Evolutionary Dynamics of Currency Substitution

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January 12, 2023

Settings

Model Setting and Background

Consider an open economy with

- Two fiat currencies
- Flexible exchange rate
- Perfect substitution

Also, the economy has

- No government intervention
- No restriction on holding

How is the exchange rate and inflation rate determined?

Kareken-Wallace Model (1981)

- Overlapping generation model (OG)
- Same property in previous slide
- Agent *save* through holding fiat money

$$
\begin{aligned}
\max \quad & \ln c_t(t) + \ln c_t(t+1) \\
s.t. \quad c_t(t) \leq & w^1 - \frac{m_1(t)}{p_1(t)} - \frac{m_2(t)}{p_2(t)} \\
c_t(t+1) \leq & w^2 + \frac{m_1(t)}{p_1(t+1)} + \frac{m_2(t)}{p_2(t+1)}
\end{aligned}
$$

- $c_t(t+1)$ — Born at t, consumption at $t+1$
- $w^1(w^2)$ — Endowment when young (old)
- $m_1(t)$ — Holding of currency 1 at t
- $p_1(t)$ — Price of good in currency 1 at t

1. No specific exchange rate and price (nondeterministic)
2. Equal return in money during equilibrium (no arbitrage)
3. Two stationary inflation rate $\pi \equiv \frac{p_i(t+1)}{p_i(t)} \quad \forall i$
   - High inflation : Stable
   - Low inflation : Unstable
4. Also a set of one-currency equilibrium. Remains to have two inflation rate.

- Since no uncertainty, an equilibrium condition

$$R(t) = \frac{p_1(t)}{p_1(t+1)} = \frac{p_2(t)}{p_2(t+1)}, \quad t \geq 1$$

- Define exchange rate $e(t) \equiv \frac{p_1(t)}{p_2(t)}$, we also have

$$e(t+1) = e(t) = e, \quad t \geq 1$$

- The agent's maximization problem yields

$$s(t) = \frac{m_1(t)}{p_1(t)} + \frac{m_2(t)}{p_2(t)} = \frac{1}{2}\left[w^1 - w^2 \frac{1}{R(t)}\right]$$

- Aggregate savings in the world

$$S(t) = N\left[w^1 - w^2 \frac{1}{R(t)}\right] = \frac{H_1(t)}{p_1(t)} + \frac{H_2(t)}{p_2(t)}$$

where $H_i(t)$ is the nominal supply of currency i at time t

Government Consumption

Government i finances purchases by issuing currency i

$$G_i = \frac{H_i(t) - H_i(t-1)}{p_i(t)}$$

i.e. finance via seignorage.

World government consumption can be derived

$$G_1 + G_2 = S(t) - S(t-1)R(t-1)$$

Exchange Rate is Indeterminate

Since what matters is real saving $s(t)$, or in aggregate form $S(t)$

$$S(0) = \frac{H_1(0) + H_2(0)e}{p_1(1)} = \frac{H_1(0) + H_2(0)\hat{e}}{\hat{p}_1(1)}$$

And

$$G_w = G_1 + G_2 = S(1) - S(0)$$

Together

$$\frac{H_1(0) + H_2(0)\hat{e}}{\hat{p}_1(1)} = S(1) - [G_1 + G_2]$$

The values of $\hat{p}_2$ can be selected to solve any pair of $\hat{e}, \hat{p}_1(1)$ that keeps the aggregate saving unaltered.

Inflation rate

Aggregate government consumption + equilibrium aggregate saving :

$$\frac{1}{R(t+1)} = \pi_w(t+1) = \frac{w^1}{w^2} + 1 - \frac{G_w}{2Nw^2} - \frac{w^1}{w^2}\frac{1}{\pi_w(t)}$$

- Stationary inflation rate comes in quadratic form
- High inflation rate is the **stable equilibrium**
- Low inflation rate is the **unstable equilibrium**

Breaking down of Currency System

- Another equilibria — single currency scheme
- One currency is not valued $1/p_i(t) = 0$
- Again two stationary equilibria, high and low inflation
- Price level constant if supply of money constant

In perfect foresight, if $G_1 = 0, G_2 > 0$

- Growth of wolrd money supply = Growth of currency 2
- As $t \to \infty$, currency 2 drive currency 1 out of agent.
- Currency of country with restrictive monetary policy cannot survive???
- ACE dynamics presents different result.

Agent-Based Model

Agent-Based Model

- 2 Populations
- Agents have rule, represented by **string**.
- Make decision @young, evaluate payoff @old
- Pass rule to next generation via genetic

$$\overbrace{1001\ldots}^{k:\ \text{consumption}} \quad \text{string} \quad \overbrace{0110\ldots}^{30-k:\ \text{fraction}}$$

$$\begin{array}{ccc} \downarrow & \text{encode} & \downarrow \\ c_{it}t & & \lambda_i(t) \end{array}$$

$$c_{it}(t) \implies s_i(t) \implies \begin{cases} s_i(t)\lambda_i(t) & \text{In currency 1} \\ s_i(t)(1-\lambda_i(t)) & \text{In currency 2} \end{cases}$$

Price of currency I

Once the demand for currency is determined, the price is thus determined

$$H(t) = \sum_{i=1}^{N} \lambda_i t s_i(t) p(t)$$

Together with $G_i = \frac{H_i(t) - H_i(t-1)}{p_i(t)}$ we get

$$p_1(t) = \frac{H_1(t-1)}{\sum_i^N \lambda_i(t) s_i(t) - G_1}$$
$$p_2(t) = \frac{H_2(t-1)}{\sum_i^N (1 - \lambda_i(t)) s_i(t) - G_2}$$

Price of currency II

If the holding of currency i does not exceed the level of defict G_i, currency i becomes valueless.

- Reduced to single-currency economy

Next period consumption

For agents born at $t-1$, once price at t is set (Determined by agents born at t with their portfolio decision) they can determine their consumption in t : $c_{it-1}(t)$

$$c_{it-1}(t) = w^2 + s_i(t-1)\bar{R}_i(t-1)$$

where

$$\bar{R}_i(t-1) = \lambda_i(t-1)\underbrace{\frac{p_1(t-1)}{p_1(t)}}_{R_1(t-1)} + (1-\lambda_i(t-1))\underbrace{\frac{p_2(t-1)}{p_2(t)}}_{R_2(t-1)}$$

Lifetime Payoff (Fitness)

Before dying, agents in generation $t-1$ recap their life and evaluate their lifetime utility, or *fitness*

$$\mu_{it-1} = \ln c_{it-1}(t-1) + \ln c_{it-1}(t)$$

The information is then used to pass on to $t+1$ generation newborns.

Genetic Algorithm(GA)

1. Reproduction — Tournament with replacement, select higher fitness
2. Crossover — Agents in mating pool exchange genes with probability p_{cross}
3. Mutation — A possibility p_{mut} for one of the position in string to mutate
4. Election — Compare potential fitness of offspring and actual fitness with parents. Highest two survives.
5. New generation $t + 1$ form, repeat.

See handout note.

Experiment

- 9 pairs of $\{G_1, G_2\} = \{0, G_2\}$
- 2 pairs of endowment$\{w^1, w^2\}$
- 2 pairs of GA parameters $\{p_{cross}, p_{mut}\}$
- 5 random seed for each cartesian tuple

$$\begin{array}{cccccc} G_1 & 0 & 0 & \dots & 0 \\ G_2 & 0.6 & 1.5 & \dots & 30 \end{array} \Bigg\| \begin{array}{ccc} w^1 & 10 & 10 \\ w^2 & 4 & 1 \end{array} \Bigg\| \begin{array}{ccc} p_{cross} & 0.6 & 0.6 \\ p_{mut} & 0.0033 & 0.033 \end{array}$$

A total of 180 simulations with $N = 30$

Results

Result

Reports

- Small deficit gap
- Large deficit gap

on

- Average fraction
- Average first consumption
- Inflation rate
- Exchange rate

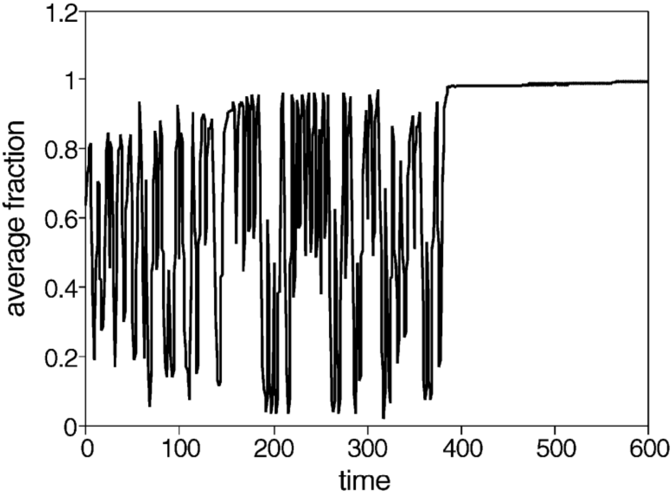

Fig. 6. Average portfolio fraction, $G_1 = 0$, $G_2 = 1.5$, $p_{\mathrm{cross}} = 0.6$, $p_{\mathrm{mut}} = 0.033$.

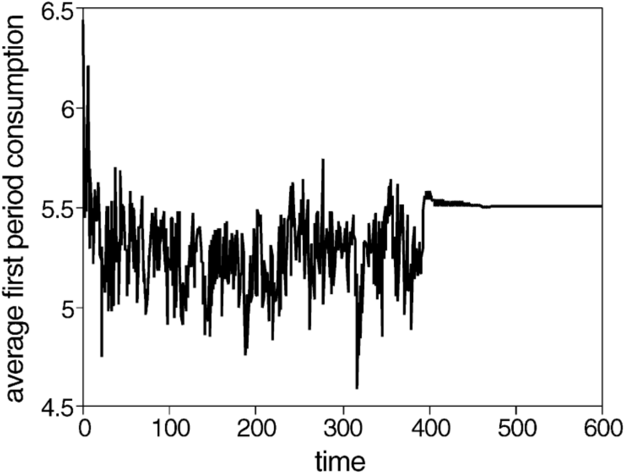

Fig. 7. Average first period consumption, $G_1 = 0$, $G_2 = 1.5$, $p_{\text{cross}} = 0.6$, $p_{\text{mut}} = 0.033$.

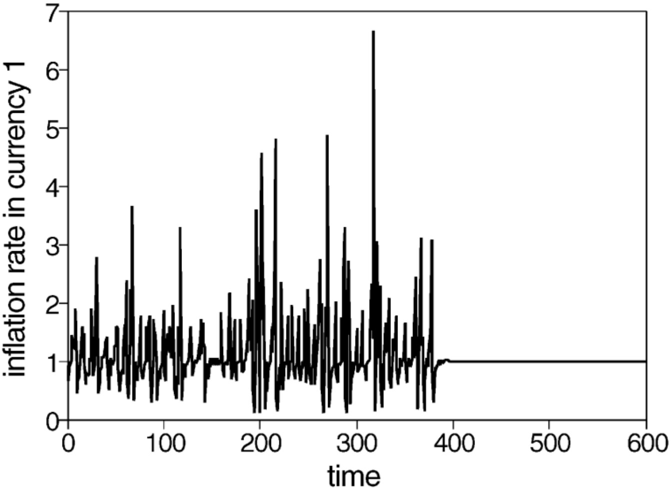

Fig. 8. Inflation rate – currency 1, $G_1 = 0$, $G_2 = 1.5$, $p_{\text{cross}} = 0.6$, $p_{\text{mut}} = 0.033$.

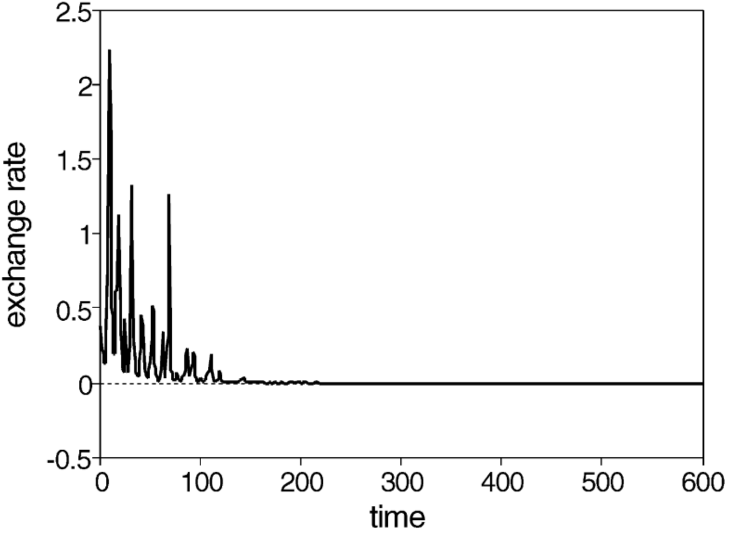

Fig. 5. Exchange rate, $G_1 = 0$, $G_2 = 1.5$, $p_{\mathrm{cross}} = 0.6$, $p_{\mathrm{mut}} = 0.033$.

Large Deficit

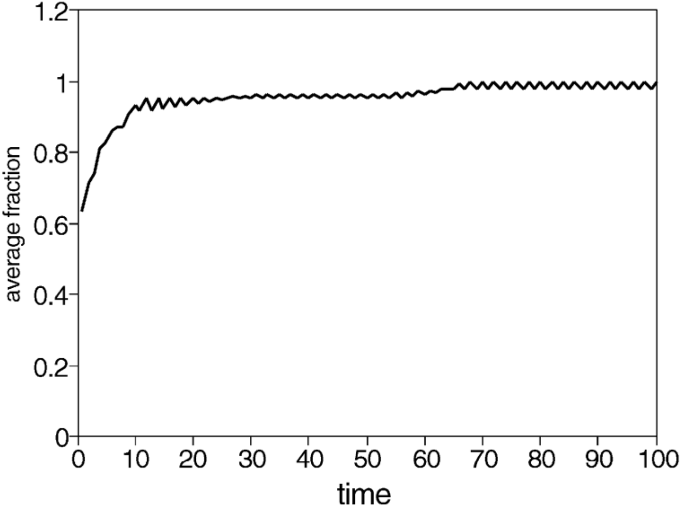

Fig. 2. Average portfolio fraction, $G_1 = 0$, $G_2 = 9$, $p_{\text{cross}} = 0.6$, $p_{\text{mut}} = 0.033$.

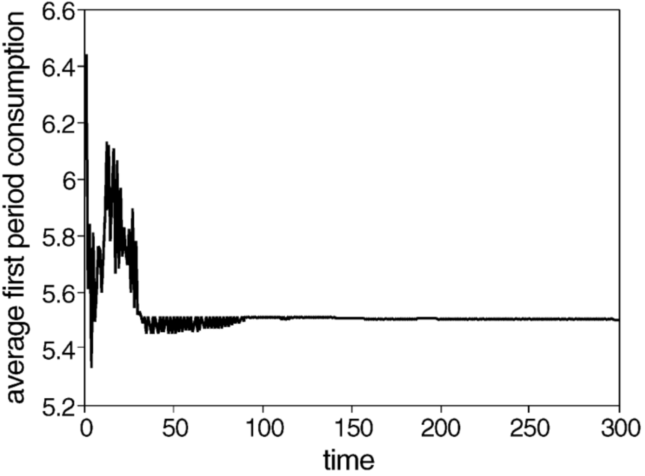

Fig. 3. Average first period consumption, $G_1 = 0$, $G_2 = 9$, $p_{\text{cross}} = 0.6$, $p_{\text{mut}} = 0.033$.

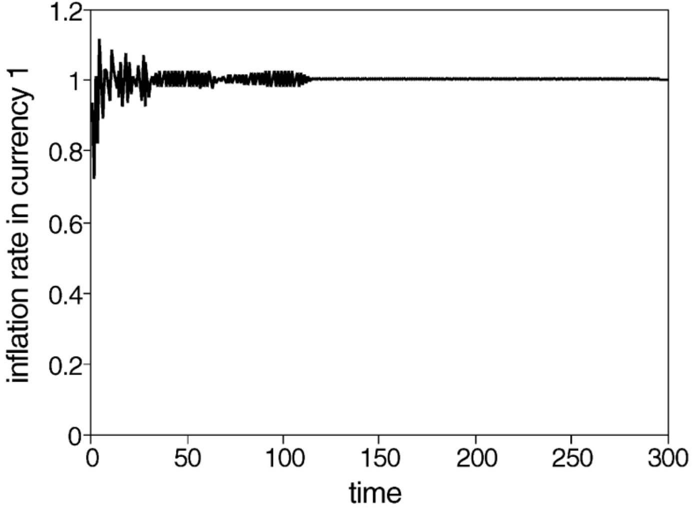

Fig. 4. Inflation rate – currency 1, $G_1 = 0$, $G_2 = 9$, $p_{\mathrm{cross}} = 0.6$, $p_{\mathrm{mut}} = 0.033$.

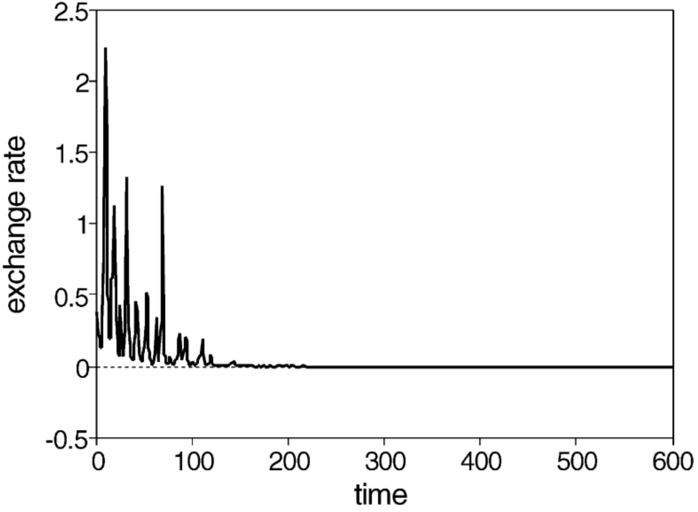

Fig. 5. Exchange rate, $G_1 = 0$, $G_2 = 1.5$, $p_{\mathrm{cross}} = 0.6$, $p_{\mathrm{mut}} = 0.033$.

Table 3
Selection of currency 1 (average number of periods)

G_2	λ	0.001	0.01	0.1
1.50		835	950	1212
15.00		21	5.2	18.2

All reduce to single-currency.

Behaviors

Condition for ineqiulibrium

$$R_1(t) > R_2(t) \iff \frac{\bar{\lambda}(t)}{\bar{\lambda}(t-1)} > 1 - \frac{G}{N\bar{s}(t)}$$

- A small deviation can cause room for arbitrage
- Polya urn scheme, more portfolio in currency 1
- If growing speed of λ decrease, might cause $R_1(t) < R_2(t)$

3 Behaviors on ABM Result

1. GA adjustment are much closer to the patterns observed in the experiments
2. There are economies in which GAs and experiments with human subjects converge, while other algorithms exhibit divergence
3. Other algorithms converge to a rational expectations equilibrium, while, at the same time, GAs and experiments with human subjects fail to converge and instead exhibit persistent fluctuations

Arifovic 1995, 1996 performed experiments in lab experiments.

1. Convergence pattern in GA is highly fluctuated.
2. GA converges to low-inflation path
3. Persistent volatility of exchange rate, instead of R.E.E result.