Beliefs: a catalyst for cooperation

Evolutionary dynamics in infinite populations

Brief description

In total there are eight different strategies.The strategies can be enumerated by the generic profile $(a_1{}^*, a_2{}^*, u^*)$where $a_i{}^*$ is the hunting strategy of the focal individual when in reality i and u^* is the preferred reality of the focal individual (objective or inter-subjective).The individuals in the tribe form a group of size G and first they need to decide which reality they choose to believe in.

Definitions and variables that will be used

there are eight possible strategies as defined by $(a_1{}^*, a_2{}^*, u^*)$, since there are two action in each reality i so

In[435]:=
```
x1 = xhh1 + xhs1 + xsh1 + xss1;
x2 = xhh2 + xhs2 + xsh2 + xss2;
```

Let us define the strategies and give them colors - This is how we will identify strategies later.

In[437]:=
```
freqnotation = {xhh1, xhh2, xhs1, xhs2, xsh1, xsh2, xss1, xss2};
colors = {RGBColor["#B22222"], RGBColor["#DC143C"],
    RGBColor["#FF4501"], RGBColor["#00BFBF"], RGBColor["#6495ED"],
    RGBColor["#F08080"], RGBColor["#02008B"], RGBColor["#008B8B"]};
```

```
In[439]:= PieChart[{1/8, 1/8, 1/8, 1/8, 1/8, 1/8, 1/8, 1/8}, SectorOrigin → {Automatic, 2},
  ChartStyle → colors[[{1, 3, 2, 4, 7, 8, 5, 6}]], ChartLabels → Placed[
    Style[ToString[#], White, 14] & /@ {1, 2, 1, 2, 1, 2, 1, 2}, "RadialCenter"]]
```

Out[439]=

Group size if G of which M is the minimum number of stag hunters required to bring down a stag. The nutritional worth of a hare is set to πH while that of a stag is πS.

```
In[440]:= G = 5; M = 4; πS = 4; πH = 1;
```

All calculations done here (compile this)

Group decides on how to define reality

```
In[441]:= f1[k_, state_] := (*0.5*) (k + KroneckerDelta[state, 1]) / G
  (*Round[(k+KroneckerDelta[state,1])/G]*);
```

The function $f_1(k, \text{state})$ returns the probability of group G choosing reality 1. The probability is a frequency dependent function

What is an individuals behaviour? Hare hunting or stag hunting?

```
In[442]:= Phare1[focal_] := KroneckerDelta[focal, "Hare"];
```

This function returns “Hare” if the focal individual is a hare hunter for a given reality.

What is the group composition?

What is the composition of the group? How many stag hunters are there?given that k of the group believe in reality 1 and the rest in reality 2

In[443]:=
```
Pcomposition[k_, state_] :=
```
$$\left(\sum_{l=0}^{k}\sum_{m=0}^{k}\sum_{n=0}^{k}\sum_{o=0}^{k}\sum_{p=0}^{G-1-k}\sum_{q=0}^{G-1-k}\sum_{r=0}^{G-1-k}\sum_{s=0}^{G-1-k}\Big(\mathrm{If}\big[l+m+n+o==k\,\&\&\,p+q+r+s==G-1-k,\ \mathrm{Multinomial}[l,m,n,o]\ \mathrm{Multinomial}[p,q,r,s]\ (x_{hh1})^{l}\ (x_{hs1})^{m}\ (x_{sh1})^{n}\ (x_{ss1})^{o}\ (x_{hh2})^{p}\ (x_{hs2})^{q}\ (x_{sh2})^{r}\ (x_{ss2})^{s}\ \mathrm{If}[state==1,\ \mathrm{HeavisideTheta}[1+n+o+r+s-M],\ \mathrm{If}[state==2,\ \mathrm{HeavisideTheta}[1+m+o+q+s-M],\ 0]],\ 0\big]\Big)\right);$$

Calculating the average payoffs

What is the composition of the group? How many stag hunters are there? given that k of the group believe in reality 1 and the rest in reality 2

Simplified version of the average payoffs

In[444]:=
```
avgpayoffsimplified[focal1_, focal2_, state_] :=
```
$$\pi H\left(\mathrm{Phare1}[focal1]\sum_{k=0}^{G-1}\left(\mathrm{Binomial}[G-1,k]\ (x_1)^{k}\ (x_2)^{G-1-k}\ \mathrm{f1}[k,state]\right)+\mathrm{Phare1}[focal2]\sum_{k=0}^{G-1}\left(\mathrm{Binomial}[G-1,k]\ (x_1)^{k}\ (x_2)^{G-1-k}\ (1-\mathrm{f1}[k,state])\right)\right)+$$
$$\pi S\left((1-\mathrm{Phare1}[focal1])\sum_{k=0}^{G-1}(\mathrm{Binomial}[G-1,k]\ \mathrm{f1}[k,state]\times \mathrm{Pcomposition}[k,1])+(1-\mathrm{Phare1}[focal2])\sum_{k=0}^{G-1}(\mathrm{Binomial}[G-1,k]\ (1-\mathrm{f1}[k,state])\ \mathrm{Pcomposition}[k,2])\right);$$

The average payoffs are stored in a table called payoffstab:

In[445]:=
```
payoffstab =
  Flatten[Table[{{foc1, foc2, flags}, avgpayoffsimplified[foc1, foc2, flags]},
     {foc1, {"Hare", "Stag"}}, {foc2, {"Hare", "Stag"}}, {flags, {1, 2}}] /.
    HeavisideTheta[0] → 1, 2] // Simplify;
```

In the following we write down the matrix which gives the average payoff of strategy i when playing with strategy j at the i,j entry of the *fullreducedmatrix*

In[446]:=
```
nullreplace = {x_hh1 → 0, x_hh2 → 0, x_hs1 → 0, x_hs2 → 0, x_sh1 → 0, x_sh2 → 0, x_ss1 → 0, x_ss2 → 0};
toreplace[pair_] :=
  nullreplace /. {(freqnotation⟦pair⟦1⟧⟧ → 0) → freqnotation⟦pair⟦1⟧⟧ → y,
    (freqnotation⟦pair⟦2⟧⟧ → 0) → freqnotation⟦pair⟦2⟧⟧ → 1 - y};
reducedmatrix = {};
Table[
  temp = Table[Drop[Drop[nullreplace, {j}], {i}], {i, 1, 7, 1}];
  AppendTo[reducedmatrix,
   Table[payoffstab⟦j, 2⟧ /. temp⟦i⟧ // Simplify, {i, 1, 7, 1}]], {j, 1, 8, 1}];
diagonals = Table[payoffstab⟦j, 2⟧ /. Drop[nullreplace, {j}] // Simplify,
   {j, 1, 8, 1}];
fullreducedmatrix = Table[Insert[reducedmatrix⟦i⟧, diagonals⟦i⟧, i], {i, 1, 8, 1}];
```

Calculating fixed points

Next we calculate the fixed points between two strategies at a time and store the results in the array allpairs

In[452]:=
```
allpairs =
  Table[If[i ≠ j, freqnotation⟦i⟧ /. Solve[freqnotation⟦i⟧ > 0 && freqnotation⟦i⟧ <
            1 && 0 == freqnotation⟦i⟧ (1 - freqnotation⟦i⟧)
             (fullreducedmatrix⟦i, j⟧ - fullreducedmatrix⟦j, i⟧) /.
           {freqnotation⟦j⟧ → 1 - freqnotation⟦i⟧}, freqnotation⟦i⟧, Reals] //
       N,] // Flatten, {i, 1, 7, 1}, {j, i + 1, 8, 1}] // Quiet;
```

Using the above information now we can simply plot the dynamics between any two strategies

Prepping the plot package

In[453]:=
```
Pairwiseplot[strat1_, strat2_] :=
  (eq = freqnotation⟦strat1⟧ (1 - freqnotation⟦strat1⟧)
      (fullreducedmatrix⟦strat1, strat2⟧ - fullreducedmatrix⟦strat2, strat1⟧) /.
     {freqnotation⟦strat2⟧ → 1 - freqnotation⟦strat1⟧};
   Plot[eq /. {freqnotation⟦strat1⟧ → x}, {x, 0, 1}, Frame → True, Filling → Axis,
    FillingStyle → {colors⟦strat2⟧, colors⟦strat1⟧}, PlotStyle → Black,
    FrameStyle → Directive[Black, Thickness[0.002]], FrameLabel →
     {freqnotation⟦strat1⟧, freqnotation⟦strat1⟧ (1 - freqnotation⟦strat1⟧)
       (f_freqnotation⟦strat1⟧ - f_freqnotation⟦strat2⟧)}, ImageSize → Medium]);
```

```
In[454]:= (*CirclePoints is an alternative to manually generating the vertex list*)
pts = CirclePoints[{1, 23 Degree}, 8];
pts = {{-Cos[23 °], -Sin[23 °]}, {-Cos[23 ° - π/4], -Sin[23 ° - π/4]},
   {-Cos[23 ° + π/4], -Sin[23 ° + π/4]}, {Sin[23 °], -Cos[23 °]},
   {Cos[23 ° + π/4], Sin[23 ° + π/4]}, {-Sin[23 °], Cos[23 °]},
   {Cos[23 °], Sin[23 °]}, {Cos[23 ° - π/4], Sin[23 ° - π/4]}};
(*Generate all edges and internal diagonals as Line objects*)
lines = Line /@ Subsets[pts, {2}];
(*List all possible pairs of lines from the set above*)
linepairs = Subsets[lines, {2}];
(*Find unique intersection points between lines in each pair*)
intersectionpts = DeleteDuplicatesBy[N] @
   Simplify@DeleteCases[RegionIntersection /@ linepairs, _EmptyRegion];
eqpts = {};
Table[If[MatchQ[allpairs[[i, j]][[1]], _Real],
   AppendTo[eqpts, {pts[[i]][[1]] + (1 - allpairs[[i, j]][[1]]) (pts[[j + i]][[1]] - pts[[i]][[1]]),
     pts[[i]][[2]] + (1 - allpairs[[i, j]][[1]]) (pts[[j + i]][[2]] - pts[[i]][[2]])}]],
  {i, 1, 7, 1}, {j, 1, Length[allpairs[[i]]], 1}];
eqpts = DeleteCases[eqpts, Null];
eqptsplot = Point[#] & /@ eqpts;
```

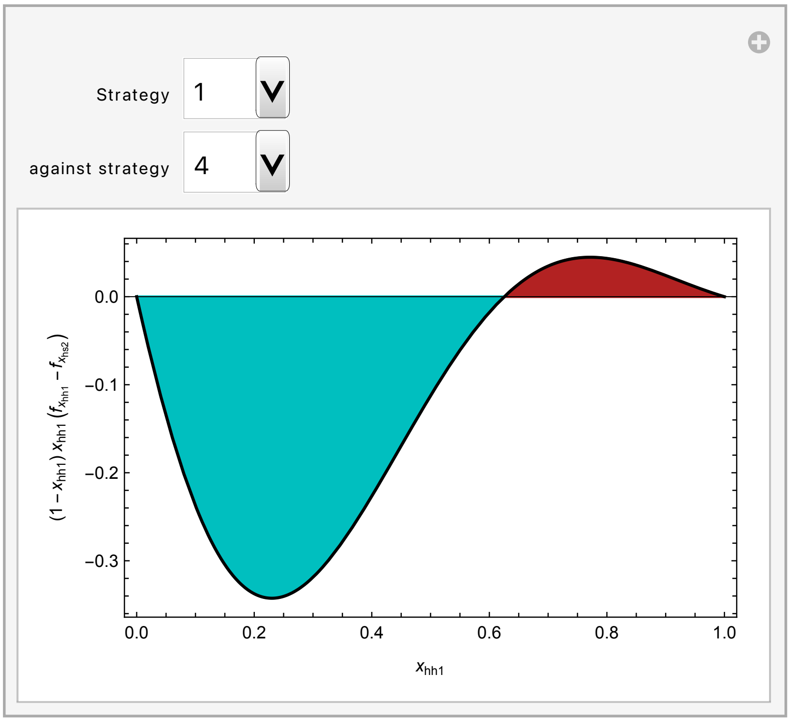
Plotting

Simply choose the two strategies you want to see the dynamics of selected strategies, the list below might be useful for coding

1. (x_{hh1}, ■); 2. (x_{hh2}, ■); 3. (x_{hs1}, ■); 4. (x_{hs2}, ■);
 5. (x_{sh1}, ■); 6. (x_{sh2}, ■); 7. (x_{ss1}, ■); 8. (x_{ss2}, ■)

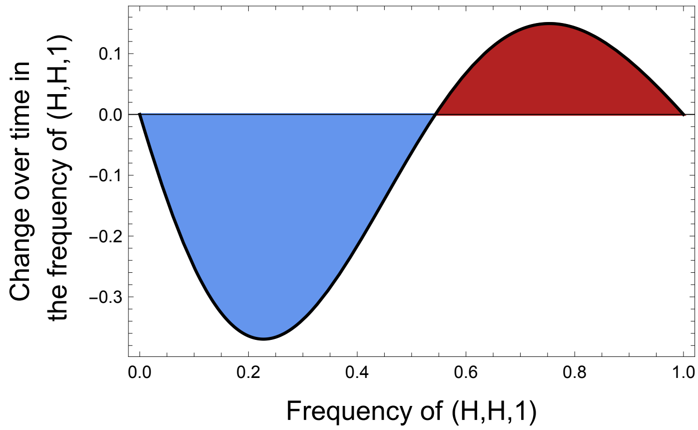
In[463]:=
```
Manipulate[Pairwiseplot[i, j], {{i, 1, "Strategy"}, {1, 2, 3, 4, 5, 6, 7, 8}},
 {{j, 4, "against strategy"}, {1, 2, 3, 4, 5, 6, 7, 8}}]
```

Out[463]=



In[464]:=
```
Show[Pairwiseplot[1, 5], Frame → True, FrameStyle → Black,
 FrameLabel → {Style["Frequency of (H,H,1)", 14, Black],
   Style["Change over time in\n the frequency of (H,H,1)", 14, Black]}]
```

Out[464]=



The dynamics of the whole population takes place in a seven dimensional simplex $\Delta_7$and we can show the dynamics as . $(x_{hh1},)$; 2. $(x_{hh2},)$; 3. $(x_{hs1},)$; 4. $(x_{hs2},)$; 5. $(x_{sh1},)$; 6. $(x_{sh2},)$; 7. $(x_{ss1},)$; 8. $(x_{ss2},)$

In[465]:=
```
Graphics[{lines, Black, PointSize[0.03], eqptsplot, White, PointSize[0.02],
  eqptsplot, PointSize[0.05], Point[pts, VertexColors → colors]}]
```

Out[465]=