TIME SERIES ANALYSIS LECTURE 1

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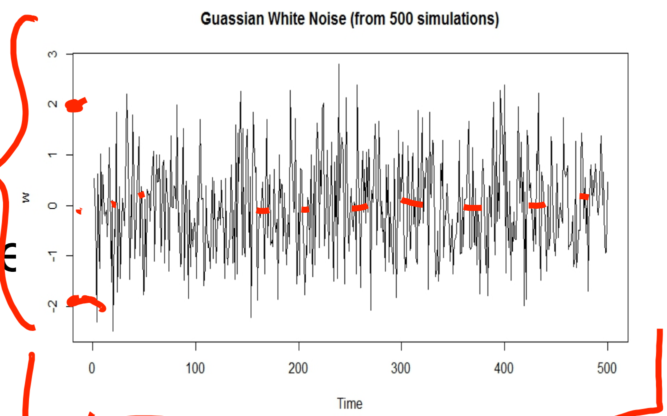
Simple Time Series Model Examples

Model 1: White Noise

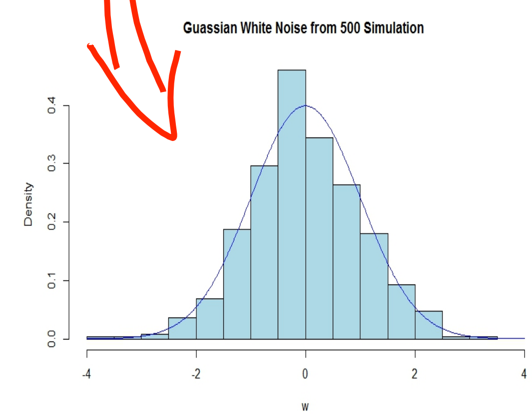
White Noise:

- The most general form of a **white noise** is a collection of uncorrelated random variables with mean **0** and variance
- To put more structure on the process, we impose assumptions on the underlying distribution. One popular set of assumptions is that each of the random variables is independently and identically distributed (**iid**) with a normal distribution with mean 0 and variance 1, leading to Gaussian WN.



Gaussian White Noise:

- Note that as the underlying distribution is a normal distribution, specifying the mean and variance completely characterize the distribution.
- The graph shows a simulated Gaussian white noise process.



Model 1: White Noise (2)

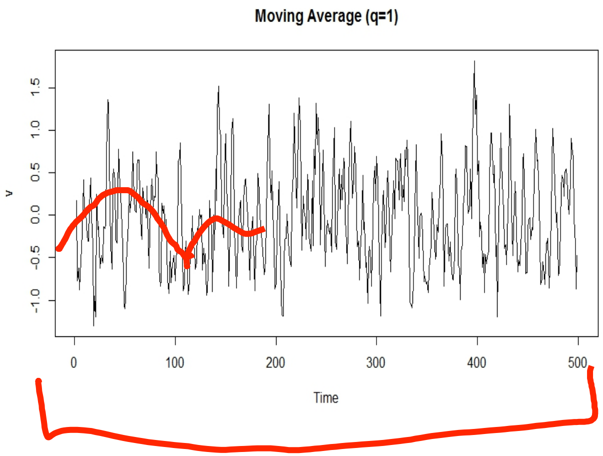
- White noise forms the most fundamental component in time series modeling and assumption testing.
- All of the time series models introduced in this course will have white noise as its stochastic components.
- As an example, a deterministic dynamic model, such as a population growth model, can be turned into a stochastic dynamic model using the addition of the white noise components.
- At this point, we will focus only on the dynamic pattern of the white noise using a time series plot and the distribution using techniques such as histogram or nonparametric kernel density.
- It is very important to note that representing a series using a histogram or density takes the time element away. It does not show the dependence of the series. And because in time series analysis, the main object of study in dependency among observations, representing a series using a histogram or density alone does not capture all of the important patterns embedded in the series.
- In later lectures, we will use other techniques to capture the dependency of a series.

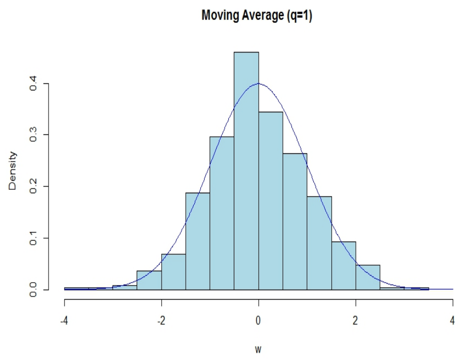
Model 2: Symmetric, Equal-Weight Moving Average Model

A **symmetric, equal-weight moving average model** takes the following general form:

The graph shows a series simulated using a centered-moving average model with

- Moving average is a common way to "smooth" (out the volatility of) a series where are Gaussian white noises introduced above.
- Moving average can be used to generate dependency among observations.

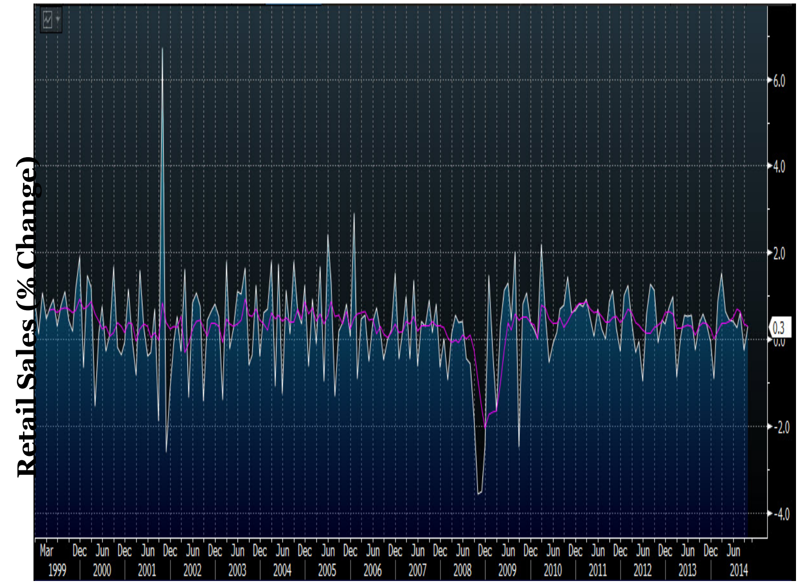




Model 2: Moving Averages: An Application

- The graph shows both the historical U.S. retail sales, measured in total monthly % change, from January 1999 to November 2014, and its six-month moving averages.
- Centered moving average is an example of a *smoothing* technique applied to the past values of the series.
- By "smoothing" out some of the variations and even seasonal effect, the underlying trend may emerge.
- In the case of adjusting for seasonality (i.e., "average-out" the seasonal effects), the length of the moving average is chosen so as to eliminate the seasonal pattern. (More on this point later.)

Retail Sales (Total Monthly % Change)



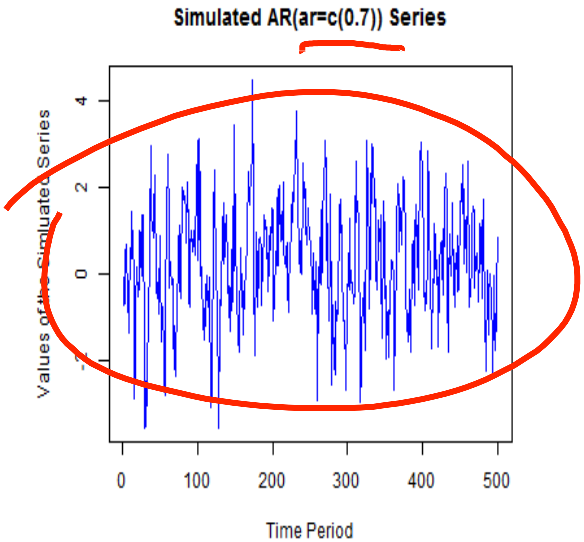
Source: Bloomberg

Model 3: Autoregressive Models

An autoregressive model with order p=2:

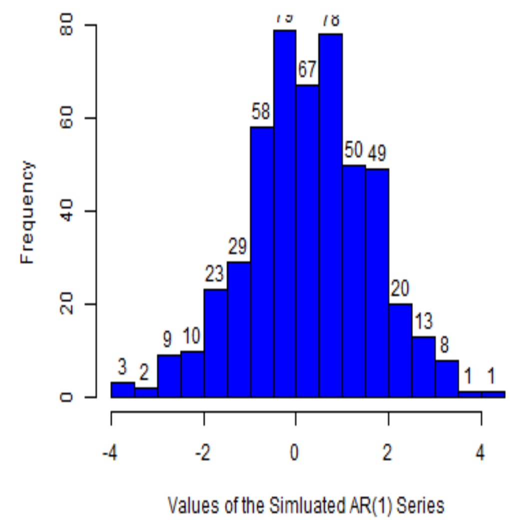
where is white noise (WN) and the are the coefficients of the model. We generally consider the series has its mean subtracted. Formally, we would write $\{x_t\}$.

$$\tilde{x}_t = x_t - \mu$$



The graph shows a simulated time series using the following zero-mean AR(1) model:

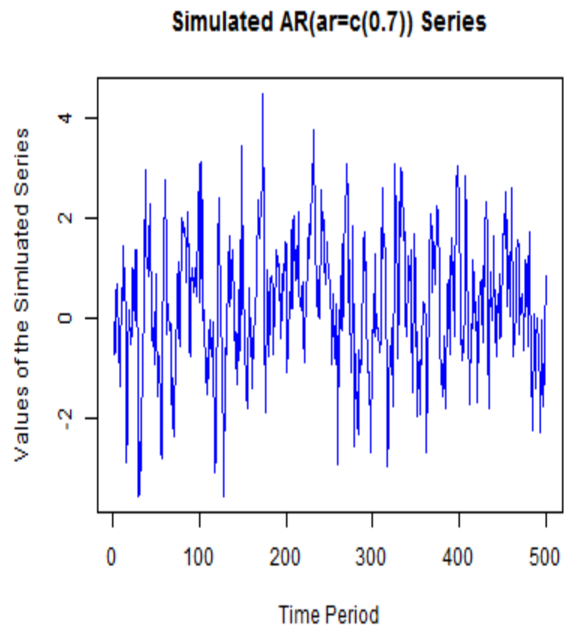
where

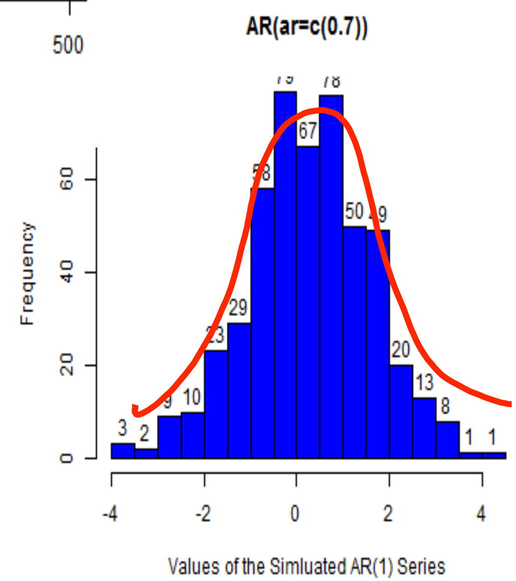


Note the simulated AR(a) series evolve around a constant level (mean = 0).

Model 3: Autoregressive Models (2)

The distribution of this simulated series looks fairly symmetric



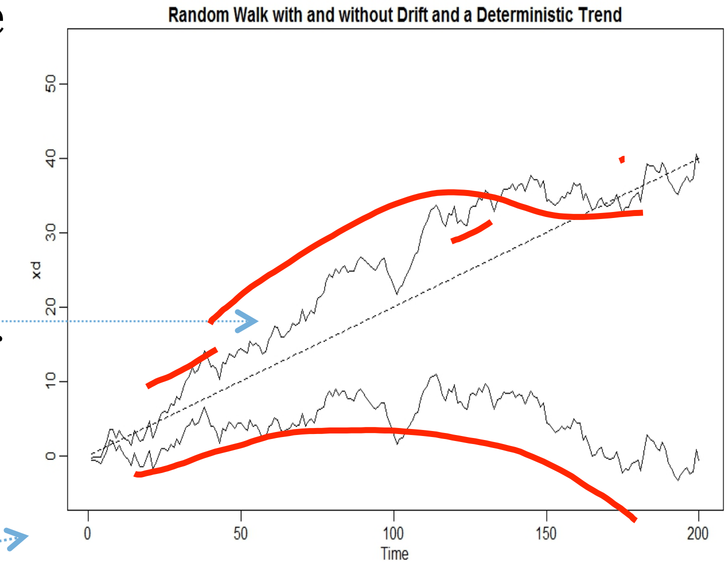


Model 4: Random Walk and Deterministic Trend

A form of **random walk (with drift)** can be expressed as + where

The graph shows three models:

1. A deterministic trend (i.e., is a deterministic function of the index . That is, knowing the value of will identify the value of



2. Random walk without drift

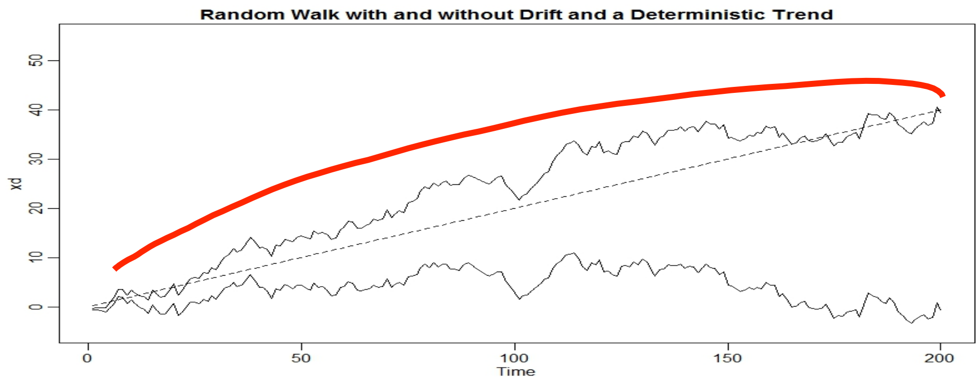
where is

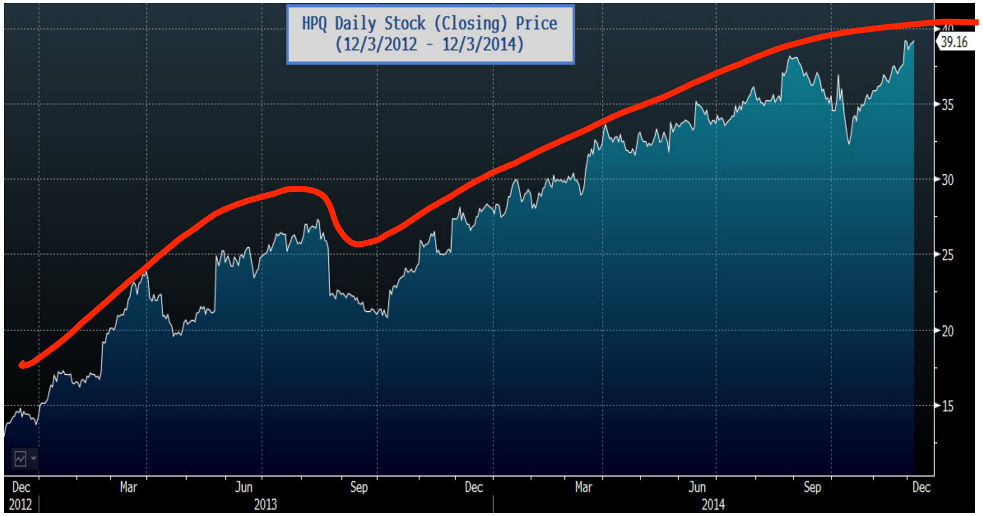
3. Random walk with drift

3.

Model 4: Random Walk and Deterministic Trend (2)

- Note the persistence of the simulated random walk with drift series (top curve in the top graph)–it gradually "drifts" upward and does not show any signs of reverting back to a the starting observed level (or a constant level).
- Observe the similarity (in the general pattern) between the simulated random walk with drift series and the Hewlett-Packard Company (HPQ) daily closing price series (from 12/3/2012 to 12/3/2014).





Source: Bloomberg

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