TIME SERIES ANALYSIS LECTURE 1

datascience@berkeley

Notion of Stationarity: Definitions

Strict and Weak Stationarity

A time series x_t is *strictly stationary* if the joint distributions $F(x_{t_1}, \ldots, x_{t_n})$ and $F(x_{t_1+m}, \ldots, x_{t_n+m})$ are the same, $\forall\ t_1, \ldots t_n$ and m. This is a very strong condition; it implies that the distribution is unchanged for any time shift!

- A weaker and more practical stationarity condition is *weakly stationary* (or *second order stationary*).
- A time series x_t is *weakly stationary* if it is mean and variance stationary and its autocovariance $Cov(x_t, x_{t+k})$ depends only the time displacement k and can be written as $\gamma(k)$.
- Second-order stationarity plays an important role in many of the time series models we will discuss in this course: If a time series is second-order stationary, once a distribution assumption, such as normality, is imposed, the series can be completely characterized by its mean and covariance structure.
- Stationarity is a very important property and plays a very crucial role in time series analysis. The most popular models that are used in practice, namely autoregressive models, are stationary models, and oftentimes if a series is not stationary, it is transformed into a stationary series and modeled it using as stationary model.

Introduction to This Section: Stationarity

- In this section, we will illustrate the concepts of stationarity using four foundational processes, in their simplest form.
- Each of these processes are either building blocks of other stochastic processes and time series models or are useful by themselves very often in practice.
- We will
    1. Define the process in a mathematical form.
    2. Study the mean, variance, autocovariance, and autocorrelation of the process.
    3. Simulate the process using R.
    4. Examine the empirical behavior of the simulated realizations.

Berkeley SCHOOL OF INFORMATION