Lotka Volterra Dynamics

Here are the growth rate and interactions parameters for the LV

In[∘]:=
```
r1 = 1; a11 = 1.3; a12 = -2;
r2 = -1.1; a21 = 2.2; a22 = -1.5;
```

Here the self interaction terms are chosen appropriately to yield the following equations

In[∘]:=
```
{y1'[t] == y1[t] (r1 + a11 y1[t] + a12 y2[t]),
 y2'[t] == y2[t] (a21 y1[t] + a22 y2[t] + r2)}
```

Out[∘]=

$$\{y_1'[t] == y_1[t]\,(1 + 1.3\, y_1[t] - 2\, y_2[t]),\ y_2'[t] == (-1.1 + 2.2\, y_1[t] - 1.5\, y_2[t])\, y_2[t]\}$$

In[∘]:=
```
predprey = NDSolve[{
    y1'[t] == y1[t] (r1 + a11 y1[t] + a12 y2[t]),
    y2'[t] == y2[t] (a21 y1[t] + a22 y2[t] + r2),
    y1[0] == 0.2, y2[0] == 0.5}, {y1, y2}, {t, 0, 100, 0.1}]
```

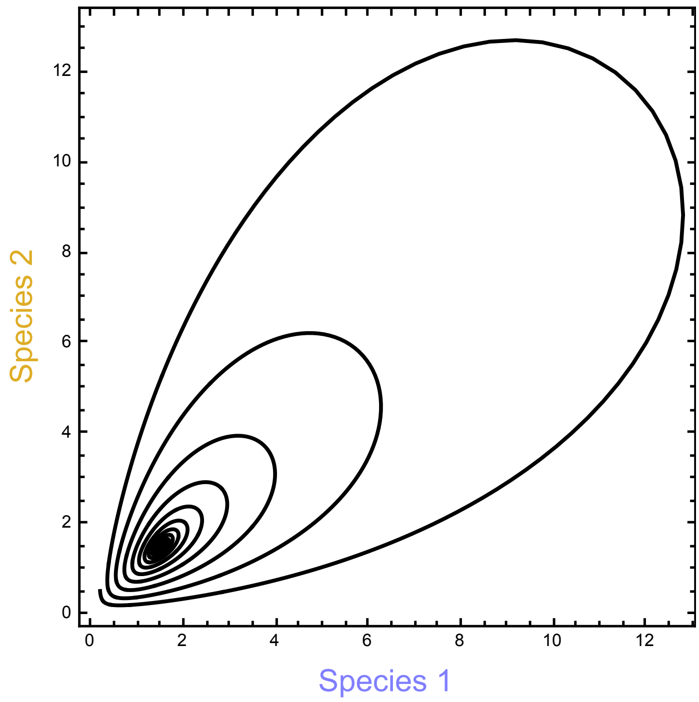
Out[∘]=

{{y1 → InterpolatingFunction[Domain: {{0., 100.}} Output: scalar],

y2 → InterpolatingFunction[Domain: {{0., 100.}} Output: scalar]}}

In[∘]:=
```
ListPlot[Table[Evaluate[{y1[t], y2[t]} /. predprey][[1]], {t, 0, 100, 0.005}],
  Joined → True, Axes → None, PlotStyle → Black,
  PlotStyle → Black, PlotStyle → Automatic, Frame → True,
  FrameStyle → Directive[Black, Thickness[0.004]], PlotRange → Full,
  FrameLabel → {Style["Species 1", 16, RGBColor["#807FFF"]],
    Style["Species 2", 16, RGBColor["#DEAD26"]]}, AspectRatio → 1]
```

Out[∘]=



Equivalent replicator dynamics

Simplex Plot

Plotting

For the two player case

In[∘]:=
```
twoplpayoffs = {{a11, a12, r1}, {a21, a22, r2}, {0, 0, 0}};
fits[x_, y_] := twoplpayoffs.{x, y, 1 - x - y};
```

```
In[•]:= π1[x_, y_, z_] := fits[x, y][[1]];

        π2[x_, y_, z_] := fits[x, y][[2]];

        π3[x_, y_, z_] := fits[x, y][[3]];
        πbar[x_, y_, z_] := x π1[x, y, z] + y π2[x, y, z] + z π3[x, y, z];
        dx[x_, y_, z_] := x (π1[x, y, z] - πbar[x, y, z]);
        dy[x_, y_, z_] := y (π2[x, y, z] - πbar[x, y, z]);
        dz[x_, y_, z_] := z (π3[x, y, z] - πbar[x, y, z]);

        Fermi[x_] := {
          dx[x[[1]], x[[2]], x[[3]]],
          dy[x[[1]], x[[2]], x[[3]]],
          dz[x[[1]], x[[2]], x[[3]]]
         }
        x =.; y =.; z = 1 - x - y;
        fa = fits[x, y][[1]];
        fb = fits[x, y][[2]];
        fc = fits[x, y][[3]];

In[•]:= x =.
        y =.
        PT // MatrixForm;
        sol = NSolve[{dx[x, y, 1 - x - y] == 0, dy[x, y, 1 - x - y] == 0}, {x, y}, Reals];

        data = {x, y} /. sol;
        relData = Select[data, Total[#] ≤ 1 && Total[#] ≥ 0 &];
        p1 = ListPlot[relData, PlotStyle → {Black, PointSize[0.03]}];
        p2 = ListPlot[relData, PlotStyle → {White, PointSize[0.015]}];
        p3 = ListPlot[{{1, 0}, {0, 1}, {0, 0}}, PlotStyle → {Red, PointSize[0.03]}];
        p4 = ListPlot[{{1, 0}, {0, 1}, {0, 0}}, PlotStyle → {White, PointSize[0.015]}];
        points = Show[p1, p2, p3, p4];
```

Plotting isoclines

```
In[•]:= eq1 = fa - fc;
        eq2 = fb - fc;

In[•]:= cplt = ContourPlot[eq1 == 0, {x, 0, 1},
           {y, 0, 1}, RegionFunction → Function[{x, y}, x + y ≤ 1],
           PlotRange → All, ContourStyle → Darker[Red]];
        cplt2 = ContourPlot[eq2 == 0, {x, 0, 1},
           {y, 0, 1}, RegionFunction → Function[{x, y}, x + y ≤ 1],
           PlotRange → All, ContourStyle → Darker[Blue]];
```

```
sp = StreamDensityPlot[{dx[x, y, 1 - x - y], dy[x, y, 1 - x - y]}, {x, 0, 1}, {y, 0, 1},
    Frame → True, ColorFunction → "Pastel", ColorFunctionScaling → False,
    StreamStyle → Darker[Gray], StreamColorFunction → None,
    StreamPoints → Fine, StreamMarkers → {"PinDart"}, StreamScale → Large,
    RegionFunction → Function[{x, y, vx, vy, n}, x + y ≤ 1 && x ≥ 0 && y ≥ 0]];
```

```
In[∘]:= trajdata = NDSolve[
    {x'[t] == dx[x[t], y[t], 1 - x[t] - y[t]], y'[t] == dy[x[t], y[t], 1 - x[t] - y[t]],
     x[0] == 0.45, y[0] == 0.08}, {x, y}, {t, 0, 50}];
parpl = ParametricPlot[Evaluate[{x[t], y[t]} /. trajdata],
    {t, 0, 50}, PlotRange → All, PlotStyle → Directive[Black, Dashed]];
```

```
In[∘]:= psim = Show[
   Graphics[GeometricTransformation[First[Show[sp]], trans]], triangle,
   Graphics[GeometricTransformation[First[Show[points]], trans]],
   Graphics[GeometricTransformation[First[Show[cplt, cplt2]], trans]],
   Graphics[GeometricTransformation[First[parpl], trans]] (*,
   Graphics[GeometricTransformation[Text["label",{0,1}],trans]]*)
  ]
```

Out[∘]=