Money as a Medium of Exchange in an Economy With Artificial Intelligent Agents

November 10, 2022

Introduction

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It turned out to have two consistent equilibria

- Fundamental equilibrium : Good 1 as the medium of Exchange
- Speculative equilibrium : Type 1 also wants good 3

This paper explores the *emergence* of such equilibria with agent-based modeling.

Notations

- $a \in \mathscr{A} = \{1, 2, ...A\}$: agent
- At time t , a holds good x_{at}
- Trade with agent $\rho_t(a)$, who holds $x_{\rho_t(a)t}$. $z_{at} \equiv (x_{at}, x_{\rho_t(a)t})$
- a produces $f(a)$
- a receives utility $u_i(i) > 0$ after consumption. (Only consumes its own type)

Decisions: Trade or Not Trade, Consume or Not Consume.

$$\lambda_{at} = \begin{cases} 1 & \text{if} \quad a \text{ wants } x_{at} \text{ for } x_{\rho_t(a)t} \\ 0 & \text{if} \quad a \text{ refuses to trade} \end{cases} \tag{1}$$

$$\gamma at = \begin{cases} 1 & \text{if} \quad a \text{ wants to consume } x^+_{at} \\ 0 & \text{if} \quad a \text{ not consume} \end{cases} \tag{3}$$

$$x_{a,t+1} = \gamma_{at} f(a) + (1-\gamma_{at})\left((1-\lambda_{at}\cdot\lambda_{\rho_t(a)t})x_{at} + \lambda_{at}\cdot\lambda_{\rho_t(a)t}x_{\rho_t(a)t}\right) \tag{4}$$

Classifier System

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3. Genetic algorithm

Encodings for a classifier system I

Code			Meaning
1	0	0	Good 1
0	1	0	Good 2
0	0	1	Good 3
0	#	#	Not good 1
#	0	#	Not good 2
#	#	0	Not good 3

1 : hold
0: not hold
#: either

Encodings for a classifier system II

Define an exchange classifier

$$\text{Holding of } a + \text{Holding of } \rho(a) + \text{Trade / Not Trade}$$

total of 3+3+1 codes

Desicion Making

$$M_e(z_{at}) = \{e : z_{at} \text{ matches the condition part of classifier } e\} \quad (5)$$

Members of $M_e(z_{at})$ form a collection of potential "bidders". e with the highest strength wins

$$\lambda_{at} = e_t(z_{at}) = \arg\max\{S_e^a(t) : e \in M_e(z_{at})\} \quad (6)$$

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Same for consumption desicion

$$M_c(z_{at}) = \{c : x_{at}^+ \text{ matches the condition part of classifier } c\} \tag{7}$$

$$\gamma_{at} = c_t(z_{at}) = \arg\max\{S_c^a(t) : c \in M_c(z_{at})\} \tag{8}$$

Along with the law of motion in Eq. 4

$$\tau_e^a(t) = 1 + \sum_{s=0}^{t} I_e^a(s) \tag{9}$$

- $\tau_e^a(t)$: the count of successful trade using the classifier e up to time t.
- $I_e^a(t)$: whether e wins the auction (between classifiers) and helps the agent successfully trade, i.e., $\lambda_{at} \cdot \lambda_{\rho_t(a)t} x_{\rho_t(a)t} = 1$

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Analogously,

$$\tau_c^a(t) = 1 + \sum_{s=0}^{t} I_c^a(s) \tag{10}$$

The strength of a classifier is a function of successful wins.

$$S_e^a(t) = S^a_{e\tau_e^a(t)}$$
$$S_c^a(t) = S^a_{c\tau_c^a(t)}$$

Bid-paying Mechanism

Winning classifier e_t^a pays its bids to the winning classifier c_{t-1}^a
Winning classifier c_t^a pays its bids to the winning classifier e_t^a

This mechanism rewards the former classifier that lead to its success, reinforcing the right decision.

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The bid is set as $b_1(e)S_e^a(t)$ for a exchange classifier, and $b_2(c)S_c^a(t)$ for a consumption classifier.

$$b_1(e) = b_{11} + b_{12}\sigma_e \tag{11a}$$

$$b_2(c) = b_{21} + b_{22}\sigma_c \tag{11b}$$

$\sigma. = \frac{1}{1+\text{number of \#'s in the classifier}}$. Higher uncertainty, lower bid.

Evolution of Strength I

$$
\begin{aligned}
S^a_{c\tau^a_c(t)} =& S^a_{c\tau^a_c(t)-1} - \frac{1}{\tau^a_c(t)-1}\Bigg[(1+b_2(c))S^a_{c\tau^a_c(t)-1} \\
&- \sum_e I^a_e(t) b_1 S^a_{e\tau^a_e(t)} - U_a(\gamma^a_{ct})\Bigg]
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
S^a_{e\tau^a_e(t)+1} =& S^a_{e\tau^a_e(t)} - \frac{1}{\tau^a_e(t)}\Bigg[(1+b_1(e))S^a_{e\tau^a_e(t)} \\
&- \sum_c I^a_c(t) b_c S^a_{c\tau^a_c(t)}\Bigg]
\end{aligned}
\tag{13}
$$

Evolution of Strength II

The external payoff after making consumption desicion

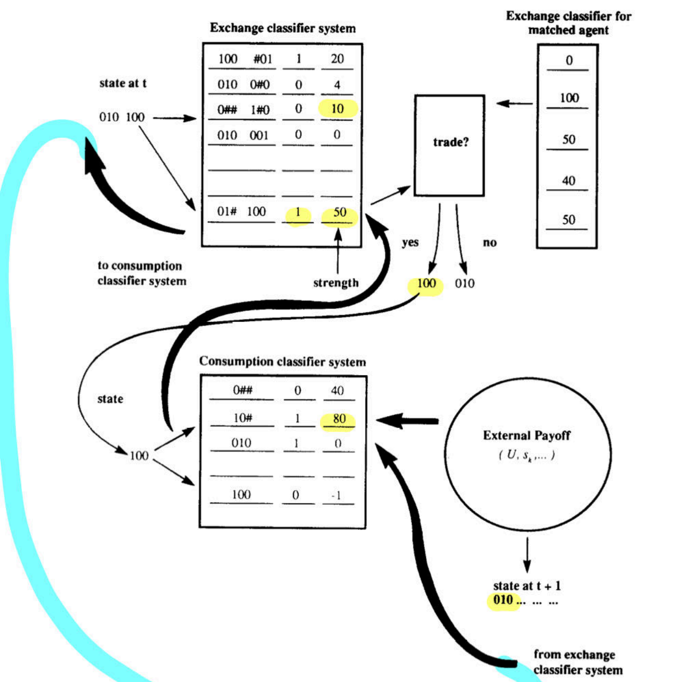
$$U_a(\gamma_{ct}^a) = \gamma_{ct}^a \left[u_i(x_{at}^+) - s(f(a)) \right] + (1 - \gamma_{ct}^a) s(x_{at}^+) \qquad (14)$$

Possible Typo

I think the authors might have a typo here. $(1 - \gamma_{ct}^a) s(x_{at}^+)$ should be a minus since it is the cost of holding.

Evolution of Strength III

- $S^a_{e\tau^a_e(t)}$ and $S^a_{c\tau^a_c(t)}$ evolves recursively
- Average of past payoff (External reward + bid from other classifier)
- minus payment (bids made to other classifier)
- Only winning classifier pays the bid (the sum term)
- Indexing with counter $\tau^a_{(\cdot)}(t)$: change is made only when successfully winning the auction.



Exchange classifier system
100 #01 1 20
010 0#0 0 4
0## 1#0 0 10
010 001 0 0
01# 100 1 50
state at t
010 100
to consumption
classifier system
strength
Exchange classifier for
matched agent
0
100
50
40
50
trade?
yes
no
100
010
Consumption classifier system
0## 0 40
10# 1 80
010 1 0
100 0 -1
state
100
External Payoff
(U, s_k,...)
state at t + 1
010 ... ...
from exchange
classifier system

Genetic Algorithm

Curse of Dimentionality

- As the variety of goods increase, the state space grows exponentially.
- Impossible to take all enumerations of classifiers into account when initializing.
- Genetic algorithm will be considered.

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Four steps for adding and deleting incumbent classifiers:

1. Creation
2. Diversification
3. Specialization
4. Generalization

Creation

Activation

There are no classifier that matches the current state z_{at}, i.e., $|M_e(z_{at})| = 0$.

Action

Assign random action to the current state z_{at} and add it to the collection of classifiers.

Diversification I

Activation

After the matched collection of classifiers $M_e(z_{at})$ are constructed.
If for all $e \in M_e(z_{at})$ have the same action.

Action

Assign opposite action to the current state z_{at} and add it to the collection of classifiers.
Assign the strength of the winning classifier to that of the new one.

There is also a deletion process

Deletion

Remove a "weak" classifier from the set of $M_e(z_{at})$. The weakness is defined jointly with the strength $S_e^a(t)$ and winning counter $\tau_e^a(t)$

Activation

After the winning bit has been determined. Activation probability decreases over time.
The winning classifier has some ambiguous position (#).

Action

Add a new classifier, which changes the #s in the condition part of the current winning classifier with some probability.
If the # is changed, it changes to the correspond value of the state.

Deletion

A weak rule is replaced by the new rule above.

Activation

Called randomly after the above variation steps are conducted.
The activation probability decreases over time.

Initialization

Draw *potential parents* and *potential exterminants*. The probability of drawing depends on some fitness criterion.

Generalizarion - Mating

1. Pick two parents to mate
2. Pick two position in the classifier
3. Pick to alter the *inner* or *outer* part of the slicing
4. Inconsistent positions are replaced with ambiguity symbol #

parent 1:	1	0	#	1	0	0	0
parent 2:	0	#	0	1	1	1	1
offspring 1:	1	0	#	1	#	0	0
offspring 2:	0	#	0	1	#	1	1

Fig. 4. The mating process for exchange classifiers who have drawn '3, 6' and 'in'.

5 Strength is set to be the average of its parents

Generalization - Exterminating

Remove one of the random selected classifier from the potential exterminants set.

I will report the simulation result next week.

Table 4

Description of the economies.

	Production					Storage cost					Utility	Initial	Equil.
	I	II	III	IV	V	1	2	3	4	5	u_i[a]	CS[b,c]	type[c]
A1.1	2	3	1	—	—	0.1	1	20	—	—	100	F	F
A1.2	2	3	1	—	—	0.1	1	20	—	—	100	R	F
A2.1	2	3	1	—	—	0.1	1	20	—	—	500	F	S
A2.2	2	3	1	—	—	0.1	1	20	—	—	500	R	S
B.1	3	1	2	—	—	1	4	9	—	—	100	F	F/S
B.2	3	1	2	—	—	1	4	9	—	—	100	R	F/S
C	2	3	1	—	—	0.1	20	70	0	—	100	R	F
D	3	4	5	1	2	1	4	9	20	30	200	R	—

[a] Utility levels u_i are set equal for $i = 1, 2, 3$.
[b] CS denotes 'classifier system'.
[c] 'F' implies fixed enumeration and 'R' implies randomly generated rules.