

TIME SERIES ANALYSIS

LECTURE 1

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Stationarity: Example 4

4. Autoregressive Model of Order 1

AR(1) An ~~AR~~AR(1) model takes the form

$$x_t = \alpha x_{t-1} + w_t$$

where w_t is a white noise series with mean zero and variance σ_w^2 .

The expected value of x_t is

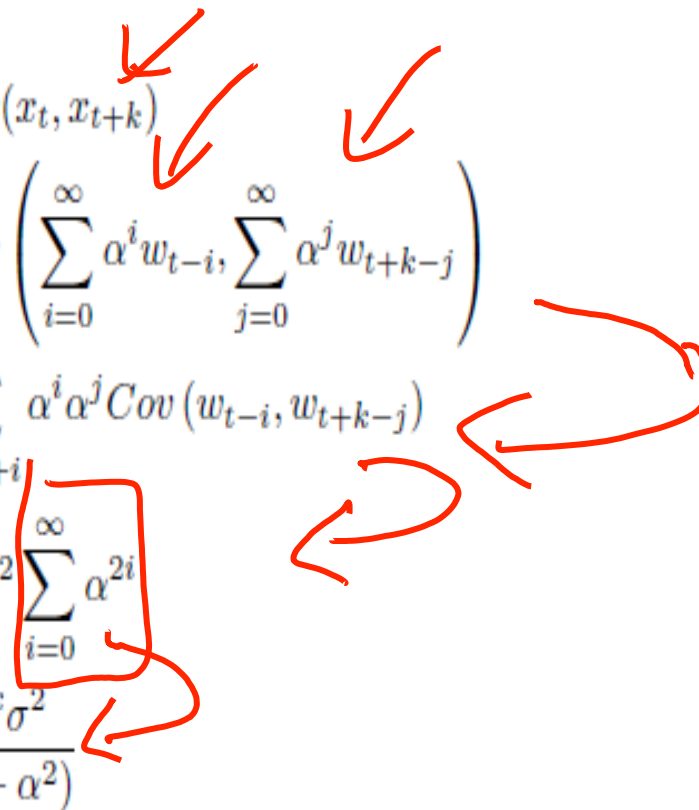
$$\begin{aligned} E(x_t) &= E(\alpha x_{t-1} + w_t) \\ &= \alpha E(x_{t-1}) + E(w_t) \\ &= \alpha E(x_{t-1}) \\ &= 0 \end{aligned}$$

because using recursive substitution, an $AR(1)$ model can be written as a linear process in terms of the sum of infinite white noises:

$$x_t = \sum_{i=0}^{\infty} \alpha^i w_{t-i}$$

4. Autoregressive Process of Order 1 (3)

With the linear process (of white noise) representation, the autocovariance can be derived as follow:



The derivation of the autocovariance for an AR(1) process is shown with several handwritten red annotations:

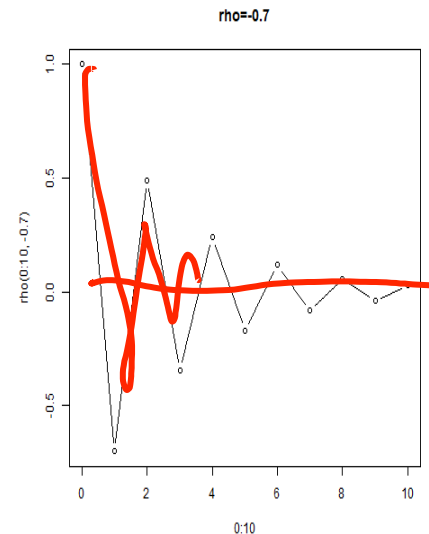
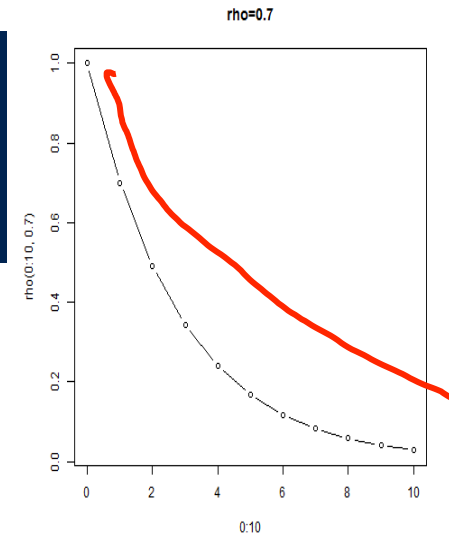
- Red arrows point from the x_{t+k} term in the first line to the $\sum \alpha^j w_{t+k-j}$ term in the second line.
- Red arrows point from the w_{t+k-j} term in the second line to the $Cov(w_{t-i}, w_{t+k-j})$ term in the third line.
- A red box highlights the summation $\sum_{i=0}^{\infty} \alpha^{2i}$ in the fourth line, with an arrow pointing to the final result.
- A large red arrow points from the third line down to the fourth line.

$$\begin{aligned}\gamma_k &= Cov(x_t, x_{t+k}) \\ &= Cov\left(\sum_{i=0}^{\infty} \alpha^i w_{t-i}, \sum_{j=0}^{\infty} \alpha^j w_{t+k-j}\right) \\ &= \sum_{j=k+i} \alpha^i \alpha^j Cov(w_{t-i}, w_{t+k-j}) \\ &= \alpha^k \sigma^2 \sum_{i=0}^{\infty} \alpha^{2i} \\ &= \frac{\alpha^k \sigma^2}{(1 - \alpha^2)}\end{aligned}$$

4. Autoregressive Process of Order 1: Simulations (1)

```
#-----
# Examine the exponential decay behavior of AR(1) correlogram
rho <- function(k, alpha) alpha^k
plot(0:10, rho(0:10, 0.7), type="b", main="rho=0.7")
plot(0:10, rho(0:10, -0.7), type="b", main="rho=-0.7")
#-----
```

- Top graph: The pattern of a theoretical AR(1) model with a positive correlation decays exponentially.
- Bottom graph: The patterns of a theoretical AR(1) model with a negative correlation oscillate between positive and negative correlation.



4. Autoregressive Process of Order 1: Simulations (1)

Recall that an AR(1) model takes the

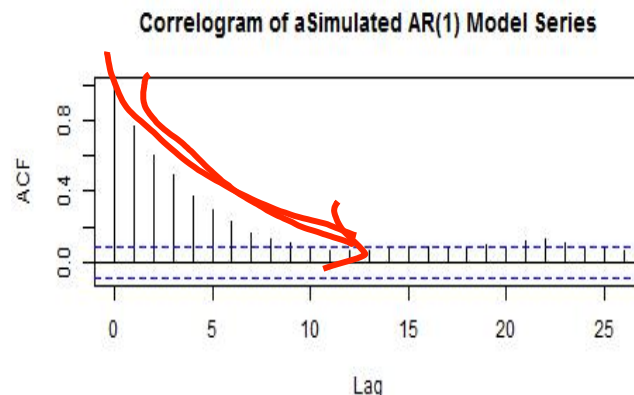
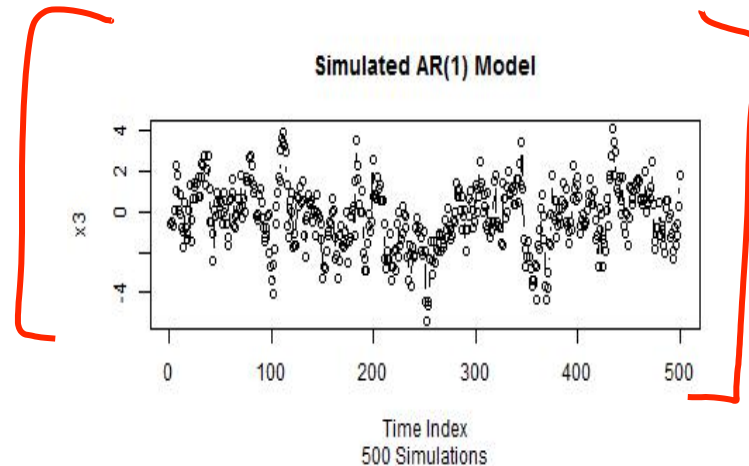
$$\tilde{x}_t = \alpha \tilde{x}_{t-1} + w_t \quad \text{form:}$$

where w_t is a white noise series with mean zero and variance σ_w^2

Consider \tilde{x}_t as series of deviations from μ , which is a parameter that determines the "level" of the process.

```
set.seed(898)
sigma_w = 1
alpha0 = 0
alpha1 = 0.8
x3 <- w <- rnorm(500,0,sigma_w)
for (t in 2:500) x3[t] <- alpha0 + alpha1*x3[t-1] + w[t]
```

- Note the exponential decay of the autocorrelation of the simulated AR(1) series.



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