Discrete Response Model
Lecture 1

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Maximum Likelihood Estimation (1)

Maximum Likelihood Estimation

Suppose the success or failure of a field goal in football can be modeled with a Bernoulli(π) distribution. Let Y = 0 if the field goal is a failure and Y = 1 if the field goal is a success. Then the probability distribution for Y is:

$$P(Y = y) = \pi^y (1-\pi)^{1-y}$$

where π denotes the probability of success.

Suppose we would like to estimate π for a 40-yard field goal. Let $y_1, \ldots, y_n$ denote a random sample of observed field goal results at 40 yards. Thus, these y_is are either 0s or 1s. Given the resulting data $(y_1,\ldots,y_n)$, the "likelihood function" measures the plausibility of different values of π:

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$$
\begin{aligned}
L(\pi \mid y_1, \ldots, y_n) &= P(Y_1 = y_1) * P(Y_2 = y_2) * \quad * P(Y_n = y_n) \\
&= \prod_{i=1}^{n} P(Y_i = y_i) \\
&= \prod_{i=1}^{n} \pi^{y_i} (1-\pi)^{1-y_i} \\
&= \pi^{\sum_{i=1}^{n} y_i} (1-\pi)^{n-\sum_{i=1}^{n} y_i} \\
&= \pi^{w} (1-\pi)^{n-w}
\end{aligned}
$$

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