Assignment 1

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Defanations

Two Dimensional Arrays Expression

To describe variants of two-dimensional arrays we write $(b : k \mapsto j \mapsto x)$ instead of $(b : k \mapsto (b[k] : j \mapsto x))$. We use this new notation to state an instance of the array-assignment axiom we saw already

$$\left\{\phi[^{(b:k\mapsto x)}/_b]\right\} b[k] := x \left\{\phi\right\}$$

for two-dimensional arrays:

$$\left\{\phi[^{(b:k\mapsto j\mapsto x)}/_b]\right\} b[k][j] := x \left\{\phi\right\}$$

String Length

A string $S \in \mathit{Letter}^*$ which is an array of letters[1].Also, string will be terminate by the null character which is a convention by the C programming language and we will follow this convention in this proof. We write $|S|$ for the number of letters in the string. Formally, we define these two nothion inductively by

$$|S\ell| = |S| + \begin{cases} 1 & \text{if } \ell \neq' \backslash 0' \\ 0 & \text{if } \ell =' \backslash 0' \end{cases}$$

Also, by the convention of C we has this defination for $S \in \mathit{string}$.

$$S[|S|] =' \backslash 0' \wedge \forall 0 \leq i < |S| \, (S[i] \neq' \backslash 0')$$

[1]The letter here is a legal charater encode with ASCII, UTF-8 or other charater encoding standard.

String Equals

To describe two string a, b $(a, b \in String)$ are equals we write $a = b$ when:

$$a = b \iff |a| = |b| \wedge \forall j \in 0..|a|\,(a[j] = b[j])$$

Similarly, we write:

$$a \neq b \iff \neg(a = b)$$

String Assign

To assign a string to another string array, we will denote as

$$a := b$$

instead of a long programme of our toy language:

$\{a, b \in String\}$
$\{I[^0/_i]\}$
$i := 0;$
$\{I\}$
while $i \leq |b|$ **do**
  $\{I \wedge i < n\}$
  $\{I[^{i+1}/_i][^{a:i\mapsto b[i]}/_a]\}$
  $a[i] := b[i];$
  $\{I[^{i+1}/_i]\}$
  $i := i + 1;$
  $\{I\}$
od;
$\{I \wedge i > |b|\}$
$\{a, b \in String \wedge a = b\}$

when our invariant is

$$I = a, b \in String \wedge 0 \leq i < (|b| + 1) \wedge \forall k \in 0..(i-1)\,(a[k] = b[k])$$

Here are the proofs of the implications:

First Implication for String assign: $a, b \in String \Rightarrow I[^0/_i]$

$a, b \in String$
$\Leftrightarrow$ $\langle$Defination of String $\rangle$
sss
$\Leftrightarrow$ $\langle$def$\rangle$
$I[^0/_i]$

Second Implication $I \wedge i < n \Rightarrow I\left[{}^{i+1}/_{i}\right]\left[{}^{a:i \mapsto b[i]}/_{a}\right]$

Third Implication $I \wedge i \geq 0 \Rightarrow a = b$

0.1 String Compare

1 Task 1