# Discrete Response Model
# Lecture 2

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# Binomial Logistic Regression Model

# The Logit Transformation

- The $\log\left(\frac{\pi_i}{1-\pi_i}\right)$ transformation is often referred to as the <u>logit transformation</u>:

$$\mathrm{logit}(\pi_i) = \beta_0 + \beta_1 x_{i1} + \quad + \beta_p x_{ip}$$

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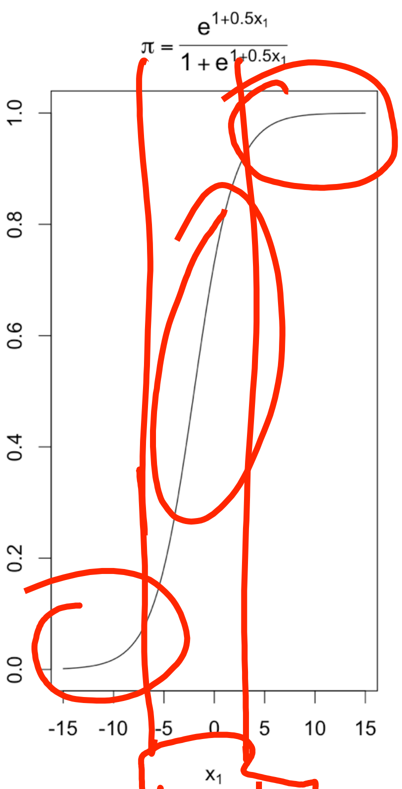
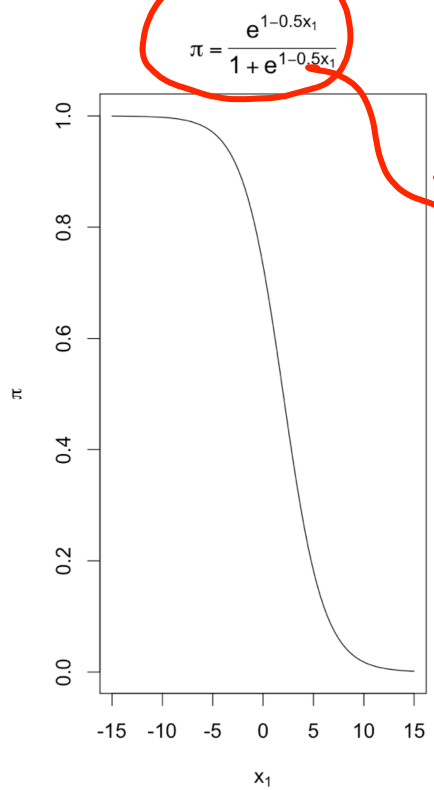
This part of the model is often referred to as the <u>linear predictor.</u>

# Visualizing the Logistic Curve

When there is only one explanatory variable, $\beta_0 = 1$, and $\beta_1 = 0.5$ (or -0.5), a plot of $\pi$ vs. x looks like the following:




```
par(mfrow = c(1,2))

beta0<-1
beta1<-0.5
curve(expr = exp(beta0+beta1*x)/(1+exp(beta0+beta1*x)),
    xlim = c(-15, 15), col = "black", main = expression(pi
    == frac(e^{1+0.5*x[1]}, 1+e^{1+0.5*x[1]})), xlab =
    expression(x[1]), ylab = expression(pi))

beta0<-1
beta1<--0.5
curve(expr = exp(beta0+beta1*x)/(1+exp(beta0+beta1*x)),
    xlim = c(-15, 15), col = "black", main = expression(pi
    == frac(e^{1-0.5*x[1]}, 1+e^{1-0.5*x[1]})), xlab =
    expression(x[1]), ylab = expression(pi))
```

# Observations:

- $0 < \pi < 1$.
- When $\beta_1 > 0$, there is a positive relationship between $x_1$ and $\pi$. When $\beta_1 < 0$, there is a negative relationship between $x_1$ and $\pi$.
- The shape of the curve is somewhat similar to the letter *s*.
- Above $\pi = 0.5$ is a mirror image of below $\pi = 0.5$.
- The slope of the curve is dependent on the value of $x_1$. This is an important property worth remembering when interpreting the coefficients of a logistic regression model.
  - We can show this mathematically by taking the derivative with respect to $x_1$:

$$\frac{d\pi}{dx_1} = \beta_1 \pi (1 - \pi)$$

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