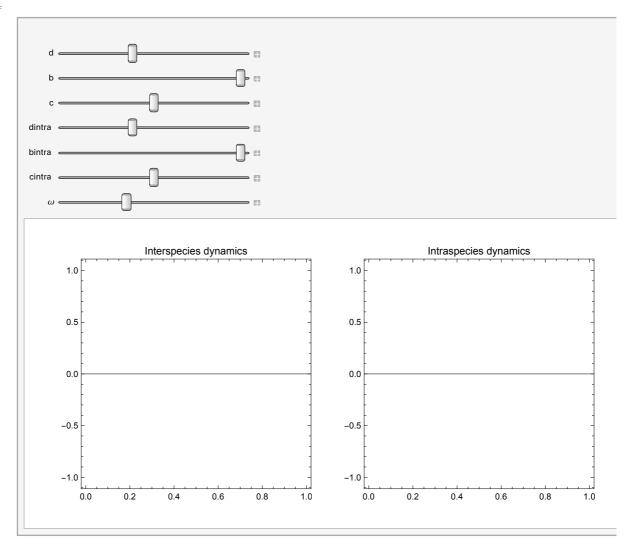
## **Fitnesses**

```
In[\cdot]:=\theta[x_{n}, thresh_{n}]:=If[x < thresh_{n}, 0, 1];
                   \piC[b_, c_, num_, thresh_] :=
                          \pi D[b, num, thresh] - \frac{c}{num} \theta[num, thresh] - \frac{c}{thresh} (1 - \theta[num, thresh]);
                 \piintraD[d_, b_, c_, num_, \omega_] := \frac{b}{d} \sum_{i=1}^{num-1} \omega^{i};
                  \piintraC[d_, b_, c_, num_, \omega] := \piintraD[d, b, c, num, \omega] - c;
In[o]:= Manipulate
                      GraphicsRow \Big[ \Big\{ Plot \Big[ \Big\{ \sum_{k=-n}^{d-1} \left( Binomial[d-1,\,k] \; x^k \; (1-x)^{d-1-k} \; \pi C[b,\,c,\,k+1,\,1] \right) \Big\} \Big] \Big\} \Big] \Big\} \Big] \Big\} \Big] \Big\} \Big] \Big] \Big] \Big] \Big] \Big] \Big] \Big] \Big[ \left\{ \sum_{k=-n}^{d-1} \left( Binomial[d-1,\,k] \; x^k \; (1-x)^{d-1-k} \; \pi C[b,\,c,\,k+1,\,1] \right) \right\} \Big] \Big\} \Big] \Big] \Big] \Big] \Big] \Big[ \left\{ \sum_{k=-n}^{d-1} \left( Binomial[d-1,\,k] \; x^k \; (1-x)^{d-1-k} \; \pi C[b,\,c,\,k+1,\,1] \right) \right\} \Big] \Big] \Big] \Big] \Big] \Big] \Big[ \left\{ \sum_{k=-n}^{d-1} \left( Binomial[d-1,\,k] \; x^k \; (1-x)^{d-1-k} \; \pi C[b,\,c,\,k+1,\,1] \right) \right\} \Big] \Big] \Big] \Big] \Big] \Big[ \left\{ \sum_{k=-n}^{d-1} \left( Binomial[d-1,\,k] \; x^k \; (1-x)^{d-1-k} \; \pi C[b,\,c,\,k+1,\,1] \right\} \Big] \Big] \Big] \Big] \Big] \Big[ \left\{ \sum_{k=-n}^{d-1} \left( Binomial[d-1,\,k] \; x^k \; (1-x)^{d-1-k} \; \pi C[b,\,c,\,k+1,\,1] \right\} \Big] \Big] \Big] \Big] \Big] \Big[ \left\{ \sum_{k=-n}^{d-1} \left( Binomial[d-1,\,k] \; x^k \; (1-x)^{d-1-k} \; \pi C[b,\,c,\,k+1,\,1] \right\} \Big] \Big] \Big] \Big] \Big] \Big[ \left\{ \sum_{k=-n}^{d-1} \left( Binomial[d-1,\,k] \; x^k \; (1-x)^{d-1-k} \; \pi C[b,\,c,\,k+1,\,1] \right\} \Big] \Big] \Big] \Big] \Big] \Big[ \left\{ \sum_{k=-n}^{d-1} \left( Binomial[d-1,\,k] \; x^k \; (1-x)^{d-1-k} \; \pi C[b,\,c,\,k+1,\,1] \right\} \Big] \Big] \Big] \Big] \Big[ \left\{ \sum_{k=-n}^{d-1} \left( Binomial[d-1,\,k] \; x^k \; (1-x)^{d-1-k} \; \pi C[b,\,c,\,k+1,\,1] \right\} \Big] \Big] \Big] \Big] \Big[ \left\{ \sum_{k=-n}^{d-1} \left( Binomial[d-1,\,k] \; x^k \; (1-x)^{d-1-k} \; \pi C[b,\,c,\,k+1,\,1] \right\} \Big] \Big] \Big] \Big] \Big[ \left\{ \sum_{k=-n}^{d-1} \left( Binomial[d-1,\,k] \; x^k \; (1-x)^{d-1-k} \; \pi C[b,\,c,\,k+1,\,1] \Big] \Big] \Big] \Big] \Big[ \left\{ \sum_{k=-n}^{d-1} \left( Binomial[d-1,\,k] \; x^k \; (1-x)^{d-1-k} \; \pi C[b,\,c,\,k+1,\,1] \right\} \Big] \Big] \Big] \Big] \Big[ \left\{ \sum_{k=-n}^{d-1} \left( Binomial[d-1,\,k] \; x^k \; (1-x)^{d-1-k} \; \pi C[b,\,k+1,\,1] \right\} \Big] \Big] \Big] \Big] \Big] \Big[ \left\{ \sum_{k=-n}^{d-1} \left( Binomial[d-1,\,k] \; x^k \; (1-x)^{d-1-k} \; \pi C[b,\,k+1,\,1] \Big] \Big] \Big] \Big] \Big] \Big[ \left\{ \sum_{k=-n}^{d-1} \left( Binomial[d-1,\,k] \; x^k \; (1-x)^{d-1-k} \; \pi C[b,\,k+1,\,1] \Big] \Big] \Big] \Big] \Big[ \left\{ \sum_{k=-n}^{d-1} \left( Binomial[d-1,\,k] \; x^k \; x^k \; (1-x)^{d-1-k} \; \pi C[b,\,k+1,\,1] \Big] \Big] \Big] \Big] \Big] \Big[ \left\{ \sum_{k=-n}^{d-1} \left( Binomial[d-1,\,k] \; x^k \; x^
                                      \sum_{k=0}^{d-1} \left( \text{Binomial}[d-1, k] \, x^k \, (1-x)^{d-1-k} \, \pi D[b, k, 1] \right) \right\}, \, \{x, 0, 1\},
                                   PlotStyle → {Blue, {Red, Dashed}}, Frame → True, AspectRatio → 1,
                                   Filling \rightarrow \{1 \rightarrow \{\{2\}, \{Directive[Lighter[Red], Opacity[0.6]], Directive[
                                                             Lighter[Blue], Opacity[0.6]]}}}, PlotLabel → "Interspecies dynamics"],
                              Plot \left[\left\{\sum_{k=0}^{dintra-1} \left(Binomial[dintra-1, k] y^{k} (1-y)^{dintra-1-k} \pi intraC[dintra, y^{k}]\right\}\right]
                                                       bintra, cintra, k+1, \omega]), \sum_{k=0}^{dintra-1} (Binomial[dintra-1, k] y^k
                                                     (1-y)^{dintra-1-k} \pi intraD[dintra, bintra, cintra, k, \omega]), \{y, 0, 1\},
                                   PlotStyle → {Blue, {Red, Dashed}}, Frame → True, AspectRatio → 1,
                                   Filling \rightarrow \{1 \rightarrow \{\{2\}, \{Directive[Lighter[Red], Opacity[0.6]], \}\}\}
                                                        Directive[Lighter[Blue], Opacity[0.6]]}}},
                                   PlotLabel → "Intraspecies dynamics"]}],
                       \{\{d, 5\}, 2, 10, 1\},\
                       {{b, 10}, 2, 10},
                       \{\{c, 3\}, 1, 5\},\
                       {{dintra, 5}, 2, 10, 1},
                       {{bintra, 10}, 2, 10},
                       {{cintra, 3}, 1, 5},
                       \{\{\omega, 0.75\}, 0.1, 2\}
```

Out[0]=



## **Dynamics**

$$\begin{split} &\text{fD[d\_, dintra\_, x\_, y\_, b\_, c\_, bintra\_, cintra\_, thresh\_, \omega\_, p\_] := \\ &p\left(\sum_{k=0}^{d-1} \left(\text{Binomial[d-1, k]} \ x^k \ (1-x)^{d-1-k} \ \pi \text{D[b, k, thresh]}\right)\right) + \\ & (1-p)\left(\sum_{k=0}^{\dim tra-1} \left(\text{Binomial[dintra-1, k]} \ y^k \right) \right); \\ & (1-y)^{\dim tra-1-k} \ \pi \text{intraD[dintra, bintra, cintra, k, } \omega]\right); \\ & \text{fC[d\_, dintra\_, x\_, y\_, b\_, c\_, bintra\_, cintra\_, thresh\_, } \omega\_, p\_] := \\ & p\left(\sum_{k=0}^{d-1} \left(\text{Binomial[d-1, k]} \ x^k \ (1-x)^{d-1-k} \ \pi \text{C[b, c, k+1, thresh]}\right)\right) + \\ & (1-p)\left(\sum_{k=0}^{\dim tra-1} \left(\text{Binomial[dintra-1, k]} \ y^k \ (1-y)^{\dim tra-1-k}\right)\right) + \\ & (1-p)\left(\sum_{k=0}^{\dim tra-1} \left(\text{Binomial[dintra-1, k]} \ y^k \ (1-y)^{\dim tra-1-k}\right)\right) + \\ & (1-p)\left(\sum_{k=0}^{\dim tra-1} \left(\text{Binomial[dintra-1, k]} \ y^k \ (1-y)^{\dim tra-1-k}\right)\right) + \\ & (1-p)\left(\sum_{k=0}^{\dim tra-1} \left(\text{Binomial[dintra-1, k]} \ y^k \ (1-y)^{\dim tra-1-k}\right)\right) + \\ & (1-p)\left(\sum_{k=0}^{\dim tra-1} \left(\text{Binomial[dintra-1, k]} \ y^k \ (1-y)^{\dim tra-1-k}\right)\right) + \\ & (1-p)\left(\sum_{k=0}^{\dim tra-1} \left(\text{Binomial[dintra-1, k]} \ y^k \ (1-y)^{\dim tra-1-k}\right)\right) + \\ & (1-p)\left(\sum_{k=0}^{\dim tra-1} \left(\text{Binomial[dintra-1, k]} \ y^k \ (1-y)^{\dim tra-1-k}\right)\right) + \\ & (1-p)\left(\sum_{k=0}^{\dim tra-1} \left(\text{Binomial[dintra-1, k]} \ y^k \ (1-y)^{\dim tra-1-k}\right)\right) + \\ & (1-p)\left(\sum_{k=0}^{\dim tra-1} \left(\text{Binomial[dintra-1, k]} \ y^k \ (1-y)^{\dim tra-1-k}\right)\right) + \\ & (1-p)\left(\sum_{k=0}^{\dim tra-1} \left(\text{Binomial[dintra-1, k]} \ y^k \ (1-y)^{\dim tra-1-k}\right)\right) + \\ & (1-p)\left(\sum_{k=0}^{\dim tra-1} \left(\text{Binomial[dintra-1, k]} \ y^k \ (1-y)^{\dim tra-1-k}\right)\right) + \\ & (1-p)\left(\sum_{k=0}^{\dim tra-1} \left(\text{Binomial[dintra-1, k]} \ y^k \ (1-y)^{\dim tra-1-k}\right)\right) + \\ & (1-p)\left(\sum_{k=0}^{\dim tra-1} \left(\text{Binomial[dintra-1, k]} \ y^k \ (1-y)^{\dim tra-1-k}\right)\right) + \\ & (1-p)\left(\sum_{k=0}^{\dim tra-1} \left(\text{Binomial[dintra-1, k]} \ y^k \ (1-y)^{\dim tra-1-k}\right)\right) + \\ & (1-p)\left(\sum_{k=0}^{\dim tra-1} \left(\text{Binomial[dintra-1, k]} \ y^k \ (1-y)^{\dim tra-1-k}\right)\right) + \\ & (1-p)\left(\sum_{k=0}^{\dim tra-1} \left(\text{Binomial[dintra-1, k]} \ y^k \ (1-y)^{\dim tra-1-k}\right)\right) + \\ & (1-p)\left(\sum_{k=0}^{\dim tra-1} \left(\text{Binomial[dintra-1, k]} \ y^k \ (1-y)^{\dim tra-1-k}\right)\right) + \\ & (1-p)\left(\sum_{k=0}^{\dim tra-1} \left(\text{Binomial[dintra-1, k]} \ y^k \$$

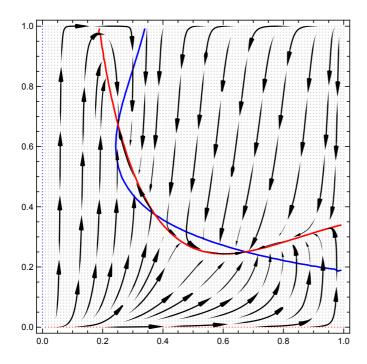
```
\piintraC[dintra, bintra, cintra, k+1, \omega]);
fbar1[d_, dintra_, x_, y_, b_, c_, bintra_, cintra_, thresh_, \omega_, p_] :=
  x fC[d, dintra, y, x, b, c, bintra, cintra, thresh, \omega, p] +
    (1-x) fD[d, dintra, y, x, b, c, bintra, cintra, thresh, \omega, p];
fbar2[d_, dintra_, x_, y_, b_, c_, bintra_, cintra_, thresh_, \omega_, p_] :=
  y fC[d, dintra, x, y, b, c, bintra, cintra, thresh, ω, p] +
    (1-y) fD[d, dintra, x, y, b, c, bintra, cintra, thresh, \omega, p];
xdot[d_{,dintra_{,x_{,y_{,b_{,c_{,dintra_{,cintra_{,thresh_{,m_{,w_{,p_{,i}}}}}}}}, m_{,w_{,p_{,i}}}] :=
  m \times (1-x) (fC[d, dintra, y, x, b, c, bintra, cintra, thresh, \omega, p] -
      fD[d, dintra, y, x, b, c, bintra, cintra, thresh, \omega, p]);
ydot[d_, dintra_, x_, y_, b_, c_, bintra_, cintra_, thresh_, n_, \omega_, p_] :=
  ny (1-y) (fC[d, dintra, x, y, b, c, bintra, cintra, thresh, <math>\omega, p] -
      fD[d, dintra, x, y, b, c, bintra, cintra, thresh, \omega, p]);
res[p , q , lim ] :=
  Quiet[NDSolve[\{x'[t] = mx[t] (fC[d1, dlintra, y[t], x[t], b, c, bintraforsp1,
            cintraforsp1, thresh1, ωforsp1, prob] - fbar1[d1, d1intra, x[t],
            y[t], b, c, bintraforsp1, cintraforsp1, thresh1, \omegaforsp1, prob]),
      y'[t] = ny[t] (fC[d2, d2intra, x[t], y[t], b, c, bintraforsp2,
            cintraforsp2, thresh2, ωforsp2, prob] - fbar2[d2, d2intra, x[t],
            y[t], b, c, bintraforsp2, cintraforsp2, thresh2, ωforsp2, prob]),
      x[0] = p, y[0] = q, \{x, y\}, \{t, \lim\}];
liseval[t_, rep_] := Evaluate[{x[t], y[t]} /. rep];
gamea = \{3, 3/4\};
gameb = \{1, 3/4\};
gamec = \{1, 4/3\};
gamed = {3, 4/3};
d1 = 5;
d2 = 5;
d1intra = 5;
d2intra = 5;
thresh1 = 1;
thresh2 = 1;
b = 2;
c = 1;
bintraforsp1 = 10;
cintraforsp1 = gameb[[1]];
\omegaforsp1 = gameb[2] // N;
bintraforsp2 = 10;
cintraforsp2 = gameb[[1]];
\omegaforsp2 = gameb[[2]] // N;
m = 1 / 8 / / N;
n = 1;
```

```
prob = 0.666;
     lim = 800;
in[0]:= Clear[blue, red, black];
     blue = {};
     red = {};
     black = {};
     For [u = 0.0, u \le 1.0, u = u + 0.01, For [v = 0.0, v \le 1.0, v = v + 0.01,
        If[
          liseval[lim, res[u, v, lim]][1][1] > 0.9999 &&
           liseval[lim, res[u, v, lim]][1][2] < 0.01,
          AppendTo[red, \{u, v\}], If[liseval[lim, res[u, v, lim]][1][1]] < 0.01 &&
             liseval[lim, res[u, v, lim]][1][2] > 0.9999,
           AppendTo[blue, {u, v}], AppendTo[black, {u, v}]]];
      ]]
     ans = Solve[
         {xdot[d1, dlintra, x, y, b, c, bintraforsp1, cintraforsp1, thresh1, m, ωforsp1,
             prob] == 0, ydot[d2, d2intra, x, y, b, c, bintraforsp2, cintraforsp2, thresh2,
             n, \omegaforsp2, prob] == 0, xdot[d1, d1intra, x, y, b, c, bintraforsp1,
             cintraforsp1, thresh1, m, \omegaforsp1, prob] = ydot[d2, d2intra, x, y, b,
             c, bintraforsp2, cintraforsp2, thresh2, n, ωforsp2, prob]}, {x, y}];
     xlis = {};
     ylis = {};
     For[i = 1, i ≤ Length[ans], i++,
      For [j = 1, j \le 2, j++,
        If [ans[i][j][1] = x,
         temp = ans[i][j][2] // N;
         If[Head[temp] == Real && temp > 0 && temp < 1, AppendTo[xlis, temp]]</pre>
        ];
        If [ans[i][j][1] = y,
         temp = ans[i][j][2] // N;
         If[Head[temp] == Real && temp > 0 && temp < 1, AppendTo[ylis, temp]]</pre>
        ]
      ]
     ]
     xlis = Flatten[Union[xlis]];
     ylis = Flatten[Union[ylis]];
     xplots = Table[ContourPlot[y == ylis[z]], \{x, 0, 1\}, \{y, 0, 1\}, AspectRatio \rightarrow 1,
          ContourStyle → {Blue, Thickness[0.003]}], {z, 1, Length[ylis]}];
     yplots = Table[ContourPlot[x == xlis[z]], \{x, 0, 1\}, \{y, 0, 1\}, AspectRatio \rightarrow 1,
          ContourStyle → {Red, Thickness[0.003]}], {z, 1, Length[xlis]}];
```

```
In[*]:= (*(*p1fun[d1_,d2_,b_,c_,thresh1_,thresh2_,m_,n_]:=ContourPlot[
         \{ydot[d2,x,y,b,c,thresh2,n]=0,xdot[d1,x,y,b,c,thresh1,m]=0\},\{y,0,1\},
         {x,0,1},ContourStyle→{{Thickness[0.009],Red},{Thickness[0.009],Blue}},
         Axes→False,Frame→True];*)
     p2fun[d1_,d2_,d1intra_,d2intra_,b_,c_,thresh1_,thresh2_,m_,n_]:=StreamPlot[{
          xdot[d1,d1intra,x,y,b,c,bintraforsp1,cintraforsp1,thresh1,m,ωforsp1,prob],
          ydot[d2,d2intra,x,y,b,c,bintraforsp2,cintraforsp2,thresh2,n,ωforsp2,
           prob]},{x,0.0001,1},{y,0.0001,1},StreamScale→0.2,Axes→False,
         StreamPoints→30,StreamStyle→{"PinDart",Directive[Black,Thick]},Frame→True,
         PlotRange \rightarrow \{\{-0.01,1.01\},\{-0.01,1.01\}\},StreamColorFunction \rightarrow (Black\&)\};
     (*diagonal=Plot[1-x,{x,0,1},
         PlotStyle→{Thickness[0.005],Green,Dashed},AspectRatio→1,Frame→True];*)
     (*sep=ContourPlot[ydot[d2,x,y,b,c,thresh2,n]==xdot[d1,x,y,b,c,thresh1,m],
         \{x,0,1\},\{y,0,1\},ContourStyle\rightarrow\{Thickness[0.005],Green,Dashed\}];*)*)
In[0]:= p2fun[d1_, d2_, d1intra_, d2intra_,
        b_, c_, thresh1_, thresh2_, m_, n_] := StreamPlot[{
         xdot[d1, d1intra, x, y, b, c, bintraforsp1, cintraforsp1, thresh1,
          m, \omegaforsp1, prob], ydot[d2, d2intra, x, y, b, c, bintraforsp2,
          cintraforsp2, thresh2, n, \omegaforsp2, prob]}, {x, 0.0001, 1},
        \{y, 0.0001, 1\}, StreamScale \rightarrow 0.2, Axes \rightarrow False, StreamPoints \rightarrow 30,
        StreamStyle \rightarrow \{"PinDart", Directive[Black, Thick]\}, Frame \rightarrow True,
        PlotRange → {{-0.01, 1.01}, {-0.01, 1.01}}, StreamColorFunction → (Black &)]
In[0]:= g1 = ListPlot[{blue, red, black},
         PlotStyle → {Lighter[Blue, 0.4], Lighter[Red, 0.4], Lighter[Gray, 0.7]},
         AspectRatio \rightarrow 1, Frame \rightarrow True, PlotRange \rightarrow {{-0.01, 1.01}, {-0.01, 1.01}}];
ln[\cdot]:= p2fun[d1, d2, d1intra, d2intra, b, c, thresh1, thresh2, m, n]
In[@]:= p2contour[d1_, d2_, d1intra_, d2intra_,
        b_, c_, thresh1_, thresh2_, m_, n_] := ContourPlot[{
         xdot[d1, d1intra, x, y, b, c,
           bintraforsp1, cintraforsp1, thresh1, m, \omegaforsp1, prob] == 0.00,
         ydot[d2, d2intra, x, y, b, c, bintraforsp2, cintraforsp2, thresh2,
           n, \omega forsp2, prob] = 0.00\}, \{x, 0.001, 0.99\}, \{y, 0.001, 0.99\},
        ContourStyle → {{Thickness[0.005], Blue}, {Thickness[0.005], Red}}]
In[a]:= p2contour[d1, d2, d1intra, d2intra, b, c, thresh1, thresh2, m, n]
```

In[o]:= Show[g1, (\*,p1fun[d1,d2,b,c,thresh1,thresh2,m,n],\*) p2contour[d1, d2, d1intra, d2intra, b, c, thresh1, thresh2, m, n], p2fun[d1, d2, d1intra, d2intra, b, c, thresh1, thresh2, m, n](\*,xplots,yplots\*), Frame → True, FrameStyle → Thickness[0.0025], ImagePadding  $\rightarrow$  17, PlotRange  $\rightarrow$  {{-0.02, 1.02}, {-0.02, 1.02}}]

Out[0]=



(\*Export["Untitled.pdf",%]\*)