TIME SERIES ANALYSIS LECTURE 1

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Stationarity: Example 2

2. A Stochastic Model With a Deterministic Linear Trend

A Stochastic Model with a Linear Trend

Consider a model with a deterministic linear trend:

$$x_t = \boxed{a + bt} + w_t$$

where $\cancel{x_t}$ w_t is a white noise with mean 0 and variance σ_w^2

The expected value of x_t is

$$\begin{aligned} E(x_t) &= E(a + bt + w_t) \\ &= a + bt + E(w_t) \\ &= a + bt \end{aligned}$$

2. A Stochastic Model With a Deterministic Linear Trend (2)

As such, a stochastic model with a linear trend is not mean stationary, as the mean changes with time. If $b > 0$, then the mean is an increasing function of the time index t. On the other hand, if $b < 0$, then the mean is an decreasing function of the time index t.

$$
\begin{aligned}
Var(x_t) &= Var(a + bt + w_t) \\
&= Var(w_t) \\
&= \sigma_w^2
\end{aligned}
$$

which is a constant. So, while the model is not mean stationary, it is variance stationary.

2. A Stochastic Model With a Deterministic Linear Trend (3)

$$\begin{aligned} Cov(x_t, x_{t-1}) &= Cov\left(a + bt + w_t, a + b(t-1) + w_{t-1}\right) \\ &= Cov\left(w_t, w_{t-1}\right) \\ &= 0 \end{aligned}$$

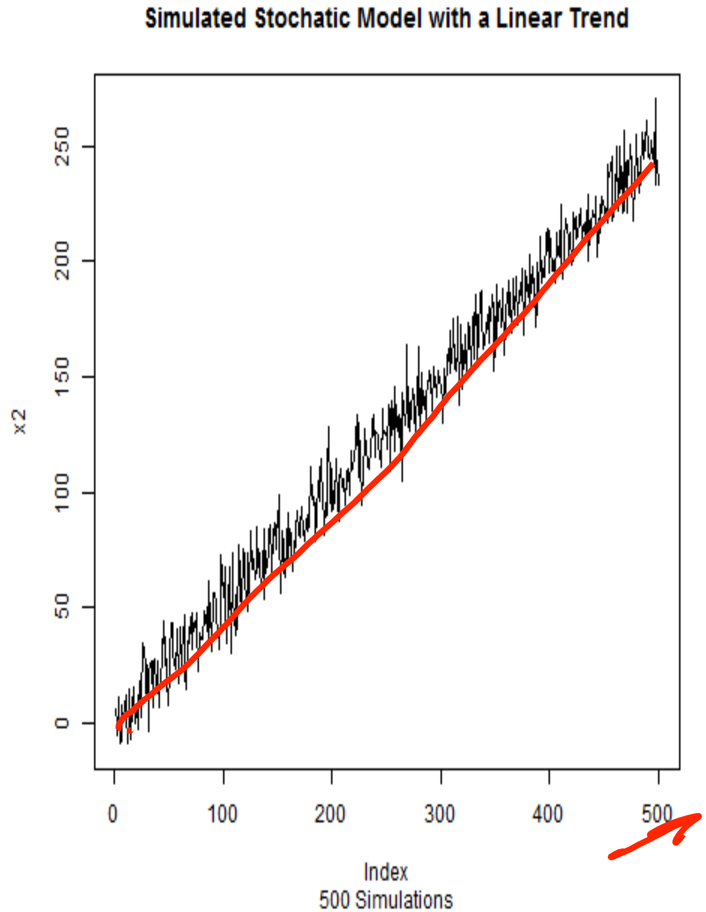
since w_t is a white noise series.

Therefore, the stochastic model with a deterministic linear trend is not (strictly or weakly) stationary, although as we will see in a few lectures, it can be easily transformed into a stationary model.

2. A Stochastic Model With a Deterministic Linear Trend: Simulations

$x_t = 1 + 0.5t + w_t$ where w_t is a series of independent Gaussian white noise with mean 0 and variance 10

```
set.seed(898)
sigma_w = 10
beta0 = 1
beta1 = 0.5
t = seq(1,500)
w <- rnorm(500,0,sigma_w)
x2 <- beta0 + beta1*t + w
cbind(t, x2, w)
summary(x2)
mean(x2)
sd(x2)
```



- The simulated series appears as a linear trend, as if the white noise does not affect the trend.
- The acf shows that the series is very persistence, meaning that is highly correlated with its lags.

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