TIME SERIES ANALYSIS LECTURE 1

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Stationarity: Example 3

3. Moving Average Model of Order 1:

MA(1) An textitMA(1) model takes the form

$$x_t = w_t + \beta w_{t-1}$$

where w_t is a white noise series with mean zero and variance σ_w^2.

The expected value of x_t is

$$\begin{aligned} E(x_t) &= E(w_t + \beta w_{t-1}) \\ &= \alpha E(w_t) + E(w_{t-1}) \\ &= 0 \end{aligned}$$

since $E(w_t) = 0 \forall t$

It is straight-forward (left as an exercise) to derive the autocorrelation function for the MA(1) model:

$$\rho_k = \begin{cases} 1 & \text{if } k = 0 \\ \frac{\sum_{i=0}^{q-k} \beta_i \beta_{i+k}}{\sum_{i=0}^{q} \beta_i^2} & \text{if } k = 1 \dots q \\ 0 & \text{if } k > q \end{cases}$$

3. Moving Average Process: Simulations (1)

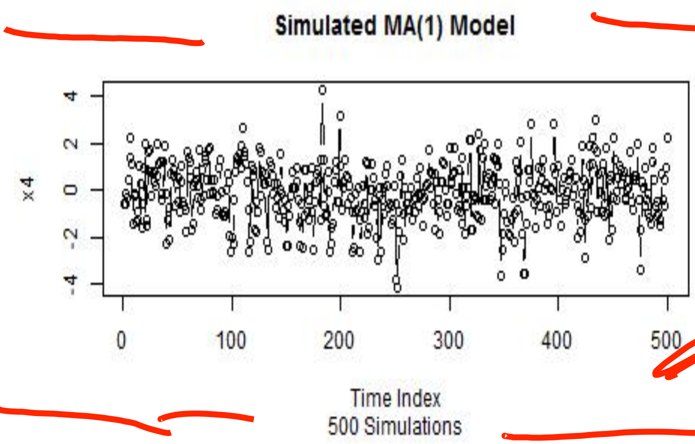
- As seen in various places in the course, a useful way to learn about the empirical patterns of a econometric model is to use simulate to simulate realizations from a theoretical model.
- In this example, we simulated from a MA(1) model 500 realization:

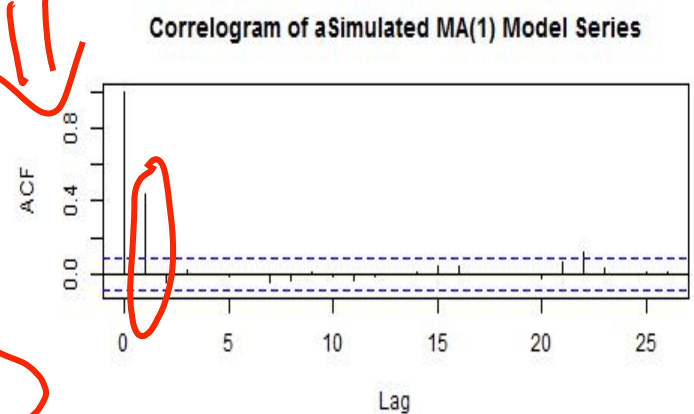
$$x_t = \omega_t + \beta\omega_{t-1}$$
$$= \omega_t + 0.8\omega_{t-1}$$

```
## 4. MA(1) Model
# Simulation
set.seed(898)
beta1 = 0.8
x4 <- w <- rnorm(500,0,sigma_w)

for (t in 2:500) x4[t] <- w[t] + beta1*w[t-1]
```

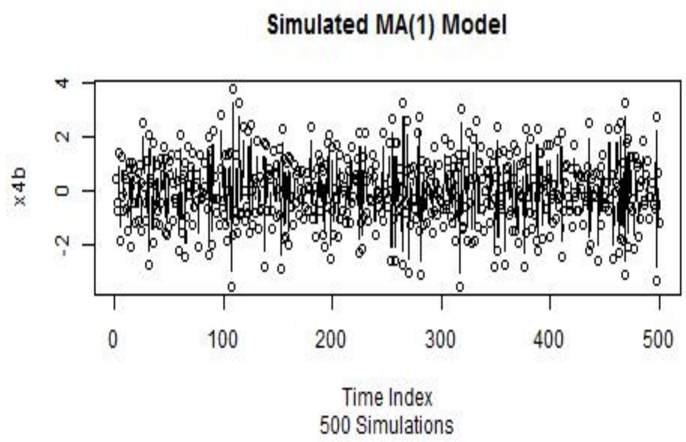
```
> summary(x4)
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-41.5400  -8.9360  -0.8613  -0.9833   7.5360  43.0900
> sd(x4)
[1] 12.28474
```

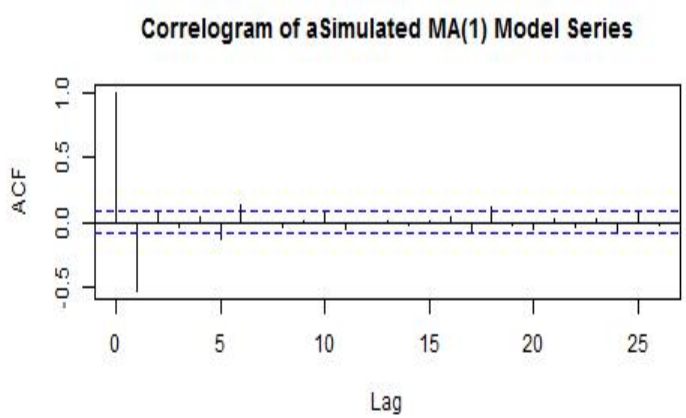




3. Moving Average Process: Simulations (2)

- A distinguish property of a MA(q) model is that its ACF drops off abruptly at q lags.
- In the case of MA(1) model, the ACF drops off to almost zero after the first lag, as shown in the correlogram.





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