Assignment 2

Ruofei HUANG(z5141448) Anqi ZHU(z5141541)

April 28, 2018

1 Task 1

1.1 Prime

A number is a prime is a nautal number greater than 1 that cannot be formed by mutiplying two smaller number[1]. Hence we can have the definition by set theory:

$$Prime = \{n | \neg \exists x \in (2..n-1)\, (x|n)\}$$

1.2 Reverse

To define a emirp, a mathmatical function to decribe a number's reverse would be helpful. By the spec in verifying reversen[2], we have this definition:

$$rev(n) = \sum_{i=0}^{c(n)} (S_i 10^i)$$

where:

$$c(n) = \lfloor log_{10}(n) \rfloor,$$
$$S = [10]^*,$$
$$n \in \mathbb{N} \wedge n = \sum_{i=0}^{c(n)} (S_i 10^{(c(n)-i)})$$

Hence, we can justify the spec of **proc** $reversen($**value** $n : \mathbb{Z},$ **result** $r : \mathbb{Z})$ as

$$r : [\text{TRUE}, r = rev(n)]$$

[1] Direct reference from Wikipedia: https://en.wikipedia.org/wiki/Prime_number

[2] A proof provided by Lecturer in Control of this course on https://www.cse.unsw.edu.au/~cs2111/18s1/lec/reverse.pdf

1.3 Emirp

An emirp is a prime number that results iin a different prime when its decimal digits are reverse[3]. Hence a definition of emirp can be construct as follow:

$$n \in Emirp \iff n \in Prime \wedge rev(n) \in Prime$$

We also construct another function to help us find the n^{th} emirp, which is as follow:

$$isEmirp(n) = \begin{cases} 0 & \text{if } n \in Prime \wedge rev(n) \in Prime \\ 1 & \text{else} \end{cases}$$

1.3.1 Procedure Call

Also for our usage in the procedure call in the main programme, we have develop a procedure to do the same thing. Hence we have this spec

$$\textbf{proc}\ ISEMIRP(\textbf{value}\ n : \mathbb{N}, \textbf{result}\ w)\cdot$$
$$\llcorner n, w : [\text{TRUE}, \begin{pmatrix} (w = 1 \wedge rev(n) \neq n \wedge n \in Prime \wedge rev(n) \in Prime) \vee \\ (w \neq 1 \wedge \neg(rev(n) \neq n \wedge n \in Prime \wedge rev(n) \in Prime)) \end{pmatrix}] \lrcorner_{(A)}$$

1.3.2 Refinement Calculatiion

$$(A) \sqsubseteq \quad \langle \textbf{c-frame} \rangle$$
$$w : [\text{TRUE}, \begin{pmatrix} (w = 1 \wedge rev(n) \neq n \wedge n \in Prime \wedge rev(n) \in Prime) \vee \\ (w \neq 1 \wedge \neg(rev(n) \neq n \wedge n \in Prime \wedge rev(n) \in Prime)) \end{pmatrix}]$$

1.3.3 Toy Language Code

1.4 Pre- and Postcondition

Our task is find the n^{th} emprip, by the previous definition of $isEmirp(n)$ we can construct the pre- and postcondition in this way:

$$\textbf{proc} EMIRP(\textbf{value}\ n : \mathbb{N}, \textbf{result}\ r)$$
$$\llcorner n : \mathbb{N}, \textbf{result}\ r : \mathbb{N}[\text{TRUE}, \sum_{i=0}^{r} isEmirp(i) = n \wedge r \in Emirp] \lrcorner_{(1)}$$

2 Task 2

[3] Another reference from Wikipedia :https://en.wikipedia.org/wiki/Emirp