COMP20007 Design of Algorithms

Hashing

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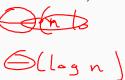
Lecture 15

Semester 1, 2020

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- Average case performance for Search, Insert and Delete: $\Theta(1)$
- Requires a hash function: $h(K) \rightarrow i \in [0, m-1]$.
- A hash function should:
 - Be efficient $\Theta(1)$.
 - Distribute keys evenly (uniformly) along the table.

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- This is the *identity* hash function: h(K) = K.
- Note that $K \in [0, m-1]$. In other words we need to know the maximum number of keys in advance.
- Sometimes this is possible: postcodes, for example.
- Many times it is not:
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 - <u>m</u> is too large (need to preallocate)
 - Unbounded integers (student IDs)
 - Non-integer keys (games)

Hashing Integers

For large/unbounded integers, an alternative function is

$$h(K) = \frac{K \mod m}{m}$$

$$17 \mod 9 = 8$$

$$20 \mod 9 = 2$$

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Hashing Integers

- For large/unbounded integers, an alternative function is $h(K) = K \mod m$
- Allow us to set the size m.
- Small m results in lots of collisions, large m takes excessive memory. Best m will vary.

Hashing Strings

- Assume A \mapsto 0, B \mapsto 1, etc. \bigcirc -2.5
- Assum 26 characters and m = 101.
- Each character can be mapped to a *binary* string of length 5 ($2^5 = 32$). > 2.6

Hashing Strings

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We can think of a string as a long binary number:

So 64 is the position of string M Y K E Y in the hash table.

Hashing Strings

We deliberately chose m to be prime.

$$13379736 = \underbrace{12 \times 32^4 + 24 \times 32^3 + 10 \times 32^2 + 4 \times 32}_{24}$$

With $\underline{m} = 32$, the hash value of any key is the last character's value! $\sim 2^{s}$

Hashing Long Strings

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Then we have

$$h(s) = (\sum_{i=0}^{|s|-1} chr(s_i) \times 32^{|s|-i-1}) \mod m$$
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where m is a prime number. For example,

$$h(V E R Y L O N G K E Y) = (21 \times 32^{10}) \cdot 4 \times 32^{9} + \cdots) \mod 101$$

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$$h(V E R Y L O N G K E Y) = (21 \times 32^{10} + 4 \times 32^{9} + \cdots) \mod 101$$

The term between parenthesis can become quite large and result in overflow.

Horner's Rule

Instead of

$$21 \times 32^{10} + 4 \times 32^{9} + 17 \times 32^{8} + 24 \times 32^{7} \cdots$$

factor out repeatedly:

$$(\cdots (21 \times 32 + 4) \times 32 + 17) \times 32 + \cdots) + 24$$

$$21 \times (32) \times 32 + 4 \times (32) =$$

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factor out repeatedly:

$$(\cdots ((21 \times 32 + 4) \times 32 + 17) \times 32 + \cdots) + 24 \mod n$$

Now utilize these properties of modular arithmetic:

$$(x + y) \mod m = ((x \mod m) + (y \mod m)) \mod m$$

 $(x \times y) \mod m = ((x \mod m) \times (y \mod m)) \mod m$

So for each sub-expression it suffices to take values modulo m.

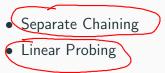
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We saw two solutions:

- Separate Chaining
- Linear Probing

Practical efficiency will depend on the table **load factor**:

$$\alpha = n/m$$
 $N = #foial of-$

Assign multiple records per cell (usually through a linked list)

• Assuming even distribution of the *n* keys.

0<<<1 <>1

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- Almost same numbers for Insert and Delete.
- Worst case $\Theta(n)$ only with a bad hash function (load factor is more of an issue).
- Requires extra memory.

Linear Probing

Populate successive empty cells.

- Much harder analysis, simplified results show:
- A sucessful search requires $(1/2) \times (1 + 1/(1 \alpha))$ operations on average.
- An unsucessful search requires $(1/2) \times (1 + 1/(1 \alpha)^2)$ operations on average.
 - 0<0<1

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- Similar numbers for Insert. Delete virtually impossible.
- Does not require extra memory.
- Worst case $\Theta(n)$ with a bad hash function and/or clusters.

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Apply a second hash function in case of collision.

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Apply a second hash function in case of collision.

• First try:
$$h(K)$$

- Second try: (h(K) + s(K)) mod m
- Third try: $(h(K) + 2s(K)) \mod m$

UN CE35 1 (10)~1=m = 0 13

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Both Linear Probing and Double Hashing are sometimes referred as *Open Addressing* methods.

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- High load factors deteriorate the performance of a hash table (for linear probing, ideally we should have $\alpha < 0.9$).
- Rehashing allocates a new table (usually around double the size) and move every item from the previous table to the new one.
- Very expensive operation, but happens infrequently.

Hash Tables:

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- Requires good collision handling.

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That being said, if hashing is applicable, a well-tuned hash table will typically outperform BSTs.

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Next lecture: what happens if records/data is too large?