# Workshop - Week 2: Modelling of Mechanical Systems

1. Consider the ordinary differential equation (ODE)

$$3\frac{d^3y}{dt^3} + 2\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 2y = 0$$

   (a) Rewrite this as a system of three first-order ODEs, defining $x_0 = y$ and

$$x_i = \frac{d^iy}{dt^i}$$

   for $i = 1,2$.

   (b) Given initial conditions of $y(0) = 1$, $\frac{dy}{dt}(0) = 0$ and $\frac{d^2y}{dt^2}(0) = 0$ Use the MATLAB function ode45 to simulate the system for 50s. You will find it useful to define an anonymous function for the ODE system using the syntax

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odefn = @(T,X) (...).
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2. Consider the dynamic model for the read/write head of a disk drive shown below:

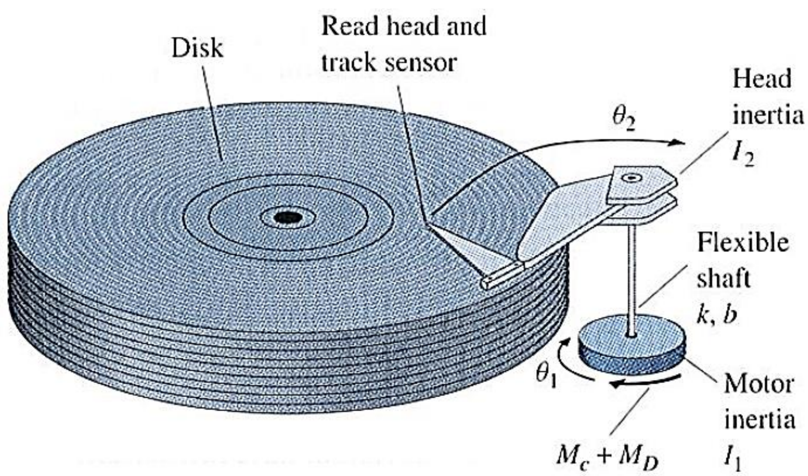



*Figure 1: Dynamic model for the read/write head of a disk drive[1]*

When the moments are summed, equated to the angular accelerations and re-arranged the following two second-order ODEs are obtained:

$$I_1\ddot{\theta}_1 + b(\dot{\theta}_1 - \dot{\theta}_2) + k(\theta_1 - \theta_2) = M_C + M_D$$
$$I_2\ddot{\theta}_2 + b(\dot{\theta}_2 - \dot{\theta}_1) + k(\theta_2 - \theta_1) = 0$$

   (a) Rewrite this as a system of four first-order ODEs.

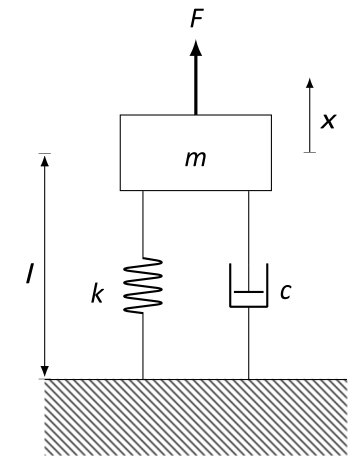
   (b) The motor and read/write head inertia values are estimated as $I_1 = 0.001kgm^2$ and $I_2 = 0.002kgm^2$ respectively. The shaft spring value is estimated as $k = 10Nmrad^{-1}$ and the shaft damping value as $b = 0.004Nmrad^{-1}s$. The input torque from the motor is set as a sinusoidal function with an amplitude $A = 10\ Nm$ and frequency $f = 100Hz$ (i.e. $M_C +$

[1] Feedback Control of Dynamic Systems, Franklin, Powell and Emami-Naeini, Pearson Prentice Hall, 2006, p. 31

$M_D = A\,sin(2\pi f t)$). Assuming that the read/write head is initially stationary (all initial conditions are zero), use the MATLAB function ode45 to simulate the system for 200ms.

Is there anything you notice about the propagation of oscillations in this system from the motor to the read/write head?

3. Consider a vibration isolation platform modelled by the mass-spring-damper system shown below:




The parameters are given by $m = 100kg$, $c = 100Nsm^{-1}$ and $k = 5 \times 10^5\ Nm^{-1}$. The length from the ground to the platform is *l* = 2m when it is at rest. The ODE describing the vertical motion of the mass is given by

$$m\ddot{x} + c\dot{x} + kx = F,$$

where $x$ denotes the distance of the mass from rest.

(a) Rewrite the second-order ODE as two first-order ODEs, in terms of the distance $x$ and velocity $v$.

(b) Assume that a load of $m_2 = 500kg$ is rigidly placed on the platform. The additional force which compresses the spring is given by $F = -m_2 g$, where $g = 9.81ms^{-2}$. Use the MATLAB function ode45 to simulate the system for 10s, assuming that the platform is initially at rest before the weight is applied to it (i.e. $x(0) = 0m$ and $v(0) = 0ms^{-1}$). From the simulation data, determine the new height of the mass from the ground when it comes to rest.

(c) The fixed load is replaced with a sinusoidal load, having amplitude $A = 1 \times 10^5\ N$ and frequency $f = 5Hz$. Rerun the simulation and determine the amplitude of steady-state oscillation (hint: the system will be close to steady-state after 7s). Notice how the resulting response has two parts, a *transient response* which decays away and a *steady-state response*. The mixture of frequencies seen in the first few seconds come from the superposition of both of these responses.

(d) Repeat the previous step for frequencies $f = 5,6,7,\ldots,15,20,30,40,50\ Hz$ and create a log-log plot of the steady-state amplitude of *x* vs frequency (hint: the function loglog will be useful). From this plot, identify the approximate frequency where the response peaks.

(e) Extension: Keeping $A = 1 \times 10^5\ N$ and setting the excitation frequency to $f = 10Hz$, create a log-log plot of the steady-state amplitude when $c = 100,200,300,\ldots,1000,1500,2000\ Nsm^{-1}$.