Two player game with two strategies

In[∘]:=
```
n = 2;
A[i_, j_] := a_{i,j};
```

In[∘]:= $fi = \sum_{j=1}^{n} A[i, j]\, x_j$

Out[∘]=

$x_1\, a_{i,1} + x_2\, a_{i,2}$

In[∘]:= $fbar = \sum_{i=1}^{n} x_i\, fi$

Out[∘]=

$x_1\,(x_1\, a_{1,1} + x_2\, a_{1,2}) + x_2\,(x_1\, a_{2,1} + x_2\, a_{2,2})$

In[∘]:=
```
strategytoedit = 1;
```

In[∘]:= `A'[i_, j_] := If[j == strategytoedit,` $a_{i,j} + c, a_{i,j}$`]`

In[∘]:= $finew = \sum_{j=1}^{n} A'[i, j]\, x_j$

Out[∘]=

$x_1\,(c + a_{i,1}) + x_2\, a_{i,2}$

In[∘]:= `Collect[finew, c] == fi + c` $x_{strategytoedit}$

Out[∘]=

True

In[∘]:= $fbarnew = \sum_{i=1}^{n} x_i\, finew$

Out[∘]=

$x_1\,(x_1\,(c + a_{1,1}) + x_2\, a_{1,2}) + x_2\,(x_1\,(c + a_{2,1}) + x_2\, a_{2,2})$

In[∘]:= $\text{Simplify}\left[\text{Collect}[fbarnew, c] == fbar + c\, x_{strategytoedit},\ \text{Assumptions} \to \left\{\sum_{i=1}^{n} x_i == 1\right\}\right]$

Out[∘]=

True

In[∘]:= $\text{Simplify}\left[\text{Collect}[finew - fbarnew, c] == fi + c\, x_{strategytoedit} - fbar - c\, x_{strategytoedit},\ \text{Assumptions} \to \left\{\sum_{i=1}^{n} x_i == 1\right\}\right]$

Out[∘]=

True

Three player game with two strategies

Number of strategies is *n* and the payoff tensor **A**

In[•]:=
```
n = 3;
A[i_, j_, k_] := a_{i,j,k};
```

The fitness of strategy *i*

In[•]:=
$$fi = \sum_{j=1}^{n}\sum_{k=1}^{n} A[i, j, k]\, x_j\, x_k$$

Out[•]=
$$x_1^2\, a_{i,1,1} + x_1\, x_2\, a_{i,1,2} + x_1\, x_3\, a_{i,1,3} + x_1\, x_2\, a_{i,2,1} + x_2^2\, a_{i,2,2} + x_2\, x_3\, a_{i,2,3} + x_1\, x_3\, a_{i,3,1} + x_2\, x_3\, a_{i,3,2} + x_3^2\, a_{i,3,3}$$

Average payoff of the population

In[•]:=
$$fbar = \sum_{i=1}^{n} x_i\, fi$$

Out[•]=
$$x_1\left(x_1^2\, a_{1,1,1} + x_1\, x_2\, a_{1,1,2} + x_1\, x_3\, a_{1,1,3} + x_1\, x_2\, a_{1,2,1} + x_2^2\, a_{1,2,2} + x_2\, x_3\, a_{1,2,3} + x_1\, x_3\, a_{1,3,1} + x_2\, x_3\, a_{1,3,2} + x_3^2\, a_{1,3,3}\right) + x_2\left(x_1^2\, a_{2,1,1} + x_1\, x_2\, a_{2,1,2} + x_1\, x_3\, a_{2,1,3} + x_1\, x_2\, a_{2,2,1} + x_2^2\, a_{2,2,2} + x_2\, x_3\, a_{2,2,3} + x_1\, x_3\, a_{2,3,1} + x_2\, x_3\, a_{2,3,2} + x_3^2\, a_{2,3,3}\right) + x_3\left(x_1^2\, a_{3,1,1} + x_1\, x_2\, a_{3,1,2} + x_1\, x_3\, a_{3,1,3} + x_1\, x_2\, a_{3,2,1} + x_2^2\, a_{3,2,2} + x_2\, x_3\, a_{3,2,3} + x_1\, x_3\, a_{3,3,1} + x_2\, x_3\, a_{3,3,2} + x_3^2\, a_{3,3,3}\right)$$

This is the strategy we choose to remove by adding a constant *c*

In[•]:=
```
strategytoedit = 1;
```

In[•]:=
```
A'[i_, j_, k_] := If[j == strategytoedit, a_{i,j,k} + c, a_{i,j,k}]
```

In[•]:=
```
Table[A'[i, j, k], {k, 1, n}, {j, 1, n}, {i, 1, n}] // MatrixForm
```

Out[•]//MatrixForm=
$$\begin{pmatrix} \begin{pmatrix} c + a_{1,1,1} \\ c + a_{2,1,1} \\ c + a_{3,1,1} \end{pmatrix} & \begin{pmatrix} a_{1,2,1} \\ a_{2,2,1} \\ a_{3,2,1} \end{pmatrix} & \begin{pmatrix} a_{1,3,1} \\ a_{2,3,1} \\ a_{3,3,1} \end{pmatrix} \\ \begin{pmatrix} c + a_{1,1,2} \\ c + a_{2,1,2} \\ c + a_{3,1,2} \end{pmatrix} & \begin{pmatrix} a_{1,2,2} \\ a_{2,2,2} \\ a_{3,2,2} \end{pmatrix} & \begin{pmatrix} a_{1,3,2} \\ a_{2,3,2} \\ a_{3,3,2} \end{pmatrix} \\ \begin{pmatrix} c + a_{1,1,3} \\ c + a_{2,1,3} \\ c + a_{3,1,3} \end{pmatrix} & \begin{pmatrix} a_{1,2,3} \\ a_{2,2,3} \\ a_{3,2,3} \end{pmatrix} & \begin{pmatrix} a_{1,3,3} \\ a_{2,3,3} \\ a_{3,3,3} \end{pmatrix} \end{pmatrix}$$

In[•]:=
$$finew = \sum_{j=1}^{n}\sum_{k=1}^{n} A'[i, j, k]\, x_j\, x_k$$

Out[•]=
$$x_1^2\,(c + a_{i,1,1}) + x_1\, x_2\,(c + a_{i,1,2}) + x_1\, x_3\,(c + a_{i,1,3}) + x_1\, x_2\, a_{i,2,1} + x_2^2\, a_{i,2,2} + x_2\, x_3\, a_{i,2,3} + x_1\, x_3\, a_{i,3,1} + x_2\, x_3\, a_{i,3,2} + x_3^2\, a_{i,3,3}$$

In[•]:=
```
Collect[finew, c] - fi == 0 // Simplify
```

Out[•]=
$$c\, x_1\,(x_1 + x_2 + x_3) == 0$$

In[∘]:= **fbarnew = $\sum_{i=1}^{n} x_i$ finew**

Out[∘]=

$$x_1 \left(x_1^2 (c + a_{1,1,1}) + x_1 x_2 (c + a_{1,1,2}) + x_1 x_3 (c + a_{1,1,3}) + x_1 x_2 a_{1,2,1} + x_2^2 a_{1,2,2} + x_2 x_3 a_{1,2,3} + x_1 x_3 a_{1,3,1} + x_2 x_3 a_{1,3,2} + x_3^2 a_{1,3,3}\right) + x_2 \left(x_1^2 (c + a_{2,1,1}) + x_1 x_2 (c + a_{2,1,2}) + x_1 x_3 (c + a_{2,1,3}) + x_1 x_2 a_{2,2,1} + x_2^2 a_{2,2,2} + x_2 x_3 a_{2,2,3} + x_1 x_3 a_{2,3,1} + x_2 x_3 a_{2,3,2} + x_3^2 a_{2,3,3}\right) + x_3 \left(x_1^2 (c + a_{3,1,1}) + x_1 x_2 (c + a_{3,1,2}) + x_1 x_3 (c + a_{3,1,3}) + x_1 x_2 a_{3,2,1} + x_2^2 a_{3,2,2} + x_2 x_3 a_{3,2,3} + x_1 x_3 a_{3,3,1} + x_2 x_3 a_{3,3,2} + x_3^2 a_{3,3,3}\right)$$

In[∘]:= **Simplify[Collect[fbarnew, c] == fbar + c $x_{\text{strategytoedit}}$, Assumptions → {$\sum_{i=1}^{n} x_i$ == 1}]**

Out[∘]=

True

In[∘]:= **Simplify[Collect[finew - fbarnew, c] == fi + c $x_{\text{strategytoedit}}$ - fbar - c $x_{\text{strategytoedit}}$, Assumptions → {$\sum_{i=1}^{n} x_i$ == 1}]**

Out[∘]=

True