1 Setups

1.1 Data Camp

I completed the course by taking the quiz.

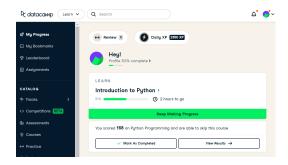


Figure 1: Quiz in datacamp

1.2 R

As shown in Figure 2

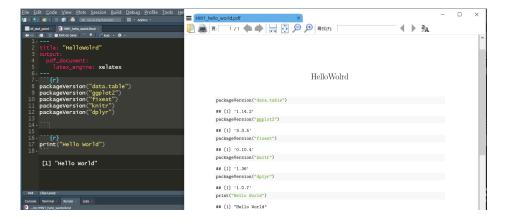


Figure 2: Settings and the libraries

1.3 Debugger

See figure 3



Figure 3: Screenshot of using debugger on a simple function

1.4 GitHub Setting

I created a repository for the whole lecture instead. See figure 4

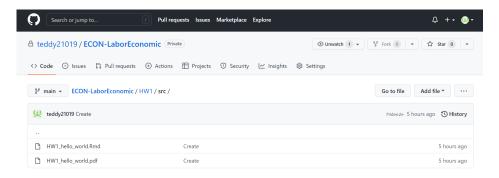


Figure 4: Github page, with the hello world files pushed on it

2 NBER Working Paper

2.1 List paper on NBER

The second paper on the list is

KFstar and Portfolio Inflows: A Focus on Latin America by Burger, John D and Warnock, Francis E and Warnock, Veronica Cacdac

2.2 Download a Paper

As in figure 5, I downloaded the paper.

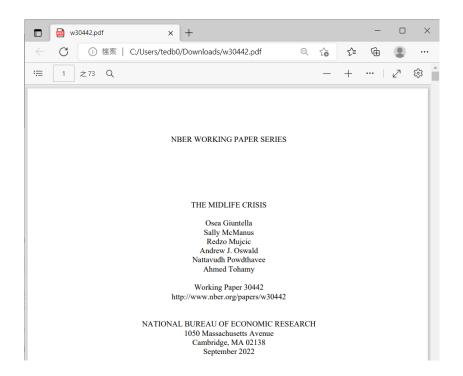


Figure 5: Screenshoe - paper downloaded

3 SRDA

3.1 SRDA Account

The account info is shown in figure 6

4 Roy Model

4.1 Derivation of Roy Model

We first start from considering the probability of migration. One migrates under the condition

$$w_{1i} > w_{0i} + C \tag{1}$$

Where $w_{0i} = \mu_{0i} + \epsilon_0$ and $w_{1i} = \mu_{1i} + \epsilon_1$, and the error terms together follow a joint-normal distribution



Figure 6: Account info of SRDA

$$\begin{pmatrix} \epsilon_0 \\ \epsilon_1 \end{pmatrix} \sim \mathcal{N} \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_0^2, \sigma_{01} \\ \sigma_{01}, \sigma_1^2 \end{pmatrix} \right) \tag{2}$$

The probability of migration is then, according to Eq. (2)

$$P(w_{1i} > w_{0i} + C) = P(v_i > \mu_0 - \mu_1 + C)$$
$$= 1 - \Phi(\frac{\mu_0 - \mu_1 + C}{\sigma_{\nu}})$$
$$= 1 - \Phi(z)$$

The expected wage of an imigrant is then Note: Subscripts i are neglected for simplicity

$$\mathbb{E}(w_0 \mid I) = \mathbb{E}(w_0 \mid v > \mu_0 - \mu_1 + C)$$

$$= \mu_0 + \mathbb{E}(\epsilon_0 \mid v > \mu_0 - \mu_1 + C)$$

$$= \mu_0 + \mathbb{E}\left(\epsilon_0 \mid \frac{\nu}{\sigma_{\nu}} > \frac{\mu_0 - \mu_1 + C}{\sigma_{\nu}}\right)$$

A linear combination of random variables following normal distribution also follows a normal distribution. Therefore $\mathbb{E}\left(\epsilon_0 \mid \frac{\nu}{\sigma_{\nu}}\right)$ is just a conditional expectation of a bivariate normal distribution consisting ϵ and ν/σ_{ν} .

Since for a bivariate normal distribution, we have

$$\mathbb{E}[X \mid Y = y] = \mu_x + \rho_{xy} \frac{\sigma_x}{\sigma_y} (y - \mu_y)$$
 (3)

Substituding $X = \epsilon_0$ and $Y = \nu$ into Eq. (3), we get

$$\mathbb{E}(\epsilon_0 \mid \nu) = \rho_{0\nu} \frac{\sigma_0}{\sigma_\nu} \nu \tag{4}$$

and hence

$$\mathbb{E}(\epsilon_0 \mid \frac{\nu}{\sigma_{\nu}}) = \left(\rho_{0\nu} \frac{1}{\sigma_{\nu}}\right) \frac{\sigma_0}{\sigma_{\nu} \frac{1}{\sigma^2}} \frac{\nu}{\sigma_{\nu}} = \frac{\rho_{0\nu} \sigma_0}{\sigma_{\nu}} \nu$$

Therefore

$$\mathbb{E}(w_0 \mid I) = \mu_0 + \mathbb{E}\left(\epsilon_0 \mid \frac{\nu}{\sigma_\nu} > \frac{\mu_0 - \mu_1 + C}{\sigma_\nu}\right)$$

$$= \mu_0 + \rho_{0\nu}\sigma_0 \mathbb{E}\left(\frac{\nu}{\sigma_\nu} \mid \frac{\nu}{\sigma_\nu} > \frac{\mu_0 - \mu_1 + C}{\sigma_\nu}\right)$$
(5)

Realize that the last expectation Eq. (5) is the expected value of a truncated normal distribution, which can rewrite it as

$$\mathbb{E}(w_0 \mid I) = \mu_0 + \rho_{0\nu}\sigma_0 \frac{\phi(z)}{1 - \Phi(z)}$$
 (6)

Since the correlation coefficient can be negative, we know nothing about this result Let us preced and solve for $\rho_{0\nu}$ further for better insight.

$$\rho_{0v} = \frac{\sigma_{0\nu}}{\sigma_0 \sigma_{\nu}}$$

$$\sigma_{0\nu} = cov(\epsilon_0, \nu) = \mathbb{E}(\epsilon_0(\epsilon_1 - \epsilon_0)) = \sigma_{01} - \sigma_0^2$$
(7)

Substitude the result from Eq. Eq. (7) and $\rho_{01} = \sigma_{01}/\sigma_0\sigma_1$, Eq. (6) now becomes

$$\mathbb{E}(w_0 \mid I) = \mu_0 + \frac{\sigma_{01} - \sigma_0^2}{\sigma_\nu} \frac{\phi(z)}{1 - \Phi(z)} = \mu_0 + \frac{\sigma_0 \sigma_1}{\sigma_\nu} \left(\rho - \frac{\sigma_0}{\sigma_1}\right) \frac{\phi(z)}{1 - \Phi(z)}$$
(8)

Similarly, for $\mathbb{E}(w_1 \mid I)$, we have

$$\mathbb{E}(w_1 \mid I) = \mathbb{E}(w_1 \mid v > \mu_0 - \mu_1 + C)$$

$$= \mu_1 + \mathbb{E}(\epsilon_1 \mid v > \mu_0 - \mu_1 + C)$$

$$= \mu_1 + \mathbb{E}\left(\epsilon_1 \mid \frac{\nu}{\sigma_{\nu}} > \frac{\mu_0 - \mu_1 + C}{\sigma_{\nu}}\right)$$

Substituding 1 for 0 in Eq. (4), we get

$$\mathbb{E}(\epsilon_1 \mid \frac{\nu}{\sigma_{\nu}}) = \frac{\rho_{1\nu}\sigma_1}{\sigma_{\nu}}\nu\tag{9}$$

Eq. (7) now becomes

$$\sigma_{1\nu} = cov(\epsilon, \nu) = \mathbb{E}(\epsilon_1(\epsilon_1 - \epsilon_0)) = \sigma_1^2 - \sigma_{01}$$
 (10)

Hence together we get

$$\mathbb{E}(w_1 \mid I) = \mu_1 + \rho_{1\nu}\sigma_1 \frac{\phi(z)}{1 - \Phi(z)} = \mu_1 + \frac{\sigma_0\sigma_1}{\sigma_\nu} \left(\frac{\sigma_1}{\sigma_0} - \rho\right) \frac{\phi(z)}{1 - \Phi(z)}$$
(11)

4.2 $Q_0 > 0$ and $Q_1 < 0$?

According to Eq. (8) and Eq. (11), this would imply that

$$\begin{cases} \rho > \frac{\sigma_0}{\sigma_1} \\ \rho > \frac{\sigma_0}{\sigma_1} \end{cases}$$

But either will exceed 1, making it mathematically imposible to happen since correlation coefficient must be bounded under 1.

Intuitively, it means that people migrate to a place where expectation wage is lower than that in local. which is weird and irrational.