Banking, Liquidity and Bank Runs in an Infinite Horizon Economy

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Introduction

Banking Distress and the Economy

1. Gertler and Kiyotaki (2011)
    - Depletion of capital
    - Ability to raise fund
    - Cost of bank credit
    - Slows the economy
2. Diamond and Dybvig (1983)
    - Liquidity mismatch
    - Inefficient asset liquidation
    - Possibility of bank run

Most models capture one, but lack the other

- Financial acceleration effect, but no bank run
- Bank run, but not connected to fundamentals

Ben Bernanke (2010) and Gorton(2010)

- Weakening financial position led to classical runs
- Usually on shadow banking sectors

Historical fact:

1. Slow run — Mar 2008, Bear Stearns. Creditors are reluctant to deposit
2. Fast run — Sep 2008, Lehman Brothers. Collapse of the entire shadow banking system.

1. Balance sheet condition affect both cost of bank credit and possibility of bank run
2. Anticipation of run can affect asset price.

Approach closer to Cole and Kehoe (2000) on *self-fulfilling debt crisis*. Therefore no *sequential service constraint*.

Model—Main Structure

Resource

- Fixed capital, allocated to bank and household

$$K_t^b + K_t^h = 1 \tag{1}$$

- Payoff for banker

$$\begin{array}{ccc} \text{date } t & & \text{date } t+1 \\ K_t^b & \rightarrow & \begin{cases} Z_{t+1} K_t^b \\ K_t^b \end{cases} \end{array} \tag{2}$$

- Payoff for HH

$$\begin{array}{ccc} \text{date } t & & \text{date } t+1 \\ \begin{cases} K_t^h \\ f(K_t^h) \end{cases} & \rightarrow & \begin{cases} Z_{t+1} K_t^h \\ K_t^h \end{cases} \end{array} \tag{3}$$

- HH needs management cost

$$f(K_t^h) = \frac{\alpha}{2} (K_t^h)^2 \tag{4}$$

Household

Household Saving

- HH holds nondurable goods $Z_t W^h$ each period.
- HH deposits their funds to (shadow) banks

$$R_{t+1} = \begin{cases} \bar{R}_{t+1} & \text{if no bank run} \\ x_{t+1}\bar{R}_{t+1} & \text{if bank run} \end{cases} \tag{5}$$

all HH is distributed equal amount, unlike DD.

Household decision I

- HH maximizes lifetime utility

$$U_t = E_t \left(\sum_{i=0}^{\infty} \beta^i \ln C_{t+i}^h \right)$$

- s.t

$$C_t^h + D_t + Q_t K_t^h + f(K_t^h) = Z_t W^h + R_t D_{t-1} + (Z_t + Q_t) K_{t-1}^h \quad (6)$$

Household decision II

- (Assuming 0 prob of bank run) F.O.C for deposit

$$E_t \Lambda_{t,t+1} R_{t+1} = 1 \tag{7}$$

- $\Lambda_{t,t+i}$ is defined by

$$\Lambda_{t,t+i} = \beta^i \frac{C_t^h}{C_{t+i}^h} \tag{8}$$

- F.O.C for holding capital

$$E_t \Lambda_{t,t+1} R_{t+1}^h = 1 \tag{9}$$

with

$$R_{t+1}^h = \frac{Z_{t+1} + Q_{t+1}}{Q_t + f'(K_t^h)} \tag{10}$$

Remark

- If HH hold capital, Eq. 9 help determine market price.
- i.e., Liquidation price during run.
- $Q_t = E_t \Lambda_{t,t+1}(Z_{t+1} + Q_{t+1}) - f'(K_t^h)$. Market price **tends to** be decreasing as K_t^h increase.
- During bank run, HH absorbs all capital from bank, asset prices drop sharply.

Bank

Finite expected lifetime

100 percent equity financing

To the extent bankers may face financial market frictions, they will attempt to save their way out of the financing constraint by accumulating retained earnings in order to move toward 100 percent equity financing.

To limit this possibility — probability σ of surviving, hence expected lifetime $\frac{1}{1-\sigma}$

1. Meaning?
2. How does limited lifetime help solve this? In reality?

The Balance Sheet

- Net worth (Assets - liabilities) for surviving bankers

$$n_t = (Z_t + Q_t)k^b_{t-1} - R_t d_{t-1} \tag{11}$$

- Net worth for new bankers — get initial endowment

$$n_t^{\text{new banker}} = w_b \tag{12}$$

- Exiting bankers consume all their net worth

$$c^b_t = n_t \tag{13}$$

- Bank finance assets holdings with new deposit + net worth (retained earnings)

$$Q_t k^b_t = d_t + n_t \tag{14}$$

- Banks choose $\{d_t, k^b_t\}$ each period to maximize its franchise value.

Agency Problem

- A banker could play honest — hold depositors' asset until realization in $t+1$, and pay deposit
- Banker would also want to divert some assets for personal use
- Assume can sell $\theta \in (0,1)$ of assets secretly

Risk of doing so?

Depositors can force a bankruptcy in the next period.
The gain from diverting fund can not exceed the franchise value.

That is,

$$\theta Q_t k_t^b \leq V_t \tag{15}$$

Franchise Value and Incentive Constraint

Franchise Value

Present discount value fo future payouts from operating honestly.

$$V_t = E_t[\beta(1-\sigma)n_{t+1} + \beta\sigma V_{t+1}] \tag{16}$$

Higher franchise value reduces excessive risk taking strategy by banks (See Demsetz, Saidenberg and Strahan, 1996)

The franchise value is constant return to scale, so the bank's optimization problem is reduced to maximizing the Tobin's Q

Tobin's Q

In this context, franchise value is its market value, and the replacement cost is defined by its net worth

$$\Psi_t \equiv V_t/n_t$$

Dividing the franchise value by its net worth, we get

$$\frac{V_t}{n_t} = E_t\left[\beta(1-\sigma)\frac{n_{t+1}}{n_t} + \beta\sigma\frac{V_{t+1}}{n_{t+1}}\frac{n_{t+1}}{n_t}\right]$$

$$\begin{aligned}\frac{n_{t+1}}{n_t} =& \frac{(Z_{t+1}+Q_{t+1})k_t^b - R_{t+1}d_t}{n_t} \\ =& \underbrace{\frac{Z_{t+1}+Q_{t+1}}{Q_t}}_{\equiv R_{t+1}^b}\underbrace{\frac{Q_t k_t^b}{n_t}}_{\phi_t} - R_{t+1}\frac{d_t}{n_t} \\ =& (R_{t+1}^b - R_{t+1})\phi_t + R_{t+1}\end{aligned} \tag{17}$$

Bank's problem — max Tobin's Q

The bank's problem become choosing leverage

$$\psi_t = \max_{\phi_t}\{\mu_t\phi_t + \nu_t\} \tag{18}$$

The incentive constraint

$$\theta Q_t k_t^b \leq V_t \implies \theta\phi_t \leq \psi_t = \mu_t\phi_t + \nu_t \tag{19}$$

$$\mu_t = E_t[\beta\Omega_{t+1}(R_{t+1}^b - R_{t+1})] \qquad \text{Excess M.V of assets over deposit} \tag{20}$$

$$\nu_t = E_t[\beta\Omega_{t+1}R_{t+1}] \qquad \text{M.C of deposit} \tag{21}$$

$$\Omega_{t+1} = (1-\sigma)\times 1 + \sigma \times \psi_{t+1} \qquad \text{M.V of net worth}$$

IC BINDING

The IC binds when $\theta\phi_t = \psi_t = \mu_t\phi_t + \nu_t$, while $\theta_t \in (0,1)$, we must satisfy $0 < \mu_t < \theta$ and

$$\phi_t = \frac{\psi_t}{\theta} = \frac{\nu_t}{\theta - \mu_t}. \tag{22}$$

When IC is binding

- Portfolio size is balanced by (cost of losing) franchise value
- Fluctuation in net worth induce fluctuation in bank lending
- *Moreover, cannot negative net worth.* Otherwise, incentive constraint that ensures the bankers will not divert is violated. How?

Aggregation and Equilibrium without Runs

- Total asset held by bank during equilibrium (because ϕ_t is equal to all banks)

$$Q_t K_t^b = \phi_t N_t \tag{23}$$

- Total net worth evolution

$$N_t = \sigma \left[(Z_t + Q_t) K_{t-1}^b - R_t D_{t-1} \right] + \underbrace{W^b}_{(1-\sigma) w^b} \tag{24}$$

- Bankers' consumption — Exiting bank consumes all

$$C_t^b = (1 - \sigma) \left[(Z_t + Q_t) K_{t-1}^b - R_t D_{t-1} \right] \tag{25}$$

- Total output

$$Y_t = Z_t + Z_t W^h + W^b \tag{26}$$

- Total output usage

$$Y_t = f(K_t^h) + C_t^h + C_t^b \tag{27}$$

Bank run

- The entire banking system run
- Same decision for all depositors

Realization of Z_t causes bank run. If depositors decide to run, bank liquidates all its capitals, causing price to be Q^*.

- Value of bank's asset during forced liquidation — $(Z_t + Q_t^*)K_{t-1}^b$
- Liability still $R_t D_{t-1}$
- if $(Z_t + Q_t^*)K_{t-1}^b < R_t D_{t-1}$ — Net worth is wiped out.

Condition for bank run

$$x_t \equiv \frac{(Z_t + Q_t^*)K_{t-1}^b}{R_t D_{t-1}} < 1 \tag{28}$$

Recovery rate x_t

The recovery rate depends on two endogenous factors

- Liquidation price Q_t^*
- Balance sheet condition

$$
\begin{aligned}
x_t \equiv & \frac{(Z_t + Q_t^*)K_{t-1}^b}{R_t D_{t-1}} \\
= & \frac{Z_t + Q_t^*}{Q_{t-1}} \frac{1}{R_t} \frac{Q_{t-1}K_{t-1}^b}{D_{t-1}} \\
= & \frac{R_t^{b*}}{R_t} \frac{Q_{t-1}K_{t-1}^b}{D_{t-1}} \\
& \because Q_{t-1}K_{t-1}^b = N_{t-1} + D_{t-1} \implies \frac{D_{t-1}}{N_{t-1}} = \frac{Q_{t-1}K_{t-1}^b}{N_{t-1}} - 1 = \phi_{t-1} - 1 \\
= & \frac{R_t^{b*}}{R_t} \frac{\phi_{t-1}}{\phi_{t-1} - 1} \qquad (29)
\end{aligned}
$$

Intuition of recovery rate

We now reduced the recovery rate into just three variables

$$x_t = \frac{R_t^{b*}}{R_t}\frac{\phi_{t-1}}{\phi_{t-1}-1}$$

Bank run equilibrium occurs if

- Realized return on bank during liquidation R_t^{b*} is too low
- Leverage multiplier is too high

R_t^{b*}, R_t, ϕ_t are all endogenous — possibility of bank run varies with macroeconomic conditions.

- Equilibrium without bank run : R_t and ϕ_t
- ? Behavior of economy : Q_t^*

Liquidation Price Q_t^* I

If banks fully liquidate all their assets

1. Households hold all capitals

$$K_t^h = 1 \quad \text{During liquidation} \tag{30}$$

2. Forward solve Q_t^* from HHs' Euler equation.

Liquidation Price Q_t^* II

From HHs' Euler eq. $E_t \Lambda t, t+1 R_{t+1}^h = 1$ with $R_{t+1}^h = \frac{Z_{t+1} Q_{t+1}}{Q_t^* + f'(K_t^h)}$
Note that $f'(K_t^h) = \alpha K_t^h = \alpha$ during liquidation period, hence $E_t \Lambda_{t,t+1} \frac{Z_{t+1} Q_{t+1}}{Q_t^* + \alpha} = 1$. Rearranging, we get

$$
\begin{aligned}
Q_t^* &= E_t[\Lambda_{t,t+1}(Z_{t+1} + Q_{t+1})] - \alpha \\
&= E_t(\Lambda_{t,t+1} Z_{t+1}) + E_t(\Lambda_{t,t+1} Q_{t+1}) - \alpha
\end{aligned}
$$

Note that with Euler's equation on other periods,

$$E_t(\Lambda_{t+1,t+2} Z_{t+2}) + E_t(\Lambda_{t+1,t+2} Q_{t+2}) = E_t Q_{t+1} + E_t f'(K_t^h + 1)$$

and

$$\Lambda_{t+1,t+2} \Lambda_{t+0,t+1} = \beta \frac{C_t^h + 1}{C_{t+2}^h} \beta \frac{C_t^h}{C_{t+1}^t} = \beta^2 \frac{C_t^h}{C_{t+2}^h} = \Lambda_{t,t+2}$$

Liquidation Price Q_t^* III

We get

$$
\begin{aligned}
&Q_t^* = E_t(\Lambda_{t,t+1} Z_{t+1}) + E_t(\Lambda_{t,t+1} Q_{t+1}) - \alpha \\
&E_t(\Lambda_{t,t+1} Q_{t+1}) - E_t(\Lambda_{t,t+2} Q_{t+2}) = E_t(\Lambda_{t,t+2} Z_{t+2}) - E_t(\Lambda_{t,t+1} f'(K_{t+1}^h)) \\
&E_t(\Lambda_{t,t+2} Q_{t+2}) - E_t(\Lambda_{t,t+3} Q_{t+3}) = E_t(\Lambda_{t,t+3} Z_{t+3}) - E_t(\Lambda_{t,t+2} f'(K_{t+2}^h)) \\
&\vdots
\end{aligned}
$$

Sum to infinite, we get

$$Q_t^* = E_t \left[\sum_{i=1}^{\infty} \Lambda_{t,t+i} (Z_{t+i} - \alpha K_{t+i}^h) - \alpha \right] \tag{31}$$

Policy Implications

- Financial acceleration effect + sunspot runs both due to solvency problem.
- Setting maximum leverage multiplier ϕ, since $x_t(\phi)$ decreases with ϕ
- Lender of the last resort policies push up Q^*_{t+1}

Results

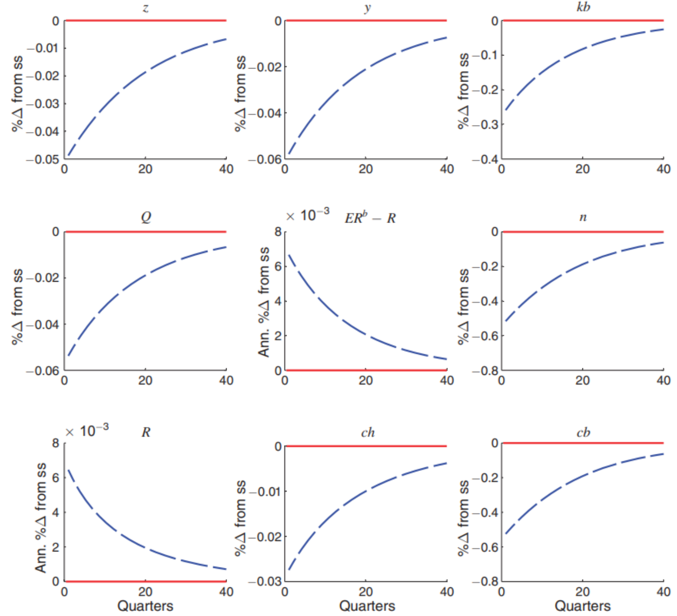

Figure 3: A Recession in the Baseline Model: No Bank Run Case

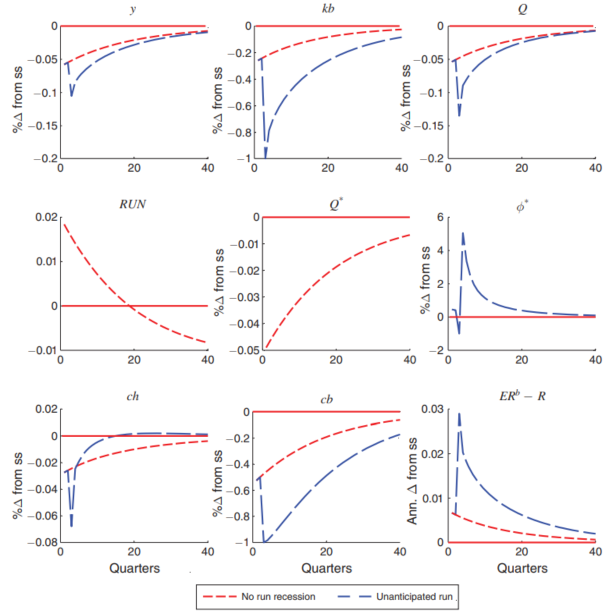

Figure 4: Ex Post Bank Run in the Baseline Model

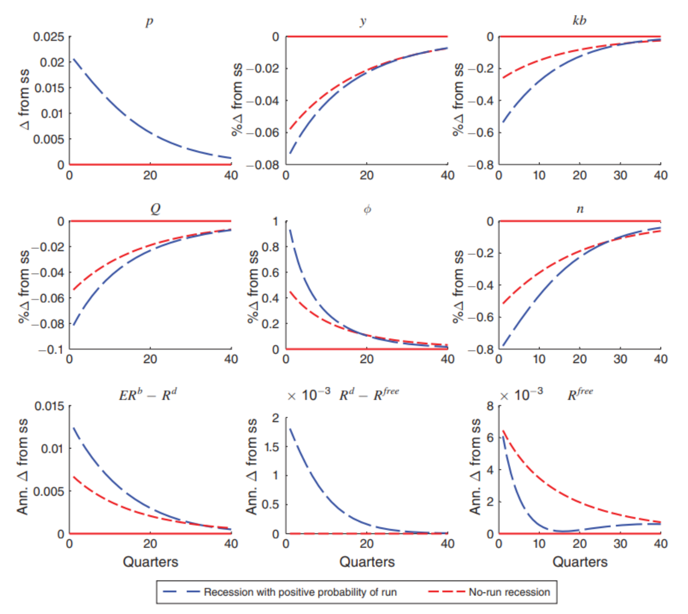

Figure 5: Recession with Positive Probability of a Run

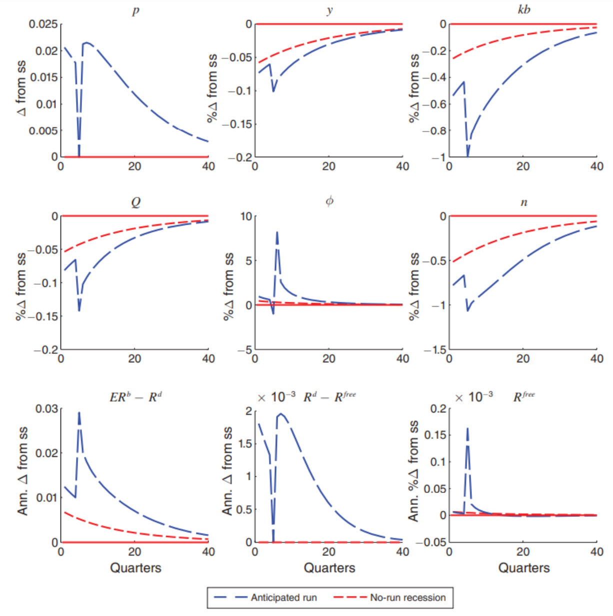

Figure 6: Recession with Positive Probability of a Run

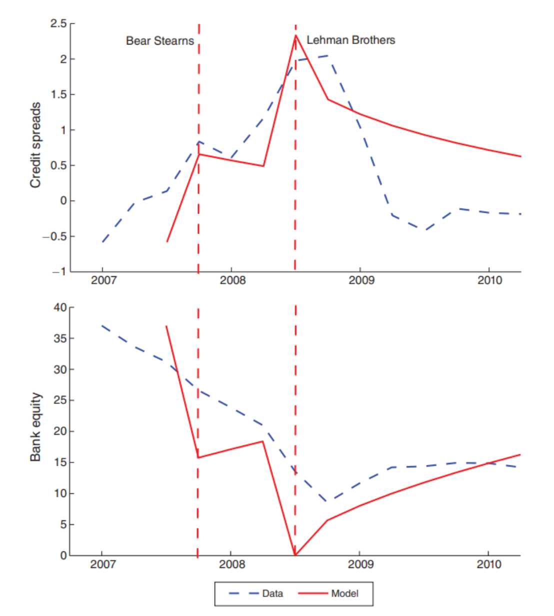

Figure 7: Recession with Positive Probability of a Run