Evolutionary Dynamics of Currency Substitution

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JANUARY 11, 2023

Settings

Model Setting and Background

Consider a open economy with

- Two fiat currency
- Flexible exchange rate
- Perfect Substitution

Also, the economy has

- No government intervention
- No restriction on holding

How is the exchange rate and inflation rate determined?

Kareken-Wallace Model (1981)

- Overlapping generation model (OG)
- Same property in previous slide
- Agent *save* through holding fiat money

$$
\begin{aligned}
\max \quad & \ln c_t(t) + \ln c_t(t+1) \\
s.t. \quad c_t(t) \leq & w^1 - \frac{m_1(t)}{p_1(t)} - \frac{m_2(t)}{p_2(t)} \\
c_t(t+1) \leq & w^2 + \frac{m_1(t)}{p_1(t+1)} + \frac{m_2(t)}{p_2(t+1)}
\end{aligned}
$$

- $c_t(t+1)$ — Born at t, consumption at $t+1$
- $w^1 (w^2)$ — Endowment when young (old)
- $m_1(t)$ — Holding of currency 1 at t
- $p_1(t)$ — Price of good in currency 1 at t

Results of the KW model

1. No specific exchange rate and price (nondeterministic)
2. Equal return in money during equilibrium (no arbitrage)
3. Two stationary inflation rate $\pi \equiv \frac{p_i(t+1)}{p_i(t)} \quad \forall i$
   - High inflation : Stable
   - Low inflation : Unstable
4. Also a set of one-currency equilibrium. Remains to have two inflation rate.

- Since no uncertainty, an equilibrium condition

$$R(t)=\frac{p_1(t)}{p_1(t+1)}=\frac{p_2(t)}{p_2(t+1)},\quad t\geq 1$$

- Define exchange rate $e(t)\equiv\frac{p_1(t)}{p_2(t)}$, we also have

$$e(t+1)=e(t)=e,\quad t\geq 1$$

- The agent's maximization problem yields

$$s(t)=\frac{m_1(t)}{p_1(t)}+\frac{m_2(t)}{p_2(t)}=\frac{1}{2}\left[w^1-w^2\frac{1}{R(t)}\right]$$

- Aggregate savings in the world

$$S(t)=N\left[w^1-w^2\frac{1}{R(t)}\right]=\frac{H_1(t)}{p_1(t)}+\frac{H_2(t)}{p_2(t)}$$

where $H_i(t)$ is the nominal supply of currency i at time t

Government Consumption

Government i finances purchases by issuing currency i

$$G_i = \frac{H_i(t) - H_i(t-1)}{p_i(t)}$$

i.e. finance via seignorage.

World government consumption can be derived

$$G_1 + G_2 = S(t) - S(t-1)R(t-1)$$

Exchange Rate is Indeterminate

Since what matters is real saving $s(t)$, or in aggregate form $S(t)$

$$S(0) = \frac{H_1(0) + H_2(0)\,e}{p_1(1)} = \frac{H_1(0) + H_2(0)\hat{e}}{\hat{p}_1(1)}$$

And

$$G_w = G_1 + G_2 = S(1) - S(0)$$

Together

$$\frac{H_1(0) + H_2(0)\hat{e}}{\hat{p}_1(1)} = S(1) - [G_1 + G_2]$$

The values of $\hat{p}_2$ can be selected to solve any pair of $\hat{e}, \hat{p}_1(1)$ that keeps the aggregate saving unaltered.

Inflation rate

Aggregate government consumption + equilibrium aggregate saving :

$$\frac{1}{R(t+1)} = \pi_w(t+1) = \frac{w^1}{w^2} + 1 - \frac{G_w}{2Nw^2} - \frac{w^1}{w^2}\frac{1}{\pi_w(t)}$$

- Stationary inflation rate comes in quadratic form
- High inflation rate is the **stable equilibrium**
- Low inflation rate is the **unstable equilibrium**

Breaking down of Currency System

- Another equilibria — single currency scheme
- One currency is not valued $1/p_i(t) = 0$
- Again two stationary equilibria, high and low inflation
- Price level constant if supply of money constant

In perfect foresight, if $G_1 = 0, G_2 > 0$

- Growth of wolrd money supply = Growth of currency 2
- As $t \to \infty$, currency 2 drive currency 1 out of agent.
- Currency of country with restrictive monetary policy cannot survive???
- ACE dynamics presents different result.