Discrete Response Model
Lecture 5

Models for Count Response, Discrete Response Model Evaluation, and Model Selection

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Poisson Probability Model

Count Response Data

Count responses can also arise from other mechanisms that have nothing to do with Bernoulli trials. Examples include:

- The number of credit cards an individual owns
- The number of arrests for a city per year
- The number of people arriving at an airport on a given day
- The number of cars stopped at the 33rd and Holdrege streets intersection
- The number of people standing in line at a specific Starbucks between 6 AM and 7 PM

For these settings, a Poisson distribution can be used to model the count responses.

Poisson PMF

Poisson PMF:

$$P(Y = y) = \frac{e^{-\mu}\mu^{y}}{y!}$$

for $y = 0, 1, 2, \ldots$, where

Y is a random variable
y denotes the possible outcomes of Y
μ is a parameter that is greater than 0

We often write $Y \sim Po(\mu)$ as shorthand notation to mean that Y has a Poisson distribution with parameter μ.

Y will denote the number of occurrences of an event.

Properties

Below are characteristics of a Poisson distribution and associated items of interest:

- One can show the mean and variance of Y to be:

  $E(Y) = \mu$ and $Var(Y) = \mu$

  Having the mean and variance BOTH equal to μ is nice, but this is also limiting. Often, the actual variability observed in a sample is GREATER than μ.

  This is referred to as overdispersion.

  If $Y_1, \ldots, Y_n$ are independent with distribution $Po(\mu)$, then $\sum_{k=1}^{n} Y_k \sim Po(\sum_{k=1}^{n} \mu = n\mu)$.

  If each Y_k had a different mean, we would have $\sum_{k=1}^{n} Y_k \sim Po(\sum_{k=1}^{n} \mu_k)$

Properties

Likelihood function:

$$L(\mu; y_1 \ldots, y_n) = \prod_{k=1}^{n} \frac{e^{-\mu} \mu^{y_k}}{y_k\,!}$$

MLE for μ is $\hat{\mu} = n^{-1} \sum_{k=1}^{n} y_k$, i.e., the sample mean

The estimated variance for $\hat{\mu}$ is

$$\mathrm{Var}(\hat{\mu}) = -\left[E\left(\frac{\partial^2 \log[L(\mu \mid Y_1, \ldots, Y_n)]}{\partial \mu^2} \right) \right]^{-1} \Bigg|_{\mu=\hat{\mu}} = \frac{\hat{\mu}}{n}$$

Hypothesis Testing and Score CI for μ

$H_0: \mu = \mu_0$ vs. $H_a: \mu \neq \mu_0$

Wald test statistic: $$Z_0 = \frac{\hat{\mu} - \mu_0}{\sqrt{\hat{\mu} / n}}$$

Score test statistic: $$Z_0 = \frac{\hat{\mu} - \mu_0}{\sqrt{\mu_0 / n}}$$

Score CI for μ

$$\left(\hat{\mu} + \frac{Z_{1-\alpha/2}^2}{2n}\right) \pm Z_{1-\alpha/2}\sqrt{\frac{\hat{\mu} + Z_{1-\alpha/2}^2 / 4n}{n}}$$

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