Hertz noticed that sparks occurred more readily in the air gap of his transmitter when ultraviolet light was directed at one of the metal balls. He did not follow up this observation, but others did. They soon discovered that the cause was electrons emitted when the frequency of the light was sufficiently high. This phenomenon is known as the **photoelectric effect** and the emitted electrons are called **photoelectrons**. It is one of the ironies of history that the same work to demonstrate that light consists of em waves also gave the first hint that this was not the whole story.

Figure 2.9 shows how the photoelectric effect was studied. An evacuated tube contains two electrodes connected to a source of variable voltage, with the metal plate whose surface is irradiated as the anode. Some of the photoelectrons that emerge from this surface have enough energy to reach the cathode despite its negative polarity, and they constitute the measured current. The slower photoelectrons are repelled before they get to the cathode. When the voltage is increased to a certain value V_0 , of the order of several volts, no more photoelectrons arrive, as indicated by the current dropping to zero. This extinction voltage corresponds to the maximum photoelectron kinetic energy.

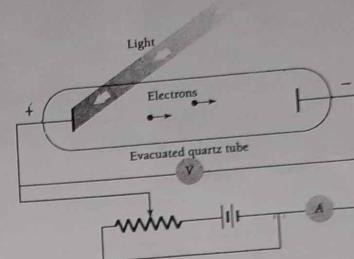


Figure 2.9 Experimental observation of the photoelectric effect.

The existence of the photoelectric effect is not surprising. After all, light waves carry energy, and some of the energy absorbed by the metal may somehow concentrate on individual electrons and reappear as their kinetic energy. The situation should be like water waves dislodging pebbles from a beach. But three experimental findings show that no such simple explanation is possible.

- 1 Within the limits of experimental accuracy (about 10^{-9} s), there is no time interval between the arrival of light at a metal surface and the emission of photoelectrons. However, because the energy in an em wave is supposed to be spread across the wavefronts, a period of time should elapse before an individual electron accumulates enough energy (several eV) to leave the metal. A detectable photoelectron current results when 10^{-6} W/m² of em energy is absorbed by a sodium surface. A layer of sodium 1 atom thick and 1 m² in area contains about 10^{19} atoms, so if the incident light is absorbed in the uppermost atomic layer, each atom receives energy at an average rate of 10^{-25} W. At this rate over a month would be needed for an atom to accumulate energy of the magnitude that photoelectrons from a sodium surface are observed to have.
- 2 A bright light yields more photoelectrons than a dim one of the same frequency, but the electron energies remain the same (Fig. 2.10). The em theory of light, on the contrary, predicts that the more intense the light, the greater the energies of the electrons.
- 3 The higher the frequency of the light, the more energy the photoelectrons have (Fig. 2.11). Blue light results in faster electrons than red light. At frequencies below a certain critical frequency ν_0 , which is characteristic of each particular metal, no electrons are emitted. Above ν_0 the photoelectrons range in energy from 0 to a maximum value that increases linearly with increasing frequency (Fig. 2.12). This observation, also, cannot be explained by the em theory of light.

Quantum Theory of Light

When Planck's derivation of his formula appeared, Einstein was one of the first—perhaps the first—to understand just how radical the postulate of energy quantization

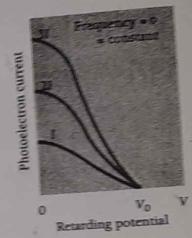


Figure 2.10 Photoelectron current is proportional to light intensity I for all retarding voltages. The stopping potential V_0 , which corresponds to the maximum photoelectron energy, is the same for all intensities of light of the same frequency ν .

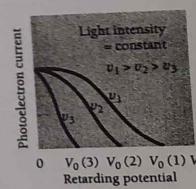


Figure 2.11 The stopping potential V_0 , and hence the maximum photoelectron energy, depends on the frequency of the light. When the retarding potential is $V = 0$, the photoelectron current is the same for light of a given intensity regardless of its frequency.

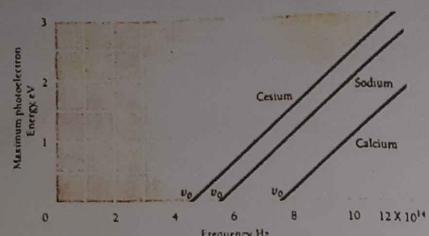


Figure 2.12 Maximum photoelectron kinetic energy KE_{max} , versus frequency of incident light for three metal surfaces.

of oscillators was "It was as if the ground was pulled from under one." A few years later, in 1905, Einstein realized that the photoelectric effect could be understood if the energy in light is not spread out over wavefronts but is concentrated in small packets, or photons. (The term photon was coined by the chemist Gilbert Lewis in 1926.) Each photon of light of frequency ν has the energy $h\nu$, the same as Planck's quantum energy. Planck had thought that, although energy from an electric oscillator apparently had to be given to em waves in separate quanta of $h\nu$ each, the waves themselves behaved exactly as in conventional wave theory. Einstein's break with classical physics was more drastic. Energy was not only given to em waves in separate quanta but was also carried by the waves in separate quanta.

The three experimental observations listed above follow directly from Einstein's hypothesis: (1) Because em wave energy is concentrated in photons and not spread out, there should be no delay in the emission of photoelectrons. (2) All photons of frequency ν have the same energy, so changing the intensity of a monochromatic light beam will change the number of photoelectrons but not their energies. (3) The higher the frequency ν , the greater the photon energy $h\nu$ and so the more energy the photoelectrons have.

What is the meaning of the critical frequency v_0 below which no photoelectrons are emitted? There must be a minimum energy ϕ for an electron to escape from a particular metal surface or else electrons would pour out all the time. This energy is called the work function of the metal, and is related to v_0 by the formula

$$\text{Work function } \phi = h\nu_0 \quad (2.7)$$

The greater the work function of a metal, the more energy is needed for an electron to leave its surface, and the higher the critical frequency for photoelectric emission to occur.

Some examples of photoelectric work functions are given in Table 2.1. To pull an electron from a metal surface generally takes about half as much energy as that needed

Table 2.1 Photoelectric Work Functions

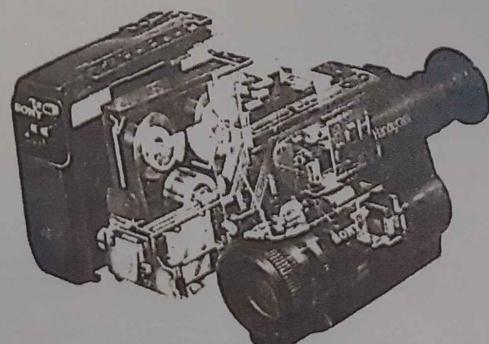
Metal	Symbol	Work Function, eV
Cesium	Cs	1.9
Francium	Fm	2.2
Sodium	Na	2.3
Lithium	Li	2.6
Calcium	Ca	3.2
Copper	Cu	4.7
Silver	Ag	4.7
Platinum	Pt	6.4

to pull an electron from a free atom of that metal (see Fig. 7.10); for instance, the ionization energy of cesium is 3.9 eV compared with its work function of 1.9 eV. Since the visible spectrum extends from about 4.3 to about 7.5×10^{14} Hz, which corresponds to quantum energies of 1.7 to 3.3 eV, it is clear from Table 2.1 that the photoelectric effect is a phenomenon of the visible and ultraviolet regions.

According to Einstein, the photoelectric effect in a given metal should obey the equation

$$\text{Photoelectric effect} \quad h\nu = KE_{max} + \phi \quad (2.8)$$

where $h\nu$ is the photon energy, KE_{max} is the maximum photoelectron energy (which is proportional to the stopping potential), and ϕ is the minimum energy needed for an



All light-sensitive detectors, including the eye and the one used in this video camera, are based on the absorption of energy from photons of light by electrons in the atoms the light falls on.

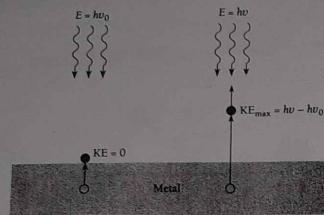


Figure 2.13 If the energy $h\nu_0$ (the work function of the surface) is needed to remove an electron from a metal surface, the maximum electron kinetic energy will be $h\nu - h\nu_0$ when light of frequency ν is directed at the surface.

electron to leave the metal. Because $\phi = h\nu_0$, Eq. (2.8) can be rewritten (Fig. 2.13)

$$\begin{aligned} h\nu &= KE_{\max} + h\nu_0 \\ KE_{\max} &= h\nu - h\nu_0 = h(\nu - \nu_0) \end{aligned} \quad (2.9)$$

This formula accounts for the relationships between KE_{\max} and ν plotted in Fig. 2.12 from experimental data. If Einstein was right, the slopes of the lines should all be equal to Planck's constant h , and this is indeed the case.

In terms of electronvolts, the formula $E = h\nu$ for photon energy becomes

$$\text{Photon energy} \quad E = \left(\frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{1.602 \times 10^{-19} \text{ J/eV}} \right) \nu = (4.136 \times 10^{-15}) \nu \text{ eV} \cdot \text{s} \quad (2.10)$$

If we are given instead the wavelength λ of the light, then since $\nu = c/\lambda$ we have

$$\text{Photon energy} \quad E = \frac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{\lambda} = \frac{1.240 \times 10^{-6} \text{ eV} \cdot \text{m}}{\lambda} \quad (2.11)$$

Example 2.2

Ultraviolet light of wavelength 350 nm and intensity 1.00 W/m² is directed at a potassium surface. (a) Find the maximum KE of the photoelectrons. (b) If 0.50 percent of the incident photons produce photoelectrons, how many are emitted per second if the potassium surface has an area of 1.00 cm²?

Solution

(a) From Eq. (2.11) the energy of the photons is, since 1 nm = 1 nanometer = 10^{-9} m,

$$E_p = \frac{1.24 \times 10^{-6} \text{ eV} \cdot \text{m}}{(350 \text{ nm})(10^{-9} \text{ m/nm})} = 3.5 \text{ eV}$$

Table 2.1 gives the work function of potassium as 2.2 eV, so

$$KE_{\max} = h\nu - \phi = 3.5 \text{ eV} - 2.2 \text{ eV} = 1.3 \text{ eV}$$

(b) The photon energy in joules is $5.68 \times 10^{-19} \text{ J}$. Hence the number of photons that reach the surface per second is

$$n_p = \frac{E/I}{E_p} = \frac{(P/A)(\lambda)}{E_p} = \frac{(1.00 \text{ W/m}^2)(1.00 \times 10^{-4} \text{ m}^2)}{5.68 \times 10^{-19} \text{ J/photon}} = 1.76 \times 10^{14} \text{ photons/s}$$

The rate at which photoelectrons are emitted is therefore

$$n_e = (0.0050)n_p = 8.8 \times 10^{11} \text{ photoelectrons/s}$$

Thermionic Emission

Einstein's interpretation of the photoelectric effect is supported by studies of thermionic emission. Long ago it was discovered that the presence of a very hot object increases the electric conductivity of the surrounding air. Eventually the reason for this effect was found to be the emission of electrons from such an object. Thermionic emission makes possible the operation of such devices as television picture tubes, in which metal filaments or specially coated cathodes at high temperature supply dense streams of electrons.

The emitted electrons evidently obtain their energy from the thermal agitation of the particles of the metal, and we would expect the electrons to need a certain minimum energy to escape. This minimum energy can be determined for many surfaces, and it is always close to the photoelectric work function for the same surfaces. In photoelectric emission, photons of light provide the energy required by an electron to escape, while in thermionic emission heat does so.

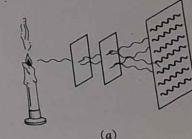
2.4 WHAT IS LIGHT?

Both wave and particle

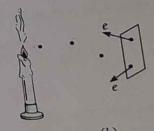
The concept that light travels as a series of little packets is directly opposed to the wave theory of light (Fig. 2.14). Both views have strong experimental support, as we have seen. According to the wave theory, light waves leave a source with their energy spread out continuously through the wave pattern. According to the quantum theory, light consists of individual photons, each small enough to be absorbed by a single electron. Yet, despite the particle picture of light it presents, the quantum theory needs the frequency of the light to describe the photon energy.

Which theory are we to believe? A great many scientific ideas have had to be revised or discarded when they were found to disagree with new data. Here, for the first time, two different theories are needed to explain a single phenomenon. This situation is not the same as it is, say, in the case of relativistic versus Newtonian mechanics, where one turns out to be an approximation of the other. The connection between the wave and quantum theories of light is something else entirely.

To appreciate this connection, let us consider the formation of a double-slit interference pattern on a screen. In the wave model, the light intensity at a place on the screen depends on E^2 , the average over a complete cycle of the square of the instantaneous magnitude E of the em wave's electric field. In the particle model, this



(a)



(b)

Figure 2.14 (a) The wave theory of light explains diffraction and interference, which the quantum theory cannot account for. (b) The quantum theory explains the photoelectric effect, which the classical theory cannot account for.

2.7 COMPTON EFFECT

Further confirmation of the photon model

According to the quantum theory of light, photons behave like particles except for their lack of rest mass. How far can this analogy be carried? For instance, can we consider a collision between a photon and an electron as if both were billiard balls?

Figure 2.22 shows such a collision: an x-ray photon strikes an electron (assumed to be initially at rest in the laboratory coordinate system) and is scattered away from its original direction of motion while the electron receives an impulse and begins to move. We can think of the photon as losing an amount of energy in the collision that is the same as the kinetic energy KE gained by the electron, although actually separate photons are involved. If the initial photon has the frequency ν associated with it, the scattered photon has the lower frequency ν' , where

$$\text{Loss in photon energy} = \text{gain in electron energy}$$

$$h\nu - h\nu' = KE \quad (2.14)$$

From Chap. 1 we recall that the momentum of a massless particle is related to its energy by the formula

$$E = pc \quad (1.25)$$

Since the energy of a photon is $h\nu$, its momentum is

Photon momentum

$$p = \frac{E}{c} = \frac{h\nu}{c} \quad (2.15)$$

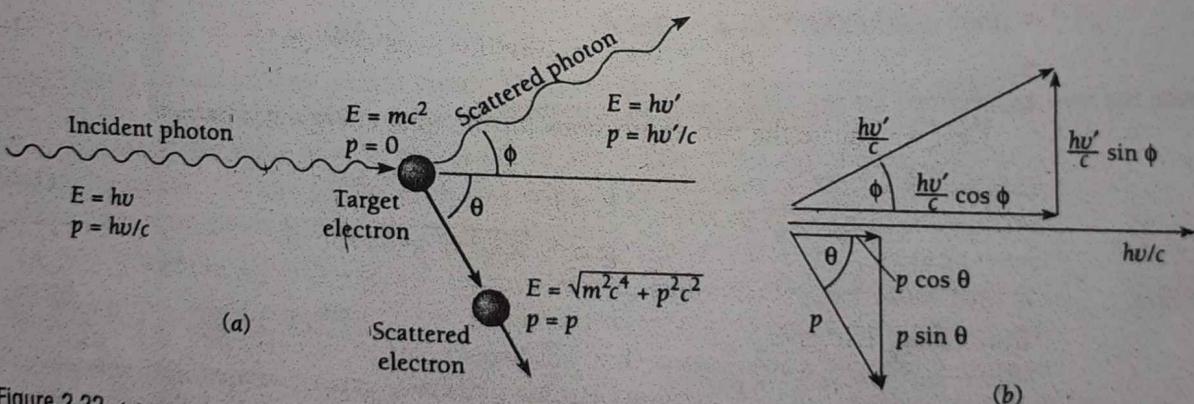


Figure 2.22 (a) The scattering of a photon by an electron is called the Compton effect. Energy and momentum are conserved in such an event, and as a result the scattered photon has less energy (longer wavelength) than the incident photon. (b) Vector diagram of the momenta and their components of the incident and scattered photons and the scattered electron.

Momentum, unlike energy, is a vector quantity that incorporates direction as well as magnitude, and in the collision momentum must be conserved in each of two mutually perpendicular directions. (When more than two bodies participate in a collision, momentum must be conserved in each of three mutually perpendicular directions.) The directions we choose here are that of the original photon and one perpendicular to it in the plane containing the electron and the scattered photon (Fig. 2.22).

The initial photon momentum is $h\nu/c$, the scattered photon momentum is $h\nu'/c$, and the initial and final electron momenta are respectively 0 and p . In the original photon

$$\text{Initial momentum} = \text{final momentum}$$

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c} \cos \phi + p \cos \theta \quad (2.16)$$

and perpendicular to this direction

$$\text{Initial momentum} = \text{final momentum}$$

$$0 = \frac{h\nu'}{c} \sin \phi - p \sin \theta \quad (2.17)$$

The angle ϕ is that between the directions of the initial and scattered photons, and θ is that between the directions of the initial photon and the recoil electron. From Eqs. (2.14), (2.16), and (2.17) we can find a formula that relates the wavelength difference between initial and scattered photons with the angle ϕ between their directions, both of which are readily measurable quantities (unlike the energy and momentum of the recoil electron).

The first step is to multiply Eqs. (2.16) and (2.17) by c and rewrite them as

$$pc \cos \theta = h\nu - h\nu' \cos \phi$$

$$pc \sin \theta = h\nu' \sin \phi$$

By squaring each of these equations and adding the new ones together, the angle θ is eliminated, leaving

$$p^2 c^2 = (h\nu)^2 - 2(h\nu)(h\nu') \cos \phi + (h\nu')^2 \quad (2.18)$$

Next we equate the two expressions for the total energy of a particle

$$E = KE + mc^2 \quad (1.20)$$

$$E = \sqrt{m^2 c^4 + p^2 c^2} \quad (1.24)$$

from Chap. 1 to give

$$(KE + mc^2)^2 = m^2 c^4 + p^2 c^2$$

$$p^2 c^2 = KE^2 + 2mc^2 KE$$

Since

$$KE = h\nu - h\nu'$$

we have

$$p^2 c^2 = (h\nu)^2 - 2(h\nu)(h\nu') + (h\nu')^2 + 2mc^2(h\nu - h\nu') \quad (2.19)$$

Substituting this value of $p^2 c^2$ in Eq. (2.18), we finally obtain

$$2mc^2(h\nu - h\nu') = 2(h\nu)(h\nu')(1 - \cos \phi) \quad (2.20)$$

This relationship is simpler when expressed in terms of wavelength λ . Dividing Eq. (2.20) by $2h^2 c^2$,

$$\frac{mc}{h} \left(\frac{\nu}{c} - \frac{\nu'}{c} \right) = \frac{\nu}{c} \frac{\nu'}{c} (1 - \cos \phi)$$

and so, since $\nu/c = 1/\lambda$ and $\nu'/c = 1/\lambda'$,

$$\frac{mc}{h} \left(\frac{1}{\lambda} - \frac{1}{\lambda'} \right) = \frac{1 - \cos \phi}{\lambda \lambda'} \quad (2.21)$$

Compton effect

$$\lambda' - \lambda = \frac{h}{mc} (1 - \cos \phi)$$

Equation (2.21) was derived by Arthur H. Compton in the early 1920s, and the phenomenon it describes, which he was the first to observe, is known as the **Compton effect**.

Equation (2.21) gives the change in wavelength expected for a photon that is scattered through the angle ϕ by a particle of rest mass m . This change is independent of the wavelength λ of the incident photon. The quantity

$$\text{Compton wavelength} \quad \lambda_C = \frac{h}{mc} \quad (2.22)$$

is called the **Compton wavelength** of the scattering particle. For an electron $\lambda_C = 2.426 \times 10^{-12}$ m, which is 2.426 pm (1 pm = 1 picometer = 10^{-12} m). In terms of λ_C , Eq. (2.21) becomes

$$\text{Compton effect} \quad \lambda' - \lambda = \lambda_C (1 - \cos \phi) \quad (2.23)$$

The Compton wavelength gives the scale of the wavelength change of the incident photon. From Eq. (2.23) we note that the greatest wavelength change possible corresponds to $\phi = 180^\circ$, when the wavelength change will be twice the Compton wavelength λ_C . Because $\lambda_C = 2.426$ pm for an electron, and even less for other particles owing to their larger rest masses, the maximum wavelength change in the Compton effect is 4.852 pm. Changes of this magnitude or less are readily observable only in x-rays: the shift in wavelength for visible light is less than 0.01 percent. The Compton effect is the chief means by which x-rays lose energy when they pass through matter.

Arthur Holly Compton (1892–1962), a native of Ohio, was educated at College of Wooster and Princeton. While at Washington University in St. Louis he found that x-rays increase in wavelength when scattered, which he explained in 1923 on the basis of the quantum theory of light. This work convinced remaining doubters of the reality of photons.

Example 2.4

X-rays of wavelength 10.0 pm are scattered from a target. (a) Find the wavelength of the x-rays scattered through 45° . (b) Find the maximum wavelength present in the scattered x-rays. (c) Find the maximum kinetic energy of the recoil electrons.

Solution

(a) From Eq. (2.23), $\lambda' - \lambda = \lambda_C(1 - \cos \phi)$, and so

$$\begin{aligned}\lambda' &= \lambda + \lambda_C(1 - \cos 45^\circ) \\ &= 10.0 \text{ pm} + 0.293\lambda_C \\ &= 10.7 \text{ pm}\end{aligned}$$

(b) $\lambda' - \lambda$ is a maximum when $(1 - \cos \phi) = 2$, in which case

$$\lambda' = \lambda + 2\lambda_C = 10.0 \text{ pm} + 4.9 \text{ pm} = 14.9 \text{ pm}$$

(c) The maximum recoil kinetic energy is equal to the difference between the energies of the incident and scattered photons, so

$$KE_{\max} = h(\nu - \nu') = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$$

where λ' is given in (b). Hence

$$\begin{aligned}KE_{\max} &= \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{10^{-12} \text{ m/pm}} \left(\frac{1}{10.0 \text{ pm}} - \frac{1}{14.9 \text{ pm}} \right) \\ &= 6.54 \times 10^{-15} \text{ J}\end{aligned}$$

which is equal to 40.8 keV.

The experimental demonstration of the Compton effect is straightforward. As in Fig. 2.23, a beam of x-rays of a single, known wavelength is directed at a target, and the wavelengths of the scattered x-rays are determined at various angles ϕ . The results, shown in Fig. 2.24, exhibit the wavelength shift predicted by Eq. (2.21), but at each angle the scattered x-rays also include many that have the initial wavelength. This is not hard to understand. In deriving Eq. (2.21) it was assumed that the scattering particle is able to move freely, which is reasonable since many of the electrons in matter

Particle Properties of Waves

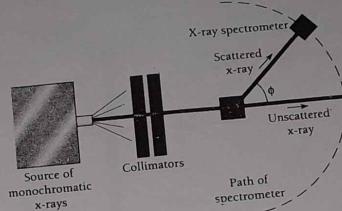


Figure 2.23 Experimental demonstration of the Compton effect.

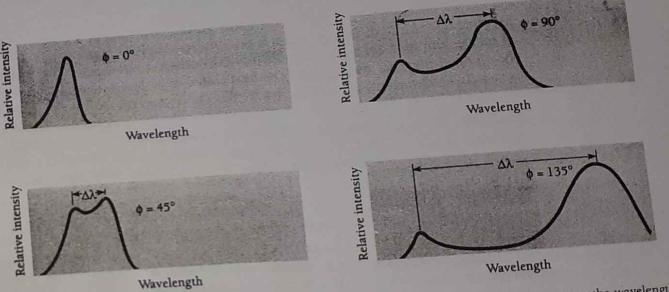


Figure 2.24 Experimental confirmation of Compton scattering. The greater the scattering angle, the greater the wavelength change, in accord with Eq. (2.21).

are only loosely bound to their parent atoms. Other electrons, however, are very tightly bound and when struck by a photon, the entire atom recoils instead of the single electron. In this event the value of m to use in Eq. (2.21) is that of the entire atom, which is tens of thousands of times greater than that of an electron, and the resulting Compton shift is accordingly so small as to be undetectable.

2.8 PAIR PRODUCTION

Energy into matter

As we have seen, in a collision a photon can give an electron all of its energy (the photoelectric effect) or only part (the Compton effect). It is also possible for a photon to materialize into an electron and a positron, which is a positively charged electron. In this process, called pair production, electromagnetic energy is converted into matter.

Far in the past people began to suspect that matter, despite appearing continuous, has a definite structure on a microscopic level beyond the direct reach of our senses. This suspicion did not take on a more concrete form until a little over a century and a half ago. Since then the existence of atoms and molecules, the ultimate particles of matter in its common forms, has been amply demonstrated, and their own ultimate particles, electrons, protons, and neutrons, have been identified and studied as well. In this chapter and in others to come our chief concern will be the structure of the atom, since it is this structure that is responsible for nearly all the properties of matter that have shaped the world around us.

Every atom consists of a small nucleus of protons and neutrons with a number of electrons some distance away. It is tempting to think that the electrons circle the nucleus as planets do the sun, but classical electromagnetic theory denies the possibility of stable electron orbits. In an effort to resolve this paradox, Niels Bohr applied quantum ideas to atomic structure in 1913 to obtain a model which, despite its inadequacies and later replacement by a quantum-mechanical description of greater accuracy and usefulness, still remains a convenient mental picture of the atom. Bohr's theory of the hydrogen atom is worth examining both for this reason and because it provides a valuable transition to the more abstract quantum theory of the atom.

4.1 THE NUCLEAR ATOM

An atom is largely empty space

Most scientists of the late nineteenth century accepted the idea that the chemical elements consist of atoms, but they knew almost nothing about the atoms themselves. One clue was the discovery that all atoms contain electrons. Since electrons carry negative charges whereas atoms are neutral, positively charged matter of some kind must be present in atoms. But what kind? And arranged in what way?

One suggestion, made by the British physicist J. J. Thomson in 1898, was that atoms are just positively charged lumps of matter with electrons embedded in them, like raisins in a fruitcake (Fig. 4.1). Because Thomson had played an important role in discovering the electron, his idea was taken seriously. But the real atom turned out to be quite different.

The most direct way to find out what is inside a fruitcake is to poke a finger into it, which is essentially what Hans Geiger and Ernest Marsden did in 1911. At the suggestion of Ernest Rutherford, they used as probes the fast alpha particles emitted by certain radioactive elements. Alpha particles are helium atoms that have lost two electrons each, leaving them with a charge of $+2e$.

Geiger and Marsden placed a sample of an alpha-emitting substance behind a lead screen with a small hole in it, as in Fig. 4.2, so that a narrow beam of alpha particles was produced. This beam was directed at a thin gold foil. A zinc sulfide screen, which gives off a visible flash of light when struck by an alpha particle, was set on the other side of the foil with a microscope to see the flashes.

It was expected that the alpha particles would go right through the foil with hardly any deflection. This follows from the Thomson model, in which the electric charge inside an atom is assumed to be uniformly spread through its volume. With only weak electric forces exerted on them, alpha particles that pass through a thin foil ought to be deflected only slightly, 1° or less.

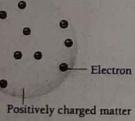


Figure 4.1 The Thomson model of the atom. The Rutherford scattering experiment showed it to be incorrect.

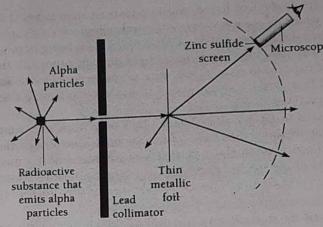


Figure 4.2 The Rutherford scattering experiment.

What Geiger and Marsden actually found was that although most of the alpha particles indeed were not deviated by much, a few were scattered through very large angles. Some were even scattered in the backward direction. As Rutherford remarked, "It was as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you."

Alpha particles are relatively heavy (almost 8000 electron masses) and those used in this experiment had high speeds (typically 2×10^7 m/s), so it was clear that powerful forces were needed to cause such marked deflections. The only way to



Ernest Rutherford (1871–1937), a native of New Zealand, was on his family's farm digging potatoes when he learned that he had won a scholarship for graduate study at Cambridge University in England. "This is the last potato I will ever dig," he said, throwing down his spade. Thirteen years later he received the Nobel Prize in chemistry.

At Cambridge, Rutherford was a research student under J. J. Thomson, who would soon announce the discovery of the electron. Rutherford's own work was on the newly found phenomenon of radioactivity, and he quickly distinguished between alpha and beta particles, two of the emissions of radioactive materials. In 1898 he went to McGill University in Canada, where he found that alpha particles are the nuclei of helium atoms and that the radioactive decay of an element gives rise to another element. Working with the chemist Frederick Soddy and others, Rutherford traced the successive transformations of radioactive elements, such as uranium and radium, until they end up as stable lead.

In 1907 Rutherford returned to England as professor of physics at Manchester, where in 1911 he showed that the nuclear model of the atom was the only one that could explain the observed scattering of alpha particles by thin metal foils. Rutherford's last important discovery, reported in 1919, was the disintegration of nitrogen nuclei when bombarded with alpha particles, the first example of the artificial transmutation of elements into other elements. After other similar experiments, Rutherford suggested that all nuclei contain hydrogen nuclei, which he called protons. He also proposed that a neutral particle was present in nuclei as well.

In 1919 Rutherford became director of the Cavendish Laboratory at Cambridge, where under his stimulus great strides in understanding the nucleus continued to be made. James Chadwick discovered the neutron there in 1932. The Cavendish Laboratory was the site of the first accelerator for producing high-energy particles. With the help of this accelerator, fusion reactions in which light nuclei unite to form heavier nuclei were observed for the first time.

Rutherford was not infallible: only a few years before the discovery of fission and the building of the first nuclear reactor, he dismissed the idea of practical uses for nuclear energy as "moonshine." He died in 1937 of complications of a hernia and was buried near Newton in Westminster Abbey.

The maximum KE found in alpha particles of natural origin is 7.7 MeV, which is 1.2×10^{-12} J. Since $1/4\pi\epsilon_0 = 9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$,

$$R = \frac{(2)(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.6 \times 10^{-19} \text{ C})^2 Z}{1.2 \times 10^{-12} \text{ J}} \\ = 3.8 \times 10^{-16} Z \text{ m}$$

The atomic number of gold, a typical foil material, is $Z = 79$, so that

$$R(\text{Au}) = 3.0 \times 10^{-14} \text{ m}$$

The radius of the gold nucleus is therefore less than 3.0×10^{-14} m, well under 10^{-4} the radius of the atom as a whole.

In more recent years particles of much higher energies than 7.7 MeV have been artificially accelerated, and it has been found that the Rutherford scattering formula does indeed eventually fail to agree with experiment. These experiments and the information they provide on actual nuclear dimensions are discussed in Chap. 11. The radius of the gold nucleus turns out to be about $\frac{1}{4}$ of the value of $R(\text{Au})$ found above.

Neutron Stars

The density of nuclear matter is about $2.4 \times 10^{17} \text{ kg/m}^3$, which is equivalent to 4 billion tons per cubic inch. As discussed in Sec. 9.11, neutron stars are stars whose atoms have been so compressed that most of their protons and electrons have fused into neutrons, which are the most stable form of matter under enormous pressures. The densities of neutron stars are comparable to those of nuclei: a neutron star packs the mass of one or two suns into a sphere only about 10 km in radius. If the earth were this dense, it would fit into a large apartment house.

4.2 ELECTRON ORBITS

The planetary model of the atom and why it fails

The Rutherford model of the atom, so convincingly confirmed by experiment, pictures a tiny, massive, positively charged nucleus surrounded at a relatively great distance by enough electrons to render the atom electrically neutral as a whole. The electrons cannot be stationary in this model, because there is nothing that can keep them in place against the electric force pulling them to the nucleus. If the electrons are in motion, however, dynamically stable orbits like those of the planets around the sun are possible (Fig. 4.5).

Let us look at the classical dynamics of the hydrogen atom, whose single electron makes it the simplest of all atoms. We assume a circular electron orbit for convenience, though it might as reasonably be assumed to be elliptical in shape. The centripetal force

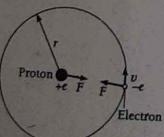


Figure 4.5 Force balance in the hydrogen atom.

$$F_c = \frac{mv^2}{r}$$

holding the electron in an orbit r from the nucleus is provided by the electric force

$$F_e = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$$

between them. The condition for a dynamically stable orbit is

$$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} \quad (4.3)$$

The electron velocity v is therefore related to its orbit radius r by the formula

$$\text{Electron velocity} \quad v = \frac{e}{\sqrt{4\pi\epsilon_0 mr}} \quad (4.4)$$

The total energy E of the electron in a hydrogen atom is the sum of its kinetic and potential energies, which are

$$\text{KE} = \frac{1}{2} mv^2 \quad \text{PE} = -\frac{e^2}{4\pi\epsilon_0 r}$$

(The minus sign follows from the choice of $\text{PE} = 0$ at $r = \infty$, that is, when the electron and proton are infinitely far apart.) Hence

$$E = \text{KE} + \text{PE} = \frac{mv^2}{2} - \frac{e^2}{4\pi\epsilon_0 r}$$

Substituting for v from Eq. (4.4) gives

$$\begin{aligned} E &= \frac{e^2}{8\pi\epsilon_0 r} - \frac{e^2}{4\pi\epsilon_0 r} \\ \text{Total energy of} \\ \text{hydrogen atom} \quad E &= -\frac{e^2}{8\pi\epsilon_0 r} \end{aligned} \quad (4.5)$$

The total energy of the electron is negative. This holds for every atomic electron and reflects the fact that it is bound to the nucleus. If E were greater than zero, an electron would not follow a closed orbit around the nucleus.

Actually, of course, the energy E is not a property of the electron alone but is a property of the system of electron + nucleus. The effect of the sharing of E between the electron and the nucleus is considered in Sec. 4.7.

Example 4.1

Experiments indicate that 13.6 eV is required to separate a hydrogen atom into a proton and an electron; that is, its total energy is $E = -13.6$ eV. Find the orbital radius and velocity of the electron in a hydrogen atom.

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Solution

Since $13.6 \text{ eV} = 2.2 \times 10^{-18} \text{ J}$, from Eq. (4.5)

$$r = -\frac{e^2}{8\pi\epsilon_0 E} = -\frac{(1.6 \times 10^{-19} \text{ C})^2}{(8\pi)(8.85 \times 10^{-12} \text{ F/m})(-2.2 \times 10^{-18} \text{ J})} = 5.3 \times 10^{-11} \text{ m}$$

An atomic radius of this magnitude agrees with estimates made in other ways. The electron's velocity can be found from Eq. (4.4):

$$v = \frac{e}{\sqrt{4\pi\epsilon_0 mr}} = \frac{1.6 \times 10^{-19} \text{ C}}{\sqrt{(4\pi)(8.85 \times 10^{-12} \text{ F/m})(9.1 \times 10^{-31} \text{ kg})(5.3 \times 10^{-11} \text{ m})}} = 2.2 \times 10^6 \text{ m/s}$$

Since $v \ll c$, we can ignore special relativity when considering the hydrogen atom.

The Failure of Classical Physics

The analysis above is a straightforward application of Newton's laws of motion and Coulomb's law of electric force—both pillars of classical physics—and is in accord with the experimental observation that atoms are stable. However, it is not in accord with electromagnetic theory—another pillar of classical physics—which predicts that accelerated electric charges radiate energy in the form of em waves. An electron pursuing a curved path is accelerated and therefore should continuously lose energy, spiraling into the nucleus in a fraction of a second (Fig. 4.6).

But atoms do not collapse. This contradiction further illustrates what we saw in the previous two chapters: The laws of physics that are valid in the macroworld do not always hold true in the microworld of the atom.

Is Rutherford's Analysis Valid?

An interesting question comes up at this point. When he derived his scattering formula, Rutherford used the same laws of physics that prove such dismal failures when applied to atomic stability. Might it not be that this formula is not correct and that in reality the atom does not resemble Rutherford's model of a small central nucleus surrounded by distant electrons? This is not a trivial point. It is a curious coincidence that the quantum-mechanical analysis of alpha particle scattering by thin foils yields precisely the same formula that Rutherford found.

To verify that a classical calculation ought to be at least approximately correct, we note that the de Broglie wavelength of an alpha particle whose speed is $2.0 \times 10^7 \text{ m/s}$

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(6.6 \times 10^{-27} \text{ kg})(2.0 \times 10^7 \text{ m/s})} = 5.0 \times 10^{-15} \text{ m}$$

As we saw in Sec. 4.1, the closest an alpha particle with this wavelength ever gets to a gold nucleus is $3.0 \times 10^{-14} \text{ m}$, which is six de Broglie wavelengths. It is therefore just reasonable to regard the alpha particle as a classical particle in the interaction. We are correct in thinking of the atom in terms of Rutherford's model, though the dynamics of the atomic electrons—which is another matter—requires a nonclassical approach.

Classical physics fails to provide a meaningful analysis of atomic structure because it approaches nature in terms of "pure" particles and "pure" waves. In reality particles and waves have many properties in common, though the smallness of Planck's constant makes the wave-particle duality imperceptible in the macroworld. The usefulness of classical physics decreases as the scale of the phenomena under study decreases, and we must allow for the particle behavior of waves and the wave behavior of particles to understand the atom. In the rest of this chapter we shall see how the Bohr atomic model, which combines classical and modern notions, accomplishes part of the latter task. Not until we consider the atom from the point of view of quantum mechanics, which makes no compromise with the intuitive notions we pick up in our daily lives, will we find a really successful theory of the atom.

4.3 ATOMIC SPECTRA

Each element has a characteristic line spectrum

Atomic stability is not the only thing that a successful theory of the atom must account for. The existence of spectral lines is another important aspect of the atom that finds no explanation in classical physics.

We saw in Chap. 2 that condensed matter (solids and liquids) at all temperatures emits em radiation in which all wavelengths are present, though with different intensities. The observed features of this radiation were explained by Planck without reference to exactly how it was produced by the radiating material or to the nature of the material. From this it follows that we are witnessing the collective behavior of a great many interacting atoms rather than the characteristic behavior of the atoms of a particular element.

At the other extreme, the atoms or molecules in a rarefied gas are so far apart on the average that they only interact during occasional collisions. Under these circumstances we would expect any emitted radiation to be characteristic of the particular atoms or molecules present, which turns out to be the case.

When an atomic gas or vapor at somewhat less than atmospheric pressure is suitably "excited," usually by passing an electric current through it, the emitted radiation has a spectrum which contains certain specific wavelengths only. An idealized arrangement for observing such atomic spectra is shown in Fig. 4.7; actual spectrometers use diffraction

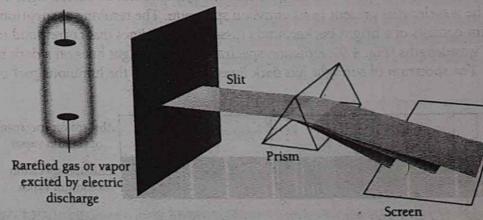


Figure 4.7 An idealized spectrometer.

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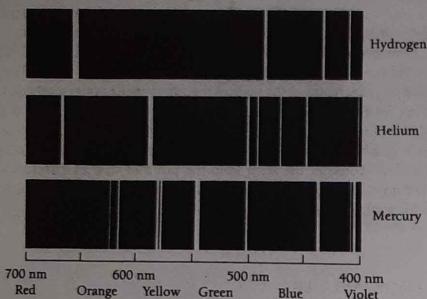


Figure 4.8 Some of the principal lines in the emission spectra of hydrogen, helium, and mercury.



Gas atoms excited by electric currents in these tubes radiate light of wavelengths characteristic of the gas used.

gratings. Figure 4.8 shows the **emission line spectra** of several elements. Every element displays a unique line spectrum when a sample of it in the vapor phase is excited. Spectroscopy is therefore a useful tool for analyzing the composition of an unknown substance.

When white light is passed through a gas, the gas is found to absorb light of certain of the wavelengths present in its emission spectrum. The resulting **absorption line spectrum** consists of a bright background crossed by dark lines that correspond to the missing wavelengths (Fig. 4.9); emission spectra consist of bright lines on a dark background. The spectrum of sunlight has dark lines in it because the luminous part of the



Figure 4.9 The dark lines in the absorption spectrum of an element correspond to bright lines in its emission spectrum.

sun, which radiates very nearly like a blackbody heated to 5800 K, is surrounded by an envelope of cooler gas that absorbs light of certain wavelengths only. Most other stars have spectra of this kind.

The number, intensity, and exact wavelengths of the lines in the spectrum of an element depend upon temperature, pressure, the presence of electric and magnetic fields, and the motion of the source. It is possible to tell by examining its spectrum not only what elements are present in a light source but much about their physical state. An astronomer, for example, can establish from the spectrum of a star which elements its atmosphere contains, whether they are ionized, and whether the star is moving toward or away from the earth.

Spectral Series

A century ago the wavelengths in the spectrum of an element were found to fall into sets called **spectral series**. The first such series was discovered by J. J. Balmer in 1885 in the course of a study of the visible part of the hydrogen spectrum. Figure 4.10 shows the **Balmer series**. The line with the longest wavelength, 656.3 nm, is designated H_{α} , the next, whose wavelength is 486.3 nm, is designated H_{β} , and so on. As the wavelength decreases, the lines are found closer together and weaker in intensity until the **series limit** at 364.6 nm is reached, beyond which there are no further separate lines but only a faint continuous spectrum. Balmer's formula for the wavelengths of this series is

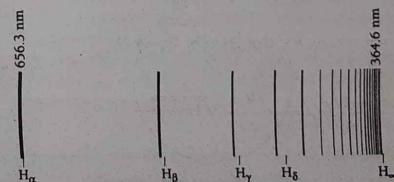
$$\text{Balmer} \quad \frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n^2} \right) \quad n = 3, 4, 5, \dots \quad (4.6)$$

The quantity R , known as the **Rydberg constant**, has the value

$$\text{Rydberg constant} \quad R = 1.097 \times 10^7 \text{ m}^{-1} = 0.01097 \text{ nm}^{-1}$$

The H_{α} line corresponds to $n = 3$, the H_{β} line to $n = 4$, and so on. The series limit corresponds to $n = \infty$, so that it occurs at a wavelength of $4/R$, in agreement with experiment.

The Balmer series contains wavelengths in the visible portion of the hydrogen spectrum. The spectral lines of hydrogen in the ultraviolet and infrared regions fall into several other series. In the ultraviolet the **Lyman series** contains the wavelengths given by the formula

Figure 4.10 The Balmer series of hydrogen. The H_{α} line is red, the H_{β} line is blue, the H_{γ} and H_{δ} lines are violet, and the other lines are in the near ultraviolet.

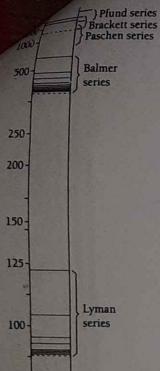


Figure 4.11 The spectral series of hydrogen. The wavelengths in each series are related by simple formulas.

$$\text{Lyman} \quad \frac{1}{\lambda} = R \left(\frac{1}{1^2} - \frac{1}{n^2} \right) \quad n = 2, 3, 4, \dots \quad (4.7)$$

In the infrared, three spectral series have been found whose lines have the wavelengths specified by the formulas

$$\text{Paschen} \quad \frac{1}{\lambda} = R \left(\frac{1}{3^2} - \frac{1}{n^2} \right) \quad n = 4, 5, 6, \dots \quad (4.8)$$

$$\text{Brackett} \quad \frac{1}{\lambda} = R \left(\frac{1}{4^2} - \frac{1}{n^2} \right) \quad n = 5, 6, 7, \dots \quad (4.9)$$

$$\text{Pfund} \quad \frac{1}{\lambda} = R \left(\frac{1}{5^2} - \frac{1}{n^2} \right) \quad n = 6, 7, 8, \dots \quad (4.10)$$

These spectral series of hydrogen are plotted in terms of wavelength in Fig. 4.11; the Brackett series evidently overlaps the Paschen and Pfund series. The value of R is the same in Eqs. (4.6) to (4.10).

These observed regularities in the hydrogen spectrum, together with similar regularities in the spectra of more complex elements, pose a definitive test for any theory of atomic structure.

4.4 THE BOHR ATOM

Electron waves in the atom

The first theory of the atom to meet with any success was put forward in 1913 by Niels Bohr. The concept of matter waves leads in a natural way to this theory, as de Broglie found, and this is the route that will be followed here. Bohr himself used a different approach, since de Broglie's work came a decade later, which makes his achievement all the more remarkable. The results are exactly the same, however.

We start by examining the wave behavior of an electron in orbit around a hydrogen nucleus. (In this chapter, since the electron velocities are much smaller than c , we will assume that $\gamma = 1$ and for simplicity omit γ from the various equations.) The de Broglie wavelength of this electron is

$$\lambda = \frac{h}{mv}$$

where the electron velocity v is that given by Eq. (4.4):

$$v = \frac{e}{\sqrt{4\pi\epsilon_0 mr}}$$

Hence

$$\text{Orbital electron wavelength} \quad \lambda = \frac{h}{e} \sqrt{\frac{4\pi\epsilon_0 r}{m}} \quad (4.11)$$

By substituting 5.3×10^{-11} m for the radius r of the electron orbit (see Example 4.1), we find the electron wavelength to be

$$\lambda = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{1.6 \times 10^{-19} \text{ C}} \sqrt{\frac{(4\pi)(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(5.3 \times 10^{-11} \text{ m})}{9.1 \times 10^{-31} \text{ kg}}} \\ = 33 \times 10^{-11} \text{ m}$$

This wavelength is exactly the same as the circumference of the electron orbit,

$$2\pi r = 33 \times 10^{-11} \text{ m}$$

The orbit of the electron in a hydrogen atom corresponds to one complete electron wave joined on itself (Fig. 4.12)!

The fact that the electron orbit in a hydrogen atom is one electron wavelength in circumference provides the clue we need to construct a theory of the atom. If we consider the vibrations of a wire loop (Fig. 4.13), we find that their wavelengths always fit an integral number of times into the loop's circumference so that each wave joins smoothly with the next. If the wire were perfectly elastic, these vibrations would continue indefinitely. Why are these the only vibrations possible in a wire loop? If a fractional number of wavelengths is placed around the loop, as in Fig. 4.14, destructive

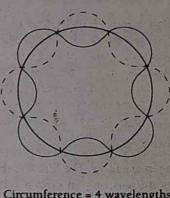
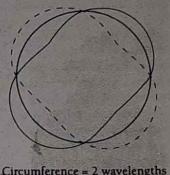


Figure 4.12 The orbit of the electron in a hydrogen atom corresponds to a complete electron de Broglie wave joined on itself.

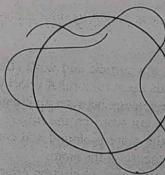


Figure 4.14 A fractional number of wavelengths cannot persist because destructive interference will occur.

Figure 4.13 Some modes of vibration of a wire loop. In each case a whole number of wavelengths fit into the circumference of the loop.



Niels Bohr (1885–1962) was born and spent most of his life in Copenhagen, Denmark. After receiving his doctorate at the university there in 1911, Bohr went to England to broaden his scientific horizons. At Rutherford's laboratory in Manchester, Bohr was introduced to the just-discovered nuclear model of the atom, which was in conflict with the existing principles of physics. Bohr realized that it was "hopeless" to try to make sense of the atom in the framework of classical physics alone, and he felt that the quantum theory of light must somehow be the key to understanding atomic structure.

Back in Copenhagen in 1913, a friend suggested to Bohr that Balmer's formula for one set of the spectral lines of hydrogen might be relevant to his quest. "As soon as I saw Balmer's formula the whole thing was immediately clear to me," Bohr said later. To construct his theory, Bohr began with two revolutionary ideas. The first was that an atomic electron can circle its nucleus only in certain orbits, and the other was that an atom emits or absorbs a photon of light when an electron jumps from one permitted orbit to another.

What is the condition for a permitted orbit? To find out, Bohr used as a guide what became known as the correspondence principle: When quantum numbers are very large, quantum effects should not be conspicuous, and the quantum theory must then give the same results as classical physics. Applying this principle showed that the electron in a permitted orbit must have an angular momentum that is a multiple

of $\hbar = h/2\pi$. A decade later Louis de Broglie explained this quantization of angular momentum in terms of the wave nature of a moving electron.

Bohr was able to account for all the spectral series of hydrogen, not just the Balmer series, but the publication of the theory aroused great controversy. Einstein, an enthusiastic supporter of the theory (which "appeared to me like a miracle—and appears to me as a miracle even today," he wrote many years later), nevertheless commented on its bold mix of classical and quantum concepts, "One ought to be ashamed of the successes [of the theory] because they have been earned according to the Jesuit maxim, 'Let not thy left hand know what the other doeth.'" Other noted physicists were more deeply disturbed: Otto Stern and Max von Laue said they would quit physics if Bohr were right. (They later changed their minds.) Bohr and others tried to extend his model to many-electron atoms with occasional success—for instance, the correct prediction of the properties of the then-unknown element hafnium—but real progress had to wait for Wolfgang Pauli's exclusion principle of 1925.

In 1916 Bohr returned to Rutherford's laboratory, where he stayed until 1919. Then an Institute of Theoretical Physics was created for him in Copenhagen, and he directed it until his death. The institute was a magnet for quantum theoreticians from all over the world, who were stimulated by the exchange of ideas at regular meetings there. Bohr received the Nobel Prize in 1922. His last important work came in 1939, when he used an analogy between a large nucleus and a liquid drop to explain why nuclear fission, which had just been discovered, occurs in certain nuclei but not in others. During World War II Bohr contributed to the development of the atomic bomb at Los Alamos, New Mexico. After the war, Bohr returned to Copenhagen, where he died in 1962.

interference will occur as the waves travel around the loop, and the vibrations will die out rapidly.

By considering the behavior of electron waves in the hydrogen atom as analogous to the vibrations of a wire loop, then, we can say that

An electron can circle a nucleus only if its orbit contains an integral number of de Broglie wavelengths.

This statement combines both the particle and wave characters of the electron since the electron wavelength depends upon the orbital velocity needed to balance the pull of the nucleus. To be sure, the analogy between an atomic electron and the standing waves of Fig. 4.13 is hardly the last word on the subject, but it represents an illuminating step along the path to the more profound and comprehensive, but also more abstract, quantum-mechanical theory of the atom.

It is easy to express the condition that an electron orbit contain an integral number of de Broglie wavelengths. The circumference of a circular orbit of radius r is $2\pi r$, and so the condition for orbit stability is

$$(4.12)$$

$$\text{Condition for orbit stability} \quad n\lambda = 2\pi r_n \quad n = 1, 2, 3, \dots$$

where r_n designates the radius of the orbit that contain n wavelengths. The integer n is called the **quantum number** of the orbit. Substituting for λ , the electron wavelength given by Eq. (4.11), yields

$$\frac{nh}{e} \sqrt{\frac{4\pi\epsilon_0 r_n}{m}} = 2\pi r_n$$

and so the possible electron orbits are those whose radii are given by

$$\text{Orbital radii in Bohr atom} \quad r_n = \frac{n^2 h^2 \epsilon_0}{\pi m e^2} \quad n = 1, 2, 3, \dots \quad (4.13)$$

The radius of the innermost orbit is customarily called the **Bohr radius** of the hydrogen atom and is denoted by the symbol a_0 :

$$\text{Bohr radius} \quad a_0 = r_1 = 5.292 \times 10^{-11} \text{ m}$$

The other radii are given in terms of a_0 by the formula

$$r_n = n^2 a_0 \quad (4.14)$$

4.5 ENERGY LEVELS AND SPECTRA

A photon is emitted when an electron jumps from one energy level to a lower level

The various permitted orbits involve different electron energies. The electron energy E_n is given in terms of the orbit radius r_n by Eq. (4.5) as

$$E_n = -\frac{e^2}{8\pi\epsilon_0 r_n}$$

Substituting for r_n from Eq. (4.13), we see that

$$\text{Energy levels} \quad E_n = -\frac{me^4}{8\epsilon_0^2 h^2} \left(\frac{1}{n^2} \right) = \frac{E_1}{n^2} \quad n = 1, 2, 3, \dots \quad (4.15)$$

$$E_1 = -2.18 \times 10^{-18} \text{ J} = -13.6 \text{ eV}$$

The energies specified by Eq. (4.15) are called the **energy levels** of the hydrogen atom and are plotted in Fig. 4.15. These levels are all negative, which signifies that the electron does not have enough energy to escape from the nucleus. An atomic electron can have only these energies and no others. An analogy might be a person on a ladder, who can stand only on its steps and not in between.

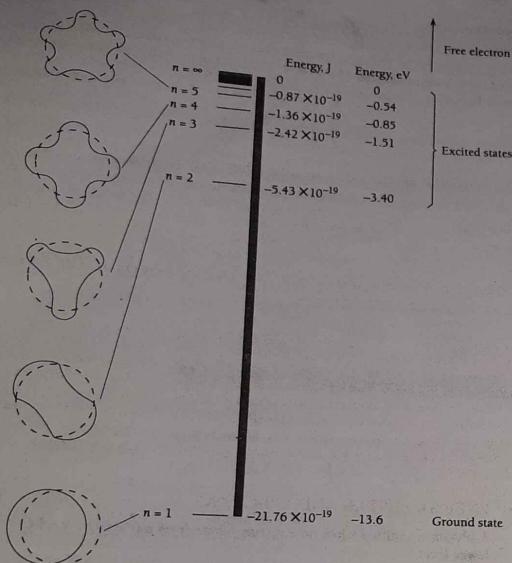


Figure 4.15 Energy levels of the hydrogen atom.

The lowest energy level E_1 is called the **ground state** of the atom, and the higher levels E_2, E_3, E_4, \dots are called **excited states**. As the quantum number n increases, the corresponding energy E_n approaches closer to 0. In the limit of $n = \infty$, $E_\infty = 0$ and the electron is no longer bound to the nucleus to form an atom. A positive energy for a nucleus-electron combination means that the electron is free and has no quantum conditions to fulfill, such a combination does not constitute an atom, of course.

The work needed to remove an electron from an atom in its ground state is called its **ionization energy**. The ionization energy is accordingly equal to $-E_1$, the energy that must be provided to raise an electron from its ground state to an energy of $E = 0$, when it is free. In the case of hydrogen, the ionization energy is 13.6 eV since the ground-state energy of the hydrogen atom is -13.6 eV. Figure 7.10 shows the ionization energies of the elements.

Example 4.2

An electron collides with a hydrogen atom in its ground state and excites it to a state of $n = 3$. How much energy was given to the hydrogen atom in this inelastic (KE not conserved) collision?

Solution

From Eq. (4.15) the energy change of a hydrogen atom that goes from an initial state of quantum number n_i to a final state of quantum number n_f is

$$\Delta E = E_f - E_i = \frac{E_1}{n_f^2} - \frac{E_1}{n_i^2} = E_1 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

Here $n_i = 1$, $n_f = 3$, and $E_1 = -13.6$ eV, so

$$\Delta E = -13.6 \left(\frac{1}{3^2} - \frac{1}{1^2} \right) \text{ eV} = 12.1 \text{ eV}$$

Example 4.3

Hydrogen atoms in states of high quantum number have been created in the laboratory and observed in space. They are called **Rydberg atoms**. (a) Find the quantum number of the Bohr orbit in a hydrogen atom whose radius is 0.0100 mm. (b) What is the energy of a hydrogen atom in this state?

Solution

(a) From Eq. (4.14) with $r_n = 1.00 \times 10^{-3}$ m,

$$n = \sqrt{\frac{r_n}{a_0}} = \sqrt{\frac{1.00 \times 10^{-3} \text{ m}}{5.29 \times 10^{-11} \text{ m}}} = 435$$

(b) From Eq. (4.15),

$$E_n = \frac{E_1}{n^2} = \frac{-13.6 \text{ eV}}{(435)^2} = -7.19 \times 10^{-5} \text{ eV}$$

Rydberg atoms are obviously extremely fragile and are easily ionized, which is why they are found in nature only in the near-vacuum of space. The spectra of Rydberg atoms range down to radio frequencies and their existence was established from radio telescope data.

Origin of Line Spectra

We must now confront the equations developed above with experiment. An especially striking observation is that atoms exhibit line spectra in both emission and absorption. Do such spectra follow from our model?

The presence of discrete energy levels in the hydrogen atom suggests the connection. Let us suppose that when an electron in an excited state drops to a lower state, the lost energy is emitted as a single photon of light. According to our model, electrons cannot exist in an atom except in certain specific energy levels. The jump of an electron from one level to another, with the difference in energy between the levels being given off all at once in a photon rather than in some more gradual manner, fits in well with this model.

so that

Frequency of
photon

$$\nu = \frac{-E_1}{h} \left(\frac{2p}{n^3} \right) \quad (4.20)$$

When $p = 1$, the frequency ν of the radiation is exactly the same as the frequency of rotation f of the orbital electron given in Eq. (4.19). Multiples of this frequency are radiated when $p = 2, 3, 4, \dots$. Hence both quantum and classical pictures of the hydrogen atom make the same predictions in the limit of very large quantum numbers. When $n = 2$, Eq. (4.19) predicts a radiation frequency that differs from that given by Eq. (4.20) by almost 300 percent. When $n = 10,000$, the discrepancy is only about 0.01 percent.

The requirement that quantum physics give the same results as classical physics in the limit of large quantum numbers was called by Bohr the **correspondence principle**. It has played an important role in the development of the quantum theory of matter.

Bohr himself used the correspondence principle in reverse, so to speak, to look for the condition for orbit stability. Starting from Eq. (4.19) he was able to show that stable orbits must have electron orbital angular momenta of

Condition for
orbital stability

$$mvnr = \frac{nh}{2\pi} \quad n = 1, 2, 3, \dots \quad (4.21)$$

Since the de Broglie electron wavelength is $\lambda = h/mv$, Eq. (4.21) is the same as Eq. (4.12), $n\lambda = 2\pi r$, which states that an electron orbit must contain an integral number of wavelengths.