TIME SERIES ANALYSIS LECTURE 1

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Stationarity: Example 4

4. Autoregressive Model of Order 1

AR(1) An AR(1) model takes the form

$$x_t = \alpha x_{t-1} + w_t$$

where w_t is a white noise series with mean zero and variance σ_w^2.

The expected value of x_t is

$$\begin{aligned} E(x_t) &= E(\alpha x_{t-1} + w_t) \\ &= \alpha E(x_{t-1}) + E(w_t) \\ &= \alpha E(x_{t-1}) \\ &= 0 \end{aligned}$$

because using recursive substitution, an *AR(1)* model can be written as a linear process in terms of the sum of infinite white noises:

$$\boxed{x_t = \sum_{i=0}^{\infty} \alpha^i w_{t-i}}$$

4. Autoregressive Process of Order 1 (3)

With the linear process (of white noise) representation, the autocovariance can be derived as follow:

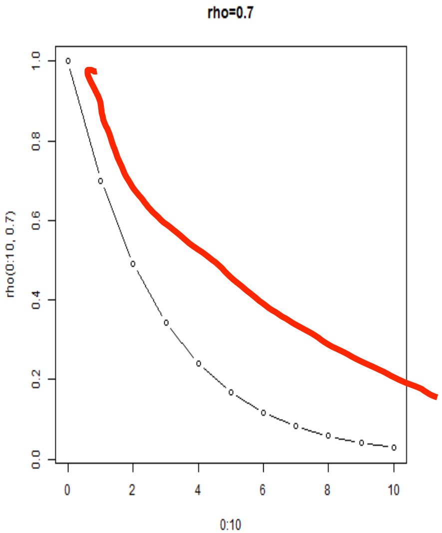
$$
\begin{aligned}
\gamma_k &= Cov(x_t, x_{t+k}) \\
&= Cov\left(\sum_{i=0}^{\infty} \alpha^i w_{t-i}, \sum_{j=0}^{\infty} \alpha^j w_{t+k-j}\right) \\
&= \sum_{j=k+i} \alpha^i \alpha^j Cov\left(w_{t-i}, w_{t+k-j}\right) \\
&= \alpha^k \sigma^2 \boxed{\sum_{i=0}^{\infty} \alpha^{2i}} \\
&= \frac{\alpha^k \sigma^2}{(1-\alpha^2)}
\end{aligned}
$$

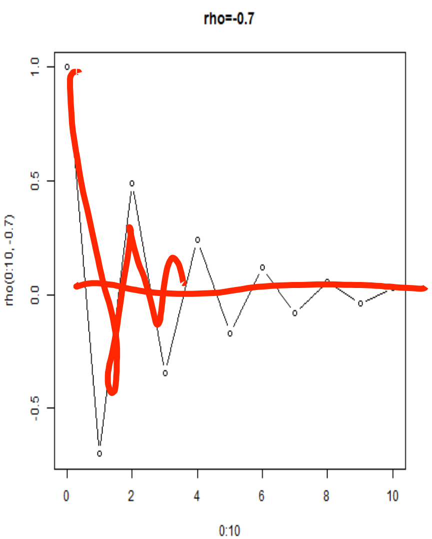
4. Autoregressive Process of Order 1: Simulations (1)

```
#------------------------------------------------------------------
# Examine the exponential decay behavior of AR(1) correlogram
rho <- function(k, alpha) alpha^k
plot(0:10, rho(0:10, 0.7), type="b", main="rho=0.7")
plot(0:10, rho(0:10, -0.7), type="b", main="rho=-0.7")
#------------------------------------------------------------------
```

- Top graph: The pattern of a theoretical AR(1) model with a positive correlation decays exponentially.

- Bottom graph: The patterns of a theoretical AR(1) model with a negative correlation oscillate between positive and negative correlation.





4. Autoregressive Process of Order 1: Simulations (1)

Recall that an AR(1) model takes the form:

$$\tilde{x}_t = \alpha \tilde{x}_{t-1} + w_t$$

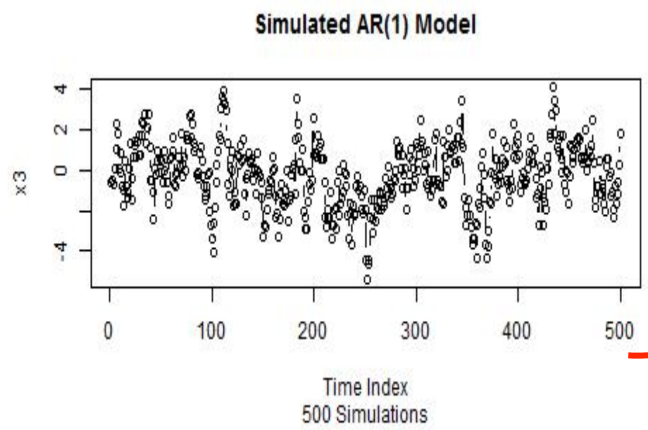
where w_t is a white noise series with mean zero and variance σ_w^2

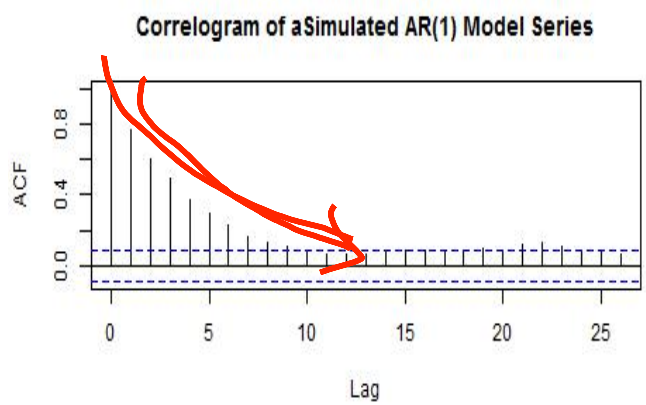
Consider $\tilde{x}_t$ as series of deviations from μ, which is a parameter that determines the "level" of the process.

```
set.seed(898)
sigma_w = 1
alpha0 = 0
alpha1 = 0.8
x3 <- w <- rnorm(500,0,sigma_w)

for (t in 2:500) x3[t] <- alpha0 + alpha1*x3[t-1] + w[t]
```

- Note the exponential decay of the autocorrelation of the simulated AR(1) series.





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