(Pages : 4)

Reg. No. : 328-12-112-021

Name : Sandeep . P

2295

Fourth Semester B.Sc. Degree Examination, June 2014
(Career Related First Degree Programme under CBCSS)
Group 2(a) : Physics and Computer Applications
Complementary Course MM 1431.6 : Mathematics – IV
LINEAR TRANSFORMATIONS, VECTOR INTEGRATION AND COMPLEX ANALYSIS

Time : 3 Hours

Max. Weights : 30

SECTION – A

All the first **16** questions are **compulsory**. **Four** consecutive questions beginning with the **first** form a bunch. **Each** bunch carries **1** weight.

1. Test whether $T : \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T(x, y) = (x + c, y, 0)$, where c is a nonzero constant, is linear or not.

2. Define the nullspace of a linear transformation.

3. If U and W are vector spaces with zero elements 0_U and 0_W, respectively, then Kernel of the identity linear map $T : U \to W$ is

4. State True or False: $\{(1, 0), (0, \sqrt{2})\}$ is a basis for the vector space $\mathbb{R}^2$ over the field $\mathbb{R}$.

5. Show that $\int_C f(x, y)dx = 0$ along any line segment parallel to the y-axis.

6. Show that $F(x, y, z) = yz\, i + zx\, j + xy\, k$ is solenoidal.

7. State Stokes's theorem.

8. Give an example of a piecewise smooth surface.

9. Let a and b are constants, with $a \neq 0$; and z is a complex variable. Then give the value of $\lim_{z \to z_0} (az + b)$.

P.T.O.

10. State Euler's formula.

11. Give the principal amplitude of 1.

12. Define entire function.

13. When an arc is called a simple closed arc ?

14. Show by Cauchy-Goursat theorem that for $f(z) = \frac{z^2}{z-3}$, $\int_C f(z)dz = 0$, where C is the unit circle $|z| = 1$.

15. Define the principal part of a Laurent series.

16. Give the value of $\int_C \frac{dz}{z-1}$, where C is the circle $|z| = 3$.

SECTION – B

Answer **any 8** questions from among the questions **17** to **28**. These questions carry **1** weightage **each**.

17. If R_θ is the rotation matrix of order 2×2 associated with θ, then $R_\theta R_\theta^t$ is

18. Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by
$T(x, y) = (3x + 4y, 2x - 5y)$.
Find the matrix of T with respect to the basis $S = \{(1, 2), (2, 3)\}$ for $\mathbb{R}^2$.

19. Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by
$T(x_1, x_2) = (3x_1 - x_2, 4x_1 + 2x_2)$.
Show that T is invertible.

20. If $F(t) = (t - t^2)\,i + 2t^3 j - 3k$, find $\int_1^2 F(t)dt$.

21. Use the Divergence Theorem to find the outward flux of the vector field $\vec{F}(x,y,z) = x^3\bar{i} + y^3\bar{j} + z^2\bar{k}$ across the surface of the region that is enclosed by the circular cylinder $x^2 + y^2 = 9$ and the planes $z = 0$ and $z = 2$.

22. Find the total work done in moving a particle in a force field given by $\vec{F} = (3x^2 + 6y)\bar{i} - 14yz\bar{j} + 20xz^2\bar{k}$ along the straight line from (0, 0, 0) to (1, 0, 0)

23. Show that

$(-1 + i)^7 = -8(1 + i)$.

24. Verify Cauchy-Riemann equations for the function $f(z) = z^2$.

25. If $f(z) = u(x, y) + iv(x, y)$ be an analytic function. Then prove that $v(x, y) = a$ constant implies $f(z)$ is a constant.

26. Show that the function $v = xy$ is harmonic.

27. Find $\int_C \bar{z}\, dz$, where C is the upper half of the circle $|z| = 1$ from $z = -1$ to $z = 1$.

28. Using Cauchy's integral formula, evaluate $\int_C \frac{z^2}{z-2}\, dz$, where C is the circle $|z| = 3$.

SECTION – C

Answer **any 5** questions from the questions **29** to **36**. These questions carry **2** weights each.

29. Find the standard matrix A for the dilation transformation $T(x) = 5x$ for $x \in \mathbb{R}^2$

30. Let $T:\mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation defined by

$T(x, y, z) = (4x + 2y, -y + 3z)$.

Find the matrix of T, if the bases of $\mathbb{R}^3$ and $\mathbb{R}^2$ are respectively

$B_1 = \{(1, 1, 1), (1, 0, 0), (0, 0, 1)\}$ and $B_2 = \{(1, 3), (2, 5)\}$.

31. Using Green's theorem, evaluate the integral

$\oint_C xydy - y^2dx$,

where C is the square cut from the first quadrant by the lines $x = 1$ and $y = 1$.

32. Evaluate the value of the integral

$I = \int_C \bar{z}\, dz$

where C is the right-hand half $z = 2e^{i\theta}$ $\left(-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}\right)$ of the circle $|z| = 2$, from $z = -2i$ to $z = 2i$.

33. Using Cauchy's integral formula, integrate $\frac{z^2+1}{z^2-1}$ in the counter clock wise sense around a circle of radius 1 with centre at the point $z = \frac{1}{2}$.

34. Prove that if two functions u and v are conjugate harmonic of each other, both u and v must be constant functions.

35. Verify that $u(x, y) = \frac{x}{x^2 + y^2}$ is harmonic and find its conjugate. Also give the associated analytic function.

36. Use an antiderivative to evaluate the integral

$$\int_C z^{1/2}\, dz$$

where the integrand is the branch

$$z^{1/2} = \sqrt{r}\, e^{i\theta/2} \; (r > 0, 0 < \theta < 2\pi)$$

of the square root function and C is any contour from $z = -3$ to $z = 3$ which except for its end points, lies below the x-axis.

SECTION – D

Answer **any 2** questions from among the questions **37 to 39**. These questions carry **4** weights **each**.

37. a) Let T be the linear transformation whose standard matrix is

$$A = \begin{bmatrix} 1 & -4 & 8 & 1 \\ 0 & 2 & -1 & 3 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$. Does T map $\mathbb{R}^4$ onto $\mathbb{R}^3$? Is T a one-to-one mapping?

b) Let F be the rotation through an angle of $\frac{\pi}{2}$ and let $a = (2, 1)$. Find the coordinates of F(x) relative to the standard basis $\{(1, 0), (0, 1)\}$.

38. a) Verify Green's theorem in the plane for $\oint_C (xy\,dx + x^2 dy)$, where C is the curve enclosing the region bounded by the parabola $y = x^2$ and the line $y = x$.

b) Evaluate $\iint_S F.n\,dS$, where $F = zi + xj - 3yzk$ and S is the surface of the cylinder $x^2 + y^2 = 9$ included in the first octant between $z = 0$ and $z = 4$.

39. Find an analytic function whose real part is $e^x(x \cos y - y \sin y)$ and which takes the value e at $z = 1$.

(Pages : 4) **E – 3480**

Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, July 2018

Career Related First Degree Programme under CBCSS

Group 2(a) : Complementary Course for Physics and Computer Applications

MM 1431.6 : MATHEMATICS – IV – Linear Transformations, Vector Integration and Complex Analysis

(2013 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **10** questions are **compulsory**. They carry **1** mark **each**.

1. Write the matrix representation of contraction with factor k on $\mathbb{R}^2$.
2. Find the reflection of (–1, 2) about the line y = x.
3. Show that the transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(a, b, c) = (4a - 2b, bc, c)$ is not linear.
4. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that maps each vector into its orthogonal projection on the x-axis. What is the matrix representation of T with respect to the standard basis ?
5. What is the physical interpretation of divergence of a vector field $\vec{F}$?
6. State Stoke's theorem.
7. Let $z = x + iy$, find $\mathrm{Im}\,[(1 + i)^8 z^2]$.
8. Express $f(z) = 2iz + 6\bar{z}$ in the form $(x, y) + iv(x, y)$.
9. State Cauchy's Integral formula for derivatives.
10. Evaluate $\int_0^{\pi} e^{it}\, dt$.

P.T.O.

SECTION – II

Answer **any 8** questions from among the questions **11** to **22**. These questions carry **2** marks **each**.

11. Find the transformation from $\mathbb{R}^3$ to $\mathbb{R}^2$ that has the matrix representation $\begin{bmatrix} 1 & 2 & 5 \\ 4 & -1 & 2 \end{bmatrix}$ with respect to the standard basis of $\mathbb{R}^3$ and the basis $\{(1, 1), (1, -1)\}$ of $\mathbb{R}^2$.

12. Consider a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$ given by $T(a, b) = (-a, a + b, a - b)$. Find the matrix representation of this transformation with respect to the basis $\{(2, 1), (1, 7)\}$ of $\mathbb{R}^2$ and the standard basis of $\mathbb{R}^3$.

13. Find the coordinates of the vector $(2, 1, 3, 4)$ of $\mathbb{R}^4$ relative to the basis $\{(1, 1, 0, 0), (1, 0, 1, 1), (2, 0, 0, 2), (0, 0, 2, 2)\}$.

14. Find the work done by the force field $\vec{F}(x, y, z) = xyi + yzj + xzk$ on a particle that moves along the curve $\vec{r}(t) = ti + t^2j + t^3k$, $0 \le t \le 1$.

15. Evaluate the line integral $\int_C (x^2 - y^2)\,dx + x\,dy$ along the curve $= t^3, y = t, -1 \le t \le 1$.

16. Find the divergence of $\vec{F}(x, y) = (x^2 - y)\,i + (xy - y^2)\,j$.

17. Find the principal branch of $\log(-ei)$.

18. Sketch the graph of $0 < |z - 1| < 1$.

19. Find all values of $(-8i)^{1/3}$.

20. Evaluate $\int_C \operatorname{Re} z\,dz$ where C is the shortest path from 0 to $1 + 2i$.

21. Evaluate $\int_C \frac{z+2}{z}\,dz$ where C is the circle $z = 2e^{it}$, $0 \le t \le 2\pi$.

22. Evaluate $\int_C \frac{1}{z^2 + 2z + 2}\,dz$ where C is the unit circle $|z| = 1$.

SECTION – III

Answer **any 6** questions from among the questions **23** to **31**. These questions carry **4** marks **each**.

23. Consider the linear transformation $F : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $F(x, y) = (3x + 4y, 2x - 5y)$. Find the matrix representing F relative to the basis $\{(1, 2), (2, 3)\}$ of $\mathbb{R}^2$.

24. The matrix $A = \begin{bmatrix} 1 & -2 & 1 \\ 3 & -1 & 0 \\ 1 & 4 & -2 \end{bmatrix}$ represents a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ with respect to the basis $\mathcal{B} = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$. Find the matrix that represents the linear transformation relative to the basis $S = \{(1, 1, 1), (0, 1, 1), (1, 2, 3)\}$.

25. Find a potential function of a vector field $\vec{F}(x, y) = 2xy^5 i + (1 + 5x^2y^4)j$, if the vector field is conservative.

26. Use the Divergence theorem to find the outward flux of the vector field $\vec{F}(x, y, z) = x^3 i + y^3 j + z^2 k$ across the surface of the region that is enclosed by the circular cylinder $x^2 + y^2 = 9$ and the planes $z = 0$ and $z = 2$.

27. Using Green's theorem evaluate the line integral $\oint_C y^2\,dx + x^2\,dy$ where C is the square with vertices (0, 0), (1, 0), (1, 1) and (0, 1) oriented counterclockwise.

28. Show that $\cos z = \cos x \cosh y - i \sin x \sinh y$.

29. Is $f(z) = u(x, y) + iv(x, y) = e^x(\cos y + i \sin y)$ analytic ? Explain.

30. Find the value of the integral $\int_C \frac{1}{z^3(z+4)}\,dz$ taken counterclockwise around the circle $|z| = 2$.

31. Find the principal value of $(-i)^i$.

SECTION – IV

Answer **any 2** questions from among the questions **32** to **35**. These questions carry 15 marks **each**.

32. Let $A = \begin{bmatrix} 5 & 6 \\ 3 & -2 \end{bmatrix}$ be the matrix representation of a linear transformation on $\mathbb{R}^2$.

    Find a suitable basis for $\mathbb{R}^2$ so that the matrix representation of the given linear transformation is a diagonal matrix.

33. Evaluate the surface integral $\iint_{\sigma} y^2 z^2 \, ds$ where σ is the part of the cone $z = \sqrt{x^2 + y^2}$ that lies between the planes $z = 1$ and $z = 2$.

34. a) Verify that $u(x, y) = 2x(1 - y)$ is harmonic in the whole complex plane and find a harmonic conjugate function $v(x, y)$ of u.

    b) Let $f(z) = \begin{cases} \dfrac{\bar{z}^2}{z} & \text{when } z \neq 0 \\ 0 & \text{when } z = 0 \end{cases}$

    verify whether the Cauchy-Riemann equations are satisfied at the origin.

35. Let C be the circle $|z| = 3$ in counterclockwise direction. Show that if

    $g(s) = \int_C \dfrac{2z^2 - z - 2}{z - s} \, dz$, $(|s| \neq 3)$, then $g(2) = 8\pi i$. What is the value of $g(s)$ when $|s| > 3$?

(Pages : 3)

C – 3882

Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, July 2017
(Career Related First Degree Programme Under CBCSS)
Group 2(a) : Complementary Course for Physics and Computer Applications
MM 1431.6 : MATHEMATICS – IV : LINEAR TRANSFORMATIONS, VECTOR INTEGRATION AND COMPLEX ANALYSIS
(2013 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first 10 questions are compulsory. They carry 1 mark each.

1. Define the linear transformation reflection.
2. If $T(x) = Ax$ is a linear transformation from R^2 to R^3, what is the order of A ?
3. Define matrix representation of a linear transformation.
4. Find the work done by the conservative field $F = \nabla(xyz)$ along a smooth curve joining the points (– 1, 3, 9) and (1, 6, – 4).
5. Define potential function.
6. Write the condition for $F = Mi - Nj + Pk$ to be conservative.
7. Define argument of a complex number.
8. Is complex conjugate differentiable ?
9. Write Cauchy-Riemann equations for analytic functions.
10. Find $\int_{-\pi}^{\pi} \cos z\,dz$.

P.T.O.

SECTION – II

Answer **any 8** questions from among the questions **11** to **22**. These questions carry **2** marks **each**.

11. Determine whether the transformation $T : R^2 \to R^1$ defined by $T[a\ b] = ab$ is linear or not.

12. If $T : R^2 \to R^2$ is a linear transformation which satisfies $T[1\ 1] = [5\ 6]$ and $T[1\ -1] = [7\ 8]$. Find $T[a\ b]$, for any two real numbers a and b.

13. Determine whether the linear transformation $T\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a+b \\ a-b \\ 2a+3b \end{bmatrix}$ is one-to-one.

14. Find unit normal to the surface $x^2y + 2xz = 4$ at the point $(2, -2, 3)$.

15. Find the flux of $F = (y - x)i + yj$ across the circle $x^2 + y^2 = 1$ in the xy-plane.

16. Test whether $F = (z + y)i + zj + (y + x)k$ is conservative or not.

17. Show that an analytic function is constant if its modulus is constant.

18. If $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain D, then prove that u and v satisfies Laplace's equation.

19. Find all points where $w = z^2 + \frac{1}{z^2}$, is not conformal.

20. Find an upper bound for the absolute value of the integral $\int_C z^2\,dz$, where C is the straight line from 0 to $1 + i$.

21. State Cauchy's integral theorem. What is $\int_C \cos z\,dz$, where C is any closed path.

22. Evaluate $\int_C \frac{z^2 - 6}{2z - i}\,dz$, where C is any closed path.

SECTION – III

Answer **any 6** questions from among the questions **23** to **31**. These questions carry **4** marks **each**.

23. Find the both change of coordinate matrices for the bases $C = \{t + 1, t - 1\}$ and $D = \{2t + 1, 3t + 1\}$.

24. Prove that the image of a linear transformation is a subspace of the codomain. Determine the image of the matrix $A = \begin{bmatrix} 1 & 1 & 5 \\ 2 & -1 & 1 \end{bmatrix}$.

25. Find the area of the cap cut from the hemisphere $x^2 + y^2 + z^2 = 2$ by the cylinder $x^2 + y^2 = 1$.

26. Find the flux of $F = yzj + z^2k$ outward through the surface S cut from the cylinder $y^2 + z^2 = 1$, $z \geq 0$, by the planes $x = 0$ and $x = 1$.

27. Show that $ydx + xdy + 4dz$ is exact and evaluate the integral $\int_C (ydx + xdy + 4dz)$ from $A(1, 1, 1)$ to $B(2, 3, -1)$.

28. Write a short note on logarithmic and hyperbolic functions in the complex plane.

29. Find an analytic function $f(z) = u + iv$, where $v = 2y(-1 + x)$.

30. Evaluate $\int_C \frac{e^z}{z}\,dz$ where C is the circle $|z| = 2$, counterclockwise.

31. Evaluate $\int_C \frac{dz}{z^2 + 1}$ where C is $|z - i| = 1$, counterclockwise.

SECTION – IV

Answer **any 2** questions from among the questions **32** to **35**. These questions carry **15** marks **each**.

32. Find the matrix representation for the linear transformation $T\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 11a + 3b \\ -5a - 5b \end{bmatrix}$.

33. a) Use divergence theorem to find the outward flux of the vector field $F(x, y, z) = zk$ across the sphere $x^2 + y^2 + z^2 = a^2$.
    b) Verify divergence theorem for the field $F = xi + yj + zk$ over the sphere of radius a.

34. Discuss the analyticity of exponential and trignometric functions.

35. Integrate $g(z) = \frac{1}{z^2 - 1}\tan z$, counterclockwise around the circle $|z| = \frac{3}{2}$.