HW1
Labor Economics

Chia-wei, Chen*

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*R10323045

1 Setups

1.1 Data Camp

I completed the course by taking the quiz.

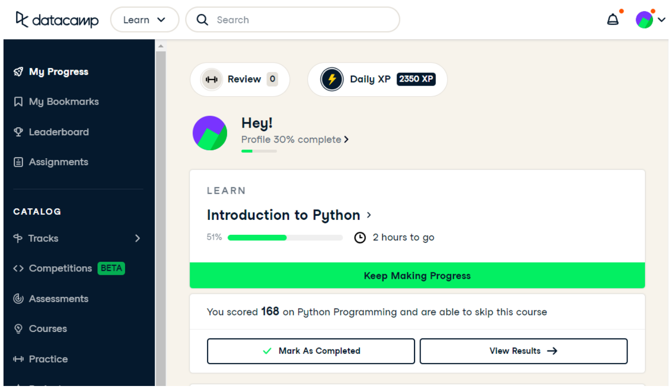

Figure 1: Quiz in datacamp

1.2 R

As shown in Figure 2

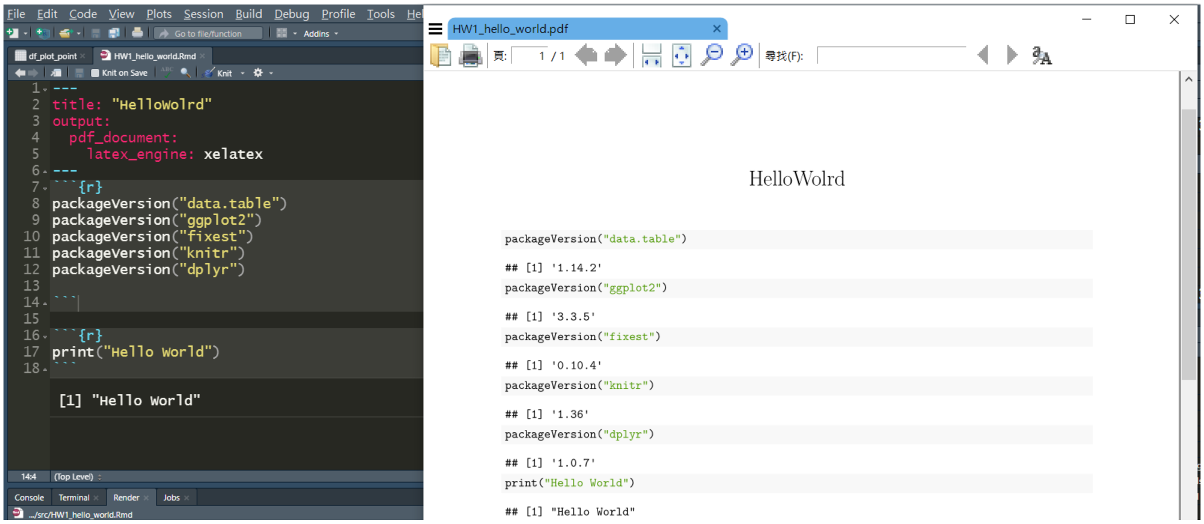

Figure 2: Settings and the libraries

1.3 Debugger

See figure 3

Figure 3: Screenshot of using debugger on a simple function

1.4 GitHub Setting

I created a repository for the whole lecture instead. See figure 4

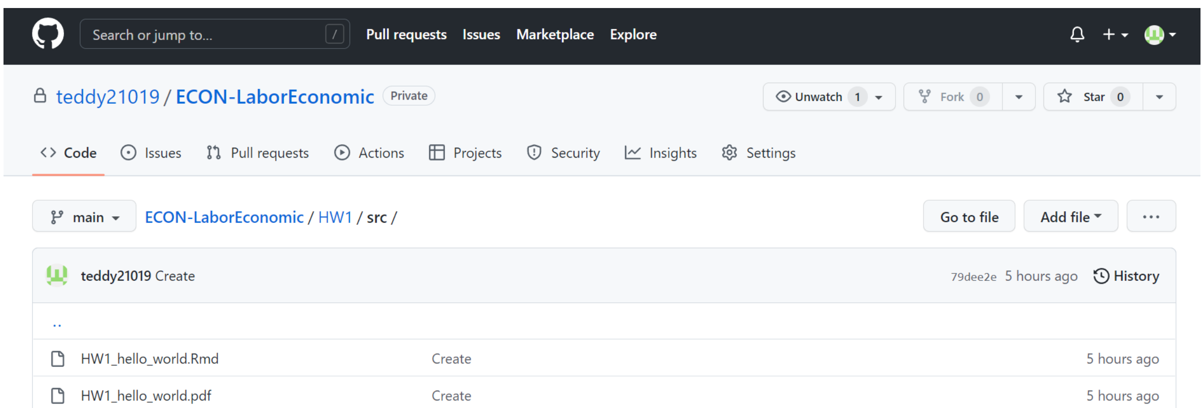

Figure 4: Github page, with the hello world files pushed on it

2 NBER Working Paper

2.1 List paper on NBER

The second paper on the list is

KFstar and Portfolio Inflows: A Focus on Latin America

by Burger, John D and Warnock, Francis E and Warnock, Veronica Cacdac

2.2 Download a Paper

As in figure 5, I downloaded the paper.

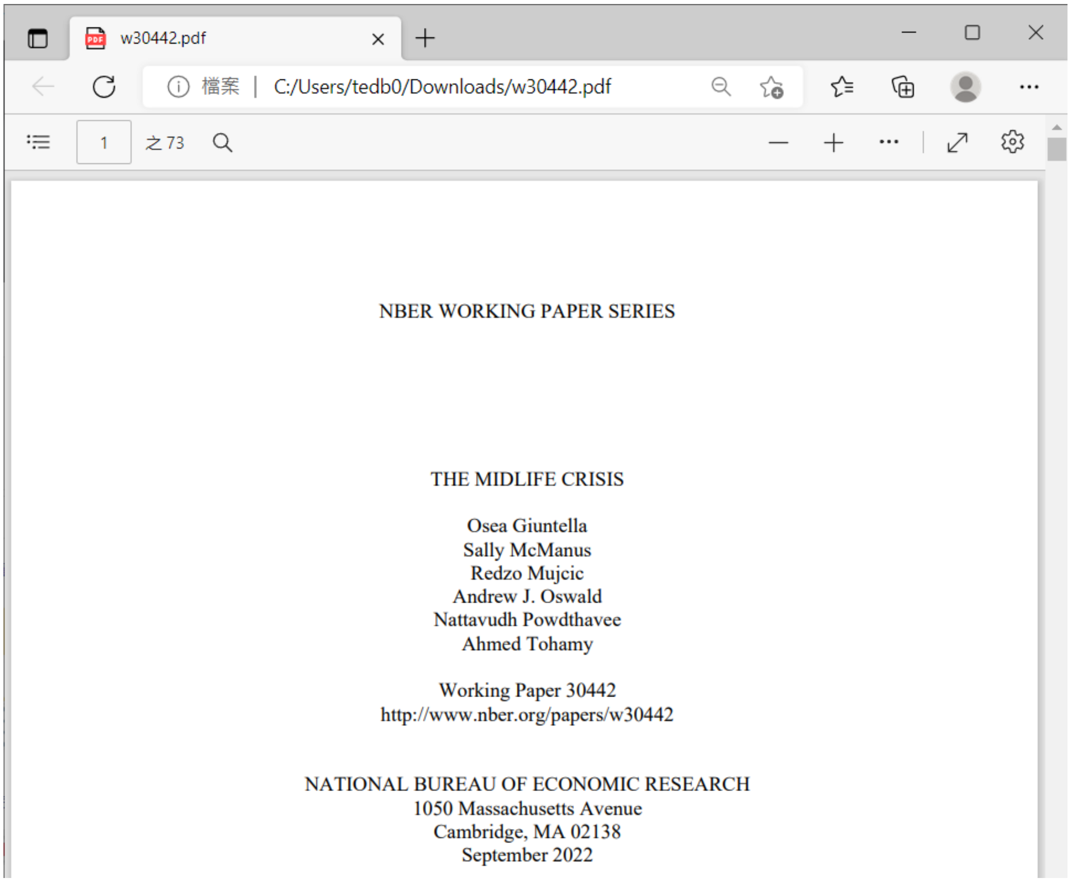

Figure 5: Screenshoe - paper downloaded

3 SRDA

3.1 SRDA Account

The account info is shown in figure 6

3.2 Plotting Rate of Working - Age from PSFD Data

Plot as well as the year is shown in figure 7

Figure 6: Account info of SRDA

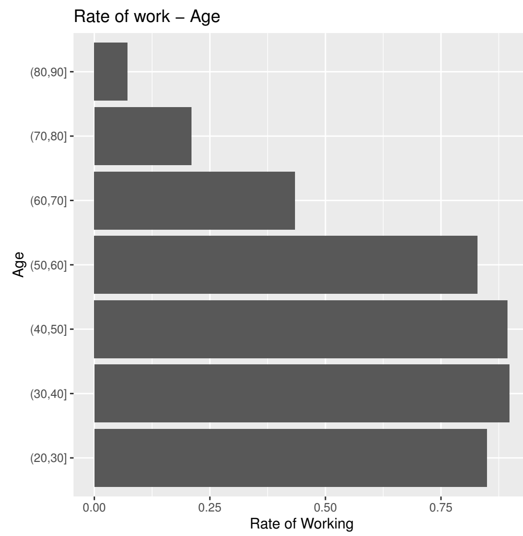

Figure 7: Rate of Working Versus the Age. Data source: PSFD 2000

4 Roy Model

4.1 Theoretical Results

4.1.1 Derivation

We first start from considering the probability of migration. One migrates under the condition

$$w_{1i} > w_{0i} + C \tag{1}$$

Where $w_{0i} = \mu_{0i} + \epsilon_0$ and $w_{1i} = \mu_{1i} + \epsilon_1$, and the error terms together follow a joint-normal distribution

$$\begin{pmatrix} \epsilon_0 \\ \epsilon_1 \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_0^2, \sigma_{01} \\ \sigma_{01}, \sigma_1^2 \end{pmatrix}\right) \tag{2}$$

The probability of migration is then, according to Eq. (2)

$$\begin{aligned} P(w_{1i} > w_{0i} + C) &= P(v_i > \mu_0 - \mu_1 + C) \\ &= 1 - \Phi(\frac{\mu_0 - \mu_1 + C}{\sigma_\nu}) \\ &= 1 - \Phi(z) \end{aligned}$$

The expected wage of an imigrant is then
Note: Subscripts i are neglected for simplicity

$$
\begin{aligned}
\mathbb{E}(w_0 \mid I) =& \mathbb{E}(w_0 \mid v > \mu_0 - \mu_1 + C) \\
=& \mu_0 + \mathbb{E}(\epsilon_0 \mid v > \mu_0 - \mu_1 + C) \\
=& \mu_0 + \mathbb{E}\left(\epsilon_0 \mid \frac{\nu}{\sigma_\nu} > \frac{\mu_0 - \mu_1 + C}{\sigma_\nu}\right)
\end{aligned}
$$

A linear combination of random variables following normal distribution also follows a normal distribution. Therefore $\mathbb{E}\left(\epsilon_0 \mid \frac{\nu}{\sigma_\nu}\right)$ is just a conditional expectation of a bivariate normal distribution consisting ϵ and ν/σ_ν.

Since for a bivariate normal distribution, we have

$$\mathbb{E}[X \mid Y = y] = \mu_x + \rho_{xy}\frac{\sigma_x}{\sigma_y}(y - \mu_y) \tag{3}$$

Substituding $X = \epsilon_0$ and $Y = \nu$ into Eq. (3), we get

$$\mathbb{E}(\epsilon_0 \mid \nu) = \rho_{0\nu}\frac{\sigma_0}{\sigma_\nu}\nu \tag{4}$$

and hence

$$\mathbb{E}(\epsilon_0 \mid \frac{\nu}{\sigma_\nu}) = \left(\rho_{0\nu}\frac{1}{\sigma_\nu}\right)\frac{\sigma_0}{\sigma_\nu\frac{1}{\sigma_\nu^2}}\frac{\nu}{\sigma_\nu} = \frac{\rho_{0\nu}\sigma_0}{\sigma_\nu}\nu$$

Therefore

$$
\begin{aligned}
\mathbb{E}(w_0 \mid I) =& \mu_0 + \mathbb{E}\left(\epsilon_0 \mid \frac{\nu}{\sigma_\nu} > \frac{\mu_0 - \mu_1 + C}{\sigma_\nu}\right) \\
&= \mu_0 + \rho_{0\nu}\sigma_0\,\mathbb{E}\left(\frac{\nu}{\sigma_\nu} \mid \frac{\nu}{\sigma_\nu} > \frac{\mu_0 - \mu_1 + C}{\sigma_\nu}\right)
\end{aligned}
\tag{5}
$$

Realize that the last expectation Eq. (5) is the expected value of a truncated normal distribution, which can rewrite it as

$$\mathbb{E}(w_0 \mid I) = \mu_0 + \rho_{0\nu}\sigma_0\frac{\phi(z)}{1 - \Phi(z)} \tag{6}$$

Since the correlation coefficient can be negative, we know nothing about this result Let us preceed and solve for $\rho_{0\nu}$ further for better insight.

$$\rho_{0v} = \frac{\sigma_{0\nu}}{\sigma_0\sigma_\nu}$$

$$\sigma_{0\nu} = cov(\epsilon_0, \nu) = \mathbb{E}(\epsilon_0(\epsilon_1 - \epsilon_0)) = \sigma_{01} - \sigma_0^2 \tag{7}$$

Substitude the result from Eq. Eq. (7) and $\rho_{01} = \sigma_{01}/\sigma_0\sigma_1$, Eq. (6) now becomes

$$\mathbb{E}(w_0 \mid I) = \mu_0 + \frac{\sigma_{01} - \sigma_0^2}{\sigma_\nu}\frac{\phi(z)}{1 - \Phi(z)} = \mu_0 + \frac{\sigma_0\sigma_1}{\sigma_\nu}\left(\rho - \frac{\sigma_0}{\sigma_1}\right)\frac{\phi(z)}{1 - \Phi(z)} \tag{8}$$

Similarily, for $\mathbb{E}(w_1 \mid I)$, we have

$$\begin{aligned}
\mathbb{E}(w_1 \mid I) =& \mathbb{E}(w_1 \mid v > \mu_0 - \mu_1 + C) \\
=& \mu_1 + \mathbb{E}(\epsilon_1 \mid v > \mu_0 - \mu_1 + C) \\
=& \mu_1 + \mathbb{E}\left(\epsilon_1 \mid \frac{\nu}{\sigma_\nu} > \frac{\mu_0 - \mu_1 + C}{\sigma_\nu}\right)
\end{aligned}$$

Substituding 1 for 0 in Eq. (4), we get

$$\mathbb{E}(\epsilon_1 \mid \frac{\nu}{\sigma_\nu}) = \frac{\rho_{1\nu}\sigma_1}{\sigma_\nu}\nu \tag{9}$$

Eq. (7) now becomes

$$\sigma_{1\nu} = cov(\epsilon_, \nu) = \mathbb{E}(\epsilon_1(\epsilon_1 - \epsilon_0)) = \sigma_1^2 - \sigma_{01} \tag{10}$$

Hence together we get

$$\mathbb{E}(w_1 \mid I) = \mu_1 + \rho_{1\nu}\sigma_1\frac{\phi(z)}{1 - \Phi(z)} = \mu_1 + \frac{\sigma_0\sigma_1}{\sigma_\nu}\left(\frac{\sigma_1}{\sigma_0} - \rho\right)\frac{\phi(z)}{1 - \Phi(z)} \tag{11}$$

4.1.2 $Q_0 > 0$ and $Q_1 < 0$?

Accordind to Eq. (8) and Eq. (11), this would imply that

$$\begin{cases} \rho > \frac{\sigma_0}{\sigma_1} \\ \rho > \frac{\sigma_0}{\sigma_1} \end{cases}$$

But either will exceed 1, making it mathematically imposible to happen since correlation coefficient must be bounded under 1.

Intuitively, it means that people migrate to a place where expectation wage is lower than that in local. which is weird and irrational.

5 Simulation

5.1 Simulation

5.1.1 The code

```
library(MASS)
library(data.table)
library(dplyr)
library(xtable)
setwd("~/111-1/Labor Economic/HW1/src")

    ## Parameters settings
N = 1e7
mu0 = 10
mu1 = 15
sigma0 = 3
sigma1 = 4.5
sigma_01 = 2
sigma_matrix = matrix(c(sigma0^2, sigma_01,
                        sigma_01, sigma1^2),
                      ncol=2)
C = 3

    ## Creating error terms, saving to data.table
wage = data.table(
  mvrnorm(n=N,mu=c(0, 0),Sigma=sigma_matrix)
)

  ## Rename
setnames(wage, "V1", "e0")
setnames(wage, "V2", "e1")

    ## Creating variables
wage[, W0 := e0 + mu0]
wage[, W1 := e1 + mu1]
wage[, Nu := e1 - e0]
wage[, I  := W1 > W0 + C]

    ## Calculating the empirical conditional mean
E_w0_I = wage[I==T, mean(W0)]
E_w1_I = wage[I==T, mean(W1)]

    ## Calculate the theoretical value

## Calculate rho_nu
```

```r
rho = sigma_01/(sigma0 * sigma1)
sigma_nu = sqrt( sigma0^2 + sigma1^2 - 2 * sigma_01 )

## calculate z
z = (mu0 - mu1 + C)/sigma_nu
IMR = dnorm(z)/(1-pnorm(z))

## According to formula
calc_Q0 = mu0 + (sigma0 * sigma1)/sigma_nu * (rho - sigma0/sigma1) * IMR
calc_Q1 = mu1 + (sigma0 * sigma1)/sigma_nu * (sigma1/sigma0 - rho) * IMR

## Compare
result = data.frame("Source" = c(E_w0_I, calc_Q0), "Host" = c(E_w1_I, calc_Q1))
rownames(result) = c("Simulation", "Theoretical")

print(
  xtable(result,
         caption = "Simulation result versus the theoretical result",
         label = "tab:sim_res",
         digits=5),
       floating = TRUE, latex.environments = "center",
  file="roy_sim.tex")
```

5.1.2 Simulation Result

	Source	Host
Simulation	9.21475	17.04457
Theoretical	9.21577	17.04459

Table 1: Simulation result versus the theoretical result

6 Roy Model is Everywhere

6.1 Example in Applied Economics

A phenomenal example is that published by Heckman (1976). He estimates the labor supply and wage for females. Not only does the model handles the self-selection problem, it also provides a faster method for estimation, compared to his similar work in (see Heckman, 1974)

6.2 Write Down a Research Question

I found the concept of self selection extremely useful in explaining "Who take courses that are not in its own department?"

Similar to immigrants and wages, I found an analogy between students and grades. Back when I was in the department of physics, some classmates tend to take calculus and linear algebra in the department of mathematics, instead of the equivalent required course in the department of physics. Those courses are typically hard and has a relatively higher proportion of people failing.

Meanwhile, people like me tend to take the so called "sweet courses", which guarantees a good grade, with little need for effort.

Assume that in average, taking courses in one's own department (Denoted 0) will get a grade μ_0 , while taking courses in other department (Denoted 1) will get a grade μ_1. Also assume for now that the personalities of teachers are irrelevant, as well as the interest of individual student; that is, they only seek for a better score.

6.3 Explanation

Taking a direct analogy from the immigrants' problem, we know that

Case 1. Students that take other courses are both getting better score than average. This refers to $Q_0 > 0$ and $Q_1 > 0$. This happens when $\sigma_1 > \sigma_0$ and $\rho > \frac{\sigma_0}{\sigma_1}$

If the courses are relevant, for example, *Advanced Linear Algebra* and *Applied Mathematics(I)* [1], and the distribution of grades in *Advance Linear Algebra* is more diverse, then students that are smarter take the other course, mainly because in a course that everyone takes A+(The easier *Applied Mathematics*), a small mistake might cause the grade to drop, while in the harder *Advanced Linear Algebra*, almost all the other classmates are noobs, and hence even a relatively big mistake in the exam doesn't change ones' good result much.

Case 2. Students that take other courses suck at both courses. This refers to $Q_0 < 0$ and $Q_1 < 0$. This happens when $\sigma_1 < \sigma_0$ and $\rho > \frac{\sigma_1}{\sigma_0}$

In contrast to case 1, if taking course in the other department which is relevant has a more steady grade (obviously not F), then students not performing well in either class will take the one in which the grade don't fluctuate too much, preventing an unexpected failure that unfortunately might postpone its graduation year.

Case 3. Students that take other courses that they are good at. This refers to $Q_0 < 0$ and $Q_1 > 0$. This happens when $\rho < \min \frac{\sigma_1}{\sigma_0}, \frac{\sigma_0}{\sigma_1}$

[1] Mainly covering linear algebra as well

If the two courses are less irrelevant, like *Classical Electrodynamic Theory* and *Macroeconomics Theory*, then the students that take the course in the department of economics are the students that perform better in the other field of knowledge.

That's how I ended up here.

Case 4. Students that take other courses that they are not good at. This refers to $Q_0 > 0$ and $Q_1 < 0$. This is mathematically impossible. Refer to section 4.1.2.

These are the irrational students, trying to challenge their limits and not thinking thoroughly.

References

Heckman, J. (1974). Shadow prices, market wages, and labor supply. *Econometrica*, 42(4):679–694.

Heckman, J. J. (1976). *The Common Structure of Statistical Models of Truncation, Sample Selection and Limited Dependent Variables and a Simple Estimator for Such Models*, pages 475–492. NBER.