

#### **COMP 4161**

Data61 Advanced Course

### **Advanced Topics in Software Verification**

Miki Tanaka, Johannes Åman Pohjola, June Andronick, Christine Rizkallah

### **Binary Search**

### (java.util.Arrays)



```
1:
      public static int binarySearch(int[] a, int key) {
2:
           int low = 0:
           int high = a.length - 1;
4:
5:
           while (low <= high) {
               int mid = (low + high) / 2;
               int midVal = a[mid]:
7:
8.
9:
               if (midVal < key)
10:
                    low = mid + 1
11:
                else if (midVal > key)
                    high = mid - 1;
12:
13:
                else
14:
                    return mid; // key found
15:
16:
            return -(low + 1); // key not found.
17:
        }
```

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            return -(low + 1); // key not found.
17:
        7-
```

```
6: int mid = (low + high) / 2;
```

http://googleresearch.blogspot.com/2006/06/ extra-extra-read-all-about-it-nearly.html

# **Organisatorials**



**When** Tue 10:00 – 12:00

Wed 10:00 - 12:00

Where Tue: Electrical Engineering G04 (K-G17-G04)

Wed: UNSW Business School 205 (K-E12-205)

http://www.cse.unsw.edu.au/~cs4161/



#### The trustworthy systems verification team

→ Functional correctness and security of the seL4 microkernel Security↔ Isabelle/HOL model ↔ Haskell model ↔ C code ↔ Binary



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- → 10 000 LOC / 500 000 lines of proof; about 25 person years of effort



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Open Source http://sel4.systems

https://ts.data61.csiro.au/projects/TS/cogent.pml

https://cakeml.org



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We are always embarking on exciting new projects. We offer

- → summer student scholarship projects
- → honours and PhD theses
- → research assistant and verification engineer positions



→ how to use a theorem prover



- → how to use a theorem prover
- → background, how it works



- → how to use a theorem prover
- → background, how it works
- → how to prove and specify



- → how to use a theorem prover
- → background, how it works
- → how to prove and specify
- → how to reason about programs



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- → background, how it works
- → how to prove and specify
- → how to reason about programs

# Health Warning Theorem Proving is addictive

### **Prerequisites**



This is an advanced course. It assumes knowledge in

- → Functional programming
- → First-order formal logic

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The following program should make sense to you:

```
\begin{array}{lll} \mathsf{map} \ f \ [] & = & [] \\ \mathsf{map} \ f \ (\mathsf{x} : \mathsf{xs}) & = & \mathsf{f} \ \mathsf{x} : \ \mathsf{map} \ \mathsf{f} \ \mathsf{xs} \end{array}
```

### **Prerequisites**



#### This is an advanced course. It assumes knowledge in

- → Functional programming
- → First-order formal logic

The following program should make sense to you:

map f 
$$[]$$
 =  $[]$   
map f (x:xs) = f x : map f xs

You should be able to read and understand this formula:

$$\exists x. (P(x) \longrightarrow \forall x. P(x))$$



→ Intro & motivation, getting started



- → Intro & motivation, getting started
- → Foundations & Principles
  - Lambda Calculus, natural deduction
  - Higher Order Logic, Isar (part 1)
  - Term rewriting



- → Intro & motivation, getting started
- → Foundations & Principles
  - Lambda Calculus, natural deduction
  - Higher Order Logic, Isar (part 1)
  - Term rewriting
- → Proof & Specification Techniques
  - Inductively defined sets, rule induction
  - Datatypes, recursion, induction, Isar (part 2)
  - · Hoare logic, proofs about programs, invariants
  - C verification
  - Practice, questions, examp prep



	Rough timeline
→ Intro & motivation, getting started	[today]
<ul> <li>→ Foundations &amp; Principles</li> <li>Lambda Calculus, natural deduction</li> <li>Higher Order Logic, Isar (part 1)</li> <li>Term rewriting</li> </ul>	[1,2] [3 <sup>a</sup> ] [4]
<ul> <li>Proof &amp; Specification Techniques</li> <li>Inductively defined sets, rule induction</li> <li>Datatypes, recursion, induction, Isar (part 2)</li> <li>Hoare logic, proofs about programs, invariants</li> <li>C verification</li> <li>Practice, questions, examp prep</li> </ul>	[5] [6, 7 <sup>b</sup> ] [8] [9] [10 <sup>c</sup> ]

<sup>&</sup>lt;sup>a</sup>a1 due; <sup>b</sup>a2 due; <sup>c</sup>a3 due





→ attend lectures



- → attend lectures
- → try Isabelle early



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- → redo all the demos alone



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- → try the exercises/homework we give, when we do give some



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#### → DO NOT CHEAT

- Assignments and exams are take-home. This does NOT mean you can work in groups. Each submission is personal.
- For more info, see Plagiarism Policy<sup>a</sup>

a https://student.unsw.edu.au/plagiarism

### **Credits**



some material (in using-theorem-provers part) shamelessly stolen from







Tobias Nipkow, Larry Paulson, Markus Wenzel





David Basin, Burkhardt Wolff

Don't blame them, errors are ours



to prove



#### to prove

→ from Latin probare (test, approve, prove)

# DATA DATA (Merriam-Webster)

#### to prove

- → from Latin probare (test, approve, prove)
- → to learn or find out by experience (archaic)



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- → to establish the existence, truth, or validity of (by evidence or logic) prove a theorem, the charges were never proved in court



#### to prove

- → from Latin probare (test, approve, prove)
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### pops up everywhere

- → politics (weapons of mass destruction)
- → courts (beyond reasonable doubt)
- → religion (god exists)
- → science (cold fusion works)

# What is a mathematical proof?



In mathematics, a proof is a demonstration that, given certain axioms, some statement of interest is necessarily true. (Wikipedia)

**Example:**  $\sqrt{2}$  is not rational.

Proof:

# What is a mathematical proof?



In mathematics, a proof is a demonstration that, given certain axioms, some statement of interest is necessarily true. (Wikipedia)

**Example:**  $\sqrt{2}$  is not rational.

Proof: assume there is  $r \in \mathbb{Q}$  such that  $r^2 = 2$ .

Hence there are mutually prime p and q with  $r = \frac{p}{q}$ .

Thus  $2q^2 = p^2$ , i.e.  $p^2$  is divisible by 2.

2 is prime, hence it also divides p, i.e. p = 2s.

Substituting this into  $2q^2 = p^2$  and dividing by 2 gives  $q^2 = 2s^2$ . Hence, a is also divisible by 2. Controllisting. Only

q is also divisible by 2. Contradiction. Qed.

### Nice, but...

DATA SIRO

- → still not rigorous enough for some
  - what are the rules?
  - what are the axioms?
  - how big can the steps be?
  - what is obvious or trivial?
- → informal language, easy to get wrong
- → easy to miss something, easy to cheat

### Nice, but...



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Theorem. A cat has nine tails.

**Proof.** No cat has eight tails. Since one cat has one more tail than no cat, it must have nine tails.

### What is a formal proof?



A derivation in a formal calculus

### What is a formal proof?



#### A derivation in a formal calculus

**Example:**  $A \wedge B \longrightarrow B \wedge A$  derivable in the following system

Rules: 
$$\frac{X \in S}{S \vdash X}$$
 (assumption)  $\frac{S \cup \{X\} \vdash Y}{S \vdash X \longrightarrow Y}$  (impl)

$$\frac{S \vdash X \quad S \vdash Y}{S \vdash X \land Y} \text{ (conjl)} \quad \frac{S \cup \{X, Y\} \vdash Z}{S \cup \{X \land Y\} \vdash Z} \text{ (conjE)}$$

### What is a formal proof?



#### A derivation in a formal calculus

**Example:**  $A \wedge B \longrightarrow B \wedge A$  derivable in the following system

Rules: 
$$\frac{X \in S}{S \vdash X}$$
 (assumption)  $\frac{S \cup \{X\} \vdash Y}{S \vdash X \longrightarrow Y}$  (impl)  $\frac{S \vdash X \quad S \vdash Y}{S \vdash X \land Y}$  (conjl)  $\frac{S \cup \{X, Y\} \vdash Z}{S \cup \{X \land Y\} \vdash Z}$  (conjE)

#### Proof:

1. 
$$\{A, B\} \vdash B$$
 (by assumption)  
2.  $\{A, B\} \vdash A$  (by assumption)

2. 
$$\{A, B\} \vdash A$$
 (by assumption)  
3.  $\{A, B\} \vdash B \land A$  (by conjl with 1 and 2)

3. 
$$\{A, B\} \vdash B \land A$$
 (by conjl with 1 and 2)  
4.  $\{A \land B\} \vdash B \land A$  (by conjE with 3)  
5.  $\{\} \vdash A \land B \longrightarrow B \land A$  (by impl with 4)

### What is a theorem prover?



#### Implementation of a formal logic on a computer.

- → fully automated (propositional logic)
- → automated, but not necessarily terminating (first order logic)
- → with automation, but mainly interactive (higher order logic)

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#### Implementation of a formal logic on a computer.

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- → automated, but not necessarily terminating (first order logic)
- → with automation, but mainly interactive (higher order logic)
- → based on rules and axioms
- → can deliver proofs

There are other (algorithmic) verification tools:

- → model checking, static analysis, ...
- → usually do not deliver proofs
- → See COMP3153: Algorithmic Verification



→ Analysing systems/programs thoroughly



- → Analysing systems/programs thoroughly
- → Finding design and specification errors early



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- → Finding design and specification errors early
- → High assurance (mathematical, machine checked proof)



- → Analysing systems/programs thoroughly
- → Finding design and specification errors early
- → High assurance (mathematical, machine checked proof)
- → it's not always easy
- → it's fun

## Main theorem proving system for this course





#### Isabelle

→ used here for applications, learning how to prove



A generic interactive proof assistant



#### A generic interactive proof assistant

#### → generic:

not specialised to one particular logic (two large developments: HOL and ZF, will mainly use HOL)



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#### → interactive:

more than just yes/no, you can interactively guide the system



#### A generic interactive proof assistant

- → generic:
  - not specialised to one particular logic (two large developments: HOL and ZF, will mainly use HOL)
- → interactive: more than just yes/no, you can interactively guide the system
- → proof assistant: helps to explore, find, and maintain proofs

### Why Isabelle?



- → free
- → widely used systems
- → active development
- → high expressiveness and automation
- → reasonably easy to use

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- → free
- → widely used systems
- → active development
- → high expressiveness and automation
- → reasonably easy to use
- → (and because we know it best ;-))







#### No. because:

1 hardware could be faulty



- hardware could be faulty
- ② operating system could be faulty



- ① hardware could be faulty
- ② operating system could be faulty
- ③ implementation runtime system could be faulty



- ① hardware could be faulty
- ② operating system could be faulty
- ③ implementation runtime system could be faulty
- compiler could be faulty



- ① hardware could be faulty
- ② operating system could be faulty
- 3 implementation runtime system could be faulty
- ④ compiler could be faulty
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- 6 logic could be inconsistent



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- ② operating system could be faulty
- 3 implementation runtime system could be faulty
- ④ compiler could be faulty
- ⑤ implementation could be faulty
- 6 logic could be inconsistent
- ① theorem could mean something else



No. but:



#### No. but:

probability for

→ OS and H/W issues reduced by using different systems



#### No. but:

- → OS and H/W issues reduced by using different systems
- → runtime/compiler bugs reduced by using different compilers



#### No. but:

- → OS and H/W issues reduced by using different systems
- → runtime/compiler bugs reduced by using different compilers
- → faulty implementation reduced by having the right prover architecture



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- → inconsistent logic reduced by implementing and analysing it



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- → wrong theorem reduced by expressive/intuitive logics



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probability for

- → OS and H/W issues reduced by using different systems
- → runtime/compiler bugs reduced by using different compilers
- → faulty implementation reduced by having the right prover architecture
- → inconsistent logic reduced by implementing and analysing it
- → wrong theorem reduced by expressive/intuitive logics

No guarantees, but assurance immensly higher than manual proof



Soundness architectures careful implementation

PVS



Soundness architectures

careful implementation PVS

LCF approach, small proof kernel HOL4

Isabelle

## If I prove it on the computer, it is correct, right?



Sound	Iness	archi	tectures

careful implementation PVS

LCF approach, small proof kernel HOL4

Isabelle

explicit proofs + proof checker Coq

Twelf Isabelle

HOI 4

#### Meta Logic



#### Meta language:

The language used to talk about another language.

#### Meta Logic



#### Meta language:

The language used to talk about another language.

#### **Examples:**

English in a Spanish class, English in an English class

#### Meta Logic



#### Meta language:

The language used to talk about another language.

#### **Examples:**

English in a Spanish class, English in an English class

#### Meta logic:

The logic used to formalize another logic

#### Example:

Mathematics used to formalize derivations in formal logic

### Meta Logic – Example



Formulae:  $F ::= V \mid F \longrightarrow F \mid F \wedge F \mid False$ 

Syntax: V := [A - Z]

Derivable:  $S \vdash X$  X a formula, S a set of formulae

### Meta Logic – Example



Formulae:  $F ::= V \mid F \longrightarrow F \mid F \wedge F \mid False$ 

Syntax: V := [A - Z]

Derivable:  $S \vdash X$  X a formula, S a set of formulae

$$\begin{array}{ccc} & \log \operatorname{ic} & / & \operatorname{meta\ logic} \\ & & & \\ & & & \\ \hline & & \\ \hline$$

#### Isabelle's Meta Logic









**Syntax:**  $\bigwedge x$ . F (F another meta level formula) in ASCII: !!x. F





**Syntax:**  $\bigwedge x$ . F (F another meta level formula) in ASCII: !!x. F

→ universal quantifier on the meta level

→ used to denote parameters

→ example and more later





**Syntax:**  $A \Longrightarrow B$  (A, B other meta level formulae) in ASCII:  $A \Longrightarrow B$ 





**Syntax:**  $A \Longrightarrow B$  (A, B other meta level formulae) in ASCII:  $A \Longrightarrow B$ 

#### Binds to the right:

$$A \Longrightarrow B \Longrightarrow C = A \Longrightarrow (B \Longrightarrow C)$$

#### **Abbreviation:**

$$[\![A;B]\!] \Longrightarrow C = A \Longrightarrow B \Longrightarrow C$$

- → read: A and B implies C
- → used to write down rules, theorems, and proof states



**mathematics:** if x < 0 and y < 0, then x + y < 0



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**formal logic:**  $\vdash x < 0 \land y < 0 \longrightarrow x + y < 0$ 

variation:  $x < 0; y < 0 \vdash x + y < 0$ 



**mathematics:** if x < 0 and y < 0, then x + y < 0

**formal logic:**  $\vdash x < 0 \land y < 0 \longrightarrow x + y < 0$  variation:  $x < 0; y < 0 \vdash x + y < 0$ 

variation:  $x < 0; y < 0 \vdash x + y < 0$ 

Isabelle:lemma " $x < 0 \land y < 0 \longrightarrow x + y < 0$ "variation:lemma " $\llbracket x < 0; y < 0 \rrbracket \Longrightarrow x + y < 0$ "



**mathematics:** if x < 0 and y < 0, then x + y < 0

**formal logic:**  $\vdash x < 0 \land y < 0 \longrightarrow x + y < 0$ 

variation: x < 0;  $y < 0 \vdash x + y < 0$ 

**Isabelle:** lemma " $x < 0 \land y < 0 \longrightarrow x + y < 0$ "

variation: lemma " $[x < 0; y < 0] \Longrightarrow x + y < 0$ "

variation: lemma

assumes "x < 0" and "y < 0" shows "x + y < 0"

## Example: a rule



logic:  $\frac{X}{X \wedge Y}$ 

#### Example: a rule



logic: 
$$\frac{X}{X \wedge Y}$$

variation: 
$$\frac{S \vdash X \quad S \vdash Y}{S \vdash X \land Y}$$

#### Example: a rule



logic: 
$$\frac{X}{X \wedge Y}$$

variation: 
$$\frac{S \vdash X \quad S \vdash Y}{S \vdash X \land Y}$$

**Isabelle:** 
$$[\![X;Y]\!] \Longrightarrow X \wedge Y$$

# Example: a rule with nested implication



$$\begin{array}{ccc} & X & Y \\ \vdots & \vdots \\ X \vee Y & Z & Z \end{array}$$

logic:

## Example: a rule with nested implication



$$\begin{array}{ccc} X & Y \\ \vdots & \vdots \\ X \lor Y & Z & Z \end{array}$$

logic:

$$\frac{S \cup \{X\} \vdash Z \quad S \cup \{Y\} \vdash Z}{S \cup \{X \lor Y\} \vdash Z}$$

variation:

## Example: a rule with nested implication



$$\begin{array}{ccc} X & Y \\ \vdots & \vdots \\ X \lor Y & Z & Z \\ \hline Z \end{array}$$

logic:

$$\frac{S \cup \{X\} \vdash Z \quad S \cup \{Y\} \vdash Z}{S \cup \{X \lor Y\} \vdash Z}$$

variation:

**Isabelle:** 
$$[\![X \lor Y; X \Longrightarrow Z; Y \Longrightarrow Z]\!] \Longrightarrow Z$$





**Syntax:**  $\lambda x. F$  (F another meta level formula)

in ASCII: %x. F





**Syntax:**  $\lambda x. F$  (F another meta level formula)

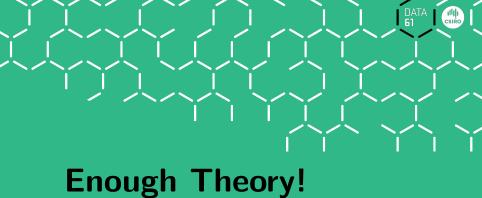
in ASCII: %x. F

→ lambda abstraction

→ used for functions in object logics

→ used to encode bound variables in object logics

→ more about this in the next lecture



**Getting started with Isabelle** 



Isabelle - generic, interactive theorem prover



**Isabelle** – generic, interactive theorem prover **Standard ML** – logic implemented as ADT



HOL, ZF – object-logics

Isabelle – generic, interactive theorem prover

Standard ML – logic implemented as ADT



Prover IDE (jEdit) – user interface

HOL, ZF – object-logics

Isabelle – generic, interactive theorem prover

Standard ML – logic implemented as ADT



**Prover IDE (jEdit)** – user interface **HOL**, **ZF** – object-logics **Isabelle** – generic, interactive theorem prover **Standard ML** – logic implemented as ADT

User can access all layers!

### **System Requirements**



- → Linux, Windows, or MacOS X (10.8 +)
- → Standard ML (PolyML implementation)
- → Java (for jEdit)

Premade packages for Linux, Mac, and Windows + info on: http://mirror.cse.unsw.edu.au/pub/isabelle/

#### **Documentation**



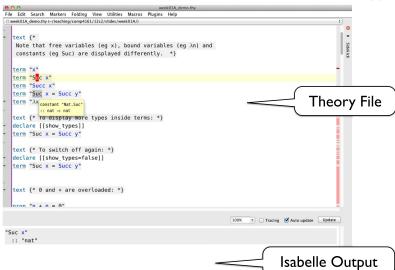
Available from http://isabelle.in.tum.de

- → Learning Isabelle
  - Concrete Semantics Book
  - Tutorial on Isabelle/HOL (LNCS 2283)
  - Tutorial on Isar
  - Tutorial on Locales
- → Reference Manuals
  - Isabelle/Isar Reference Manual
  - Isabelle Reference Manual
  - Isabelle System Manual
- → Reference Manuals for Object-Logics

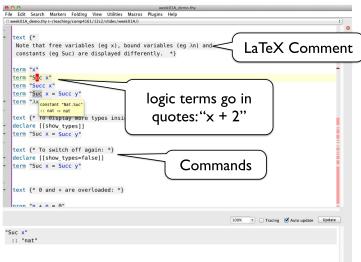


```
File Edit Search Markers Folding View Utilities Macros Plugins Help
week01A demo.thy (~/teaching/comp4161/12s2/slides/week01A/)
  text {*
   Note that free variables (eg x), bound variables (eg \lambdan) and
   constants (eg Suc) are displayed differently. *}
  term "x"
  term "Suc x"
  term "Succ x"
  term "Suc x = Succ y"
  term "\u03bax constant "Nat.Suc"
  text {* To display more types inside terms: *}
  declare [[show types]]
  term "Suc x = Succ y"
  text {* To switch off again: *}
  declare [[show types=false]]
  term "Suc x = Succ y"
  text {* 0 and + are overloaded: *}
  prop "n + n = 0"
                                                                                 ▼ Tracing ✓ Auto update Update
"Suc x"
:: "nat"
```

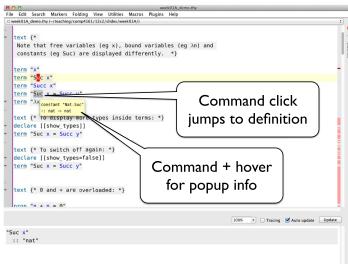




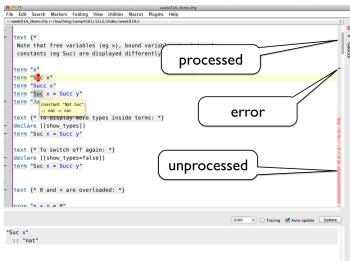














#### **Exercises**



- → Download and install Isabelle from http://mirror.cse.unsw.edu.au/pub/isabelle/
- → Step through the demo files from the lecture web page
- → Write your own theory file, look at some theorems in the library, try 'find\_theorems'
- → How many theorems can help you if you need to prove something containing the term "Suc(Suc x)"?
- → What is the name of the theorem for associativity of addition of natural numbers in the library?