
Functions

```
In[*]:=  $\theta[x\_ , thresh\_ ] := \text{If}[x < thresh, 0, 1];$   
 $\pi D[b\_ , num\_ , thresh\_ ] := b \theta[num, thresh] + 1;$   
 $\pi C[b\_ , c\_ , num\_ , thresh\_ ] :=$   
 $\pi D[b, num, thresh] - \frac{c}{num} \theta[num, thresh] - \frac{c}{thresh} (1 - \theta[num, thresh]);$   
 $\pi intra D[d\_ , b\_ , c\_ , num\_ , \omega\_ ] := \frac{b}{d} \sum_{i=0}^{num-1} \omega^i;$   
 $\pi intra C[d\_ , b\_ , c\_ , num\_ , \omega\_ ] := \pi intra D[d, b, c, num, \omega] - c;$   
 $fD[d\_ , dintra\_ , x\_ , y\_ , b\_ , c\_ , bintra\_ , cintra\_ , thresh\_ , \omega\_ , p\_ ] :=$   
 $p \left( \sum_{k=0}^{d-1} \left( \text{Binomial}[d-1, k] x^k (1-x)^{d-1-k} \pi D[b, k, thresh] \right) \right) + (1-p)$   
 $\left( \sum_{k=0}^{d-1} \left( \text{Binomial}[d-1, k] y^k (1-y)^{d-1-k} \pi intra D[dintra, bintra, cintra, k, \omega] \right) \right);$   
 $fC[d\_ , dintra\_ , x\_ , y\_ , b\_ , c\_ , bintra\_ , cintra\_ , thresh\_ , \omega\_ , p\_ ] :=$   
 $p \left( \sum_{k=0}^{d-1} \left( \text{Binomial}[d-1, k] x^k (1-x)^{d-1-k} \pi C[b, c, k+1, thresh] \right) \right) + (1-p)$   
 $\left( \sum_{k=0}^{d-1} \left( \text{Binomial}[d-1, k] y^k (1-y)^{d-1-k} \pi intra C[dintra, bintra, cintra, k+1, \omega] \right) \right);$   
 $fbar1[d\_ , dintra\_ , x\_ , y\_ , b\_ , c\_ , bintra\_ , cintra\_ , thresh\_ , \omega\_ , p\_ ] :=$   
 $x fC[d, dintra, y, x, b, c, bintra, cintra, thresh, \omega, p] +$   
 $(1-x) fD[d, dintra, y, x, b, c, bintra, cintra, thresh, \omega, p];$   
 $fbar2[d\_ , dintra\_ , x\_ , y\_ , b\_ , c\_ , bintra\_ , cintra\_ , thresh\_ , \omega\_ , p\_ ] :=$   
 $y fC[d, dintra, x, y, b, c, bintra, cintra, thresh, \omega, p] +$   
 $(1-y) fD[d, dintra, x, y, b, c, bintra, cintra, thresh, \omega, p];$   
 $xdot[d\_ , dintra\_ , x\_ , y\_ , b\_ , c\_ , bintra\_ , cintra\_ , thresh\_ , m\_ , \omega\_ , p\_ ] :=$   
 $m x (1-x) (fC[d, dintra, y, x, b, c, bintra, cintra, thresh, \omega, p] -$   
 $fD[d, dintra, y, x, b, c, bintra, cintra, thresh, \omega, p]);$   
 $ydot[d\_ , dintra\_ , x\_ , y\_ , b\_ , c\_ , bintra\_ , cintra\_ , thresh\_ , n\_ , \omega\_ , p\_ ] :=$   
 $n y (1-y) (fC[d, dintra, x, y, b, c, bintra, cintra, thresh, \omega, p] -$   
 $fD[d, dintra, x, y, b, c, bintra, cintra, thresh, \omega, p]);$   
  
 $In[*]:= \text{checkinternal}[point\_ ] :=$   
 $\text{If}[point[[1]] > 0 \&\& point[[1]] < 1 \&\& point[[2]] > 0 \&\& point[[2]] < 1, 1, 0];$   
  
 $In[*]:= \text{plorange} = \{\{-0.02, 1.02\}, \{-0.02, 1.02\}\};$ 
```

Parameters

```
In[*]:= ClearAll[n,  $\omega$ ,  $\omega1$ ,  $\omega2$ , r1, r2, d, p]
```

```

In[ ]:= gamea = {3, 3 / 4};
        gameb = {1, 3 / 4};
        gamec = {1, 4 / 3};
        gamed = {3, 4 / 3};

d1 = 5;
d2 = 5;
d1intra = 5;
d2intra = 5;
thresh1 = 1;
thresh2 = 1;
b = 2;
c = 1;
bintraforasp1 = 10;
cintraforasp1 = gameb[[1]];
ωforasp1 = gameb[[2]] // N;

bintraforasp2 = 10;
cintraforasp2 = gameb[[1]];
ωforasp2 = gameb[[2]] // N;

m = 1 / 8 // N;
n = 1;
lim = 100;

```

Functions for fluctuations

```

a = 10;
diff = 0;
num[t_] :=  $\frac{\sin[a t - \text{diff}] + 1}{2}$ ; (*If[Sin[a t]>0,  $\frac{1}{1+A \sin[a t]}$ , 1-A Sin[a t]]*)
mean = Mean[Table[num[t], {t, 0, 20 π, 0.0001}]];
(*geomean=GeometricMean[Table[num[t], {t, 0, 20 π, 0.0001}]]*)

```

Out[]=

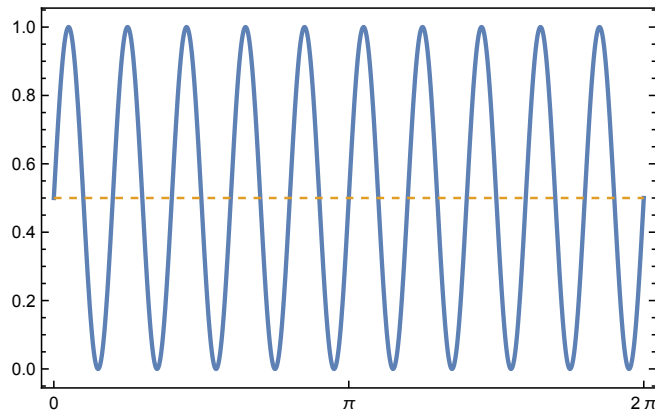
0.5

```

In[ ]:= Plot[{num[t], mean}, {t, 0, 2  $\pi$ }, PlotStyle →
  {{Thickness[0.007]}, {Thickness[0.004], Dashed}, {Thickness[0.004]}},
  PlotRange → {Automatic, Automatic}, Frame → True, FrameStyle → Thickness[0.0025],
  FrameTicks → {{Automatic, Automatic}, {{0,  $\pi$ , 2  $\pi$ , 15  $\pi$ , 20  $\pi$ }, None}},
  ImagePadding → 20]

```

Out[]:=



Analysis

```



In[ ]:= plotlis = {};
plotdata = {};
tlim = 100;
For[i = 0.1, i ≤ 0.9, i = i + 0.1,
  For[j = 0.1, j ≤ 0.9, j = j + 0.1,
    sinlistcheck = {};
    s = Quiet[NDSolve[{
      x'[t] == m x[t] (fC[d1, d1intra, y[t], x[t], b, c, bintraforsp1, cintraforsp1,
        thresh1, ωforsp1, num[t]] - fbar1[d1, d1intra, x[t], y[t], b,
        c, bintraforsp1, cintraforsp1, thresh1, ωforsp1, num[t]]),
      y'[t] == n y[t] (fC[d2, d2intra, x[t], y[t], b, c, bintraforsp2,
        cintraforsp2, thresh2, ωforsp2, num[t]] - fbar2[d2, d2intra, x[t],
        y[t], b, c, bintraforsp2, cintraforsp2, thresh2, ωforsp2, num[t]]),
      x[0] == i, y[0] == j}, {x, y}, {t, 0, tlim}, MaxStepSize → 0.005]];
    tabsin = Table[{Evaluate[x[t] /. s][[1]], Evaluate[y[t] /. s][[1]]}, {t, 0, tlim}];
    AppendTo[plotdata, tabsin];
  ]
]

```

```

In[*]:= res[p_, q_, lim_] :=
  Quiet[NDSolve[{x'[t] == m x[t] (fC[d1, d1intra, y[t], x[t], b, c, bintraforsp1,
    cintraforsp1, thresh1, ωforsp1, num[t]] - fbar1[d1, d1intra, x[t],
    y[t], b, c, bintraforsp1, cintraforsp1, thresh1, ωforsp1, num[t]]),
    y'[t] == n y[t] (fC[d2, d2intra, x[t], y[t], b, c, bintraforsp2,
    cintraforsp2, thresh2, ωforsp2, num[t]] - fbar2[d2, d2intra, x[t],
    y[t], b, c, bintraforsp2, cintraforsp2, thresh2, ωforsp2, num[t]]),
    x[0] == p, y[0] == q}, {x, y}, {t, lim}]];
liseval[t_, rep_] := Evaluate[{x[t], y[t]} /. rep];

```

 **ReplaceAll** : {rep} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing. 

```

In[*]:= Clear[blue, red, black];
blue = {};
red = {};
black = {};
For[u = 0.0, u ≤ 1.0, u = u + 0.01, For[v = 0.0, v ≤ 1.0, v = v + 0.01,
  If[
    liseval[lim, res[u, v, lim]][[1]][[1]] > 0.9999 &&
    liseval[lim, res[u, v, lim]][[1]][[2]] < 0.001,
    AppendTo[red, {u, v}], If[liseval[lim, res[u, v, lim]][[1]][[1]] < 0.001 &&
      liseval[lim, res[u, v, lim]][[1]][[2]] > 0.9999,
      AppendTo[blue, {u, v}], AppendTo[black, {u, v}]]];
]]

```

```

In[ ]:= g1 = ListPlot[{blue, red, black},
  PlotStyle → {Lighter[Blue, 0.8], Lighter[Red, 0.4], Lighter[Gray, 0.7]},
  AspectRatio → 1, Frame → True, PlotRange → {{-0.01, 1.01}, {-0.01, 1.01}}];

plotcolors = {};
For[i = 1, i ≤ Length[plotdata], i++,
  If[Last[plotdata[[i]]][1] ≥ 0.99 && Last[plotdata[[i]]][2] ≤ 0.01,
    AppendTo[plotcolors, {Darker[Red], Thickness[0.003]}],
    If[Last[plotdata[[i]]][2] ≥ 0.99 && Last[plotdata[[i]]][1] ≤ 0.01,
      AppendTo[plotcolors, {Darker[Blue], Thickness[0.003]}],
      AppendTo[plotcolors, {Black, Thickness[0.003]}]
    ]
  ];
]
Show[g1,
  ListPlot[{blue, red}, PlotStyle → {Lighter[Blue, 0.8], Lighter[Red, 0.8]},
    AspectRatio → 1, Frame → True, PlotRange → plotrange],
  ListPlot[plotdata, Joined → True, InterpolationOrder → 2, AspectRatio → 1,
    PlotRange → plotrange, PlotStyle → plotcolors, Frame → True],
  Frame → True, FrameStyle → Thickness[0.0025], ImagePadding → 17]

```

Out[]:=

