SGDs

Game1

```
payoffmat1 = {{0, -1, 2}, {2, 0, -1}, {-1, 2, 0}};
```

```
sgd1fitness[strat_, x1_, x2_, x3_] := {payoffmat1.{x1, x2, x3}}[[1, strat]];
```

```
sgd1avgfit[x1_, x2_, x3_] := {x1, x2, x3}.(payoffmat1.{x1, x2, x3});
```

```
Solve[(sgd1fitness[1, x, y, 1 - x - y] - sgd1fitness[2, x, y, 1 - x - y]) == 0 &&
  (sgd1fitness[1, x, y, 1 - x - y] - sgd1fitness[3, x, y, 1 - x - y]) == 0, {x, y}]
```

Out[•]= $\left\{\left\{x \to \frac{1}{3}, y \to \frac{1}{3}\right\}\right\}$

```
tlim = 50;
```

```
sgd1sol[init_] := NDSolve[{
    x1'[t] == x1[t]
      (sgd1fitness[1, x1[t], x2[t], x3[t]] - sgd1avgfit[x1[t], x2[t], x3[t]]),
    x2'[t] == x2[t] (sgd1fitness[2, x1[t], x2[t], x3[t]] -
        sgd1avgfit[x1[t], x2[t], x3[t]]),
    x3'[t] == x3[t] (sgd1fitness[3, x1[t], x2[t], x3[t]] -
        sgd1avgfit[x1[t], x2[t], x3[t]]),
    x1[0] == init[[1]], x2[0] == init[[2]], x3[0] == 1 - init[[1]] - init[[2]]},
  {x1, x2, x3}, {t, 0, tlim}]
```

```
TA = \begin{pmatrix} -\frac{1}{2} & -1 \\ \frac{\sqrt{3}}{2} & 0 \end{pmatrix} ;
```

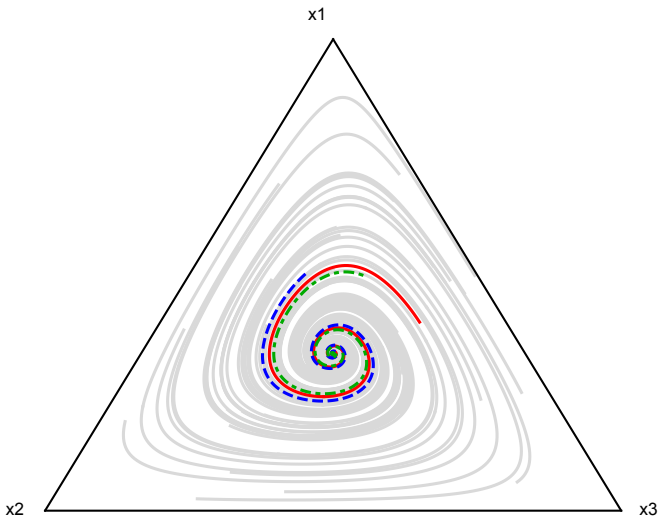
```
new[r_] := TA.{r[[1]], r[[2]]} + {1, 0} ;
list1 = Map[new[#] &,
   Flatten[Table[Evaluate[{x1[t], x2[t], x3[t]} /. sgd1sol[{0.4, 0.15}]],
     {t, 0, tlim, 0.05}], 1]];
list2 = Map[new[#] &, Flatten[Table[Evaluate[
      {x1[t], x2[t], x3[t]} /. sgd1sol[{0.5, 0.3}]], {t, 0, tlim, 0.05}], 1]];
list3 = Map[new[#] &, Flatten[Table[Evaluate[{x1[t], x2[t], x3[t]} /.
       sgd1sol[{0.5, 0.2}]], {t, 0, tlim, 0.05}], 1]];
(*list2=Map[new[#]&,Flatten[Table[Evaluate[z/.sgd1sol[init_]],
     {t,0,tlim,0.05}],1]];*)
```

```
raninits = Table[temp = RandomReal[1, 3];
   temp / Total[temp][[1 ;; 2]], {i, 1, 50}];
```

```
rantimeseries =
  Table[Flatten[Table[Evaluate[{x1[t], x2[t], x3[t]} /. sgd1sol[raninits[[i]]],
     {t, 0, tlim, 0.05}], 1], {i, 1, Length[raninits]}];
```

```
In[•]:= ranlist = Table[Map[new[#] &, rantimeseries[[i]]], {i, 1, Length[raninits], 1}];

In[•]:= randoms = ListPlot[ranlist, Joined → True,
          PlotRange → {{-0.1, 1.1}, {-0.05, √3/2 + 0.1}}, PlotStyle → LightGray,
          Axes → None, AspectRatio → 0.8, Epilog → {Text[x1, new[{1.05, 0, 0}]],
            Text[x2, new[{0, 1.05, 0}]], Text[x3, new[{0, -0.05, 1}]]}];

In[•]:= Show[randoms, ListPlot[
          {{{0, 0}, {1/2, √3/2}}, {{1, 0}, {1/2, √3/2}}, {{0, 0}, {1, 0}}, list1, list2, list3},
          Joined → True, PlotRange → {{-0.1, 1.1}, {-0.05, √3/2 + 0.1}},
          PlotStyle → {{Black, Thickness[0.003]}, {Black, Thickness[0.003]},
            {Black, Thickness[0.003]}, Red, {Blue, Dashed}, {Darker[Green], DotDashed}},
          Axes → None, AspectRatio → 0.8, Epilog → {Text[x1, new[{1.05, 0, 0}]],
            Text[x2, new[{0, 1.05, 0}]], Text[x3, new[{0, -0.05, 1}]]}]]
```

Out[•]=

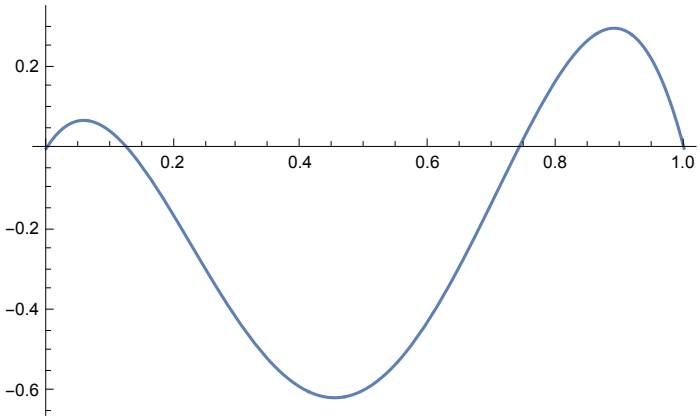


Game2

```
In[•]:= payoffmat2 = {{10, 1, 5.5}, {4, 10, 3}};

In[•]:= sgd2fitness[strat_, x1_, x2_] := {payoffmat2.{x1^2, 2 x1 x2, x2^2}}[[1, strat]];

In[•]:= sgd2avgfit[x1_, x2_] := {x1, x2}.(payoffmat2.{x1^2, 2 x1 x2, x2^2});
```

In[•]:=
```
Plot[x (1 - x) (sgd2fitness[1, x, 1 - x] - sgd2fitness[2, x, 1 - x]), {x, 0, 1}]
```

Out[•]=



In[•]:=
```
Solve[x (1 - x) (sgd2fitness[1, x, 1 - x] - sgd2fitness[2, x, 1 - x]) == 0, x]
```

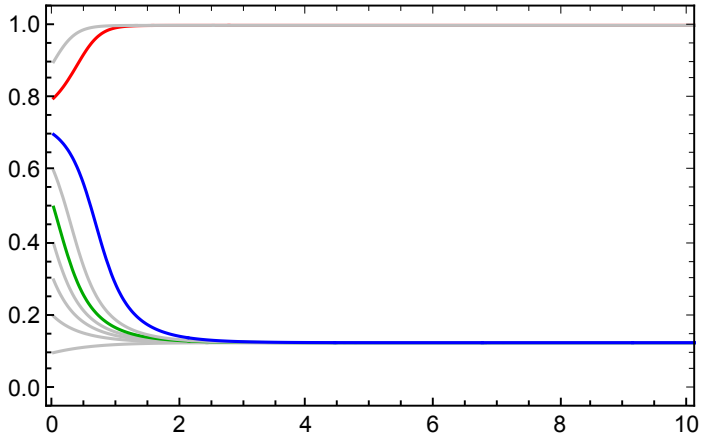
Out[•]=
```
{{x → 0}, {x → 0.127395}, {x → 0.74053}, {x → 1}}
```

In[•]:=
```
tlim = 50;
```

In[•]:=
```
sgd2sol[init_] := NDSolve[{
    x1'[t] == x1[t] (sgd2fitness[1, x1[t], x2[t]] - sgd2avgfit[x1[t], x2[t]]),
    x2'[t] == x2[t] (sgd2fitness[2, x1[t], x2[t]] - sgd2avgfit[x1[t], x2[t]]),
    x1[0] == init, x2[0] == 1 - init}, {x1, x2}, {t, 0, tlim}]
```

In[•]:=
```
sgd2pl = Plot[{
    Evaluate[{x1[iter]} /. sgd2sol[0.1]],
    Evaluate[{x1[iter]} /. sgd2sol[0.2]],
    Evaluate[{x1[iter]} /. sgd2sol[0.3]],
    Evaluate[{x1[iter]} /. sgd2sol[0.4]],
    Evaluate[{x1[iter]} /. sgd2sol[0.5]],
    Evaluate[{x1[iter]} /. sgd2sol[0.6]],
    Evaluate[{x1[iter]} /. sgd2sol[0.7]],
    Evaluate[{x1[iter]} /. sgd2sol[0.8]],
    Evaluate[{x1[iter]} /. sgd2sol[0.9]]}, {iter, 0, 30},
  PlotRange → {{-0.1, 10.1}, {-0.05, 1.05}}, Frame → True,
  PlotStyle → {Lighter[Gray, 0.5], Lighter[Gray, 0.5], Lighter[Gray, 0.5],
    Lighter[Gray, 0.5], Darker[Green], Lighter[Gray, 0.5], Blue,
    Red, Lighter[Gray, 0.5]}, Frame → True, FrameTicksStyle →
   Directive[Black, 12], FrameStyle → Directive[Thickness[0.003]]]
```

Out[•]=



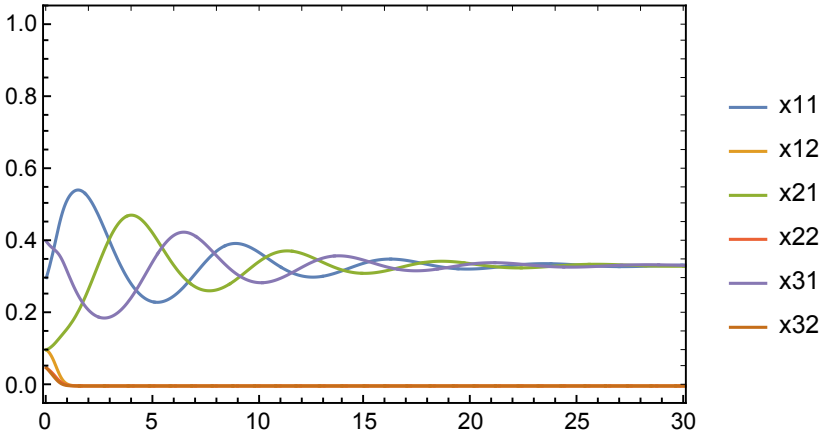
MGD

This is a function for

```
In[•]:= payoffmgd[x11_, x12_, x21_, x22_, x31_, x32_] :=
  {(sgd1fitness[1, x11 + x12, x21 + x22, x31 + x32] -
      sgd1avgfit[x11 + x12, x21 + x22, x31 + x32]) + (sgd2fitness[1, x11 + x21 + x31,
       x12 + x22 + x32] - sgd2avgfit[x11 + x21 + x31, x12 + x22 + x32]),
   (sgd1fitness[1, x11 + x12, x21 + x22, x31 + x32] - sgd1avgfit[x11 + x12,
       x21 + x22, x31 + x32]) + (sgd2fitness[2, x11 + x21 + x31, x12 + x22 + x32] -
      sgd2avgfit[x11 + x21 + x31, x12 + x22 + x32]),
   (sgd1fitness[2, x11 + x12, x21 + x22, x31 + x32] - sgd1avgfit[x11 + x12,
       x21 + x22, x31 + x32]) + (sgd2fitness[1, x11 + x21 + x31, x12 + x22 + x32] -
      sgd2avgfit[x11 + x21 + x31, x12 + x22 + x32]),
   (sgd1fitness[2, x11 + x12, x21 + x22, x31 + x32] - sgd1avgfit[x11 + x12,
       x21 + x22, x31 + x32]) + (sgd2fitness[2, x11 + x21 + x31, x12 + x22 + x32] -
      sgd2avgfit[x11 + x21 + x31, x12 + x22 + x32]),
   (sgd1fitness[3, x11 + x12, x21 + x22, x31 + x32] - sgd1avgfit[x11 + x12,
       x21 + x22, x31 + x32]) + (sgd2fitness[1, x11 + x21 + x31, x12 + x22 + x32] -
      sgd2avgfit[x11 + x21 + x31, x12 + x22 + x32]),
   (sgd1fitness[3, x11 + x12, x21 + x22, x31 + x32] - sgd1avgfit[x11 + x12,
       x21 + x22, x31 + x32]) + (sgd2fitness[2, x11 + x21 + x31, x12 + x22 + x32] -
      sgd2avgfit[x11 + x21 + x31, x12 + x22 + x32])};

In[•]:= mgdfitness[strat_, x11_, x12_, x21_, x22_, x31_, x32_] :=
  {payoffmgd[x11, x12, x21, x22, x31, x32]}[[1, strat]];

In[•]:= mgdavgfit[x11_, x12_, x21_, x22_, x31_, x32_] :=
  {x11, x12, x21, x22, x31, x32}.payoffmgd[x11, x12, x21, x22, x31, x32];

In[•]:= tlim = 100;
mgdsol[init_] := NDSolve[{
   x11'[t] == x11[t] (mgdfitness[1, x11[t], x12[t], x21[t], x22[t], x31[t], x32[t]] -
       mgdavgfit[x11[t], x12[t], x21[t], x22[t], x31[t], x32[t]]),
   x12'[t] == x12[t] (mgdfitness[2, x11[t], x12[t], x21[t], x22[t], x31[t],
        x32[t]] - mgdavgfit[x11[t], x12[t], x21[t], x22[t], x31[t], x32[t]]),
   x21'[t] == x21[t] (mgdfitness[3, x11[t], x12[t], x21[t], x22[t], x31[t],
        x32[t]] - mgdavgfit[x11[t], x12[t], x21[t], x22[t], x31[t], x32[t]]),
   x22'[t] == x22[t] (mgdfitness[4, x11[t], x12[t], x21[t], x22[t], x31[t],
        x32[t]] - mgdavgfit[x11[t], x12[t], x21[t], x22[t], x31[t], x32[t]]),
   x31'[t] == x31[t] (mgdfitness[5, x11[t], x12[t], x21[t], x22[t], x31[t],
        x32[t]] - mgdavgfit[x11[t], x12[t], x21[t], x22[t], x31[t], x32[t]]),
   x32'[t] == x32[t] (mgdfitness[6, x11[t], x12[t], x21[t], x22[t], x31[t],
        x32[t]] - mgdavgfit[x11[t], x12[t], x21[t], x22[t], x31[t], x32[t]]),
   x11[0] == init[[1]], x12[0] == init[[2]], x21[0] == init[[3]], x22[0] == init[[4]],
   x31[0] == init[[5]], x32[0] == 1 - init[[1]] - init[[2]] - init[[3]] - init[[4]] - init[[5]]},
  {x11, x12, x21, x22, x31, x32}, {t, 0, tlim}]

In[•]:= mgdinits = {{0.3, 0.1, 0.1, 0.05, 0.4, 0.05},
   {0.4, 0.1, 0.2, 0.1, 0.1, 0.1}, {0.2, 0.3, 0.1, 0.1, 0.2, 0.1}};
```

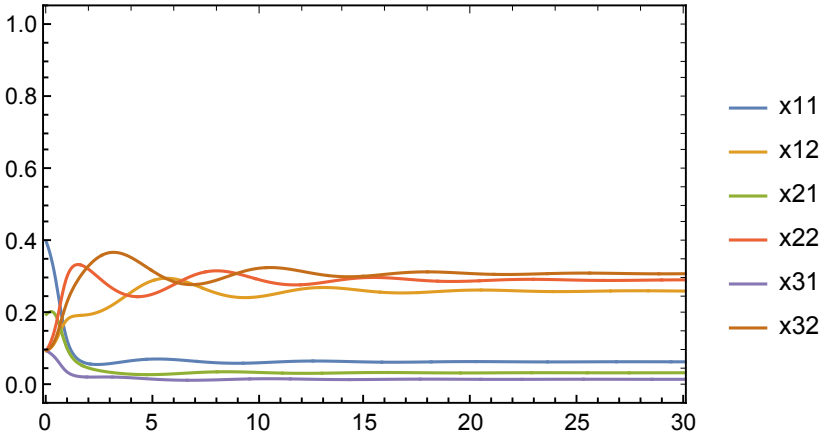
In[•]:=
```
Plot[
 Evaluate[
  {x11[t], x12[t], x21[t], x22[t], x31[t], x32[t]} /. mgdsol[mgdinits[[1]]]],
{t, 0, 30}, PlotRange → {{-0.1, 30.1}, {-0.05, 1.05}},
Frame → True, PlotStyle → Automatic
(*{Red,Lighter[Red,0.6],Red,Lighter[Red,0.6],Red,Lighter[Red,0.6]}*),
Frame → True, FrameTicksStyle → Directive[Black, 12],
FrameStyle → Directive[Thickness[0.003]],
PlotLegends → {"x11", "x12", "x21", "x22", "x31", "x32"}]
```

Out[•]=



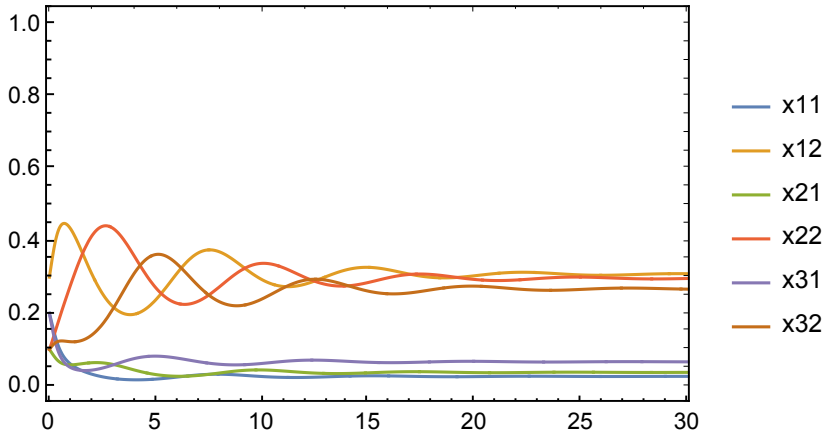
In[•]:=
```
Plot[
 Evaluate[
  {x11[t], x12[t], x21[t], x22[t], x31[t], x32[t]} /. mgdsol[mgdinits[[2]]]],
{t, 0, 30}, PlotRange → {{-0.1, 30.1}, {-0.05, 1.05}}, Frame → True,
Frame → True, FrameTicksStyle → Directive[Black, 12],
FrameStyle → Directive[Thickness[0.003]], PlotStyle → Automatic,
PlotLegends → {"x11", "x12", "x21", "x22", "x31", "x32"}]
```

Out[•]=



In[•]:=
```
Plot[
 Evaluate[
  {x11[t], x12[t], x21[t], x22[t], x31[t], x32[t]} /. mgdsol[mgdinits[[3]]]],
 {t, 0, 30}, PlotRange → {{-0.1, 30.1}, {-0.05, 1.05}}, Frame → True,
 Frame → True, FrameTicksStyle → Directive[Black, 12],
 FrameStyle → Directive[Thickness[0.003]], PlotStyle → Automatic,
 PlotLegends → {"x11", "x12", "x21", "x22", "x31", "x32"}]
```

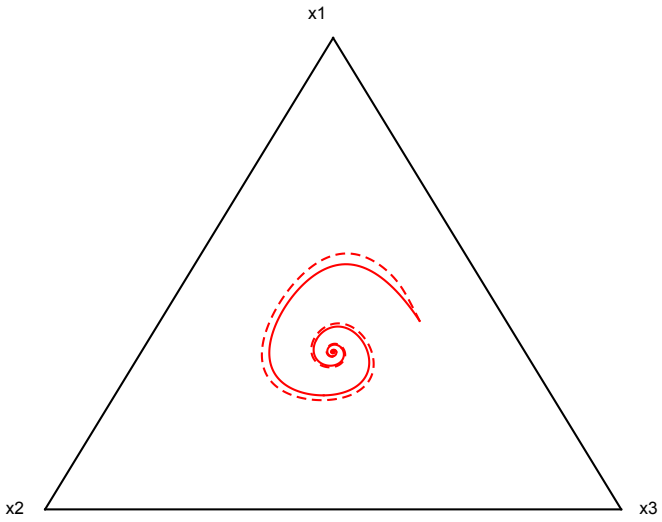
Out[•]=



In[•]:=
```
recoverlist1 = Map[new[#] &,
    Flatten[Table[Evaluate[{x11[t] + x12[t], x21[t] + x22[t], x31[t] + x32[t]} /.
         mgdsol[{0.3, 0.1, 0.1, 0.05, 0.4, 0.05}]], {t, 0, tlim, 0.05}], 1]];
recoverlist2 = Map[new[#] &, Flatten[Table[Evaluate[
       {x11[t] + x12[t], x21[t] + x22[t], x31[t] + x32[t]} /.
        mgdsol[{0.4, 0.1, 0.2, 0.1, 0.1, 0.1}]], {t, 0, tlim, 0.05}], 1]];
recoverlist3 = Map[new[#] &, Flatten[Table[Evaluate[
       {x11[t] + x12[t], x21[t] + x22[t], x31[t] + x32[t]} /.
        mgdsol[{0.2, 0.3, 0.1, 0.1, 0.2, 0.1}]], {t, 0, tlim, 0.05}], 1]];
```

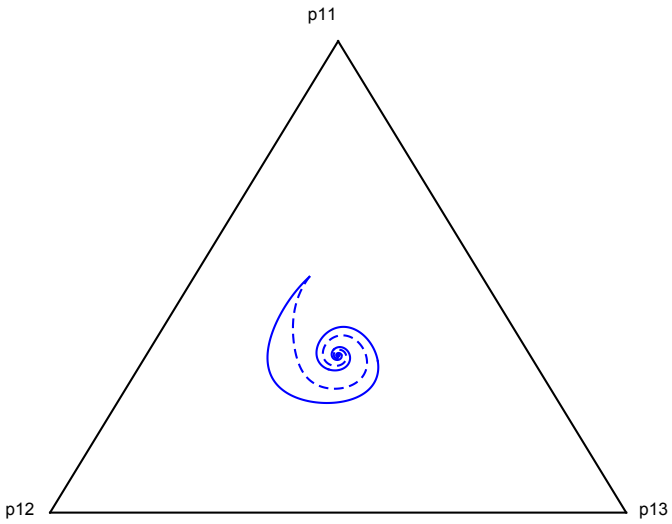
In[•]:=
```
ListPlot[{{{0, 0}, {1/2, Sqrt[3]/2}}, {{1, 0}, {1/2, Sqrt[3]/2}}, {{0, 0}, {1, 0}}, list1,
  recoverlist1}, Joined → True, PlotRange → {{-0.1, 1.1}, {-0.05, Sqrt[3]/2 + 0.1}},
 PlotStyle → {{Black, Thickness[0.003]}, {Black, Thickness[0.003]}, {Black,
    Thickness[0.003]}, {Red, Thickness[0.003]}, {Red, Thickness[0.003], Dashed}},
 Axes → None, AspectRatio → 0.8, Epilog → {Text[x1, new[{1.05, 0, 0}]],
   Text[x2, new[{0, 1.05, 0}]], Text[x3, new[{0, -0.05, 1}]]}]
```

Out[•]=



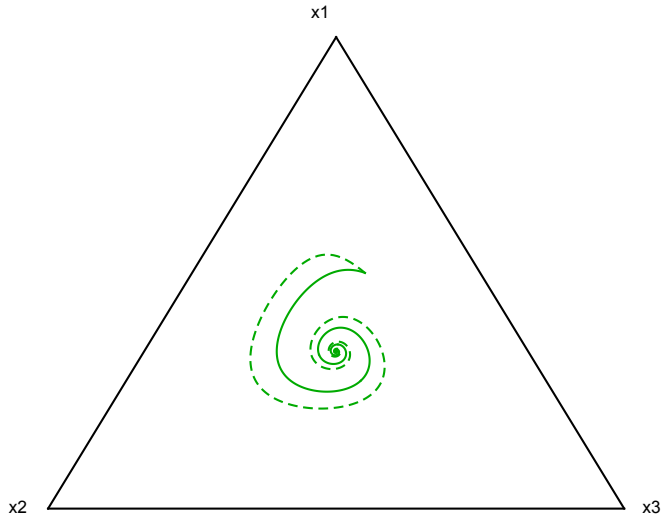
In[•]:=
```
ListPlot[{{{0, 0}, {1/2, Sqrt[3]/2}}, {{1, 0}, {1/2, Sqrt[3]/2}}, {{0, 0}, {1, 0}}, list2,
  recoverlist2}, Joined → True, PlotRange → {{-0.1, 1.1}, {-0.05, Sqrt[3]/2 + 0.1}},
 PlotStyle → {{Black, Thickness[0.003]}, {Black, Thickness[0.003]},
   {Black, Thickness[0.003]}, {Blue, Thickness[0.003]},
   {Blue, Thickness[0.003], Dashed}}, Axes → None,
 AspectRatio → 0.8, Epilog → {Text[p11, new[{1.05, 0, 0}]],
   Text[p12, new[{0, 1.05, 0}]], Text[p13, new[{0, -0.05, 1}]]}]
```

Out[•]=



In[•]:=
```
ListPlot[{{{0, 0}, {1/2, Sqrt[3]/2}}, {{1, 0}, {1/2, Sqrt[3]/2}}, {{0, 0}, {1, 0}}, list3,
  recoverlist3}, Joined → True, PlotRange → {{-0.1, 1.1}, {-0.05, Sqrt[3]/2 + 0.1}},
 PlotStyle → {{Black, Thickness[0.003]}, {Black, Thickness[0.003]},
   {Black, Thickness[0.003]}, {Darker[Green], Thickness[0.003]},
   {Darker[Green], Thickness[0.003], Dashed}}, Axes → None,
 AspectRatio → 0.8, Epilog → {Text[x1, new[{1.05, 0, 0}]],
   Text[x2, new[{0, 1.05, 0}]], Text[x3, new[{0, -0.05, 1}]]}]
```

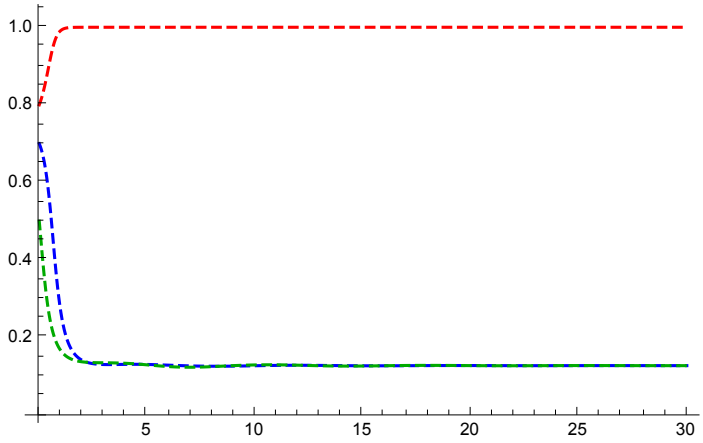
Out[•]=



In[•]:=
```
recover1sgd2 =
  Flatten[Table[Evaluate[{x11[t] + x21[t] + x31[t], +x12[t] + x22[t] + x32[t]} /.
      mgdsol[{0.3, 0.1, 0.1, 0.05, 0.4, 0.05}]], {t, 0, 30, 0.05}], 1];
recover2sgd2 = Flatten[Table[Evaluate[{x11[t] + x21[t] + x31[t], +x12[t] + x22[t] +
      x32[t]} /. mgdsol[{0.4, 0.1, 0.2, 0.1, 0.1, 0.1}]], {t, 0, 30, 0.05}], 1];
recover3sgd2 = Flatten[Table[Evaluate[{x11[t] + x21[t] + x31[t], +x12[t] + x22[t] +
      x32[t]} /. mgdsol[{0.2, 0.3, 0.1, 0.1, 0.2, 0.1}]], {t, 0, 30, 0.05}], 1];
```

In[•]:=
```
recoveredsgd2pl =
 ListPlot[{recover1sgd2[[All, 1]], recover2sgd2[[All, 1]], recover3sgd2[[All, 1]]},
  Joined → True, DataRange → {0, 30},
  PlotStyle → {{Red, Dashed}, {Blue, Dashed}, {Darker[Green], Dashed}}]
```

Out[•]=



In[•]:=
```
Show[sgd2pl, recoveredsgd2pl, Frame → True,
 FrameTicksStyle → Directive[Black, 12],
 FrameStyle → Directive[Thickness[0.003]]]
```

Out[•]=