Dynamic Bank Runs: An Agent-based Approach

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Introduction

Diamond and Dybvig (1984)

1. Influential paper about the formation and run of a bank
2. Three period model
3. Multiple equilibria

There are things that DD model is hard to handle:

1. Dynamic process
2. Wealth accumulation

This paper tackles these aspects using ABM.

Grasselli and Ismail (2013)

1. Multibank environment to study financial contagion
2. Model only adverse liquidity shock, not by depositors' coordination problem.

Modification

- Bank run triggered by depositors' strategic decision in a coordination game (DD84)
- Complex adaptive system - Memory + rule selection (GI13)
- Cycle of DD, 3 period in each cycle
- Endogenous bank formation
- Account for the effect from social networks (spacial relation)

Findings

1. Number of bank run decreases with reserve size
2. Decreases with threshold of withdrawal
3. Tradeoff between financial stability and concentration of banking industry (lots of small banks causes more runs)

Model

Timeline in one cycle

Subperiod 0	Subperiod 1	Subperiod 2
Agent draw random preference $U_i \sim [0,1]$	Liquidity shock $\rho_i = \bar{U}_i + (-1)^{b_i} \frac{\epsilon_i}{2}$	Holding illiquid asset - Receive R
Agent endowed with 1	Nondepositor - Search to trade asset.	Depositors - Receive r_2
$\bar{U}_i < 0.5$ - Impatient, invest in liquid asset	Depositors - Choose to withdraw r_1 or not	
$\bar{U}_i > 0.5$ - Patient, invest in illiquid asset	Liquid asset - Repay 1, Illiquid asset - Repay r	
Alternatively - Become depositor	Bank - Face withdrawal, decide to default	

Preference Shock

There is an initial preference shock at t=0 (subperiod is denoted as t)

$$U_i \sim \text{Unif}[0,1]$$

Denote its realization as $\bar{U}_i$
A new preference shock in t=1

$$\rho_i = \bar{U}_i + (-1)^{b_i}\frac{\epsilon_i}{2}$$

$b_i \sim$ Bernoulli(0.5), $\epsilon_i \sim \text{Unif}[0,1]$
For both U and ρ, > 0.5 represents patient, vise versa.

Rate of return

Type	t=1	t=2
Liquid Asset	1	
Illiquid Asset	r	R
Deposit	r_1	r_2

$$r < 1 < r_1 < r_2 < R$$

The rate of returns are publicly known to everybody.

Bargain happens between asset holders that have inconsistent intertemporal preference.

- Impatient in t=0, patient in t=1 (Positive preference shock)
- Patient in t=0, impatient in t=1 (Negative preference shock)

Might not find a partner in his social network *v*. The Moore neighborhood in this case.

- Decision is made in t=0, depending on the impatient agents in his social network *v*.
- Unknown proportion of impatient agents $w \in \{0, \frac{1}{9}, \dots, 1\}$

Become a banker if per capita present value must provide is less than endowment:

$$f(w_i) = w_i r_1 + (1 - w_i)\frac{r_2}{R} \leq 1$$

or

$$w_i \leq \frac{R - r_2}{Rr_1 - r_2}$$

Not honoring / discouraging of creation

For each value $Q \in (1, r_1)$, there exist $\omega \in [w^*, 1]$ such that $f(\omega) = Q$. Where $f(w^*) = 1$.

There are realizations of w that discourage the creating or incentive to default.

Within the per capita present value the bank must provide

$$f(w_i) = w_i r_1 + (1 - w_i)\frac{r_2}{R}$$

- $x_i = (1 - w_i)\frac{r_2}{R}$ - Investment in illiquid assets.
- $y_i = w_i r_1$ - Investment in liquid assets.
- $1 - x_i - y_i$ - Added to reserve

Investment decision of Bank

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Intuition

The bank ensures $y_i = w_i r_1$ of liquid assets to provide withdrawal in first period, and ensures $Rx_i = (1 - w_i)r_2$ of illiquid assets to provide withdrawal in the second period. The rest are kep as reserves.

Becoming a client

$T(v, pyf)$: If the agent evaluates that it is advantageous, he opens an account in a bank in the immediate neighborhood; if there is none in this condition, he becomes a client of the same bank of one of his neighbors.

- v — Agent's social network
- pyf — Result of the comparison of payoffs.

pyf is determined by a learning rule.

Decision of becoming a client

- Follows Grasselli and Ismail (2013)
- Agents have memory of 5 cycles.
- Memory information has three states.
  - N — If the *budget constraint* remains unchanged after the shock
  - B — There was a change but no partner was found.
  - G — There was a change and someone to bargain with was found
- Total of 7 predictors

Predictors I

1. k will be the same as k-1
2. k will be the same as t-2
3. ... t-3
4. ... t-4
5. ... t-5
6. k will be equal to mode of last 3 previous cycle
7. k will be equal to mode of last 5 previous cycle

Each predictor maps to a forecast of one of the 3 states, (I) denote $\Theta = [\theta_1, \theta_2, \ldots, \theta_7]$, where $\theta_i \in \{N, B, G\}$

In the decision to become a bank customer, the agent can map the return of each predictor to a situation in which he deposits or not his cash in bank, obtaining, respectively, the vectors $\underset{1\times 7}{\Pi_d}$ *and* Π_n

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How is the return calculated?

Note that $\bar{U}_i$ is initialized during decision

1. Do agents have to consider the probability of being patient when deposit??
2. Will all elements of Π_d be the same?
3. Do agents consider present value on t=1?

Decision

$$A^* = \arg\max_{A \in \{d,n\}} \Pi_A \cdot \Phi$$

$\underset{1\times 7}{\Phi}$ is the weight, called "force", of the predictor vectors.

That is, the agent decides to become a client of its neighbor's bank, or the same as his neighbor, if $\Pi_d \cdot \Phi > \Pi_n \cdot \Phi$

The bank is chosen as its neighbor's bank.

Law of Motion for the Weight

For each of the predictor, $+1$ to the corresponding weight if correctly forecasted, and -1 if the not.

$$\phi_{j,t+1} = (-1)^{\mathbb{1}\{\theta_{j,t}=\hat{\theta}_t\}} + \phi_{j,t}$$

Where $\hat{\theta}_t \in \{N, B, G\}$ is the true realization state at time t.

Withdrawal

Sequential service constraint

Closer clients withdraw first.
Random assign as the tie-breaking rule.

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Withdrawal period (normal case)

1. Impatient depositors ($\rho < 0.5$) — withdraw at t=1, get r_1
2. Patient depositors ($\rho < 0.5$) — withdraw at t=2, get r_2

The key of bank run is the allowance of patient clients to imitate the decision of neighbors (in its social network)

Imitation Rule

If $\rho > \frac{1}{2}$ but more than *n* neighbors in his social network *v* intend to withdraw in now (t=1), then the agent withdraws.

Bank's Behavior

The bank pays r_1 in t=1, and pays r_2 in t=2, the rest is saved as reserves.

The expected proportion of impatient agents in the next cycle follows an adaptive rule

$$w_i^k = w_i^{k-1} + \alpha(\bar{w}_i - w_i^{k-1})$$

Where $\bar{w}_i$ is the actual proportion of impatient clients.

Fail of a Bank

As mentioned before, closest bank clients withdraw first.

Order of assests used to pay

1. Liquid assets
2. Reserve
3. Illiquid assets

If the bank exhausts all its resources, the remaining clients receive nothing and they break link with the bank.

Fail

# of clients ≤ 5, it fails. The remain clients are released.

Simulation

Parameters

1. “r”s — r, r_1, r_2, R
2. Impatient threshold τ

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The authors set

- $r = 0.8$
- $r_1 \in \{1.001, 1.003, 1.005, 1.007, 1.009\}$
- $r_2 = 1.03$
- $R = 1.05$
- $\tau \in \{0.4, 0.6, 0.8\}$

Each combination of parameter is simulated 100 times, each with 1,000 cycles(3 subperiod of Diamond Dybvig environment each)

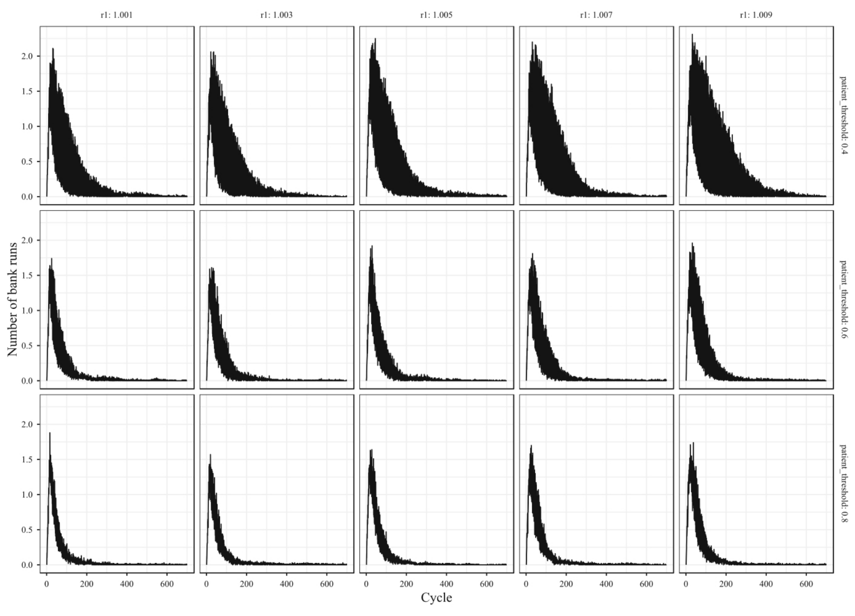

Figure 3: Average number of bank runs

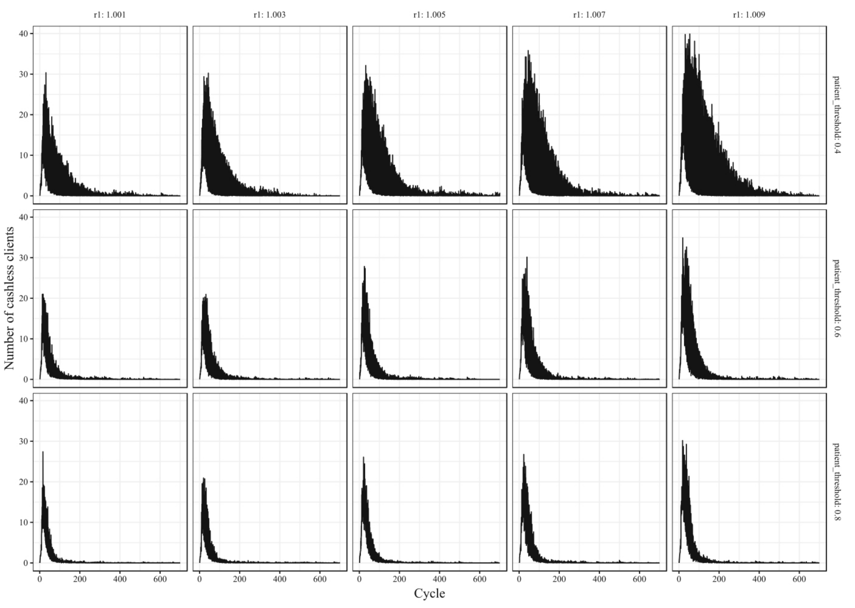

Figure 4: Average number of clients failed to withdraw

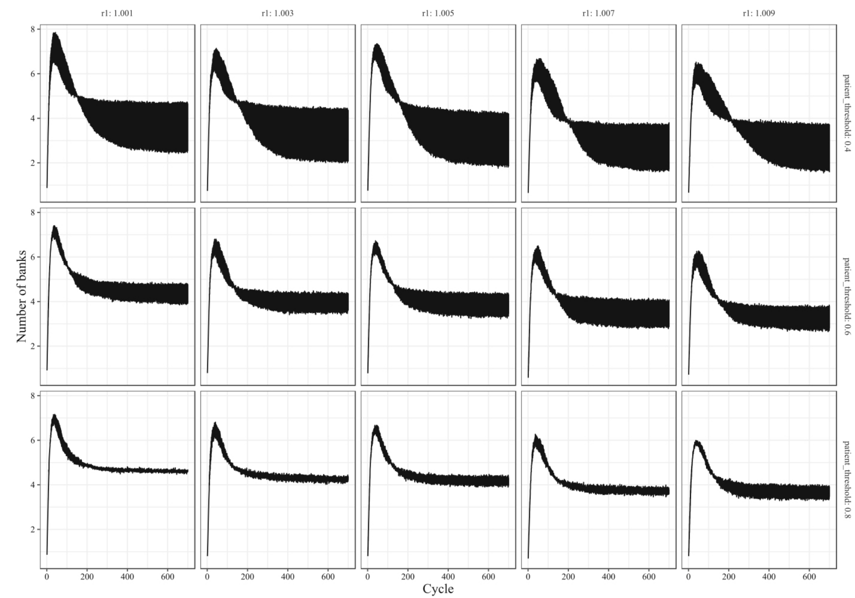

Figure 5: Average number of banks

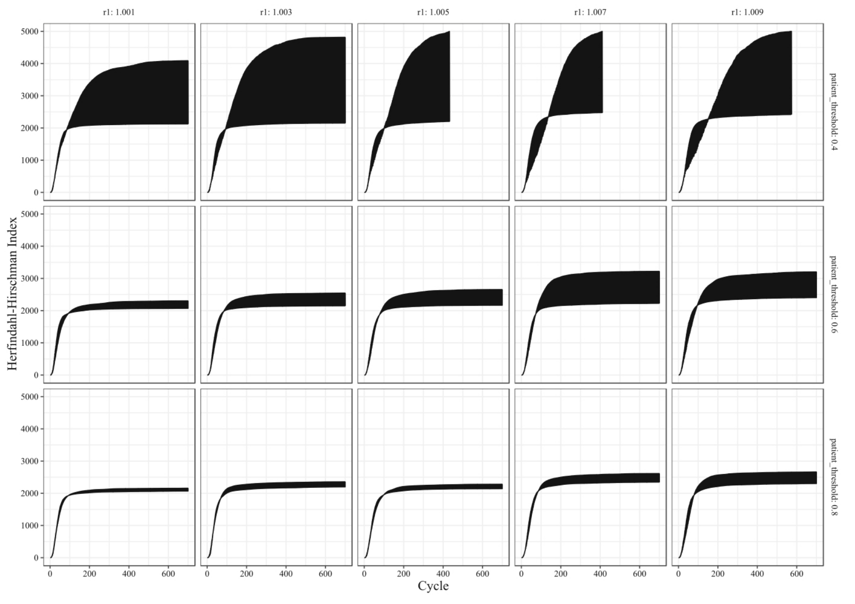

Figure 7: Average Herfindahl-Hirschman index (Sum of market share squared)

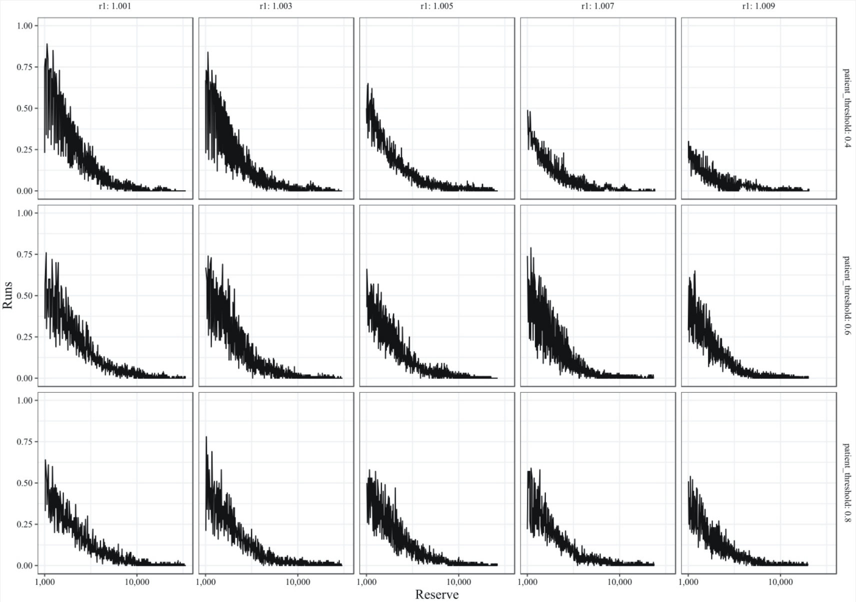

Figure 8: Bank reserve versus number of runs

Extension

Allowing Wealth Accumulation

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If we allow depositors to accumulate wealth:

1. Bank reserve large, harder for bank run to occur.
2. Withdrawal amount increases, easier for bank run to occur.

The extension checks if this scenario is possible.

Modification — Wealth

Some agents might never withdraw in t=1

- Wealth follows a geometric progression growth

- ω_i — available wealth
- 1 unit of endowment at each t=0
- Agents allocate ω_i in asset market or deposit
- Spends ω_i each cycle — (Sort of) Hand to mouth setting

Allocation decision of ω_i

Agents not clients of bank

- Impatient — Spent all in t=1. $\omega_{i,t+1} = 0$
- Patient — No spending in t=1, receive $R\omega_i$ in t=2, spend ω_i

Agents who are clients of bank

- Impatient — received $r_1\omega_i$ in t=1, spend ω_i in t=1
- Patient — No spending in t=1, receive $r_2\omega_i$ in t=2, spend ω_i

Banks' allocation

The authors did not mentioned the modification on the formation and allocation of banks.
The orders of offering money is the same

1. Liquid assests
2. Reserves
3. Illiquid assets

If the bank has not enough to pay, some clients receive nothing, and the bank fails.

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- Bank run emerges from simple imitation rule, which considers only limited knowledge from the environment(neighbors), not a global information.
- Endogenously selection of becoming clients of bank.

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Criticisms

1. Consumption smoothing

Adjustment and Improvements

1. Existing large banks. Endogenous bank formation is not necessary in most applications.
2. Exogenous bank-depositor network — Spacial network is not enough.
3. Consumption and preference shock should be endogenous and follow a transition process.

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Criticisms

1. Consumption smoothing
2. Expected value of a forecast not described clearly

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3. Consumption and preference shock should be endogenous and follow a transition process.