

Workshop - Week 2: Solutions

$$1) a) \quad X = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix}$$

$$x_0 = y$$

$$x_1 = \frac{dy}{dt}$$

$$x_2 = \frac{d^2y}{dt^2}$$

$$\frac{d^3y}{dt^3} = -\frac{2}{3}y - \frac{5}{3}\frac{dy}{dt} - \frac{2}{3}\frac{d^2y}{dt^2}$$

System of

$$\begin{cases} \dot{x}_0 = x_1 \end{cases}$$

1st-order

$$\begin{cases} \dot{x}_1 = x_2 \end{cases}$$

ODEs

$$\begin{cases} \dot{x}_2 = -\frac{2}{3}x_0 - \frac{5}{3}x_1 - \frac{2}{3}x_2 \end{cases}$$

$$2) a) \quad X = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$x_0 = \theta_1$$

$$x_1 = \dot{\theta}_1$$

$$x_2 = \theta_2$$

$$x_3 = \dot{\theta}_2$$

$$\ddot{\theta}_1 = \frac{M_c + M_0}{I_1} - \frac{k}{I_1} \theta_1 - \frac{b}{I_1} \dot{\theta}_1 + \frac{k}{I_1} \theta_2 + \frac{b}{I_1} \dot{\theta}_2$$

$$\ddot{\theta}_2 = \frac{k}{I_2} \theta_1 + \frac{b}{I_2} \dot{\theta}_1 - \frac{k}{I_2} \theta_2 - \frac{b}{I_2} \dot{\theta}_2$$

System
of
1st-order
ODEs

$$\dot{x}_0 = x_1$$

$$\dot{x}_1 = \frac{M_c + M_0}{I_1} - \frac{k}{I_1} x_0 - \frac{b}{I_1} x_1 + \frac{k}{I_1} x_2 + \frac{b}{I_1} x_3$$

$$\dot{x}_2 = x_3$$

$$x_3 = \frac{k}{I_2} x_0 + \frac{b}{I_2} x_1 - \frac{k}{I_2} x_2 - \frac{b}{I_2} x_3$$

$$3) 9) \quad X = \begin{bmatrix} x \\ v \end{bmatrix}$$

$$x = x$$

$$v = \dot{x}$$

$$\ddot{x} = \frac{F}{m} - \frac{k}{m} x - \frac{c}{m} \dot{x}$$

System
of
1st-order
ODEs

$$\begin{cases} \dot{x} = v \\ v = \frac{F}{m} - \frac{k}{m} x - \frac{c}{m} v \end{cases}$$