Discrete Response Model
Lecture 1

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Alternative Confidence Intervals and True Confidence Level

Alternatives (in Practice)

For n < 40, use the Wilson or Jeffrey's prior interval. Below is Wilson's interval. Jeffrey's prior interval is a Bayesian-based CI.

$$\pi \pm \frac{Z_{1-\alpha/2}n^{1/2}}{n + Z^2_{1-\alpha/2}} \sqrt{\hat{\pi}(1 - \hat{\pi}) + \frac{Z^2_{1-\alpha/2}}{4n}}$$

where

$$\pi = \frac{w + Z^2_{1-\alpha/2}/2}{n + Z^2_{1-\alpha/2}}$$

Alternatives (in Practice)

For n ≥ 40, use the Agresti-Coull (Agresti and Coull, 1998) interval

The (1-α)100% confidence interval is

$$\tilde{\pi} \pm Z_{1-\alpha/2}\sqrt{\frac{\tilde{\pi}(1-\tilde{\pi})}{n+Z^2_{1-\alpha/2}}}$$

This is essentially a Wald interval where we add $Z^2_{1-\alpha/2}/2$ successes and $Z^2_{1-\alpha/2}/2$ failures to the observed data. In fact, when α = 0.05, $Z_{1-\alpha/2} = 1.96 \approx 2$. Then

$$\tilde{\pi} = \frac{w+2^2/2}{n+2^2} = \frac{w+2}{n+4}$$

Thus, two successes and two failures are added. Also, notice how

$$\tilde{\pi} = \frac{w+Z^2_{1-\alpha/2}/2}{n+Z^2_{1-\alpha/2}}$$

can be thought of as an adjusted estimate of π. For values of w close to 0, $\tilde{\pi} > \hat{\pi}$ For values of w close to n, $\tilde{\pi} < \hat{\pi}$.

True Confidence Levels for Confidence Intervals

Below is a comparison of the performance of the four confidence intervals. The values on the y-axis represent the true confidence level (coverage) of the confidence intervals. Each of the confidence intervals are supposed to be 95%!

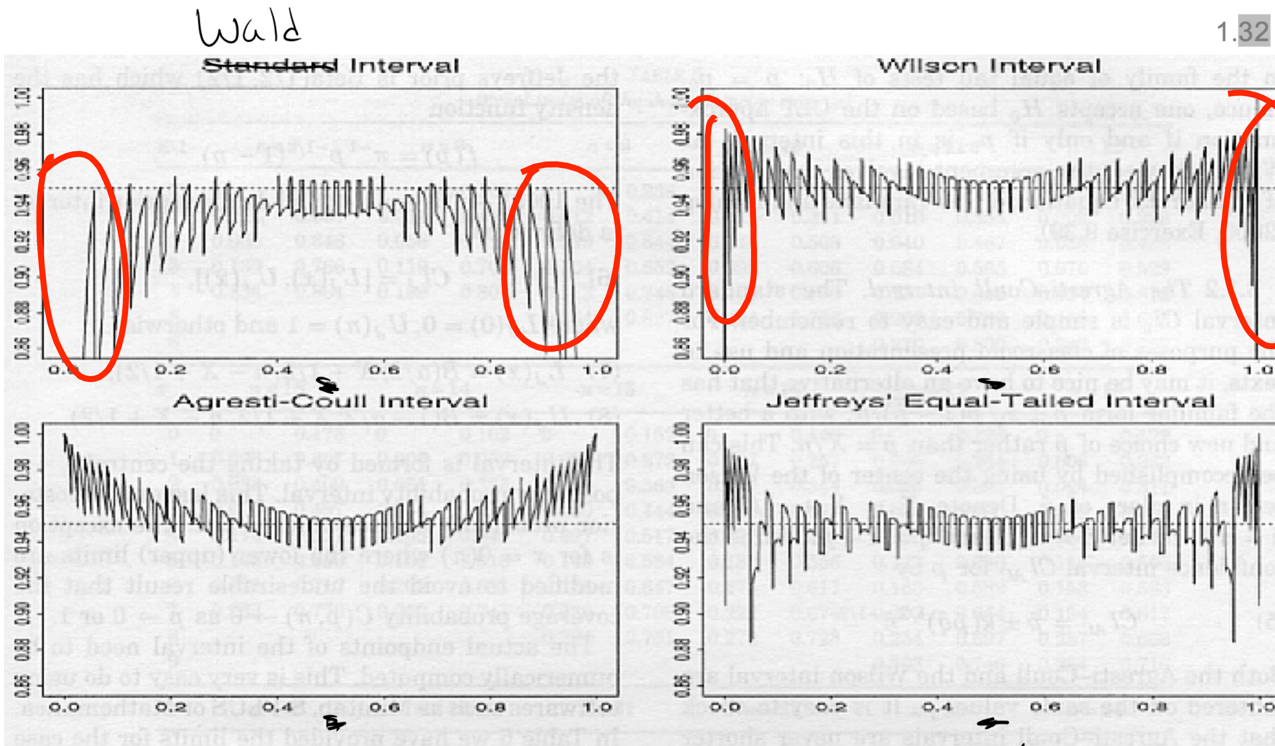

FIG. 5. *Coverage probability for n = 50.*

What Does True Confidence/Coverage Level Mean?

- Suppose a random sample of size n = 50 is taken from a population and a 95% Wald confidence interval is calculated.
- Suppose another random sample of size n = 50 is taken from the same population and a 95% Wald confidence interval is calculated.
- Repeat this process 10,000 times.
- We would expect 9,500 out of 10,000 (95%) confidence intervals to contain π.
- Unfortunately, this does not often happen. **It is guaranteed to happen only when n = ∞ for the Wald interval.**
- The true confidence or coverage level is the percent of times the confidence intervals contain or "cover" π.
- The plots show many possible values of π (0.0005 to 0.9995 by 0.0005). For example, the true confidence level using the Wald interval is approximately 0.90 for $\pi = 0.184$.

Calculate the True Confidence or Coverage Level in R

```
pi.hat<-w/n
pi.hat[1:10]
var.wald<-pi.hat*(1-pi.hat)/n
lower<-pi.hat - qnorm(p = 1-alpha/2) * sqrt(var.wald)
upper<-pi.hat + qnorm(p = 1-alpha/2) * sqrt(var.wald)
data.frame(w, pi.hat, lower, upper)[1:10,]
save<-ifelse(test = pi>lower, yes = ifelse(test =
    pi<upper, yes = 1, no = 0), no = 0)
save[1:10]
mean(save)
```

```
An estimate of the true confidence level is: 0.898
```

In this example, an estimate of the true confidence level is only 0.898 (and not 0.95)!

```
> data.frame(w, pi.hat, lower, upper)[1:20,]
    w pi.hat      lower     upper
1   9   0.18 0.07351063 0.2864894
2   9   0.18 0.07351063 0.2864894
3  10   0.20 0.08912769 0.3108723
4  11   0.22 0.10517889 0.3348211
5  12   0.24 0.12162077 0.3583792
6   9   0.18 0.07351063 0.2864894
7  11   0.22 0.10517889 0.3348211
8   5   0.10 0.01684577 0.1831542
9   8   0.16 0.05838385 0.2616161
10  6   0.12 0.02992691 0.2100731
11 16   0.32 0.19070178 0.4492982
12  8   0.16 0.05838385 0.2616161
13 11   0.22 0.10517889 0.3348211
14 10   0.20 0.08912769 0.3108723
15 15   0.30 0.17297982 0.4270202
16  5   0.10 0.01684577 0.1831542
17  7   0.14 0.04382187 0.2361781
18  8   0.16 0.05838385 0.2616161
19 11   0.22 0.10517889 0.3348211
20 10   0.20 0.08912769 0.3108723
```

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