Lotka Volterra Dynamics

Here are the growth rate and interactions parameters for the LV

In[ ]:=
```
r1 = 1; a11 = 1.3; a12 = -2;
r2 = -1.1; a21 = 2.2; a22 = -1.5;
```

Here the self interaction terms are chosen appropriately to yield the following equations

In[ ]:=
```
{y1'[t] == y1[t] (r1 + a11 y1[t] + a12 y2[t]),
 y2'[t] == y2[t] (a21 y1[t] + a22 y2[t] + r2)}
```

Out[ ]=

$$\{y_1'[t] == y_1[t]\,(1 + 1.3\, y_1[t] - 2\, y_2[t]),\ y_2'[t] == (-1.1 + 2.2\, y_1[t] - 1.5\, y_2[t])\, y_2[t]\}$$

```
int = {y1, y2} /. Solve[{y1 (r1 + a11 y1 + a12 y2) == 0 &&
      y2 (a21 y1 + a22 y2 + r2) == 0}, {y1, y2}, Assumptions → {y1 > 0 && y2 > 0}];
```

Solve : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[ ]=

```
{{1.5102, 1.48163}}
```

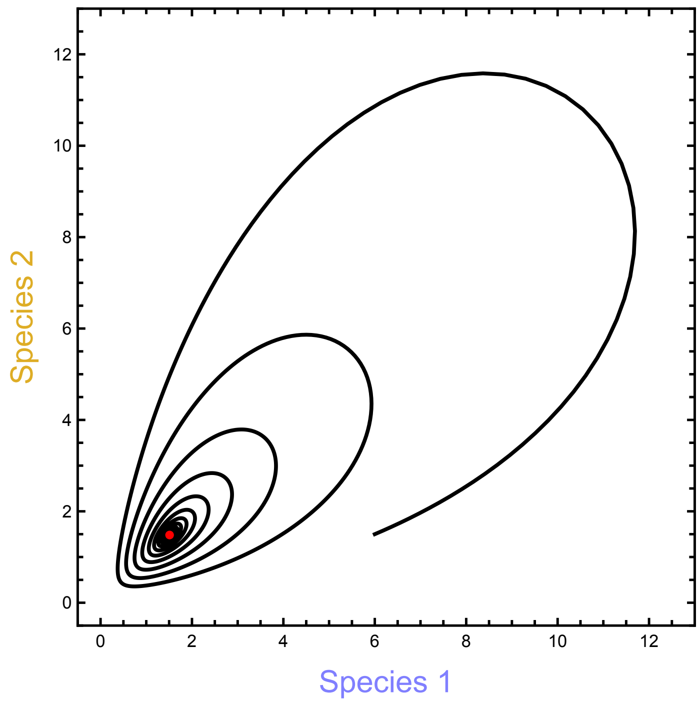
In[ ]:=
```
predprey = NDSolve[{
   y1'[t] == y1[t] (r1 + a11 y1[t] + a12 y2[t]),
   y2'[t] == y2[t] (a21 y1[t] + a22 y2[t] + r2),
   y1[0] == 6, y2[0] == 1.5}, {y1, y2}, {t, 0, 100, 0.1}]
```

Out[ ]=

```
{{y1 → InterpolatingFunction[ Domain: {{0., 100. }} Output: scalar ],
  y2 → InterpolatingFunction[ Domain: {{0., 100. }} Output: scalar ]}}
```

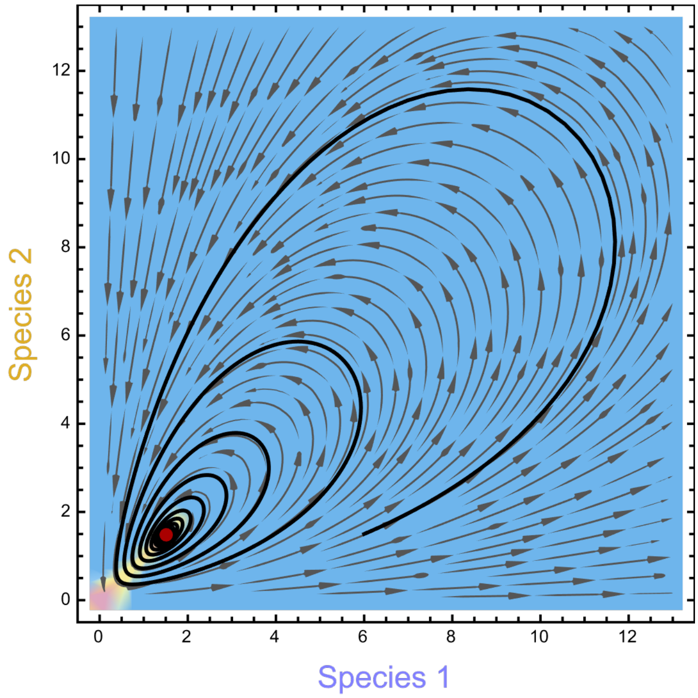
In[ ]:=
```
trajectory =
  ListPlot[Table[Evaluate[{y1[t], y2[t]} /. predprey][[1]], {t, 0, 100, 0.005}],
   Joined → True, Axes → None, PlotStyle → Black,
   PlotStyle → Black, PlotStyle → Automatic, Frame → True,
   FrameStyle → Directive[Black, Thickness[0.004]],
   PlotRange → {{-0.5, 13}, {-0.5, 13}},
   FrameLabel → {Style["Species 1", 16, RGBColor["#807FFF"]],
     Style["Species 2", 16, RGBColor["#DEAD26"]]},
   AspectRatio → 1, Epilog → {Red, PointSize[Medium], Point[int]}]
```

Out[ ]=



In[ ]:=
```
Show[StreamDensityPlot[{y1 (r1 + a11 y1 + a12 y2),
    y2 (a21 y1 + a22 y2 + r2)}, {y1, 0, 13}, {y2, 0, 13}, Frame → True,
   ColorFunction → "Pastel", ColorFunctionScaling → False,
   StreamStyle → Darker[Gray], StreamColorFunction → None,
   StreamPoints → Fine, StreamMarkers → {"PinDart"}, StreamScale → Large,
   FrameLabel → {Style["Species 1", 16, RGBColor["#807FFF"]],
     Style["Species 2", 16, RGBColor["#DEAD26"]]},
   AspectRatio → 1, PlotRange → {{-0.5, 13.5}, {-0.5, 13.5}}],
  trajectory, Epilog → {Darker[Red], PointSize[Large], Point[int]},
  FrameStyle → Directive[Black, Thickness[0.004]]]
```

Out[ ]=



Equivalent replicator dynamics

Simplex Plot

Plotting

For the two player case

In[ ]:=
```
twoplpayoffs = {{a11, a12, r1}, {a21, a22, r2}, {0, 0, 0}};
fits[x_, y_] := twoplpayoffs.{x, y, 1 - x - y};
```

In[ ]:=
```
π1[x_, y_, z_] := fits[x, y]〚1〛;

π2[x_, y_, z_] := fits[x, y]〚2〛;

π3[x_, y_, z_] := fits[x, y]〚3〛;
πbar[x_, y_, z_] := x π1[x, y, z] + y π2[x, y, z] + z π3[x, y, z];
dx[x_, y_, z_] := x (π1[x, y, z] - πbar[x, y, z]);
dy[x_, y_, z_] := y (π2[x, y, z] - πbar[x, y, z]);
dz[x_, y_, z_] := z (π3[x, y, z] - πbar[x, y, z]);

Fermi[x_] := {
   dx[x〚1〛, x〚2〛, x〚3〛],
   dy[x〚1〛, x〚2〛, x〚3〛],
   dz[x〚1〛, x〚2〛, x〚3〛]
  }
x =.; y =.; z = 1 - x - y;
fa = fits[x, y]〚1〛;
fb = fits[x, y]〚2〛;
fc = fits[x, y]〚3〛;
```

In[ ]:=
```
x =.
y =.
PT // MatrixForm;
sol = NSolve[{dx[x, y, 1 - x - y] == 0, dy[x, y, 1 - x - y] == 0}, {x, y}, Reals];
```

In[ ]:=
```
data = {x, y} /. sol;
relData = Select[data, Total[#] ≤ 1 && Total[#] ≥ 0 &];
p1 = ListPlot[relData, PlotStyle → {Black, PointSize[0.03]}];
p2 = ListPlot[relData, PlotStyle → {White, PointSize[0.015]}];
p3 = ListPlot[{{1, 0}, {0, 1}, {0, 0}}, PlotStyle → {Red, PointSize[0.03]}];
p4 = ListPlot[{{1, 0}, {0, 1}, {0, 0}}, PlotStyle → {White, PointSize[0.015]}];
points = Show[p1, p2, p3, p4];
```

Plotting isoclines

In[ ]:=
```
eq1 = fa - fc;
eq2 = fb - fc;
```

In[ ]:=
```
cplt = ContourPlot[eq1 == 0, {x, 0, 1},
    {y, 0, 1}, RegionFunction → Function[{x, y}, x + y ≤ 1],
    PlotRange → All, ContourStyle → Darker[Red]];
cplt2 = ContourPlot[eq2 == 0, {x, 0, 1},
    {y, 0, 1}, RegionFunction → Function[{x, y}, x + y ≤ 1],
    PlotRange → All, ContourStyle → Darker[Blue]];
```

In[ ]:=
```
sp = StreamDensityPlot[{dx[x, y, 1 - x - y], dy[x, y, 1 - x - y]}, {x, 0, 1}, {y, 0, 1},
   Frame → True, ColorFunction → "Pastel", ColorFunctionScaling → False,
   StreamStyle → Darker[Gray], StreamColorFunction → None,
   StreamPoints → Fine, StreamMarkers → {"PinDart"}, StreamScale → Large,
   RegionFunction → Function[{x, y, vx, vy, n}, x + y ≤ 1 && x ≥ 0 && y ≥ 0]];
```

In[ ]:=
```
trajdata = NDSolve[
   {x'[t] == dx[x[t], y[t], 1 - x[t] - y[t]], y'[t] == dy[x[t], y[t], 1 - x[t] - y[t]],
    x[0] == 0.45, y[0] == 0.08}, {x, y}, {t, 0, 50}];
parpl = ParametricPlot[Evaluate[{x[t], y[t]} /. trajdata],
   {t, 0, 50}, PlotRange → All, PlotStyle → Directive[Black, Dashed]];
```

In[ ]:=
```
psim = Show[
  Graphics[GeometricTransformation[First[Show[sp]], trans]], triangle,
  Graphics[GeometricTransformation[First[Show[points]], trans]],
  Graphics[GeometricTransformation[First[Show[cplt, cplt2]], trans]],
  Graphics[GeometricTransformation[First[parpl], trans]] (*,
  Graphics[GeometricTransformation[Text["label",{0,1}],trans]]*)
 ]
```

Out[ ]=