CPS8316 Assignment 2

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1 Question 1

Algorithm 1: Minimal External Path

Input: A set of points, S, and two points a and b to the left and right of S. ;
$hull \leftarrow convex\_hull(S)$;
Weight each edge of $hull$ using euclidean distance. ;
$Left \leftarrow$ nearest$_v ertex$(a, hull) Right $\leftarrow$ nearest$_v ertex$(b, hull) $Connectato$Left$andweightedgeusingeuclideandistance$ $Connect$btoRight$andweightedgeusingeu$
$\leftarrow$ the region of $hull$ above the line segment ab ;
$Lower \leftarrow$ the region of $hull$ below the line segment ab ;

$$U\_Path \leftarrow \sum_{i=0}^{|Upper|} edge(Upper(i), Upper(i+1) \;;$$

$$L\_Path \leftarrow \sum_{i=0}^{|Lower|} edge(Lower(i), Lower(i+1) \;;$$

return $min(U\_Path, L\_Path)$;

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2 Question 2

2.1 a

The situation that results in the most layers for a given set of points is when each nested layer is triangular. In this case, the upper bound relative to the input size of S is $\frac{|S|}{3}$

Algorithm 2: Onion Peel

Input: A set of points S, An integer i, An integer j;
$hull \leftarrow convex\_hull(S)$;
if j ¡= i **then**
  return $[hull, Onion\ Peel(S - hull, i, j - 1)]$;
else
  return $[Onion\ Peel(S - hull, i, j - 1)]$;
end

2.2 b

3 Question 3

3.1 a

Let S be a strip that contains all of the points of a given point set in general position, P. Let s be one of the lines bounding the strip which, without loss of generality, is above P. Construct a convex hull from p. For any edge of the convex hull, there must be exactly two vertices which bound it. If s passes through no points, then either all of the points are above it or below it. In the first case, this violates the assumption on s. In the second case, s violates the definition of a strip. If s passes through two points, then three cases arise: the two points are neighbours and thus define an edge, the two points are not neighbours and s passes through the convex hull, or the two points are not neighbours and s does not pass through the convex hull. In the last case, this means that there are one or more vertices below s on the convex hull. This defines a region which goes into the convex hull and then comes back to s, which violates the definition of a convex hull. In the middle case, there must be one or more points above the line. For this to be the case, the line could not have been above all of P, which violates the definition of s. In the first case, s must be co-linear and parallel with an edge. If s passes through one point, then conditions similar to when s passes through two lines arise. If s passes through P, then not all points in P were below it. If s passes through no points, then it must either result in a non-minimal width or pass through P. The first case is a contradiction, and the second case violates the definition of the strip. Finally, s cannot pass through three or more points without those points violating general position.

3.2 b

Sort the points of P by y value, with ties broken by ascending x value. Select the highest and lowest points. Construct a convex hull from the total point set. Extend lines in opposite directions from the points. If a line intersects the convex hull, find the endpoint of the edge opposite the line (since the line may only intersect the line neighbouring the originating point). When both endpoints of both lines are further than the x values of the convex hull, report the distance between the bounding lines.