Allee effect

Solutions for the Allee

In[ ]:=
```
m = 2; n = 1;
allee[y_, r_, α_, β_] := - y r (1 - α y ) (1 - β y);
Solve[allee[y, r, α, β] == 0, y]
```

Out[ ]=

$$\left\{\{y \to 0\}, \left\{y \to \frac{1}{\alpha}\right\}, \left\{y \to \frac{1}{\beta}\right\}\right\}$$

Allee to three player three strategy game

```
coeffs = CoefficientList[allee[y, r, α, β] / y, y];
B = Table[b_{i,j,k}, {i, 1, m}, {j, 1, m}, {k, 1, m}];
Table[b_{m,j,k} = 0, {i, 1, m}, {j, 1, m}, {k, 1, m}];
b_{1,2,2} = coeffs[[1]]; b_{1,1,2} = coeffs[[2]]; b_{1,2,1} = 0; b_{1,1,1} = coeffs[[3]];
payoffs = B;
fits[x_] :=
  Table[Sum_{j=1}^{m} (Sum_{k=1}^{m} (payoffs[[i, j, k]] x_j x_k)), {i, 1, 2}] /. {x_1 → x, x_2 → 1 - x};
π1[x_] := fits[x][[1]];
π2[x_] := fits[x][[2]];
πbar[x_] := x π1[x] + (1 - x) π2[x];
dx[x_] := x (π1[x] - πbar[x]);
Solve[dx[x] == 0, x]
```

Out[ ]=

$$\{-r,\ r\alpha + r\beta,\ -r\alpha\beta\}$$

Out[ ]=

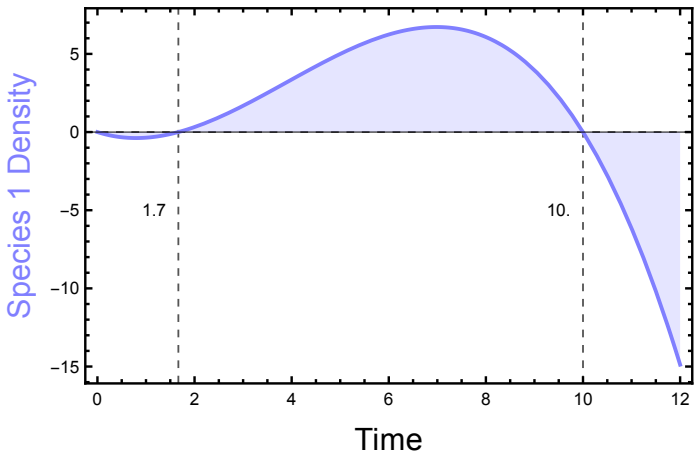
$$\left\{\{x \to 0\}, \{x \to 1\}, \left\{x \to \frac{1}{1+\alpha}\right\}, \left\{x \to \frac{1}{1+\beta}\right\}\right\}$$

Numerical plots

```
r = 1;
α = 0.6; β = 0.1;
coeffs = CoefficientList[allee[y, r, α, β] / y, y];
b_{1,2,2} = coeffs[[1]]; b_{1,1,2} = coeffs[[2]]; b_{1,2,1} = 0; b_{1,1,1} = coeffs[[3]];
payoffs = B;
intfixpt = {1/(1 + α), 1/(1 + β)};
```

In[ ]:=
```
alleeplot = Plot[allee[y, r, α, β], {y, 0, 12},
  Frame → True, FrameStyle → Directive[Black, Thickness[0.004]],
  FrameLabel → {Style["Time", 16, Black],
    Style["Species 1 Density", 16, RGBColor["#807FFF"]]},
  Epilog → {Inset[SetPrecision[1 / α // N, 2], {1 / α - 0.5, -5}],
    Inset[SetPrecision[1 / β // N, 2], {1 / β - 0.5, -5}]},
  PlotRange → {Automatic, Full}, Axes → True, Filling → Axis,
  GridLines → {{1 / α, 1 / β}, {0}},
  GridLinesStyle → Directive[Darker[Gray], Thickness[0.003], Dashed],
  PlotStyle → RGBColor["#807FFF"]
 ]
rp = Plot[dx[x], {x, 0, 1}, PlotRange → {Automatic, Automatic}, Frame → True,
  FrameStyle → Directive[Black, Thickness[0.004]], FrameLabel →
   {Style["Strategy 1 frequency", 16, RGBColor[0.957, 0.606, 0.584]], Style[
     "Change in strategy 1", 16, RGBColor[0.957, 0.606, 0.584]]}, Filling → Axis,
  FillingStyle → {RGBColor[0.946, 0.346, 0.188], RGBColor[0.957, 0.606, 0.584]},
  Epilog → {Inset[SetPrecision[intfixpt[[1]], 2], {intfixpt[[1]] - 0.05, -0.05}],
    Inset[SetPrecision[intfixpt[[2]], 2], {intfixpt[[2]] - 0.05, -0.05}]},
  GridLines → {intfixpt, {0}}, GridLinesStyle →
   Directive[Darker[Gray], Thickness[0.003], Dashed], PlotStyle → Black]
```

Out[ ]=



Out[ ]=