1 Setups

1.1 Data Camp

I completed the course by taking the quiz.

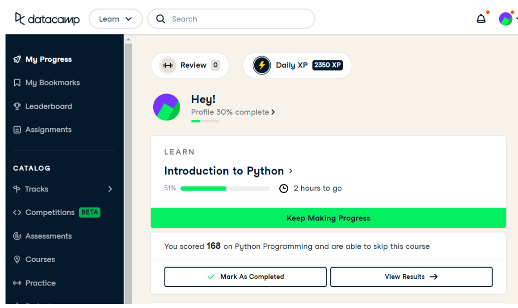

Figure 1: Quiz in datacamp

1.2 R

As shown in Figure 2

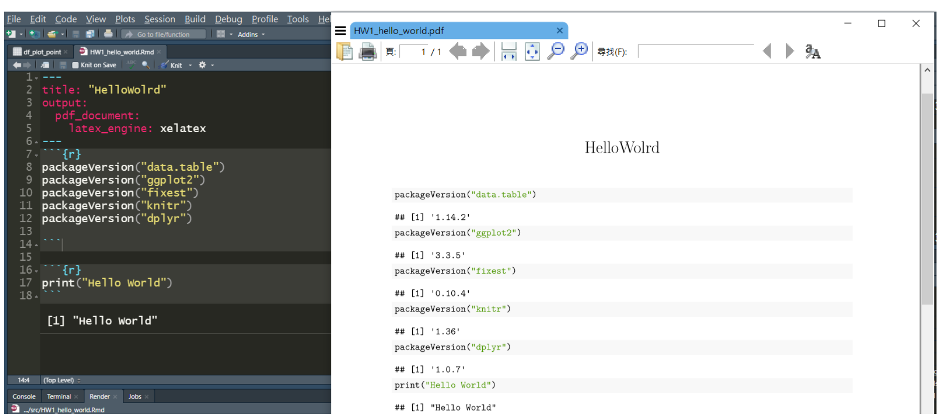

Figure 2: Settings and the libraries

1.3 Debugger

See figure 3

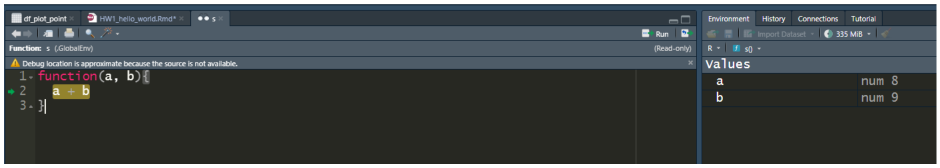

Figure 3: Screenshot of using debugger on a simple function

1.4 GitHub Setting

I created a repository for the whole lecture instead. See figure 4

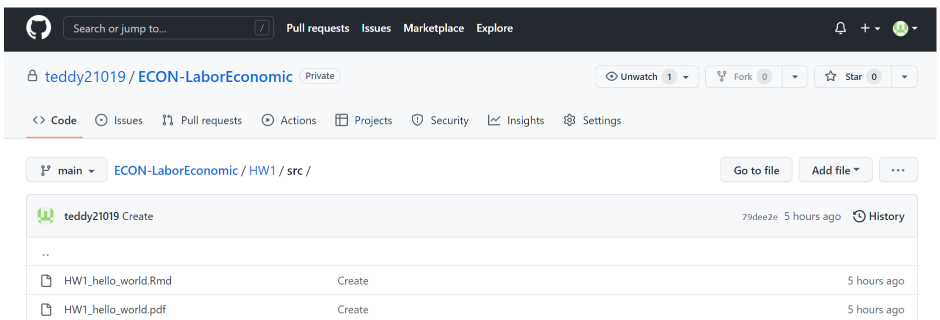

Figure 4: Github page, with the hello world files pushed on it

2 NBER Working Paper

2.1 List paper on NBER

The second paper on the list is

> KFstar and Portfolio Inflows: A Focus on Latin America
>
> by Burger, John D and Warnock, Francis E and Warnock, Veronica Cacdac

2.2 Download a Paper

As in figure 5, I downloaded the paper.

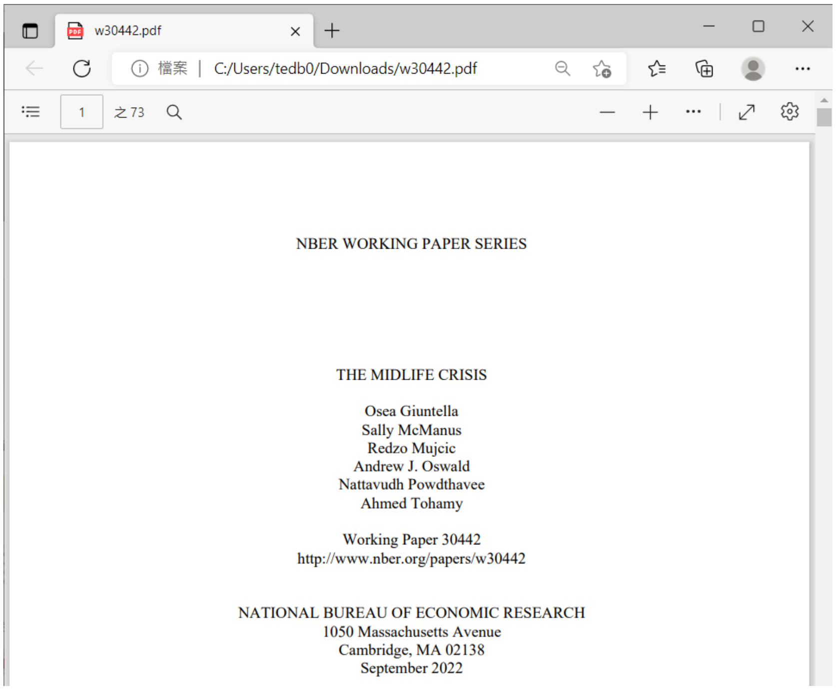

Figure 5: Screenshoe - paper downloaded

3 SRDA

3.1 SRDA Account

The account info is shown in figure 6

4 Roy Model

4.1 Theoretical Results

4.1.1 Derivation

We first start from considering the probability of migration. One migrates under the condition

$$w_{1i} > w_{0i} + C \tag{1}$$

Figure 6: Account info of SRDA

Where $w_{0i} = \mu_{0i} + \epsilon_0$ and $w_{1i} = \mu_{1i} + \epsilon_1$, and the error terms together follow a joint-normal distribution

$$\begin{pmatrix} \epsilon_0 \\ \epsilon_1 \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_0^2, \sigma_{01} \\ \sigma_{01}, \sigma_1^2 \end{pmatrix} \right) \tag{2}$$

The probability of migration is then, according to Eq. (2)

$$\begin{aligned} P(w_{1i} > w_{0i} + C) &= P(v_i > \mu_0 - \mu_1 + C) \\ &= 1 - \Phi(\frac{\mu_0 - \mu_1 + C}{\sigma_\nu}) \\ &= 1 - \Phi(z) \end{aligned}$$

The expected wage of an imigrant is then
Note: Subscripts i are neglected for simplicity

$$\begin{aligned} \mathbb{E}(w_0 \mid I) =& \mathbb{E}(w_0 \mid v > \mu_0 - \mu_1 + C) \\ =& \mu_0 + \mathbb{E}(\epsilon_0 \mid v > \mu_0 - \mu_1 + C) \\ =& \mu_0 + \mathbb{E}\left(\epsilon_0 \mid \frac{\nu}{\sigma_\nu} > \frac{\mu_0 - \mu_1 + C}{\sigma_\nu}\right) \end{aligned}$$

A linear combination of random variables following normal distribution also follows a normal distribution. Therefore $\mathbb{E}\left(\epsilon_0 \mid \frac{\nu}{\sigma_\nu}\right)$ is just a conditional expectation of a bivariate normal distribution consisting ϵ and ν/σ_ν.

Since for a bivariate normal distribution, we have

$$\mathbb{E}[X \mid Y = y] = \mu_x + \rho_{xy}\frac{\sigma_x}{\sigma_y}(y - \mu_y) \tag{3}$$

Substituding $X = \epsilon_0$ and $Y = \nu$ into Eq. (3), we get

$$\mathbb{E}(\epsilon_0 \mid \nu) = \rho_{0\nu}\frac{\sigma_0}{\sigma_\nu}\nu \tag{4}$$

and hence

$$\mathbb{E}(\epsilon_0 \mid \frac{\nu}{\sigma_\nu}) = \left(\rho_{0\nu}\frac{1}{\sigma_\nu}\right)\frac{\sigma_0}{\sigma_\nu \frac{1}{\sigma_\nu^2}}\frac{\nu}{\sigma_\nu} = \frac{\rho_{0\nu}\sigma_0}{\sigma_\nu}\nu$$

Therefore

$$\begin{aligned}\mathbb{E}(w_0 \mid I) =& \mu_0 + \mathbb{E}\left(\epsilon_0 \mid \frac{\nu}{\sigma_\nu} > \frac{\mu_0 - \mu_1 + C}{\sigma_\nu}\right) \\ &= \mu_0 + \rho_{0\nu}\sigma_0\,\mathbb{E}\left(\frac{\nu}{\sigma_\nu} \mid \frac{\nu}{\sigma_\nu} > \frac{\mu_0 - \mu_1 + C}{\sigma_\nu}\right)\end{aligned} \tag{5}$$

Realize that the last expectation Eq. (5) is the expected value of a truncated normal distribution, which can rewrite it as

$$\mathbb{E}(w_0 \mid I) = \mu_0 + \rho_{0\nu}\sigma_0\frac{\phi(z)}{1 - \Phi(z)} \tag{6}$$

Since the correlation coefficient can be negative, we know nothing about this result Let us preceed and solve for $\rho_{0\nu}$ further for better insight.

$$\rho_{0v} = \frac{\sigma_{0\nu}}{\sigma_0\sigma_\nu}$$

$$\sigma_{0\nu} = cov(\epsilon_0, \nu) = \mathbb{E}(\epsilon_0(\epsilon_1 - \epsilon_0)) = \sigma_{01} - \sigma_0^2 \tag{7}$$

Substitude the result from Eq. Eq. (7) and $\rho_{01} = \sigma_{01}/\sigma_0\sigma_1$, Eq. (6) now becomes

$$\mathbb{E}(w_0 \mid I) = \mu_0 + \frac{\sigma_{01} - \sigma_0^2}{\sigma_\nu}\frac{\phi(z)}{1 - \Phi(z)} = \mu_0 + \frac{\sigma_0\sigma_1}{\sigma_\nu}\left(\rho - \frac{\sigma_0}{\sigma_1}\right)\frac{\phi(z)}{1 - \Phi(z)} \tag{8}$$

Similarily, for $\mathbb{E}(w_1 \mid I)$, we have

$$
\begin{aligned}
\mathbb{E}(w_1 \mid I) =& \mathbb{E}(w_1 \mid v > \mu_0 - \mu_1 + C) \\
=& \mu_1 + \mathbb{E}(\epsilon_1 \mid v > \mu_0 - \mu_1 + C) \\
=& \mu_1 + \mathbb{E}\left(\epsilon_1 \mid \frac{\nu}{\sigma_\nu} > \frac{\mu_0 - \mu_1 + C}{\sigma_\nu}\right)
\end{aligned}
$$

Substituding 1 for 0 in Eq. (4), we get

$$
\mathbb{E}(\epsilon_1 \mid \frac{\nu}{\sigma_\nu}) = \frac{\rho_{1\nu}\sigma_1}{\sigma_\nu}\nu \tag{9}
$$

Eq. (7) now becomes

$$
\sigma_{1\nu} = cov(\epsilon_, \nu) = \mathbb{E}(\epsilon_1(\epsilon_1 - \epsilon_0)) = \sigma_1^2 - \sigma_{01} \tag{10}
$$

Hence together we get

$$
\mathbb{E}(w_1 \mid I) = \mu_1 + \rho_{1\nu}\sigma_1 \frac{\phi(z)}{1 - \Phi(z)} = \mu_1 + \frac{\sigma_0\sigma_1}{\sigma_\nu}\left(\frac{\sigma_1}{\sigma_0} - \rho\right)\frac{\phi(z)}{1 - \Phi(z)} \tag{11}
$$

4.1.2 $Q_0 > 0$ and $Q_1 < 0$?

Accordind to Eq. (8) and Eq. (11), this would imply that

$$
\begin{cases}
\rho > \frac{\sigma_0}{\sigma_1} \\
\rho > \frac{\sigma_0}{\sigma_1}
\end{cases}
$$

But either will exceed 1, making it mathematically imposible to happen since correlation coefficient must be bounded under 1.

Intuitively, it means that people migrate to a place where expectation wage is lower than that in local. which is weird and irrational.

5 Simulation

5.1 Simulation

5.1.1 The code

```r
library(MASS)
library(data.table)
library(dplyr)
library(xtable)
setwd("~/111-1/Labor Economic/HW1/src")

    ## Parameters settings
N = 1e7
mu0 = 10
mu1 = 15
sigma0 = 3
sigma1 = 4.5
sigma_01 = 2
sigma_matrix = matrix(c(sigma0^2, sigma_01,
                        sigma_01, sigma1^2),
                      ncol=2)
C = 3

    ## Creating error terms, saving to data.table
wage = data.table(
  mvrnorm(n=N,mu=c(0, 0),Sigma=sigma_matrix)
)

  ## Rename
setnames(wage, "V1", "e0")
setnames(wage, "V2", "e1")

    ## Creating variables
wage[, W0 := e0 + mu0]
wage[, W1 := e1 + mu1]
wage[, Nu := e1 - e0]
wage[, I  := W1 > W0 + C]

    ## Calculating the empirical conditional mean
E_w0_I = wage[I==T, mean(W0)]
E_w1_I = wage[I==T, mean(W1)]

    ## Calculate the theoretical value

## Calculate rho_nu
rho = sigma_01/(sigma0 * sigma1)
sigma_nu = sqrt( sigma0^2 + sigma1^2 - 2 * sigma_01 )

## calculate z
z = (mu0 - mu1 + C)/sigma_nu
```

```
IMR = dnorm(z)/(1-pnorm(z))

## According to formula
calc_Q0 = mu0 + (sigma0 * sigma1)/sigma_nu * (rho - sigma0/sigma1) * IMR
calc_Q1 = mu1 + (sigma0 * sigma1)/sigma_nu * (sigma1/sigma0 - rho) * IMR

## Compare
result = data.frame("Wage in source" = c(E_w0_I, calc_Q0), "Wage in host" = c(E_w1_I, calc_Q1))
rownames(result) = c("Simulation", "Theoretical")

print(
  xtable(result, caption = "Simulation result versus the theoretical result"),
       floating = TRUE, latex.environments = "center",
  file="roy_sim.tex")
```

5.1.2 Simulation Result

	Wage.in.source	Wage.in.host
1	9.22	17.05
2	9.22	17.04

Table 1: Simulation result versus the theoretical result