(1) $E(W) = 10$

$\sigma(W) = 4$

$V = 0.5 \cdot W + U$

$$cov(W, V) = cov(W, 0.5 \cdot W + U)$$

$$= cov(W, 0.5W) + cov(W, U)$$

$$= 0.5\, cov(W, W) + \emptyset$$

$$= 0.5\, \sigma^2(W) = 0.5 \times 16 = 8$$

$$cov(V, V) = \sigma^2(V)$$

$$= \sigma^2(0.5 \cdot W + U)$$

$$= 0.25\, \sigma^2(W) + \sigma^2(U)$$

$$= 0.25 \cdot 16 + 1$$

$$= 4 + 1 = 5$$

The variance - covariance matrix is

	W	V
W	16	8
V	8	5

(2) (a) $\boxed{E(Y|X) = \frac{X}{2}}$

since $f(x) = \frac{1}{x}$ for $0 \le y \le x$

(b)

$$E(Y) = E(E(Y|X))$$

$$= E\left(\frac{X}{2}\right)$$

$$= \int_0^1 \frac{x}{2} \cdot 1 \cdot dx$$

$$= \frac{x^2}{4}\Big|_0^1$$

$$\boxed{E(Y) = \frac{1}{4}}$$

(c) $E(XY) = E\left(E(XY|X)\right) = E(XE(Y|X)) = E\left(\frac{X^2}{2}\right)$

$$= \int_0^1 \frac{x^2}{2} dx = \frac{x^3}{6}\Big|_0^1 = \frac{1}{6}$$

$$\boxed{E(XY) = \frac{1}{6}}$$

(d) $Cov(X,Y) = E(XY) - E(X)E(Y) = \frac{1}{6} - \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{6} - \frac{1}{8}$

$$\boxed{cov(XY) = \frac{1}{24}}$$

(3) a

$$E(5M + 5N)$$

$$= 5\,E(M) + 5\,E(N)$$

$$= 5 \cdot \frac{10}{2} + 5 \cdot \frac{10}{2}$$

$$= \frac{25 + 50}{2} = \frac{75}{2}$$

$$\boxed{E(5M + 5N) = \frac{75}{2}}$$

|a|

3 (b) Variance

$$V(5M+5N) = 25\,V(M) + 25\,V(N)$$

$$= 25 \cdot \frac{1}{12} 5^2 + 25 \cdot \frac{10^2}{12}$$

$$\boxed{V(5M+5N) = \frac{3125}{12} = 260.14}$$

3(c) $$E(5M-5N) = 5\,E(M) - 5\,E(N)$$

$$= 5 \cdot \frac{5}{2} - 5 \cdot \frac{10}{2}$$

$$= \frac{25-10}{2} = \frac{15}{2}$$

$$\boxed{E(5M-5N) = \frac{15}{2}}$$

3(d) $$V(5M-5N) = 5^2\,V(M) + (-5)^2\,V(N)$$

$$\boxed{V(5M-5N) = \frac{3125}{12} = 260.42}$$

(4)

$$\text{cov}(X,Y) = E(XY) - E(X)E(Y)$$

$$= E\left[X(aX+b)\right] - E(X)\cdot E(aX+b)$$

$$= E(aX^2+bX) - E(X)\cdot\left[aE(X)+b\right]$$

$$= aE(X^2) - aE(X)^2$$

$$\text{cov}(X,Y) = a\,\sigma^2(X)$$

$$\sigma(Y) = \sigma(aX+b) = |a|\,\sigma(X)$$

$$\text{CORR}(X,Y) = \frac{\text{cov}(X,Y)}{\sigma(X)\sigma(Y)}$$

$$= \frac{a\,\sigma^2(X)}{|a|\,\sigma^2(X)}$$

$$\boxed{\text{CORR}(X,Y) = \frac{a}{|a|} = \begin{cases} 1 & a>0 \\ -1 & a<0 \end{cases}} \quad \text{Q.E.D.}$$