Time Series Analysis

Lecture 5

Vector Autoregressive (VAR) Models

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Unit Root Nonstationarity and Dickey–Fuller Test: Definition and Intuition

Unit Roots

Recall that a stochastic process has a unit root if 1 is a root of the process's characteristic equation. Such a process is nonstationary.

To detect the existence of unit roots, unit root tests can be used to determine if a series should be first differenced or regressed on a deterministic function of time to render the series stationary.

Dickey–Fuller Test

An **augmented Dickey–Fuller test (ADF)** is a test for a unit root in a time series. It is an augmented version of the original Dickey–Fuller test for a larger and more complicated set of time series models.

The test procedure for the ADF test is the same as the DF test, but it is applied to the following model:

$$\Delta y_t = \boxed{\alpha + \beta t} + \boxed{\gamma y_{t-1}} + \delta_1 \Delta y_{t-1} + \cdots + \delta_{p-1} \Delta y_{t-p+1} + \varepsilon_t,$$

The unit root test is then carried out under the null hypothesis $\boxed{\gamma = 0}$ against the alternative hypothesis of $\gamma < 0$.

Dickey–Fuller Test: Intuition

The intuition behind the test is that if the series is not integrated, then the lagged level of the series (y_{t-1}) will provide no relevant information in predicting the change in y_t besides the one obtained in the lagged changes (Δy_{t-k}).

In that case the $\gamma = 0$ null hypothesis is not rejected.

Dickey–Fuller Test (2)

Once a value for the test statistic

$$DF_\tau = \frac{\hat{\gamma}}{SE(\hat{\gamma})}$$

is computed, it can be compared to the relevant critical value for the Dickey–Fuller Test.

If the test statistic is less (this test is nonsymmetrical so we do not consider an absolute value) than the (larger negative) critical value, then the null hypothesis of $\gamma = 0$ is rejected and no unit root is present.

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