Time Series Analysis

Lecture 5

Vector Autoregressive (VAR) Models

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Cointegration Definition

Cointegration

Definition: Two non-stationary time series $\{x_t\}$ and $\{y_t\}$ are **cointegrated** if some linear combinations $ax_t + by_t$, where a and b are constants, is a stationary series.

Consider two random walks:

$$x_t = \mu_t + w_{x,t}$$
$$y_t = \mu_t + w_{y,t}$$

where $w_{x,t}$ and $w_{y,t}$ are independent white noise and the series $\{\mu_t\}$ is given by

$$\boxed{\mu_t = \mu_{t-1} + w_t}$$

with $\{w_t\}$ being a zero-mean white noise processes.

Cointegration

As they are both random walks, they are non-stationary processes.

However, their difference $\{x_t - y_t\}$

$$x_t - y_t = (\mu_t + w_{x,t}) - (\mu_t + w_{y,t})$$
$$= w_{x,t} - w_{y,t}$$

is just the difference (or a linear combination with $a = 1$ and $b = -1$ as defined above in the cointegration definition) between two independent white noise processes. This newly formed cointegrated series is covariance stationary. As an exercise, derive the mean and variance of this series.

Exercise: Derive the mean, variance, and covariance of the cointegrated series $x_t - y_t$.

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