1 Setups

1.1 Data Camp

I completed the course by taking the quiz.

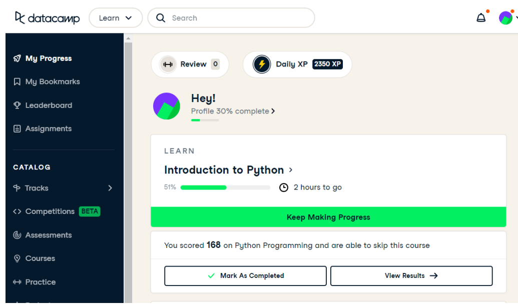

Figure 1: Quiz in datacamp

1.2 R

As shown in Figure 2

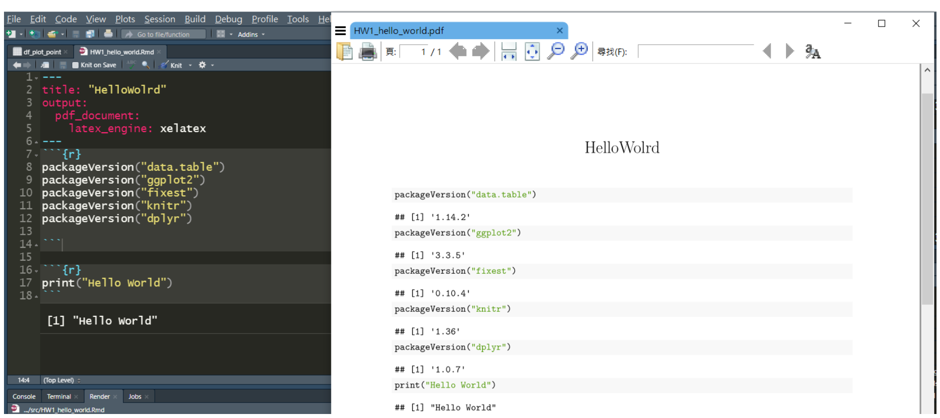

Figure 2: Settings and the libraries

1.3 Debugger

See figure 3

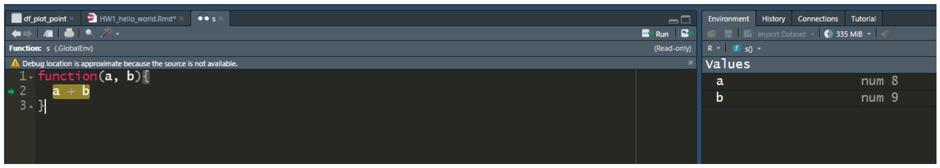

Figure 3: Screenshot of using debugger on a simple function

1.4 GitHub Setting

I created a repository for the whole lecture instead. See figure 4

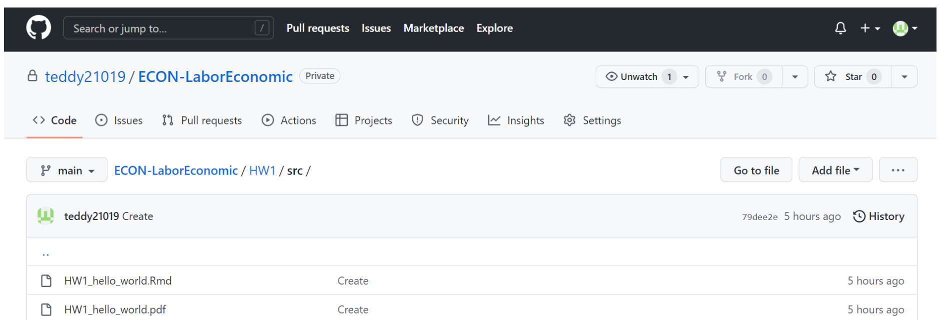

Figure 4: Github page, with the hello world files pushed on it

2 NBER Working Paper

2.1 List paper on NBER

The second paper on the list is

> KFstar and Portfolio Inflows: A Focus on Latin America
>
> by Burger, John D and Warnock, Francis E and Warnock, Veronica Cacdac

2.2 Download a Paper

As in figure 5, I downloaded the paper.

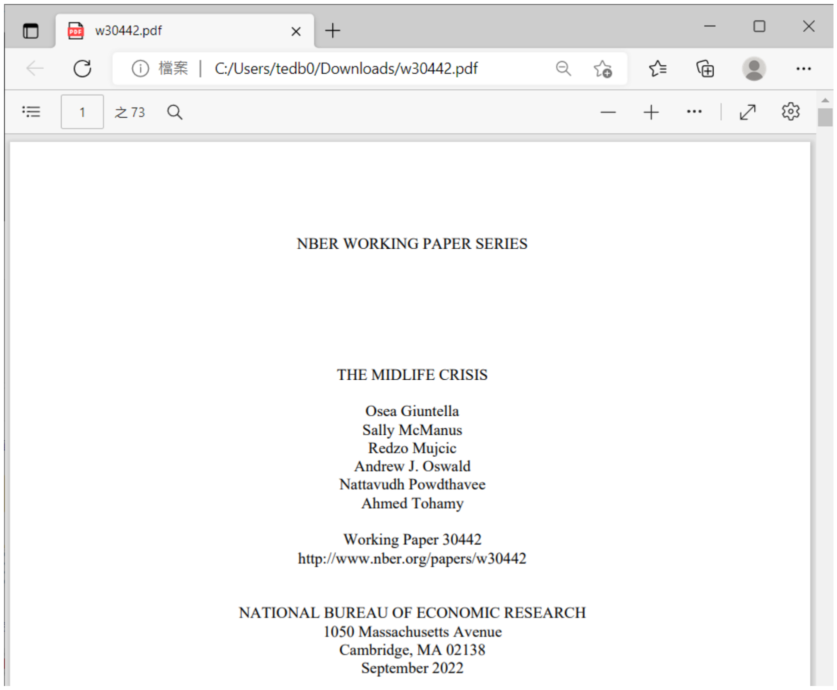

Figure 5: Screenshoe - paper downloaded

3 SRDA

3.1 SRDA Account

The account info is shown in figure 6

4 Roy Model

4.1 Derivation of Roy Model

We first start from considering the probability of migration. One migrates under the condition

$$w_{1i} > w_{0i} + C \tag{1}$$

Where $w_{0i} = \mu_{0i} + \epsilon_0$ and $w_{1i} = \mu_{1i} + \epsilon_1$, and the error terms together follow a joint-normal distribution

Figure 6: Account info of SRDA

$$\begin{pmatrix}\epsilon_0\\ \epsilon_1\end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}\sigma_0^2, \sigma_{01}\\ \sigma_{01}, \sigma_1^2\end{pmatrix}\right) \tag{2}$$

The probability of migration is then

$$\begin{aligned} P(w_{1i} > w_{0i} + C) &= P(v_i > \mu_0 - \mu_1 + C) \\ &= 1 - \Phi(\frac{\mu_0 - \mu_1 + C}{\sigma_\nu}) \\ &= 1 - \Phi(z) \end{aligned} \tag{3}$$

The expected wage of an imigrant is
Note: Subscripts i are neglected for simplicity

$$\begin{aligned} \mathbb{E}(w_0 \mid I) =& \mathbb{E}(w_0 \mid v > \mu_0 - \mu_1 + C) \\ =& \mu_0 + \mathbb{E}(\epsilon_0 \mid v > \mu_0 - \mu_1 + C) \\ =& \mu_0 + \mathbb{E}\left(\epsilon_0 \mid \frac{\nu}{\sigma_\nu} > \frac{\mu_0 - \mu_1 + C}{\sigma_\nu}\right) \end{aligned}$$

A linear combination of random variables following normal distribution also follows a normal distribution. Therefore $\mathbb{E}\left(\epsilon_0 \mid \frac{\nu}{\sigma_\nu}\right)$ is just a conditional expectation of a bivariate normal distribution consisting ϵ and ν/σ_ν.

Since for a bivariate normal distribution, we have

$$\mathbb{E}[X \mid Y = y] = \mu_x + \rho_{xy}\frac{\sigma_x}{\sigma_y}(y - \mu_y) \tag{4}$$

Substituding, we get

$$\mathbb{E}(\epsilon_0 \mid \nu) = \rho_{0\nu}\frac{\sigma_0}{\sigma_\nu}\nu$$

and hence

$$\mathbb{E}(\epsilon_0 \mid \frac{\nu}{\sigma_\nu}) = \left(\rho_{0\nu}\frac{1}{\sigma_\nu}\right)\frac{\sigma_0}{\sigma_\nu\frac{1}{\sigma_\nu^2}}\frac{\nu}{\sigma_\nu} = \frac{\rho_{0\nu}\sigma_0}{\sigma_\nu}\nu \tag{5}$$

Therefore

$$\begin{aligned}\mathbb{E}(w_0 \mid I) =& \mu_0 + \mathbb{E}\left(\epsilon_0 \mid \frac{\nu}{\sigma_\nu} > \frac{\mu_0 - \mu_1 + C}{\sigma_\nu}\right) \\ &= \mu_0 + \rho_{0\nu}\sigma_0\,\mathbb{E}\left(\frac{\nu}{\sigma_\nu} \mid \frac{\nu}{\sigma_\nu} > \frac{\mu_0 - \mu_1 + C}{\sigma_\nu}\right)\end{aligned} \tag{6}$$

Realize that the last expectation in Eq. 6 is the expected value of a truncated normal distribution, which can rewrite it as

$$\mathbb{E}(w_0 \mid I) = \mu_0 + \rho_{0\nu}\sigma_0\frac{\phi(z)}{1 - \Phi(z)} \tag{7}$$

Since the correlation coefficient can be negative, we know nothing about this result Let us preceed and solve for $\rho_{0\nu}$ further for better insight.

$$\rho_{0v} = \frac{\sigma_{0\nu}}{\sigma_0\sigma_\nu}$$

$$\sigma_{0\nu} = cov(\epsilon_0, \nu) = \mathbb{E}(\epsilon_0(\epsilon_1 - \epsilon_0)) = \sigma_{01} - \sigma_0^2 \tag{8}$$

Substitude the result from Eq. 8 and $\rho_{01} = \sigma_{01}/\sigma_0\sigma_1$, Eq. 7 now becomes

$$\mathbb{E}(w_0 \mid I) = \mu_0 + \frac{\sigma_{01} - \sigma_0^2}{\sigma_\nu}\frac{\phi(z)}{1 - \Phi(z)} = \mu_0 + \frac{\sigma_0\sigma_1}{\sigma_\nu}\left(\rho - \frac{\sigma_0}{\sigma_1}\right)\frac{\phi(z)}{1 - \Phi(z)} \tag{9}$$

Similarily, for $\mathbb{E}(w_1 \mid I)$, we have

$$
\begin{aligned}
\mathbb{E}(w_1 \mid I) =& \mathbb{E}(w_1 \mid v > \mu_0 - \mu_1 + C) \\
=& \mu_1 + \mathbb{E}(\epsilon_1 \mid v > \mu_0 - \mu_1 + C) \\
=& \mu_1 + \mathbb{E}\left(\epsilon_1 \mid \frac{\nu}{\sigma_\nu} > \frac{\mu_0 - \mu_1 + C}{\sigma_\nu}\right)
\end{aligned}
$$

Substituding 1 for 0 in Eq. 5, we get

$$\mathbb{E}(\epsilon_1 \mid \frac{\nu}{\sigma_\nu}) = \frac{\rho_{1\nu}\sigma_1}{\sigma_\nu}\nu \tag{10}$$

Eq. 8 now becomes

$$\sigma_{1\nu} = cov(\epsilon_,\nu) = \mathbb{E}(\epsilon_1(\epsilon_1 - \epsilon_0)) = \sigma_1^2 - \sigma_{01} \tag{11}$$

Hence together we get

$$\mathbb{E}(w_1 \mid I) = \mu_1 + \rho_{1\nu}\sigma_1 \frac{\phi(z)}{1 - \Phi(z)} = \mu_1 + \frac{\sigma_0\sigma_1}{\sigma_\nu}\left(\frac{\sigma_1}{\sigma_0} - \rho\right)\frac{\phi(z)}{1 - \Phi(z)} \tag{12}$$

4.2 $Q_0 > 0$ and $Q_1 < 0$?

This would imply that

$$\begin{cases} \rho > \frac{\sigma_0}{\sigma_1} \\ \rho > \frac{\sigma_0}{\sigma_1} \end{cases}$$

But either will exceed 1, making it mathematically imposible to happen since correlation coefficient must be bounded under 1.

Intuitively, it means that people migrate to a place where expectation wage is lower than that in local. which is weird and irrational.