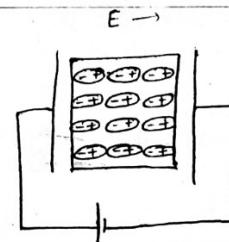


Optical, Magnetic & Dielectric properties of Materials

Basic formulas

- Dipole moment $p = qd$; $q \rightarrow$ one of the charges,
 $d \rightarrow$ distance b/w the charges
 - When a dipole is placed in an external field, the torque experienced is $\tau = p \times E$; $E \rightarrow$ Applied electric field.
 - The potential energy $V = -p \cdot E = -pE \cos\theta$.
 - Polarization $P \rightarrow$ dipole moment / unit volume.
 $P = N_p$; $N \rightarrow$ No. of molecules per unit volume
 - $D = \epsilon_0 E + P$ $D \rightarrow$ Electric displacement vector
 $E_0 \rightarrow$ External field, the field outside the dielectric
 - $D = \epsilon E$ $E = E_0 - \frac{P}{\epsilon_0}$
- On comparing both equations $E = E_0 - \frac{P}{\epsilon_0}$
- shows the effect of the polarization is to modify the field inside the medium
- Relative dielectric constant $\epsilon_r = \epsilon/\epsilon_0$

- To find dielectric constant (a quantity measuring the ability of a substance to store electrical energy than ϵ)
- The plates of a capacitor are connected to a battery which charges the plates. When there is no dielectric inside the capacitor, the electric field produced by the charges is E_0 . $E_0 = \frac{V_0}{L}$ $V_0 \rightarrow$ potential difference $L \rightarrow$ Distance b/w the plates



If a dielectric slab is now inserted b/w the plates, the field E_0 becomes E due to polarization

$$\therefore E = \frac{V}{L} \quad V \rightarrow \text{new potential difference}$$

$$\text{But } \epsilon_r = \frac{E_0}{E}$$

$$\therefore \epsilon_r = \frac{V_0}{V}$$

Thus we get dielectric constant by measuring the potential differences across the capacitor, with & without the presence of the dielectric & taking their ratio.



(2) The dielectric constant and polarizability.

When a dielectric field, it becomes polarized i.e., within a small volume of substance the geometric sum of the electric dipole moment vectors of the molecules becomes non zero. The polarization 'P' is thus defined as the dipole moment / unit volume. If the no. of molecules / unit volume is N and if each has a moment p , then polarization

$$P = Np$$

Also dipole moment is proportional to E

$$p = \alpha E$$

where α is called polarizability of the molecule.

$$P = N\alpha E \quad \text{--- (1)}$$

We have the displacement vector,

$$D = \epsilon_0 E + P$$

$$= \epsilon_0 E + N\alpha E = (\epsilon_0 + N\alpha)E$$

$$= \epsilon_0 \left(1 + \frac{N\alpha}{\epsilon_0}\right)E \quad \text{--- (2)}$$

$$\text{Also we have } D = \epsilon E \quad \text{--- (3)}$$

Comparing (2) & (3);

$$\epsilon_0 = \left(1 + \frac{N\alpha}{\epsilon_0}\right)$$

The electric susceptibility χ of a medium is defined by the relation, $P = \epsilon_0 \chi E \quad \text{--- (4)}$

Comparing this eqn with (1)

$$\chi = \frac{N\alpha}{\epsilon_0}$$

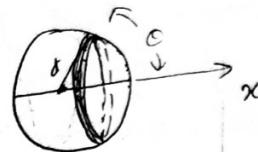
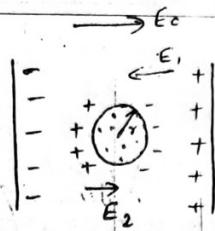
$$\therefore \boxed{\epsilon_0 = 1 + \chi}$$

Local field at an atom

The electric field acting at an atom in a dielectric is known as the polarizing field or the local field, E_{loc}

different from the applied external field. In order (3) to evaluate this, it is necessary to calculate the total field acting on a certain typical dipole. This was first of all calculated by Lorentz. The reference dipole at which the field is to be calculated is imagined to be surrounded by a spherical cavity, called Lorentz sphere of radius r sufficiently large as compared to the dimension of the dipole. The local field acting on the reference dipole is given by the sum,

$$E_{loc} = E_0 + E_1 + E_2 + E_3 \quad (1)$$



E_0 → uniform applied external field

E_1 → field due to the polarizing charges lying on the external surface of the dielectric medium, known as depolarization field.

E_2 → field due to polarization charges lying on the surface of Lorentz sphere called as Lorentz cavity field.

E_3 → field due to other dipoles lying within the sphere.

E_1 :-

E_1 is due to the polarization charges on the external surfaces and hence depends on the geometric shape of the dielectrics. This acts in a direction opposite to the external field. For an infinite slab,

$$E_1 = -\frac{P}{\epsilon_0} \quad P \rightarrow \text{polarisation}$$

E_2 :-

The polarization charges on the surface of Lorentz sphere produce this field.

- (4) The charge density arises from the bound charges and is determined by the normal component of polarization.

$\therefore \sigma = \text{ Dipole moment}$

$$\therefore E_2 = \int \frac{1}{4\pi\epsilon_0} \frac{\sigma}{r^2} dA / \delta$$

Because of the symmetry of the sphere, the σ component of this is non-zero, so we have to evaluate only the component of the field along the direction of P.

$$\therefore E_{2,x} = \int \frac{1}{4\pi\epsilon_0} \frac{P \cos\theta}{r^2} \cos\theta dA$$

$$= \int \frac{1}{4\pi\epsilon_0} \frac{P \cos^2\theta}{r^2} \cos\theta dA$$

$$= \iint \frac{1}{4\pi\epsilon_0} \frac{P \cos^2\theta}{r^2} (r^2 \sin\theta d\phi d\theta)$$

$$= 2\pi \int \frac{1}{4\pi\epsilon_0} \frac{P r^2 \cos^2\theta \sin\theta d\theta}{r^2}$$

$$= \frac{1}{2\epsilon_0} P \int \cos^2\theta \sin\theta d\theta$$

$$\theta = 0; x = 1$$

$$\theta = \pi; x = -1$$

put $\cos\theta = x$

$$\therefore -\sin\theta dx$$

$$\therefore E_2 = \frac{P}{2\epsilon_0} \int x^2 dx = \frac{P}{2\epsilon_0} \left(\frac{x^3}{3} \right)_0^1$$

$$E_2 = \frac{P}{2\epsilon_0 \cdot 3} = \frac{P}{3\epsilon_0} \quad \text{--- (3)}$$

E_3

This field, which is due to other dipoles in the cavity, may be evaluated by summing the fields of the individual dipoles. It depends on the crystal structure of the solid. For cubic structure it is;

$$E_3 = 0. \quad \text{--- (4)}$$

(For other structures E_3 may not vanish)

on substituting all the above.

$$E_{loc} = E_0 + E_1 + E_2 + E_3$$

$$= E_0 - \frac{P}{\epsilon_0} + \frac{P}{3\epsilon_0} + 0$$

$$E_{loc} = E_0 - \frac{2P}{3\epsilon_0}$$

or we have $E_0 = E + \frac{P}{\epsilon_0}$

$$E = E_0 + E_1$$

$$\therefore E_{loc} = E + \frac{1}{3} \frac{P}{\epsilon_0}$$

which shows the E_{loc} is different from E . The above eqn is called Lorentz relation. The difference b/w E , the Maxwell's field and the Lorentz field E_{loc} may be understood as follows. The field E is macroscopic in nature and is an average field. In the present situation, this is constant throughout the medium. On the other hand, E_{loc} is a microscopic field and is periodic in nature.



This is quite large at molecular sites indicating that the molecules are more effectively polarized than they are under the average field.

Now the induced dipole moments can be

written as,

$$p_s \propto E_{loc}$$

: Polarisation.

$$P_s = N \epsilon_0 E_{loc}$$

$$P_s = N \epsilon_0 \left[E + \frac{P}{3\epsilon_0} \right]$$

$$P_s \left(1 - \frac{N\epsilon_0}{3\epsilon_0} \right) = N \epsilon_0 E$$

$$P_s = \frac{N \epsilon_0 E}{\left(1 - \frac{N\epsilon_0}{3\epsilon_0} \right)}$$

\div by $\epsilon_0 E$

$$\frac{P_s}{\epsilon_0 E} = \frac{N \epsilon_0 / \epsilon_0}{\left(1 - \frac{N\epsilon_0}{3\epsilon_0} \right)}$$

$$\chi_e = \frac{P}{\epsilon_0 E}$$

but we have $\epsilon_r = 1 + \chi_e = 1 + \frac{P}{\epsilon_0 E}$

(6)

$$\frac{1+p}{\epsilon_0 E} = \frac{1 + N\alpha/\epsilon_0}{(1 - N\alpha/3\epsilon_0)}$$

$$\epsilon_r = \frac{1 + \frac{N\alpha}{3\epsilon_0} + \frac{N\alpha}{\epsilon_0}}{(1 - \frac{N\alpha}{3\epsilon_0})}$$

$$\epsilon_r = \frac{1 + 2N\alpha/3\epsilon_0}{1 - N\alpha/3\epsilon_0}$$

$$\text{or } \epsilon_0 - \epsilon_r \left(\frac{N\alpha}{3\epsilon_0} \right) = 1 + 2 \frac{N\alpha}{3\epsilon_0}$$

$$\epsilon_r - 1 = \frac{N\alpha}{3\epsilon_0} (\epsilon_0 + 2)$$

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N\alpha}{3\epsilon_0}$$

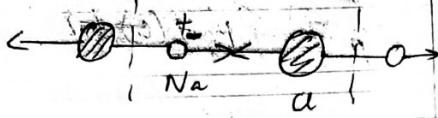
This is known as Clausius-Mosotti equation. This eqn can be used to determine the electrical polarizabilities of the atoms if the dielectric constant is known. Further, since according to the eqn,

Sources of polarizability

- 1) Dipolar polarizability
- 2) Ionic polarizability
- 3) Electronic polarizability

- 1) If the molecule has a permanent moment, i.e., a moment even in the absence of an electric field, it is called dipolar molecule. e.g. H_2O . But in a dipolar substance, since the individual molecules are at randomly oriented, the net polarization vanishes. When a field is applied to the substance the molecular dipoles tend to align with the field and this results in a net nonvanishing polarization, called dipolar polarizability.

If the molecule contains ionic bonds such as in NaCl , the field tends to stretch the lengths of these bonds. The effect of this change in length is to produce a net dipole moment in the unit cell. Since the polarization here is due to the relative displacements of oppositely charged ions, it is called ionic polarizability.

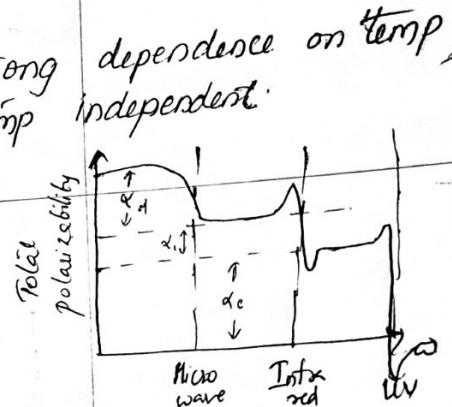


3) Electronic polarizability arises as a result of the displacement of e^- s in an atom relative to the nucleus. The -vely charged e^- s move antiparallel to the field creating an electric dipole parallel to the field. This contribution is common in all solids, since all are made up of atoms.

$$\alpha = \alpha_e + \alpha_i + \alpha_d$$

- Dipolar polarizability exhibits strong dependence on temp, while the other contributions are temp independent.

If we look at the polarizability versus frequency curve, we find that in the optical frequency range the dielectric const arises almost entirely from the electronic polarizability.



Dipolar polarizability / Orientation polarizability

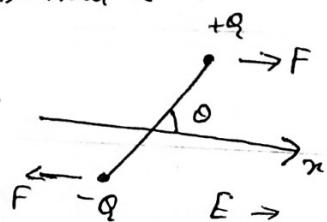
Suppose that a dipolar system is placed in a uniform \vec{E} . The torque exerted by the field on the dipole having a dipole moment \vec{p} , is given by

$$Z \cdot p \times E$$

The potential energy of the dipole in the field is

$$U = -\vec{p} \cdot \vec{E} = -pE \cos \theta$$

where θ is the angle made by the dipole with the x -axis. The dipole is no longer oriented randomly. The probability of finding it along the 0 -direction is given by the



8) distribution function of Boltzmann statistics,

$$f = e^{-\frac{E}{kT}} = e^{\frac{pE \cos \theta}{kT}}$$

\therefore The average moment contribution / dipole lying within the solid angle can be determined by integrating over all angles from parallel alignment $\theta=0$, to antiparallel alignment $\theta=\pi$, so that,

$$\bar{p} = \frac{\int_0^\pi p_x f(\theta) d\Omega}{\int_0^\pi f(\theta) d\Omega}$$

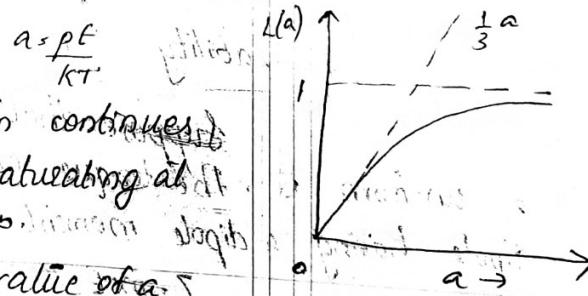
where $p_x = p \cos \theta$, $d\Omega = 2\pi \sin \theta d\theta$.

$$\therefore \bar{p} = \frac{\int_0^\pi p \cos \theta \exp(pE \cos \theta / kT) 2\pi \sin \theta d\theta}{\int_0^\pi \exp(pE \cos \theta / kT) 2\pi \sin \theta d\theta}$$

$$\text{Sub: } \frac{pE}{kT} = a; \cos \theta = t \\ -\sin \theta d\theta = dt$$

$$\therefore \bar{p} = p \int_{-1}^{+1} \frac{e^{ta}}{\int_{-1}^{+1} e^{ta} dt} \exp\left(\frac{1}{a} - \frac{t}{a}\right)$$

where $L(a) = \frac{1}{a} - \frac{t}{a}$, called the Langevin fn.



As a increases, the \bar{p}_x continues to increase, eventually saturating at the value unity as $a \rightarrow \infty$.

For small values of a ,

$$\operatorname{coth} a = \frac{1 + a + a^2}{a^3 - \frac{1}{45}}$$

$$\therefore \operatorname{coth} a \approx \frac{1 + a}{a^3}$$

$$\therefore L(a) = \operatorname{coth} \frac{1}{a} \approx \frac{1}{a^3}$$

Thus for small values of field \bar{p}_x increases linearly while at very high field, \bar{p}_x saturates at the maximum value p .

$$\bar{P}_x = p \cdot L(a) = \frac{p^2 E}{3kT}$$

Net dipole moment is directly proportional to field, and inversely proportional to the temp.

$$\therefore \alpha_d = \frac{\bar{P}_x}{E} = \frac{p^2}{3kT}$$

If α_0 is the sum of electronic + ionic polarizabilities, then the total polarizability is $\alpha = \alpha_0 + \frac{p^2}{3kT}$

\Rightarrow Langevin- Debye eqn

- a) Assuming that there are 10^{27} molecules/m³ in HCl vapour, calculate the orientational polarization at room temp if the vapour is subjected to an E of 10^6 V/m. The dipole moment of HCl molecule is 3.46×10^{-30} C-m.

$$N = 10^{27} \text{ molecules/m}^3$$

$$k = 1.38 \times 10^{-23} \text{ J K}^{-1}$$

$$E = 10^6 \text{ V/m}$$

$$T = 300 \text{ K}$$

$$p = 3.46 \times 10^{-30} \text{ Cm}$$

$$\alpha_d = \frac{p^2}{3kT} = 9.64 \times 10^{-40} \text{ F m}^2$$

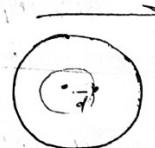
$$P = Np = N \alpha_d E = 9.64 \times 10^{-7} \text{ C/m}^3$$

The classical theory of electronic polarizability.

To obtain α_e , assume that the atom is represented by a nucleus of charge ze and homogeneous -ve charge distribution inside a sphere of radius r_e . When the atom is placed in a static homogeneous external field E , the force exerted on the nucleus & on the e_s is oppositely directed. Thus the nucleus is displaced over a distance d .

We have the volume charge density

$$\rho = \frac{ze}{4\pi R^3}$$



$$\text{Charge of } e \text{ in size} = \frac{4\pi r_e^3}{3} \rho z e = \frac{4\pi r_e^3}{3} \frac{ze}{4\pi R^3} z e = \frac{4\pi r_e^3 z^2 e^2}{3 R^3}$$

$$= \frac{Zed^3}{R^3}$$

The coulomb's force of attraction is given by,

$$F = \frac{E(Ze)}{4\pi\epsilon_0 d^2 R^3} \cdot \frac{(Ze)^2 d}{4\pi\epsilon_0 R^3}$$

: Under equilibrium condition, this force is equal to the force exerted by E .

$$ZeE = \frac{(Ze)^2 d}{4\pi\epsilon_0 R^3}$$

$$d = \frac{4\pi\epsilon_0 R^3 E}{Ze}$$

$$\therefore \text{The dipole moment } p = Zed = 4\pi\epsilon_0 R^3 E$$

$$\therefore \underline{\underline{d_e = 4\pi\epsilon_0 R^3}}$$

Piezoelectricity

When a non centrosymmetric crystal is subjected to a mechanical stress, the ions are displaced from each other in an asymmetric manner and the crystal becomes electrically polarized. This is called piezoelectric effect. The inverse effect of it is also observed. (Electrostriction) The piezoelectric effect is often used to convert electrical energy into mechanical energy and vice-versa.

Quartz is the most familiar piezoelectric substance and the one most frequently used in transducers.

The microscopic origin of piezoelectricity lies in the displacement of ionic charges within the crystal. In the absence of strain, the distribution of the charges at their lattice sites is symmetric, so the internal \bar{E} is zero. But when the crystal is strained, the charges are displaced. If the charge distribution is no longer symmetric, then an \bar{E} develops and it is this field which operates in the piezoelectric effect.

These are used in delay lines also, i.e. when an electric signal is converted into a mechanical wave, it travels through a quartz rod at the velocity of sound, which, since it is much less than the velocity of light, leads to considerable delay of the signal.

Magnetic properties of Materials

20 copys

- Magnetic dipole moment $M_m \cdot q_{md}$
- $E = M_m \times B$
- $V = -M_m \cdot B = -M_m B \cos \theta$
- Magnetic induction / Magnetic flux density - B (T or Nm/A)
- Magnetic field strength / Magnetic field density - H (A/m)
- Magnetisation - M .
- In vacuum, $B = \mu_0 H$; $\mu_0 = 4\pi \times 10^{-7} \text{ Nm}^{-1} \text{ A}^{-1}$
(permeability of free space)

When a substance is placed in a magnetic field, it gets magnetized and hence a magnetization is produced

$$B = \mu_0 H + \mu_0 M = \mu_0 (H+M)$$

→ 1st term is due to external field

→ 2nd term due to Magnetisation.

$$M = \chi H \quad (\chi \rightarrow \text{Magnetic susceptibility})$$

$$\therefore B = \mu_0 H + \mu_0 \chi H = \mu_0 (1+\chi) H$$

$$= \mu H$$

$$\mu = \mu_0 (1+\chi)$$

or relative permeability $\mu_r = \frac{\mu}{\mu_0} = (1+\chi)$

- Q) The applied magnetic field in copper is 10^6 A/m . If the magnetic susceptibility of copper is -0.8×10^{-5} , calculate the flux density and the magnetization in copper.

$$\left[B \text{ (Nm/A) or T} ; H \text{ (A/m)} \right]$$

$$H = 10^6 \text{ A/m}$$

$$\chi = -0.8 \times 10^{-5}$$

$$\therefore M = \chi H = -0.8 \times 10^{-5} \times 10^6 = -8 \text{ A/m}$$

$$B = \mu_0 (H+M) = 4\pi \times 10^{-7} (-8 + 10^6)$$

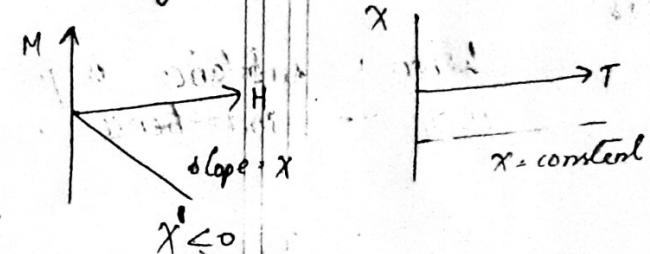
$$\underline{\underline{= 1.25 T}}$$

Classification of magnetic materials

Magnetic materials can be classified according to χ values and the way in which these vary with the magnetic field strength and temperature.

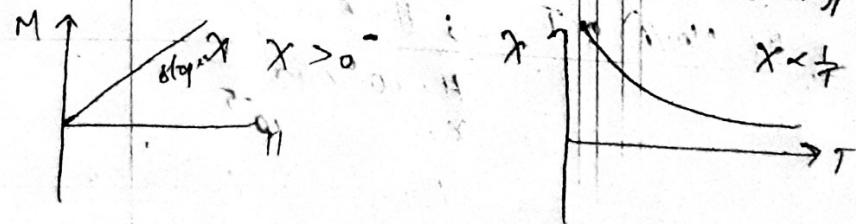
(a) Diamagnetic

Diamagnetic substances are composed of atoms which have no net magnetic moments (i.e., all the orbital shells are filled and there are no unpaired e^- s). But when the atom is placed in a magnetic field, a magnetic moment is induced, and a negative magnetization is produced and thus χ is negative. χ is independent of temp. e.g.: inert gases, metals, water, glass, marble etc.



(b) Paramagnetism

In this class of materials, the atoms or ions have a net magnetic moment due to unpaired e^- s in partially filled orbitals. But in the absence of an external field, these moments are randomly oriented and \therefore the substance exhibits no net magnetic moment. However when placed in a magnetic field, the moments tend to align along the direction of field, producing a net magnetization. Since the moments line up in the direction of the field which help enhance the external field, the paramagnetic susceptibility is greater than zero. Also they weakly attract the line of force. As thermal energy randomizes the alignment of the dipoles, the χ decreases with the T in temp.



e.g.: Iron, aluminium, platinum, potassium etc.

magnetic (Ferromagnetism is the phenomenon of spontaneous magnetization). These substances possess permanent magnetic moments even in the absence of an external magnetic field. Since the ferromagnetic susceptibility is very large and +ve, the ferromagnetic substance strongly attracts the lines of forces. Ferromagnetism exists only below a certain temp T_c , (curie temp), above which the substance becomes paramagnetic. e.g. iron, cobalt, nickel.

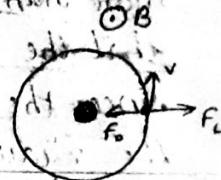
d) Ferri magnetic

e) Antiferro magnetic:

Langevin diamagnetism

Consider an \vec{e} rotating about the nucleus in a circular orbit, and let a magnetic field be applied \perp to the plane of the paper. In the absence of any external field, the centripetal force acting on the \vec{e} due to the nucleus

$$F_0 = \frac{mv^2}{r}, \text{ now } v = \omega r \quad (1)$$



The magnetic moment of the \vec{e} is,

$$\mu_m = IA = \frac{e \cdot \pi r^2}{t} \times \frac{e \omega r^2}{2\pi} \quad (2)$$

$$\mu_m = \frac{e \omega r^2}{2} \quad (2)$$

When the field is applied, an additional force starts to act on the \vec{e} , the Lorentz force,

$$F_L = e(v \times B)$$

$$q = -e$$

$$\theta = 90^\circ$$

Resultant force is

$$F_0 - F_L = mw^2 r$$

$$mw^2 r - eBrw = mw^2 r$$

$$w - w_0 = 0$$

On solving,

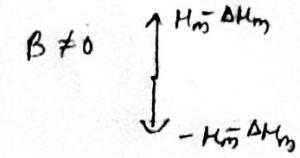
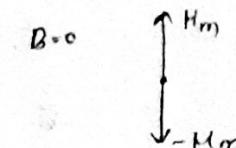
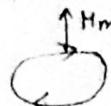
$$w = \pm \frac{eB}{2m} + w_0 - w_L$$

The change in frequency $w_L = eB/2m$ is called the precessional frequency or Larmour frequency.

This shows that the rotation of the e^- has slowed down. This reduction in frequency produces corresponding change in the magnetic moment.

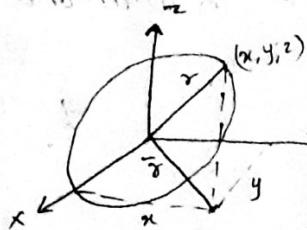
$$\Delta H_m = \frac{e A \omega r^2}{2} = \frac{e^2 r^2 B}{4m}$$

Now consider the case of 2 e^- s rotating in opposite directions



When no magnetic field is applied, the orbital magnetic moment of the e^- s being equal and opposite will mutually cancel out. But in the presence of the external \vec{B} , the magnetic moment in a direction of the field \downarrow , while that in opposite direction \uparrow , giving rise to a resultant magnetic moment in a direction opposite to the direction of B .

While considering an atom we have more than one e^- . In deriving the eqn, we have assumed that the plane of the orbit is \perp to the field. However since the orbit can have any orientation w.r.t the field the radius r in \vec{r} of the orbit should be replaced by \vec{r} , where \vec{r} is the projection of the \vec{r} on the plane \perp to the magnetic field.



Let the mean value of the radii of all possible orbits be \bar{r} then,

$$\bar{r}^2 = x^2 + y^2 + z^2$$

For a spherically symmetric atom

$$x^2 + y^2 + z^2 = \frac{1}{3} r^2$$

$$\text{Moreover } \bar{r}^2 = \frac{x^2 + y^2}{3} = \frac{2}{3} r^2$$

$$\therefore \frac{-ze^2 \bar{r}^2}{6m} B \quad (\text{for } z \text{ es})$$

If N is the no. of atoms / unit volume, then
the Magnetization is $M = NM\mu_m$

$$\chi_{dia} = \frac{M}{H} = \frac{NM\mu_m N_0}{B} \quad (B = \mu_0 H)$$

$$\chi_{\text{dia}} = \frac{ze^2 N B \mu_0}{6mB}$$

$$\chi_{\text{dia}} = -\frac{\mu_0 ze^2 N \sigma^2}{6m}$$

$\propto N$, which shows χ depends on the size of the atoms. Also $\propto r^2$ means the outer es make the largest contribution to the diamagnetic susceptibility.

- a) Estimate the order of the diamagnetic susceptibility.
 Diamagnetism of copper by assuming that only one es/atom makes the contribution. The radius of the copper atom is 1A° and the lattice parameter is 3.608 A° .

$$r = 1\text{A}^\circ = 10^{-10}\text{ m}$$

$$a = 3.608 \times 10^{-10}\text{ m}$$

Cu is a fcc crystal, so it has 4 atoms/unit cell.

The no. of es /unit volume is

$$N = \frac{0}{a^3} = \frac{4}{(3.608 \times 10^{-10})^3} = 8.5 \times 10^{28}/\text{m}^3$$

$$\begin{aligned} \chi_{\text{dia}} &= -\frac{\mu_0 ze^2 N r^2}{6m} \\ &= -4 \times 10^{-7} \times (1.6 \times 10^{-19})^2 \times 8.5 \times 10^{28} \times (10^{-10})^2 \\ &= -5 \times 10^{-6} \end{aligned}$$

Paramagnetism

Paramagnetism is the magnetic analogue of the electrical orientational polarizability and is due to the permanent magnetic moment of the atoms.

Classical theory

The potential energy of a magnetic dipole in a magnetic field is, $V = -\mu \cdot B$. The energy is least when the moment is parallel to the field and thus the moment tends to line up with the field. The effect of temp is to

randomize the direction of the dipole. The result of these two competing processes is that some magnetization is produced. We can solve this as the same way as we solve electrical polarization, which leads to

$$\bar{\mu} = \mu_m L(x)$$

$L(x) = \text{tanh } \frac{x}{x}$, Langevin function

$$x = \frac{\mu_m B}{kT}$$

$$\text{for } x \ll 1, L(x) = \frac{x}{3}$$

$$\bar{\mu} = \frac{\mu_m^2 B}{3kT}$$

$$\text{A Magnetization } M = \frac{N\bar{\mu}}{V} = \frac{NM^2 B}{3kT}$$

$$\therefore \chi = \frac{M}{H} = \frac{NM^2}{V} = \frac{M_0 N M^2}{3kT} = \frac{C}{T}$$

which is called Curie's law

$$\text{where } C = \frac{M_0 N M^2}{3k} = \text{Curie constant}$$

Thus χ is inversely proportional to T & is true.

a) A paramagnetic salt contains 10^{28} ions/m³ with magnetic moment of one Bohr magneton. Calculate the χ_{para} and the magnetization produced in a uniform magnetic field of 10^6 A/m at room temp.

$$N = 10^{28} \text{ m}^{-3}$$

$$M_0 = \mu_B = 9.27 \times 10^{-24} \text{ A m}^2$$

$$H = 10^6 \text{ A/m}$$

$$T = 300 \text{ K}$$

$$\chi = \frac{M_0 N M^2}{3kT} = \frac{4\pi \times 10 \times 10 \times (9.27 \times 10^{-24})^2}{3 \times 1.38 \times 10^{-23} \times 300} = 0.87 \times 10^{-4}$$

$$M = \chi H = 0.87 \times 10^{-4} \times 10^6 = 8.7 \text{ A/m}$$

Quantum theory

We can express the magnetic moment μ of the atom in terms of total angular momentum J as,

$$\mu = g \left(-\frac{e}{2m} \right) J$$

$g \rightarrow \text{Landé } g\text{-factor}$

$$= 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

When a magnetic field is applied to the atom, a Zeeman splitting results. The zeeman energy is,

$$E = -\mu \cdot B + \frac{g e B J}{2m}$$

$$= \frac{g e B m_J}{2m}$$

$$\frac{e h}{2m} = 8 H_B$$

$$E = g H_B B m_J$$

m_J can take the values $m_J = -\frac{1}{2}, 0, +\frac{1}{2}$. The difference in energy b/w the levels is,

$$\Delta E = g H_B B$$

The lower level $m_J = -\frac{1}{2}$ corresponds to the moment μ parallel to the field, while the upper level corresponds to the moment opposite to the field.

Let N_1 and N_2 be the concentrations of atoms in the lower and upper levels resp. and $N_1 + N_2 = N$ the total no. of atoms. The difference of these concentrations produces a magnetization

$$M = g H_B (N_1 - N_2)$$

N_1 & N_2 are related as $\frac{N_2}{N_1} = e^{-\Delta E / kT}$

$$\frac{N_1}{N} = \frac{e^{-g H_B B / kT}}{e^{-g H_B B / kT} + e^{-g H_B B / kT}}$$

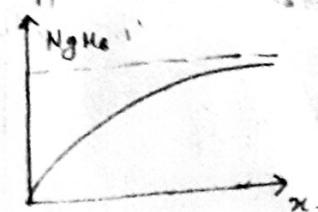
$$\frac{N_2}{N} = \frac{e^{-g H_B B / kT}}{e^{-g H_B B / kT} + e^{-g H_B B / kT}}$$

put $\frac{g H_B B}{kT} = x$

$$\therefore N_1 - N_2 = \frac{N}{e^x - e^{-x}} (e^x - e^{-x})$$

$$\therefore M = gH_B(N_1 - N_2)$$

$$N_1 H_B \left(e^{-\frac{eH_B}{kT}} - e^{\frac{eH_B}{kT}} \right) = N_1 H_B \tanh x$$



At low temp. M is proportional to B , but at higher fields M begins to saturate and eventually reaches a maximum value $N_1 H_B$, when all the dipoles turn parallel to the field. Thus for low field $x \ll 1$ and hence $\tanh x \approx x$

$$\therefore M = N_1 H_B x \quad \frac{N_1 (gH_B)^2 B}{kT} = \frac{H_0 M (gH_B)^2}{kT}$$

$$\therefore \frac{M}{H_0} = \frac{x}{\frac{kT}{H_0 N_1 (gH_B)^2}}$$

In general for $J > 1/2$, an atom with angular momentum quantum no. J has $(2J+1)$ equally spaced energy levels. The magnetization is then given by,

$$M = N J H_B \tanh(x), \quad x = \frac{9J H_B B}{kT}$$

$$\text{Boltzmann factor } B_J = \frac{(2J+1)}{2J} \left(\frac{e^{\frac{eH_B}{kT}} - e^{-\frac{eH_B}{kT}}}{2} \right) = \frac{1}{2J} \coth\left(\frac{eH_B}{2kT}\right)$$

and so the magnetization is $M = N J H_B B_J$

$$\text{if } M = N J H_B B_J \text{ has } 9/2 \text{ and } 13/2 \text{ bars} \quad \frac{1}{2J} \coth\left(\frac{eH_B}{2kT}\right)$$

$$\therefore M = N J (J+1) \frac{9}{2} H_B B$$

$$x = \frac{3kT}{H_0 N J (J+1) g^2 H_B^2}$$

$$x = \frac{H_0 N p^2 g^2 H_B^2}{3kT}$$

$$x = \frac{H_0 M H_{eff}^2}{3kT}$$

effective moment of the atom is $H_{eff} = p H_B$, $p = g(J(J+1))$
is called effective no. of Bohr Magnetons for the atoms.