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Source: *The Review of Economic Studies*, Jan., 2000, Vol. 67, No. 1 (Jan., 2000), pp. 91-116

Published by: Oxford University Press

Stable URL: <https://www.jstor.org/stable/2567030>

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Self-Fulfilling Debt Crises

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First version received December 1996; final version accepted April 1999 (Eds.)

We characterize the values of government debt and the debt's maturity structure under which financial crises brought on by a loss of confidence in the government can arise within a dynamic, stochastic general equilibrium model. We also characterize the optimal policy response of the government to the threat of such a crisis. We show that when the country's fundamentals place it inside the crisis zone, the government may be motivated to reduce its debt and exit the crisis zone because this leads to an economic boom and a reduction in the interest rate on the government's debt. We show that this reduction can be gradual if debt is high or the probability of a crisis is low. We also show that, while lengthening the maturity of the debt can shrink the crisis zone, credibility-inducing policies can have perverse effects.

1. INTRODUCTION

Financial crises brought on by a loss of confidence in the government can arise suddenly and threaten countries with huge losses. We analyse the conditions that make these crises possible and what policy makers should do in the face of this kind of risk. To address these issues, we examine optimal government policy in a dynamic, stochastic general equilibrium model in which self-fulfilling crises can arise. Because of the government's need to roll over its debt, a liquidity crunch induced by the inability to sell new debt can lead to a self-fulfilling default. We show that, if fundamentals like the level of the government's debt, its maturity structure, and the private capital stock, lie within a particular range (the crisis zone), then the probability of default is determined by the beliefs of market participants. We show that a government is motivated to reduce its debt and exit the crisis zone because doing so leads to an economic boom and a reduction in the interest rate on the government's debt.

The distinguishing feature of this paper is that it examines optimal policy within an environment in which not only can crises occur in the first period, but crises can occur in subsequent periods with positive probability. This is important since we show that previously proposed policies that seek to avert a crisis by reacting contemporaneously, such as pegging the interest rate on the government debt or lengthening the maturity of the debt being sold once a crisis has started, are ineffectual in our model. Instead, it is only preemptive policies, which seek to remove the conditions that make future crises possible, that can be effective.

To examine optimal debt policy, we construct a time-consistent equilibrium of our model with a (relatively) easy-to-characterize Markov structure in which a crisis can occur with a positive probability whenever the level and maturity structure of the government's debt and the capital stock are in the crisis zone. We show that within the crisis zone, the

country's economic activity is depressed in proportion to the probability that a crisis will take place. We show that the government may find it optimal to reduce its debt so that it exits the crisis zone, since this leads to investment, output, consumption, and government revenue all rising, while the interest rate on the government debt falls. We also show that, if the probability that the agents in equilibrium assign to a crisis is high, then the government wants to adjust its debt so as to exit the crisis zone immediately, but that as the level of the government's debt increases and the probability that the agents assign to a crisis decreases, the number of periods that this transition out of the crisis zone takes can become arbitrarily large.

We consider other policy remedies besides simply lowering the debt. One is lengthening the maturity of the debt in advance of a crisis, which has the intuitive effect of shrinking the crisis zone. For any given debt level, there exists a sufficiently long maturity structure to preclude a future crisis. In addition, we show that such credibility enhancing policy remedies as increasing the dependence on foreign trade, which work through increasing the costs of default, can be counterproductive. We also characterize the types of government that are likely to have nonempty crisis zones. We are surprised to find that a government that cares sufficiently more about private than government consumption or is sufficiently farsighted is guaranteed to have a crisis zone.

The specific features of the model are motivated by the crisis in Mexico during 1994–95. In the Mexican crisis, the fear of a government default led to an inability of the government to issue new debt, which in turn seemed about to confirm the fears of a default until the United States intervened with a rescue package. The crisis occurred despite the fact that the Mexican government's fiscal behaviour in terms of standard measures, such as its debt/GDP ratio, appears to have been responsible with respect to both its own past behaviour and the behaviour of many other governments that, at least as yet, have not experienced similar crises. At the time of the crisis, however, the average maturity of Mexico's debt had become very short. When the crisis occurred, the Mexican government found itself unable to sell either dollar-indexed *tesobonos* or domestically denominated debt, which seems hard to explain on the basis of currency risk, but easy to explain on the basis of default risk. (See Cole and Kehoe (1996) for a fuller discussion of the details of the Mexican crisis.)

The model follows the optimal policy literature in having an infinite horizon and an altruistic government (as in, for example, Chamley (1986)). Hence, neither the policy prescriptions nor the possibility of a crisis is driven by arcane features of government preferences. Because it is the lack of ability of the government to commit *ex ante* to a debt repayment policy that makes crises of confidence possible, we follow Lucas and Stokey (1983) and Chari and Kehoe (1990) in examining the optimal time-consistent policy of the government.

The model in this paper is also related to the literature on how the government's inability to commit to future policy choices can generate multiple equilibria. The first work that makes this point within the context of debt repudiation is by Calvo (1988), who shows in a simple two-period model of sovereign lending that for certain parameter values, two Pareto-ranked equilibria exist. The most closely related paper is by Alesina, Prati, and Tabellini (1990). They show that, even if the government faces a discrete cost of defaulting, its need to roll over its debt can lead to two equilibria. In one equilibrium, a crisis occurs in the first period, and the government repudiates its initial debt. In the other equilibrium, the government is always able to roll over its debt and never defaults. They show that lengthening the maturity of the government's initial debt from one to two periods can in some cases remove the possibility of the crisis equilibrium. Other papers

on self-fulfilling debt crises include Giavazzi and Pagano (1990) and Detragiache (1996). Another related literature is that on self-fulfilling currency crises (see, for example, Kehoe (1996), Krugman (1996), Obstfeld (1994) and Sachs, Tornell, and Velasco (1996)).

2. THE BASIC MODEL

There is a single good in each period $t = 0, 1, \dots$. This good can be either consumed or saved as capital. Production utilizes capital and, implicitly, inelastically supplied labour. There are three types of people in the model: consumers, international bankers, and the government. We describe each of the types of people in turn.

There is a continuum with measure one of identical, infinitely lived consumers who consume, invest, and pay taxes. The individual's utility function is

$$E \sum_{t=0}^{\infty} \beta^t (c_t + v(g_t)),$$

where c_t is private consumption and g_t is government consumption. We assume that v is continuously differentiable, strictly concave, and monotonically increasing. We also assume that $v(0) = -\infty$. The consumer's budget constraint is

$$c_t + k_{t+1} \leq (1 - \theta)a_t f(k_t). \quad (1)$$

Here k_t is the consumer's individual capital stock; a_t is a multiplicative productivity factor that depends on whether or not the government has ever defaulted; θ , $0 < \theta < 1$, is the constant proportional tax on domestic income; and f is a continuously differentiable, concave, and monotonically increasing production function that satisfies $f(0) = 0$, $f'(0) = \infty$, and $f'(\infty) = 0$. The consumer is endowed with k_0 units of capital in period 0.

There is also a continuum with measure one of identical, infinitely lived international bankers. The individual banker is risk neutral and has the utility function

$$E \sum_{t=0}^{\infty} \beta^t x_t,$$

where x_t is the banker's private consumption. Each banker is endowed with \bar{x} units of the consumption good in each period. Any of the endowment that is not spent on government bonds, along with the proceeds from the previous period's purchase of government debt, is consumed later in the period. When the banker decides how much of the new debt to buy, he faces the budget constraint $q_t b_{t+1} \leq \bar{x}$, where q_t is the price of one-period government bonds that pay b_{t+1} in period $t+1$ if the government decides to repay its debts and 0 if the government decides not to repay. Subsequently, when the banker decides how much to consume, he faces the constraint

$$x_t \leq \bar{x} + z_t b_t - q_t b_{t+1}. \quad (2)$$

We use $z_t \in \{0, 1\}$ to denote the government's default decision: $z_t = 1$ is repayment and $z_t = 0$ is default. In addition, the bankers face a constraint $b_{t+1} \geq -A$, where $A > 0$, that rules out Ponzi schemes, but does not otherwise bind in equilibrium.

It is worth noting that, since the consumers are risk neutral in their own consumption, we can neglect the possibility of their borrowing from, or lending to, the international bankers. Allowing the consumers to be risk averse in their own consumption would complicate the analysis substantially since consumers could be induced by their future income risk to accumulate assets in the form both of capital and, especially, of savings with

the international bankers. This would not change the essential features of the analysis, however.

There is a single government, which is benevolent in the sense that its objective is to maximize the utility of the consumers. In every period, the government chooses its new borrowing level B_{t+1} , whether or not to default on its old debt z_t , and the level of government consumption g_t , subject to the budget constraint

$$g_t + z_t B_t \leq a_t \theta f(k_t) + q_t B_{t+1}. \quad (3)$$

The government is the only strategic agent in the model, and, when the government makes its decisions, it takes into account these decisions' effects on the price that it receives for its debt, the level of the aggregate capital stock K_t , government revenue, and the level of private consumption. In addition, we assume that, if the government defaults on its current debt by setting $z_t = 0$, then productivity drops from $a_t = 1$ to $a_t = \alpha < 1$ from period t onward. We also assume that the government loses access to international borrowing and lending after default. As we shall see, we do not need to impose a borrowing constraint on the government to rule out Ponzi schemes, because, if the government tries to sell too much new debt B_{t+1} , its price q_t falls to zero.

In each period, the value of an exogenous sunspot variable ζ_t is realized. This variable is independently and uniformly distributed on the interval $[0, 1]$. As we shall see, this specification of the sunspot variable allows us to easily construct equilibria in which the probability of a crisis is arbitrary.

The market-clearing condition for the government's debt is $b_{t+1} = B_{t+1}$. We assume that the international bankers behave competitively in making their choice of b_{t+1} . Because consumers are also competitive, we need to distinguish between the individual decision k_{t+1} and the aggregate value K_{t+1} : consumers do not think that their individual actions affect the aggregate state in the next period, thereby affecting prices or the actions of the government. In equilibrium, because all consumers are identical, $k_{t+1} = K_{t+1}$. We assume, of course, that $k_0 = K_0$ and $b_0 = B_0$.

Timing. The timing of actions within each period is the following.

1. The sunspot variable ζ_t is realized, and the aggregate state is $s_t = (B_t, K_t, a_{t-1}, \zeta_t)$.
2. The government, taking the price schedule $q_t = q(s_t, B_{t+1})$ as given, chooses B_{t+1} .
3. The international bankers, taking q_t as given, choose b_t .
4. The government chooses whether or not to default, z_t , and how much to consume, g_t .
5. The consumers, taking a_t as given, choose c_t and k_{t+1} .

An important feature of our model is the timing of the government's decisions within a period. This timing enables the government to issue new debt before retiring the old debt, while having a maturity of one period on the debt. The need to roll over old debt into new is what allows us to have a self-fulfilling crisis. We could also obtain this feature by having longer-maturity debt that is only partially rolled over in any period. Later we discuss the role of maturity on our results. Variants of the model can be obtained by having consumers make their choices at different points within the period and by delaying the imposition of the default penalty. Although these alternative timing assumptions would change the quantitative results of the model, they would not change its qualitative features.

Another important feature of our model is our assumption that productivity falls after a default. Two well known stories can rationalize this assumption. One story is that a disruption of the country's ability to engage in international trade reduces the value of output. Another story is that the government's reputation is damaged by its decision to default, which hurts the government's ability to operate, and, consequently, output falls. See Cole and Kehoe (1997) for more on the reputation spillover story and Cole and Kehoe (1998) for an explicit discussion of how to incorporate these stories into this model.

For expositional convenience, we assume that the government loses all access to international borrowing and lending after default. In fact, it is easy to see in the results presented later that the government would not want to borrow again after a default; it would want to lend to smooth post-default government spending. Since allowing for this possibility would change none of our qualitative results but would complicate notation, we ignore it.

Finally, we make the simplifying assumption of a constant tax rate because we think that the period over which crises can occur is short compared with that over which tax policy can be changed. Allowing the government to choose the tax rate for the next period in the current period could be incorporated into the model without changing the qualitative conclusions, but at the expense of increasing the model's complexity.

3. A RECURSIVE EQUILIBRIUM

The government cannot commit itself either to honouring its debt obligations or to following a fixed borrowing and spending path. We therefore define a recursive equilibrium in which there is no commitment and the agents choose their actions sequentially. Within any period, the aggregate state $s = (B, K, a_{-1}, \zeta)$, the government's policy choices (B', g, z) , the price of bonds q , and the choices of individual consumers (c, k') determine the equilibrium.

To define a recursive equilibrium, we work backward given the timing of actions in each period. The state of an individual agent consists of the aggregate state, any individual state variables, and any relevant variables that have already been chosen within the period. Variables are relevant if they affect the agent's maximization problem or the next period's state. The solution to an agent's maximization problem is given by a value function that provides the maximum attainable value of the agent's utility function given his state and the policy functions that provide the maximizing choices of the agent's choice variables in the current period given his state. In an equilibrium, agents solve their own problems by correctly predicting what the policies of the other agents will be.

We start the definition of a recursive equilibrium with consumers because they move last. When an individual consumer acts, he knows the following: his individual capital holding k , the aggregate state $s = (B, K, a_{-1}, \zeta)$, the government's offering of new debt B' , the price that bankers are willing to pay for this debt q , and the government's spending and default decisions, g and z , respectively. (In equilibrium, of course, $k = K$.) Since the consumer's choices only depend on the price q through its effect on g and z , we define the state of the individual consumer as (k, s, B', g, z) . We denote by $B'(s)$, $g(s, B', q)$, and $z(s, B', q)$ the government's policy functions, by $q(s, B')$ the price function, and by $K'(s, B', g, z)$ the function that describes the evolution of the aggregate capital stock, all yet to be defined. The representative consumer's value function is defined by the following functional equation. (Subsequently, we provide analytical solutions to this problem and

to the other agents' dynamic programming problems.)

$$\begin{aligned}
 V_c(k, s, B', g, z) = \max_{c, k'} & c + v(g) + \beta EV_c(k', s', B'(s'), g', z'), \\
 \text{subject to} & \\
 c + k' \leq & (1 - \theta)a(s, z)f(k), \\
 c, k' \geq & 0, \\
 s' = & (B', K'(s, B', g, z), a(s, z), \zeta'), \\
 g' = & g(s', B'(s'), q(s', B'(s'))), \\
 z' = & z(s', B'(s'), q(s', B'(s'))). \tag{4}
 \end{aligned}$$

The consumers' policy functions are $c(k, s, B', g, z)$ and $k'(k, s, B', g, z)$. As we have explained, the production parameter a satisfies $a(s, z) = 1$ if $a_{-1} = 1$ and $z = 1$, that is, if the government has not defaulted in the past and has not defaulted this period. Otherwise, $a(s, z) = \alpha$.

When an individual banker acts, he knows his individual holdings of government debt b , the aggregate state s , and the government's offering of new debt B' . We therefore define the state of an individual banker as (b, s, B') . The representative banker's value function is defined by the functional equation

$$\begin{aligned}
 V_b(b, s, B') = \max_{b'} & \bar{x} + z(s, B', q(s, B'))b - q(s, B')b' + \beta EV_b(b', s', B'(s')), \\
 \text{subject to} & \\
 q(s, B')b' \leq & \bar{x}, \\
 b' \geq & -A, \\
 s' = & (B', K'(s, B', g, z), a(s, z), \zeta'). \tag{5}
 \end{aligned}$$

The banker's policy function is denoted by $b'(b, s, B')$. The policy function is actually a set-valued correspondence. Because bankers are assumed to be risk neutral and to behave competitively, they are relatively passive. They purchase the amount of bonds offered by the government as long as the price of these bonds satisfies

$$q(s, B') = \beta Ez(s', B'(s'), q(s', B'(s'))). \tag{6}$$

In other words, the bankers purchase the bonds as long as the expected gross return on these bonds is $1/\beta$. In fact, we could simply reinterpret our model as a small open economy in which the price function satisfies this relationship, and we could drop consideration of the bankers completely.

The government makes decisions at two points in time. At the beginning of the period, when the government chooses B' , the government's state is simply the initial state s . The government knows how the price that its debt will bring, $q(s, B')$, depends on this state and on the level of new borrowing. The government also knows what its own optimizing choices $g(s, B', q(s, B'))$ and $z(s, B', q(s, B'))$ will be later. The government also realizes that it can affect consumption c , investment K' , and the production parameter a through its choices. The government's objective is to maximize the welfare of consumers. The welfare of the bankers does not enter the government's objective. Putting the utility of bankers into the objective function of the government or, equivalently, allowing domestic

consumers to hold government bonds would change the quantitative, but not the qualitative, features of the model. The government's value function is defined by the functional equation

$$\begin{aligned} V_g(s) &= \max_{B'} c(K, s, B', g, z) + v(g) + \beta EV_g(s'), \\ &\text{subject to} \\ g &= g(s, B', q(s, B')), \\ z &= z(s, B', q(s, B')), \\ s' &= (B', K'(s, B', g, z), a(s, z), \zeta'). \end{aligned} \quad (7)$$

We denote the government's debt policy by $B'(s)$.

Later in the period, the government makes its default choice z , which in turn determines the level of productivity a and, through its budget constraint, the level of government spending g . Given the government's initial value function, $V_g(s)$, we can define the policy functions $g(s, B', q)$ and $z(s, B', q)$ as the solutions to the problem

$$\begin{aligned} &\max_{g, z} c(K, s, B', g, z) + v(g) + \beta EV_g(s'), \\ &\text{subject to} \\ g + zB &\leq \theta a(s, z)f(K) + qB', \\ z &= 0 \text{ or } z = 1, \\ g &\geq 0, \\ s' &= (B', K'(s, B', g, z), a(s, z), \zeta'). \end{aligned} \quad (8)$$

Having developed these concepts, we can now define a recursive equilibrium for our model economy.

Definition of an equilibrium. An *equilibrium* is a list of value functions V_c for the representative consumer, V_b for the representative banker, and V_g for the government; policy functions c and k' for the consumer, b' for the banker, and B', g , and z for the government; a price function q ; and an equation of motion for the aggregate capital stock K' such that:

1. Given B', g , and z , V_c is the value function for the solution to the representative consumer's problem, and c and k' are the maximizing choices;
2. Given B', q , and z , V_b is the value function for the solution to the representative banker's problem, and the value of B' chosen by the government solves the problem when $b = B$;
3. Given q, c, K', g , and z , V_g is the value function for the solution to the government's first problem (7), and B' is the maximizing choice. Furthermore, given c, K', V_g , and B', g and z solve the government's second problem (8);
4. $B'(s) \in b'(B, s, B')$;
5. $K'(s, B', g, z) = k'(K, s, B', g, z)$.

Our definition of an equilibrium is similar to Chari and Kehoe's (1990, 1993) definition of a sustainable equilibrium and Stokey's (1991) definition of a credible equilibrium. Our definition differs, however, in two respects: First, we have restricted ourselves to a

recursive—that is, Markov—equilibrium, and hence the agents' future conditional plans can be derived from their policy functions. Second, we have allowed the private agents to condition their actions within the period on the government's new borrowing level, and hence we do not have to assume that the government sets the prices and the private agents set the quantities. Our notion of a recursive equilibrium is restrictive. If, for example, we were to include the time period in the definition of the state, we could construct equilibria in which crises are only possible in periods whose dates are prime numbers.

We turn next to characterizing a class of crisis equilibria in which a crisis can occur for certain values of the fundamentals (B, K) . In characterizing these equilibria, we frequently specify the equilibrium outcomes of the state variables, $(B_t, K_t, a_{t-1}, \zeta_t)$, given initial values $(B_0, K_0, 1, \zeta_0)$. It is important to notice, however, that to fully specify an equilibrium, we need to specify functions, not realizations. The government, for example, solves its problems (7) and (8) taking the price function for bonds and the policy functions of consumers as given.

4. THE CRISIS ZONE

A self-fulfilling crisis arises when there are two possible equilibrium outcomes, one in which the government is able to sell new debt at a positive price and chooses to repay the old debt and another in which the government is unable to sell new debt at a positive price and defaults on the existing debt. Self-fulfilling crises are possible in these equilibria for certain values of the fundamentals (K, B) ; the realization of the sunspot variable determines which of these two outcomes ensues. In the class of equilibria we examine, the probability of a crisis, for those (K, B) for which a crisis can occur, is independent of time and the level of B and K . We index equilibria in this class by the probability of a crisis in each period π .

In equilibrium, if $\zeta \leq \pi$ and B is greater than a crucial level $\bar{b}(K)$, then international bankers predict that the government will default on its debt. Bankers are therefore not willing to pay a positive price for the new debt offered and thus provoke a crisis. If however, $\zeta > \pi$, then international bankers predict that the government will repay its debt, and they are therefore willing to roll over the debt and thus allow the government to honour its commitments. Because ζ is uniformly distributed on the unit interval, π is both the crucial value of ζ and the probability that $\zeta \leq \pi$. If B is less than or equal to the crucial level $\bar{b}(K)$, however, then no crises can occur, no matter what the realization of ζ : since bankers know that the government will repay its debt whether or not it is able to sell new debt, bankers purchase the new debt. Finally, if B is greater than another crucial value $\bar{B}(K, \pi)$, then the government defaults in the current period and the bankers, knowing this, make no loans.

There are three regions of the state space that are described by the two functions $\bar{b}(K)$ and $\bar{B}(K, \pi)$. The function $\bar{b}(K)$ is the highest level of debt for which no crisis can occur given K . The function $\bar{B}(K, \pi)$ describes the highest level of debt for which an equilibrium with positive lending is possible, given K and π . We label the region of the state space in which $\bar{b}(K) < B \leq \bar{B}(K, \pi)$ the *crisis zone* since for values of fundamentals in this region of the state space crises occur with positive probability. In the crisis zone the level of the capital stock and the price of government debt are depressed by the threat of a crisis. We label the region of the state space in which $B \leq \bar{b}(K)$ the *no-crisis zone* since crises do not occur in this region. In the no-crisis zone the level of the capital stock and the price of government debt are high. We label the region in which $B \geq \bar{B}(K, \pi)$ the *default*

zone since immediate default is the only outcome. In what follows we explicitly characterize the functions that determine these regions, $\bar{b}(K)$ and $\bar{B}(K, \pi)$, as well as the policy and payoff functions of all the agents, the price function q , and the transition function for the capital stock K' .

A. Characterizing the optimal behaviour of private agents

To construct sunspot equilibria we characterize the optimal behaviour of the bankers and consumers. This is relatively straightforward to do since it depends solely on the probability they assign to the government defaulting in the next period. The relevant probabilities here are 0, 1, and π . For each value of these probabilities we can characterize the bankers' and consumers' behaviour and from that characterization infer the equilibrium price function q and the transition function for capital K' .

The bankers' optimal behaviour depends upon the probabilities that they assign to the government's future repayment decision z' . If bankers expect that $z' = 0$, then they are not willing to buy any debt unless the price is 0 (in which case their purchases are irrelevant). If bankers expect that $z' = 1$, then they are willing to buy any amount of the government debt up to \bar{x} at price β . If bankers expect default to occur with probability π they are willing to purchase whatever amount of bonds the government offers up to \bar{x} at price $q = \beta(1 - \pi)$.

The consumers' optimal policy depends solely on what they expect the value of the productivity parameter a will be next period. Just as with the bankers, there are three cases. We start first with the case where consumers assign a probability π to a crisis occurring in the next period, in which case $a = \alpha$, and a probability $1 - \pi$ that there will not be a crisis, in which case $a = 1$. We can characterize optimal consumer behaviour for this case by solving a variational problem. In particular, if default has not occurred in period t , the representative consumer, given $k_t, k_{t+2}^n, k_{t+2}^d$, chooses c_t, k_{t+1} and c_{t+1}^n, c_{t+1}^d (where c_{t+1}^n and c_{t+1}^d are the choices of consumption in periods $t+1$ contingent on the government not defaulting and defaulting, respectively) to solve

$$\begin{aligned} & \max c_t + \beta(1 - \pi)c_{t+1}^n + \beta\pi c_{t+1}^d, \\ & \text{subject to} \\ & c_t + k_{t+1} \leq (1 - \theta)f(k_t), \\ & c_{t+1}^n + k_{t+2}^n \leq (1 - \theta)f(k_{t+1}), \\ & c_{t+1}^d + k_{t+2}^d \leq (1 - \theta)\alpha f(k_{t+1}), \\ & c_t, c_{t+1}^n, c_{t+1}^d, k_{t+1} \geq 0. \end{aligned}$$

The first-order conditions for this problem imply that the consumer's optimal policy is to set the capital stock and consumption at the levels k^π and $c^\pi(k)$ determined by the rules

$$\begin{aligned} 1 &= \beta[(1 - \pi) + \pi\alpha](1 - \theta)f'(k^\pi), \\ c^\pi(k) &= (1 - \theta)f(k) - k^\pi. \end{aligned}$$

(It is here that the risk neutrality of utility in consumption plays its role: if consumers are risk averse, then optimal investment and consumption are not stationary.)

If consumers expect that the productivity parameter will be equal to α next period, then it is optimal for them to set the capital stock for next period to a level k^d that satisfies

$$\beta(1 - \theta)\alpha f'(k^d) = 1$$

and to set their consumption to $c^d(k) = (1 - \theta)\alpha f(k) - k^d$.

Similarly, if consumers expect that the productivity parameter will be equal to 1 next period, then it is optimal for them to set the capital stock for next period to a level k^n that satisfies

$$\beta(1 - \theta)f'(k^n) = 1$$

and to set their consumption to $c^n(k) = (1 - \theta)f(k) - k^n$.

Notice that the strict concavity of f implies that $k^n > k^\pi > k^d$. (For K_0 sufficiently low, we may need to set $c^i(K_0) < 0$ for $i = d, n, \pi$; modifying the model to allow consumer borrowing to eliminate this problem would have no significant effect on our results.)

Taking as given the functions $\bar{b}(K)$ and $\bar{B}(K, \pi)$, our characterization of the behaviour of the private agents and description of the equilibrium imply that the transition rule for capital in our sunspot equilibrium is given by

$$K'(B') = \begin{cases} k^n & \text{if } B' \leq \bar{b}(k^n) \text{ and } a = 1, \\ k^\pi & \text{if } \bar{b}(k^n) < B' \leq \bar{B}(k^\pi, \pi) \text{ and } a = 1, \\ k^d & \text{otherwise,} \end{cases} \quad (9)$$

and the equilibrium price function is

$$q(B') = \begin{cases} \beta & \text{if } B' \leq \bar{b}(k^n) \text{ and } z(s, B', \beta) = 1, \\ \hat{\beta} & \text{if } \bar{b}(k^n) < B' \leq \bar{B}(k^\pi, \pi) \text{ and } z(s, B', \hat{\beta}) = 1, \\ 0 & \text{otherwise,} \end{cases} \quad (10)$$

where z is determined by whether or not the participation constraint is satisfied, and $\hat{\beta} = \beta(1 - \pi)$.

B. Defining the zones

Let $V_g^n(s, B', q)$ and $V_g^d(s, B', q)$ denote the payoff to the government of not defaulting and defaulting, respectively, in state $s = (B, K, 1, \cdot)$ after it has sold new debt B' at price q . With these definitions we can write the constraint on the government's debt that must be satisfied in any equilibrium with positive lending

$$V_g^n(s, B', q) \geq V_g^d(s, B', q). \quad (11)$$

We refer to this constraint as the *participation constraint*, since it ensures that the government wants to honour the current lending contract with the bankers given that the government is able to sell its new debt. When we set $q = \beta(1 - \pi)$, this constraint determines the upper bound on debt for which positive lending can occur in a sunspot equilibrium, $\bar{B}(K, \pi)$.

With these definitions, we can also define what needs to hold for a crisis to be possible. That is, in state s , it must be the case that the government does not weakly prefer to repay its debt even if no lending is forthcoming. To be precise, for a crisis to occur, the government must satisfy a *no-lending condition*, which says that, if $q = 0$, then the government strictly prefers to default

$$V_g^d(s, 0, 0) > V_g^n(s, 0, 0). \quad (12)$$

(Of course, $V_g^d(s, B', 0) = V_g^d(s, 0, 0)$ for any B' , because if $q = 0$, the amount of new bonds offered is irrelevant.) We use the no-lending condition to determine $\bar{b}(K)$, the upper bound debt at which the government prefers to repay and a crisis cannot occur.

C. Characterizing government policies and payoffs

We now characterize the government's payoffs and optimal policies. We find it convenient to analyse the government's behaviour in each one of the three regions: the no-crisis zone, the crisis zone, and the default zone. We start first with the hypothetical case in which the government has chosen to default despite receiving positive new loans. While this case cannot occur in equilibrium, it allows us to construct the government's payoff for this deviation, which in turn we use to determine the different regions of the state space.

After a default, the government is shut off from the international credit market and, hence, simply consumes its revenues. Therefore, government consumption is given by

$$g^d(K) = \theta \alpha f(K).$$

In every period thereafter, the capital stock is equal to k^d , private consumption is equal to $c^d(k^d)$, government consumption is $g^d(k^d)$, and the government receives a zero price on any new debt that it tries to sell. The payoff to the government if it defaults after it has sold new debt B' at price q is given by

$$V_g^d(s, B', q) = c^d(K) + v(\theta \alpha f(K) + qB') + \beta[c^d(k^d) + v(g^d(k^d))]/(1 - \beta).$$

Notice that, if $K > k^d$, the government has no incentive to sell additional debt following a default, although it would want to save so that it could smooth spending. As we have explained, we rule out this possibility only to simplify the exposition.

If $B_0 \leq \bar{b}(k_0)$, then no crisis is possible in the first period, and, if $B_1 \leq \bar{b}(k'')$, then no crisis is possible thereafter in the equilibrium we are constructing. In this case, the government is in the no-crisis zone in which $q = \beta$ and $K' = k''$ every period. We can characterize optimal government policy in the no-crisis zone for the case when the constraint that $B \leq \bar{b}(K)$ does not bind by solving the following variational problem and verifying that this constraint is satisfied. In this variational problem, given $B_t, B_{t+2}, k_t, k_{t+1}$, the government chooses g_t, g_{t+1}, B_{t+1} to solve

$$\begin{aligned} & \max \beta^t v(g_t) + \beta^{t+1} v(g_{t+1}), \\ & \text{subject to} \\ & g_t + B_t \leq \theta f(k_t) + \beta B_{t+1}, \\ & g_{t+1} + B_{t+1} \leq \theta f(k_{t+1}) + \beta B_{t+2}, \\ & g_t, g_{t+1} \geq 0. \end{aligned} \tag{13}$$

It is an immediate consequence of the first-order conditions for this problem that $g_t = g_{t+1}$. Note that, if $K \geq k''$ and $B \leq \bar{b}(k'')$, then $g_t = g_{t+1}$ implies that $B' \leq \bar{b}(k'')$ and hence that the constraint that the government stay in the no-crisis zone is not binding, and this stationary debt policy is optimal. Since, from our transition rule, $K' = k''$ if $B' \leq \bar{b}(k'')$, this implies that, if the government chooses a debt level in the no-crisis zone, it stays there.

We can now determine the government's payoff in the no-crisis zone. For simplicity we restrict attention to the case where $K_0 = k''$ and deal with the general case in the Appendix. In every period, the capital stock is equal to k'' and private consumption is equal to

$c^n(k^n)$. Government consumption is constant at $g^n(B_0) = \theta f(k^n) - (1 - \beta)B_0$, while government debt stays constant at B_0 . The government's payoff from not defaulting is

$$V_g^n((B_0, k^n, 1, \cdot), B_0, \beta) = [c^n(k^n) + v(\theta f(k^n) - (1 - \beta)B_0)] / (1 - \beta).$$

We next determine $V_g^n(s, B', q)$ and optimal government policy when $B \geq \bar{b}(K)$ and the participation constraint (11) does not bind. Suppose, to simplify the discussion, that $K_0 = k^n$ and that $B_0 > \bar{b}(k^n)$. Subsequently we show that $d\bar{b}(K)/dK > 0$. Therefore, $\bar{b}(k^n) > \bar{b}(k^\pi)$, and hence $B_0 > \bar{b}(k^\pi)$, and the government is faced with the following choices in period 0: default now; plan to run the debt down to $\bar{b}(k^n)$ or less in T periods if no crisis occurs, $T = 1, 2, \dots$; or never run the debt down. For each of these choices, we can calculate the expected payoff. The equilibrium is determined by the choice that yields the maximum expected payoff. (Of course, we must show that a maximum expected payoff exists.)

We can solve the government's variational problem to show that the government maintains a constant level of government spending if a crisis does not occur but is possible. If the government plans to run its debt down to $\bar{b}(k^n)$ in T periods, we can use the government's budget constraints to calculate that level of government spending

$$g^T(B_0) = \theta f(k^\pi) + \frac{\hat{\beta}^{T-1}(1 - \hat{\beta})\beta}{1 - \hat{\beta}^T} \bar{b}(k^n) - \frac{1 - \hat{\beta}}{1 - \hat{\beta}^T} B_0.$$

(Setting $B_T < \bar{b}(k^n)$ cannot be optimal, since it lowers g^T and—since, by the definition of T , $B_{T-1} > \bar{b}(k^n)$ —does not eliminate the possibility of a crisis before T .)

If the government chooses to never run its debt down to $\bar{b}(k^n)$, then government spending is

$$g^\infty(B_0) = \theta f(k^\pi) - (1 - \hat{\beta})B_0.$$

Notice that

$$\lim_{T \rightarrow \infty} g^T(B_0) = g^\infty(B_0).$$

We can now calculate the expected payoff of running the debt down to $\bar{b}(k^n)$ in T periods

$$\begin{aligned} V_g^T(B_0) &= (1 - \hat{\beta}^{T-1})[(1 - \pi)(c^\pi(k^\pi) + v(g^T(B_0))) + \pi V_g^d] / (1 - \hat{\beta}) \\ &\quad + \hat{\beta}^{T-1}[(1 - \pi)(c^n(k^\pi) + v(g^T(B_0))) + \pi V_g^d] \\ &\quad + \hat{\beta}^T[c^n(k^n) + v(\theta f(k^n) - (1 - \beta)\bar{b}(k^n))] / (1 - \beta), \end{aligned}$$

where the payoff in the event of a crisis is

$$V_g^d = c^d(k^\pi) + v(g^d(k^\pi)) + \beta[c^d(k^d) + v(g^d(k^d))] / (1 - \beta). \quad (14)$$

The formula for $V_g^T(B_0)$ is easy to understand: In periods $t = 0, 1, \dots, T-2$, g is constant at $g^T(B_0)$, c is constant at $c^\pi(k^\pi)$, and there is a probability π of a crisis. In period $T-1$, however, although there is a probability of a crisis, if this crisis does not occur and the government sets $B_T = \bar{b}(k^n)$, then the consumers know that no default can occur in period T and therefore set $k' = k^n$ and $c = c^n(k^\pi) = (1 - \theta)f(k^\pi) - k^n$. In other words, lowering the debt to the crucial value at which crises cannot occur leads to an investment boom in period $T-1$. In period T , consumption and government spending increase, and the equilibrium enters the no-crisis zone.

It is worth noting that, since consumers are risk neutral, there is no dynamic consistency problem involved in whether or not the consumers believe that the government is going to run its debt down to $\bar{b}(k^n)$ in period T : they set $k' = k^\pi$ until B' is actually equal to $\bar{b}(k^n)$.

To determine T , we merely choose the maximum of $V_g^1(B_0), V_g^2(B_0), \dots, V_g^\infty(B_0)$. Since

$$\lim_{T \rightarrow \infty} V_g^T(B_0) = V_g^\infty(B_0),$$

where

$$V_g^\infty(B_0) = [(1 - \pi)(c^\pi(k^\pi) + v(g^\infty(B_0))) + \pi V_g^d]/(1 - \beta),$$

it is easy to prove that this maximum exists.

D. Characterizing the zones

For simplicity we begin by considering the special case, which we refer to as a *zero-probability crisis equilibrium*, in which $\pi = 0$ and capital is k^n . In this case a level of government debt B satisfies the participation constraint (11) under a stationary debt policy if it is optimal to repay rather than default if new debt B can be sold at price β , or

$$\begin{aligned} c^n(k^n) + v(\theta f(k^n) - (1 - \beta)B) + \beta[c^n(k^n) + v(\theta f(k^n) - (1 - \beta)B)]/(1 - \beta) \\ \geq c^d(k^n) + v(\theta \alpha f(k^n) + \beta B) + \beta[c^d(k^d) + v(\theta \alpha f(k^d))]/(1 - \beta). \end{aligned} \quad (15)$$

The same level of debt B satisfies the no-lending condition (12) if it is optimal to default if the price of all new debt is 0, or

$$\begin{aligned} c^d(k^n) + v(\theta \alpha f(k^n)) + \beta[c^d(k^d) + v(\theta \alpha f(k^d))]/(1 - \beta) \\ > c^n(k^n) + v(\theta f(k^n) - B) + \beta[c^n(k^n) + v(\theta f(k^n))]/(1 - \beta). \end{aligned} \quad (16)$$

We combine these two conditions to produce

$$W_p(B) \geq \bar{W} > W_m(B), \quad (17)$$

where

$$\begin{aligned} W_p(B) = v(\theta f(k^n) - (1 - \beta)B) - v(\theta \alpha f(k^n) + \beta B) \\ + \beta[v(\theta f(k^n) - (1 - \beta)B) - v(\theta \alpha f(k^d))]/(1 - \beta) \end{aligned}$$

is the net social benefit in terms of government spending of not defaulting if the government can roll over its debt,

$$\bar{W} = c^d(k^n) - c^n(k^n) + \beta[c^d(k^d) - c^n(k^n)]/(1 - \beta)$$

is the net social cost in terms of consumption of not defaulting, and

$$W_m(B) = v(\theta f(k^n) - B) - v(\theta \alpha f(k^n)) + \beta[v(\theta f(k^n)) - v(\theta \alpha f(k^d))]/(1 - \beta)$$

is the net social benefit in terms of government spending of not defaulting if the government cannot roll over its debt.

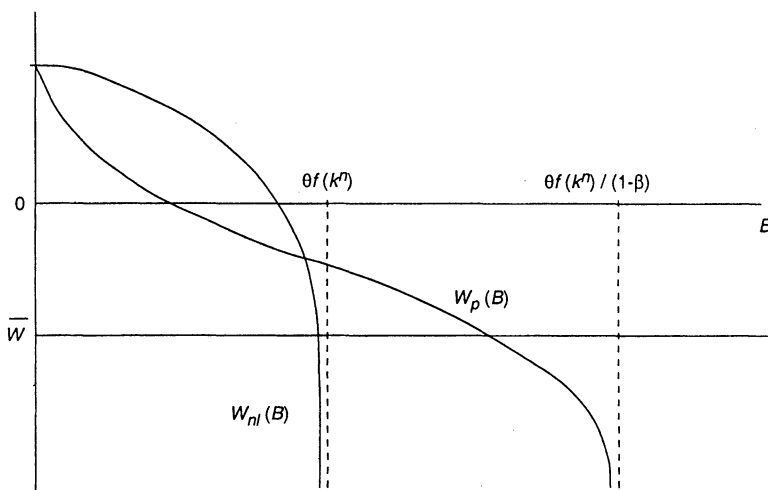


FIGURE 1
Sunspot equilibria

Figure 1 depicts the general situation: $W_p(0) = W_{nl}(0) > 0$, $W'_p(B) < 0$, $W'_{nl}(B) < 0$, $W'_p(0) < W'_{nl}(0)$, but $W_{nl}(B)$ tends to $-\infty$ as B tends to $\theta f(k^n)$, while $W_p(B)$ tends to $-\infty$ as B tends to $\theta f(k^n)/(1-\beta) > \theta f(k^n)$. Therefore, if the social cost of default in terms of private consumption \bar{W} is sufficiently large, then there is a nonempty interval of stationary levels of government debt $\bar{b}(k^n) < B \leq B^s$ for which zero-probability crises are possible, where B^s , the maximum stationary debt level, is simply the level of debt for which condition (15) holds as an equality.

Given the government's payoff from defaulting and not defaulting in the no-crisis zone, we can determine for what values of B our no-lending condition (16) is not satisfied and the government weakly prefers to repay its debt, even if it cannot sell new bonds at a positive price and hence no crisis is possible. We let $\bar{b}(K)$ be the largest value of B for which the government weakly prefers to repay its debt, even if it cannot sell new bonds at a positive price. It is easy to see that since π does not enter into the above expression, \bar{b} is independent of π . It is also easy to establish that $d\bar{b}(K)/dK > 0$ by simply differentiating both sides of (16) and noting that

$$(1-\theta)f'(K) + v'(\theta f(K) - \bar{b}(K))\theta f'(K) > (1-\theta)\alpha f'(K) + v'(\theta \alpha f(K))\theta \alpha f'(K),$$

since it must be the case that $\theta f(K) - \bar{b}(K) < \theta \alpha f(K)$.

Explicitly characterizing the participation constraint and the upper bound on debt $\bar{B}(K, \pi)$ is difficult because optimal government policy will not, in general, be stationary in the crisis zone. It is easy to characterize the upper bound on debt under a stationary debt policy, however, where the capital stock is equal to k^π . Let $B^s(\pi)$ be the largest value of B for which

$$\begin{aligned} & [c^\pi(k^\pi) + v(\theta f(k^\pi) - (1-\hat{\beta})B)]/(1-\hat{\beta}) + \beta\pi[c^d(k^\pi) + v(\theta \alpha f(k^\pi))]/(1-\hat{\beta}) \\ & \quad + \beta^2\pi[c^d(k^d) + v(\theta \alpha f(k^d))]/[(1-\beta)(1-\hat{\beta})] \\ & \geq c^d(k^\pi) + v(\theta \alpha f(k^\pi) + \hat{\beta}B) + \beta[c^d(k^d) + v(\theta \alpha f(k^d))]/(1-\beta). \end{aligned} \quad (18)$$

As π tends to 0, k^π tends to k^n , $c^\pi(k)$ tends to $c^n(k)$, and this version of the stationary participation constraint (18) tends to that in the equilibrium in which agents ignore sunspots (15).

The following proposition gives explicit conditions under which the crisis zone is nonempty.

Proposition 1. *If the economy is such that, in the zero-probability crisis equilibrium, the stationary upper bound on debt implied by (18) satisfies $B^s(0) > \bar{b}(k^n)$, then*

- (i) *for any probability π and for $K_0 = k^n$, there is a nonempty region of debt levels $\bar{b}(k^n) < B \leq \bar{B}(k^n, \pi)$ and*
- (ii) *for π sufficiently small and $K_0 = k^\pi$, there is a nonempty region of debt levels $\bar{b}(k^n) < B \leq \bar{B}(k^\pi, \pi)$ for which a crisis can occur.*

Proof. See the Appendix. ||

5. SUNSPOTS AND SELF-FULFILLING CRISES

We provide a formal characterization of optimal government policy in the crisis zone in the following proposition.

Proposition 2. *For any K_0 and $B_0 \leq B^s(\pi) - \theta(f(K_0) - f(k^\pi))$, if we denote by V_g^T the government's payoff when its policy is to lower its debt to $\bar{b}(k^n)$ in T periods while keeping g constant, then a $T \in \{1, 2, \dots, \infty\}$ that maximizes $\{V_g^1(B_0), V_g^2(B_0), \dots, V_g^\infty(B_0)\}$ exists, and the following are true:*

- (i) *If $K_0 \geq k^\pi$, as B_0 increases, $T(B_0)$ passes through critical points where it increases by one period. Furthermore, for π close enough to 0, there are necessarily regions of $B_0 \leq B^s(\pi)$ with the full range of possibilities $T(B_0) = 1, 2, \dots, \infty$;*
- (ii) *If $K_0 < k^\pi$, then the debt may increase in the first period, but afterward follows the same characterization as in (i) since $K_1 = k^\pi$ and $B_1 \leq B^s(\pi)$. The equilibrium behaviour of K and q are given by (9) and (10).*

Proof. See the Appendix. ||

If B_0 is sufficiently high, then the participation constraint binds in period 0, and the government can either default, or it can set B_1 below \bar{B} , set g_0 below $g^T(B_0)$, and then follow the policy described above starting in the next period.

We are now ready to fully characterize crisis equilibria.

Proposition 3. *For any probability $\pi > 0$ for which there exists a nonempty crisis zone $\bar{b}(k^n) < B \leq \bar{B}(k^\pi, \pi)$, there can exist an equilibrium in which the transition function for capital is given by (9), the price function on government debt is given by (10), and, depending on B_0 , the following outcomes occur.*

- (i) *If $K_0 \geq k^n$ and $B_0 \leq \bar{b}(k^n)$, then $c_0 = c^n(K_0)$ and all other equilibrium variables are stationary: $K = k^n$, $c_t = c^n(k^n)$ for $t \geq 1$, $B = B_0 - \theta(f(K_0) - f(k^n))$, $g = \theta f(k^n) - (1 - \beta)B$, and $q = \beta$. In this case no crisis occurs.*

- (ii) If $\bar{b}(K_0) \leq B_0 \leq \bar{B}(K_0, \pi)$, then a crisis occurs with probability π in the first period and every subsequent period in which $B > \bar{b}(k^n)$. If $B_0 \leq B^s(\pi) - \theta(f(K_0) - f(k^\pi))$, optimal government policy involves running down the debt to $\bar{b}(k^n)$ in $T(B_0)$ periods, while smoothing government expenditures as described in Proposition 2. If $T(B_0)$ is finite and a crisis does not occur ($\zeta > \pi$ in the first $T-1$ periods), then following period $T(B_0)$, the equilibrium outcomes are those in (i). For $B_0 > B^s(\pi) - \theta(f(K_0) - f(k^\pi))$, the equilibrium converges to the outcome described in Proposition 2 in at most two periods.
- (iii) If $K_0 < k^n$ and $B_0 \leq \bar{b}(K_0)$, then there is no possibility of a crisis in period 0, and from period 1 onward, the outcomes correspond to those described in (i) if under the government's optimal policy, $B_1 \leq \bar{b}(k^n)$ or in (ii) if not.
- (iv) If $B_0 > \bar{B}(K_0, \pi)$, then the only outcome is the default outcome in which $c_0 = c^d(K_0)$, $g_0 = \theta\alpha f(K_0)$, and all other equilibrium variables are stationary: $K = k^d$, $c = c^d(k^d)$, $B = 0$, $g = \theta\alpha f(k^d)$, and $q = 0$.

Proof. See the Appendix. ||

This characterization is greatly complicated by the need to consider all possible initial levels of the capital stock. After the first period, however, the capital stock can take on only one of three values: k^d , k^π , or k^n . We illustrate our findings for these consistent initial capital stock levels in Figure 2. If we assume that $\bar{b}(k^n) < \bar{B}(k^\pi, \pi)$, the following possibilities for crises are all possible outcomes, depending on B_0 : Case (i), $K_0 = k^n$, is illustrated by the trajectory starting from b_1 in Figure 2. Case (ii), $K_0 = k^\pi$, is illustrated by the trajectories starting in the interval $[b_2, b_5]$, where these trajectories are drawn under the

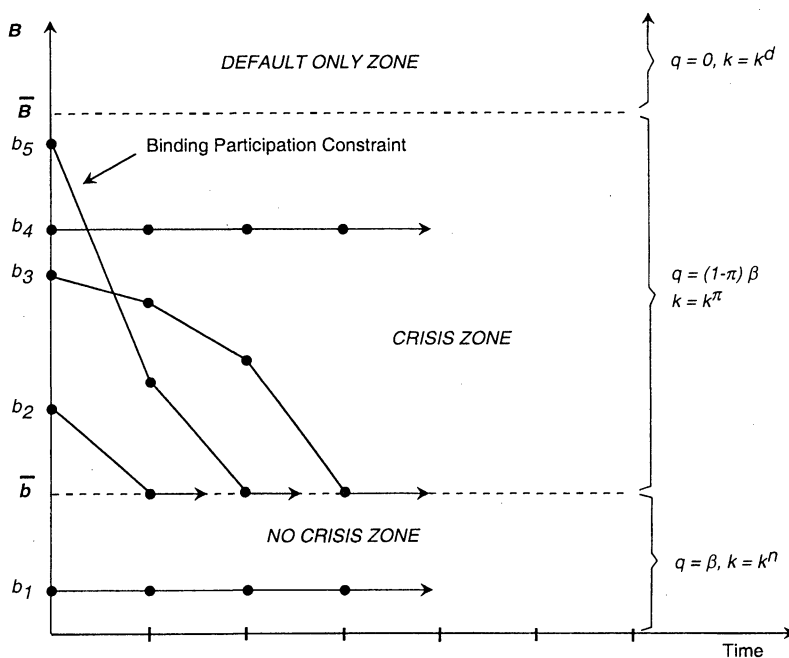


FIGURE 2
Debt trajectories if a crisis is avoided

assumption that a crisis is avoided ($\zeta < \pi$). The debt levels b_2 , b_3 , and b_4 are assumed to be less than $B^s(\pi)$; hence, the participation constraint does not bind, and the number of periods until the debt is reduced to $\bar{b}(k^n)$ is increasing in B_0 . For b_5 , the participation constraint requires that the debt drop sharply in the first period, but thereafter works like the other cases, since the debt is now below $B^s(\pi)$. Depending on the parameters, in particular on π , some types of dynamics illustrated in Figure 2 may not exist. For π large enough, for example, there is no region in which debt is constant or in which $\bar{B}(k^\pi, \pi) > B^s(\pi)$.

To understand why the government is motivated to run down its debt and thereby deviate from the standard consumption smoothing policy, notice first that, for some initial levels of the debt B larger than $\bar{b}(k^n)$, the government actually receives more when it sells new debt than if it tries to sell less debt. To see this, note that even if $B > \bar{b}(k^n)$ but close, then $\beta B < \beta \bar{b}(k^n)$. Hence, the government strictly prefers to offer $\bar{b}(k^n)$ units of debt for sale rather than B for all $\bar{b}(k^n) < B < \bar{b}(k^n)/(1 - \pi)$. Furthermore, since $\bar{b}(k^n)/(1 - \pi) \rightarrow \infty$ as $\pi \rightarrow 1$, the region for which the government receives less and, hence, prefers to run its debt down to $\bar{b}(k^n)$ in one step will come to include all of the region of the crisis zone $(\bar{b}(k^n), \bar{B}(k^n, \pi)]$ as the probability of a crisis gets large. This is an illustration of how π can affect the types of dynamics of the model.

A. Domestically initiated crises

The crises that we have examined thus far have been initiated by the self-fulfilling fears of the international bankers and, in this sense, originated within the financial markets. Among its many other types of equilibria, the model also admits an equilibrium with a different kind of crisis, which is initiated by self-fulfilling fears of the domestic investors and, in this sense, originates in the real investment sector. To see this, we need to add an additional sunspot variable ξ , which is realized after the government has sold its debt and just before the consumer's decisions. We assume that ξ is also uniformly distributed on $[0, 1]$.

Let π_ξ denote the probability of a crisis in which $q = 0$ whenever $B > \bar{b}(K)$. Now consider an outcome in which consumers' fears lead them to lower their investment level to k^{π_ξ} . If the government's new borrowing satisfies $B' > \bar{b}(k^{\pi_\xi})$, then a self-fulfilling crisis next period would be possible given that $K' = k^{\pi_\xi}$, since $\bar{b}' > 0$ and, hence, $\bar{b}(k^{\pi_\xi}) < \bar{b}(k^n)$. Therefore, consumers' fears that they could be in the crisis zone next period could be self-fulfilling, even if $K = k^n$ and they were out of the crisis zone today. In this type of equilibrium, the probability that consumers assign to a crisis can be arbitrary. If we let π_ξ denote this probability, then any value $0 < \pi_\xi \leq 1$ is possible so long as $B' > \bar{b}(k^{\pi_\xi})$.

Notice that in this new type of crisis, the consumers' fears of a default are fulfilled with probability π_ξ , and with probability $1 - \pi_\xi$, their fears turn out to be groundless. Thus, the equilibrium outcome could potentially exhibit wide swings in investment before a financial crisis that forces the government into default finally arises. Of course, the possibility of such a crisis would given the government an incentive to reduce its debt level to $\bar{b}(k^{\pi_\xi})$. Since as π_ξ tends to 1, k^{π_ξ} tends to k^d , however, only debt levels at or below $\bar{b}(k^d)$ are sufficiently low so as to ensure that a self-fulfilling crisis is not possible.

B. Temporary real cost of a default

Thus far, we have assumed that the exogenous real cost of a default, a fall in a from 1 to α , occurs forever. It is interesting to consider what would happen if this penalty were

incurred only temporarily. Assume that a falls from 1 to α for only a finite number of periods, after which it returns to 1 until a subsequent default occurs. Assume too that this interval is unaffected by whether or not a second default occurs before a returns to 1. Finally, assume that the government is allowed to re-enter the international credit market before a increases from α to 1. Because of our assumption about the default, lending only takes place in the period before α returns to 1. The qualitative characteristics of the equilibrium would be unchanged. If we expand the state space to include the number of past defaults of the government, however, it would be possible to construct sunspot equilibria in which the probability of a crisis depends on the number of past defaults by the government. Such a model would allow one possible interpretation of a reputation for repayment by the government, especially if this probability were increasing in the number of past defaults.

6. POLICY OPTIONS AND GOOD GOVERNMENTS

Thus far the only policy options that we considered for the government are with respect to its fiscal policy; that is, lowering its expenditures in order to lower its debt. We next consider a range of other policy options that have been mentioned in conjunction with sovereign debt repayment problems.

A. *The price of credibility*

Here we reconsider the standard notion that an increase in credibility is good for a government. To do so, consider a country that has raised the credibility of its commitment to repay its debts by increasing the cost of defaulting on its debt through, say, increasing its dependence on foreign trade. In our model, we can think of this as lowering α . Raising the cost of defaulting does indeed increase the level of debt that can be sustained if a crisis does not occur, $\bar{B}(k^n)$. It also raises the level of debt at which a crisis is possible, $\bar{b}(k^n)$. It does not remove the possibility of a crisis occurring, however, since the gap between $\bar{b}(k^n)$ and $\bar{B}(k^n)$ need not be removed. Hence, if this country's debt places it within the new crisis zone, the consequences of the acquisition of some additional credibility can be to make the effects of a crisis much worse. In fact, as the following proposition makes clear, a sufficiently large cost of default ensures the existence of a crisis zone.

Proposition 4. *For $K = k^n$ and positive α sufficiently close to 0, there is a nonempty interval of levels of government debt B , $\bar{b}(k^n) < B \leq B^s(0)$, for which zero-probability crises are possible.*

Proof. See the Appendix. ||

B. *The role of the maturity structure*

We can easily show that by lengthening the maturity structure of the debt, the government can increase $\bar{b}(k^n)$ and, for a long enough maturity structure, completely eliminate the possibility of crises, at least after the first period.

Suppose that the government converts a stationary quantity B of one-period bonds coming due into equal quantities B_N of bonds of maturity $1, 2, \dots, N$. Henceforth, the government redeems B_N bonds every period and sells B_N N -period bonds. If there is no

possibility of a crisis, the price of these bonds is β^N . Maintaining a constant level of government spending, we are therefore able to derive the relation

$$B_N = \frac{1 - \beta}{1 - \beta^N} B.$$

Proposition 5. *Consider an economy with one-period bonds in which a zero-probability crisis equilibrium exists for debt levels in the interval $\bar{b}(k^n) < B \leq B^s(0)$. For any level of debt B in this interval, there exists a maturity of the debt, N' , sufficiently long so that no crises are possible for any maturity $N \geq N'$.*

Proof. See the Appendix. ||

The intuition for this result is easy. As the maturity of the debt gets longer, the amount that the government needs to borrow every period get smaller. This decreases the government's incentive to default, whether or not it can roll over its debt. Furthermore, that international bankers refuse to make new loans to the government has less and less impact on the government payoff, and consequently the payoff from not defaulting if the government can roll over the debt converges to that if it cannot. Notice, however, that it is the maturity structure of the prevailing debt, and not that of the debt being issued, that determines whether or not a crisis is possible. Hence, if there is enough debt coming due to allow for a crisis, then, contrary to the suggestion of Alesina, Prati, and Tabellini (1990), the maturity structure of new debt being issued is irrelevant to the possibility of a crisis today, although it may remove the possibility of a crisis in the future.

There is an important caveat to the maturity result: It is easy for the government to increase the maturity of its debt in this model because there is no cost to doing so. In a more general framework, there may be significant costs. One well known example of such a cost arises from a worsening of the time consistency problem associated with setting the inflation tax. (See Calvo and Guidotti (1990, 1992) and Missale and Blanchard (1994) for more on models with this cost.) Notice, however, that in the case of Mexico in 1994, where *tesobonos* were dollar-indexed, this particular time consistency problem was not present.

C. Crisis zones and good governments

Here we want to examine further the relationship between government preferences and nonempty crisis zones. For simplicity we will restrict attention to a special case in which $\pi = 0$ and capital is k^n . To characterize situations in which there is a nonempty crisis zone, we parameterize the government's utility function by defining $v(g) = \gamma w(g)$, where $\gamma > 0$ determines the preference for government spending and w satisfies the same properties as v . (That is, it is a continuous differentiable, monotonically increasing, strictly concave function, with $w(0) = -\infty$.)

Proposition 6. *For positive γ sufficiently close to 0 or for $\beta < 1$ but sufficiently close to 1, there is a nonempty interval of levels of government debt B , $\bar{b}(k^n) < B \leq B^s(0)$, for which zero-probability crises are possible.*

Proof. See the Appendix. ||

Notice that, since changes in γ have no effect on consumer behaviour, we can interpret changes in γ as applying solely to the government's preferences. Notice also that we have shown that, for appropriate β and γ , the crisis zone includes a segment in which the stationary participation constraint does not bind. There is a simple intuition for the first part of this result. If the utility function puts sufficient weight on private consumption, then the high cost of default on consumption makes it possible for the government to satisfy the participation constraint with a relatively high level of debt. Such a high level of debt makes a crisis possible if the government cannot roll over its debt. One interpretation of this result is that governments that care more about private consumption than public consumption have a wider crisis zone. In Figure 1, increasing γ shifts down \bar{W}/γ while leaving $W_p(B)/\gamma$ and $W_{nl}(B)/\gamma$ fixed.

Since causality runs from expectations to interest rates in our model, Calvo's (1988) prescription that the government peg the interest rate simply results in a refusal of the private agents to buy government debt. For example, even if the government pegs the price of its debt at β , the optimal response of the bankers is to buy zero of it if they believe that $z' = 0$. This result is consistent with the Mexican government's inability to issue new debt at prices above its auctions' reservation price.

7. CONCLUSION

In this paper, we present a model in which debt crises can take two forms. In one form, fears of a default on the part of the international bankers are self-fulfilling. In the other, the fear of a future default on the part of the government leads domestic investors to reduce the level of real investment, which lowers future output and hence makes possible a subsequent financial crisis, even when a crisis would not have been possible if the level of investment had not fallen. This scenario may, in fact, capture certain important features of the 1994–95 Mexican crisis since, as is noted by the International Monetary Fund (1995), domestic investment fell before the debt crisis occurred.

Our characterization of the conditions under which crises can take place implies several lessons about what policy makers can and cannot do to avoid them. First, standard prescriptions, such as increasing the cost of default to increase credibility, may be a bad idea for the government, since they do not eliminate the possibility of a crisis, but only increase its severity. Second, since causality runs from expectations to interest rates in our model, pegging the interest rate simply results in a refusal by the private agents to buy government debt. Third, if there is enough debt coming due to allow for a crisis, then the maturity structure of new debt being issued is irrelevant to the possibility of a crisis today, although it may remove the possibility of a crisis in the future. Fourth, good governments, in the sense that they are more concerned about private consumption than government consumption or are more patient, are more likely to have a nonempty crisis zone. Overall, our model implies that the only way to avoid debt crises is to avoid the conditions on fundamentals that make them possible: in particular, relatively high levels of debt with a short maturity structure.

In subsequent work, Cole and Kehoe (1996) examine the details of the Mexican crisis. They document one surprising aspect of the Mexican crisis, that, while Mexico's debt/GDP ratio was not high by international standards, it was of very short average maturity by the time of the crisis. In a model calibrated to match the key features of the Mexican economy in 1994, the magnitude and maturity structure of Mexico's debt are consistent with Mexico being in the crisis zone. The model predicts that doubling the average maturity of this debt from roughly 200 days to 400 days, however, would have eliminated

the possibility of a crisis. Hence, the model presented here gives a plausible account of why the crisis took place when and where it did.

Recent events in Asia suggest the need to take a broad interpretation of the government's liabilities when evaluating whether it is in the crisis zone. A number of countries, including Korea, Indonesia, Malaysia, and Thailand, appear to have experienced financial problems that are similar to those experienced by Mexico, except that financial crises seem to have originated in the banking systems of these countries. Here, too, the key element allowing these crises to occur is the large amount of short-term liabilities. The threat that this maturity mismatch poses is exacerbated by the decline in the value of the local currency, since the banks' loans are denominated in local currency, while their debts are denominated in dollars. We think that a version of our model that incorporates a private banking system with explicit or implicit government provided insurance to the foreign lenders could account for these crises as well and could provide a framework within which to evaluate optimal policy toward these crises.

Finally, one additional implication that can be drawn from our work is that since debt crises are due to a coordination failure among the private lenders, an international agency that stands ready to act as a lender of last resort would be welfare enhancing. The recent bailout of Mexico and the recent bailouts in Asia suggest that policy makers perceive this need as well. It is important to note, however, that for these sorts of bailouts to be successful, the policy makers must be able to distinguish between confidence crises and crises in which the government would have an incentive to default even if it could sell new debt. Furthermore, reducing the likelihood of a crisis actually leading to a default reduces the cost to a government of being in the crisis zone, and this will in turn reduce a government's incentive to run its debt down and exit the crisis zone.

APPENDIX

A1. Proof of Proposition 1

If the government prefers not defaulting to defaulting, conditional on keeping a constant debt level, then it certainly does so under the optimal debt policy; hence, $B^s(\pi) \leq \bar{B}(k^\pi, \pi)$. As π increases, however, we can use the implicit function theorem to show that $B^s(\pi)$ decreases, making it more difficult for a nonempty interval $\bar{b}(k'') < B \leq B^s(\pi)$ to exist. To show (i), notice that $B^s(0) > \bar{b}(k'')$ implies that, if $K_0 = k''$ and $B_0 = B_1 = \bar{b}(k'')$, then the participation constraint with $q = \beta$ and $K_1 = k''$ is strictly satisfied, and hence it is also satisfied by B_0 slightly larger than $\bar{b}(k'')$. Since this holds for any π , $\bar{B}(k'', \pi) > \bar{b}(k'')$. To show (ii), notice that $B^s(0) > \bar{b}(k'')$ implies that $B^s(\pi) > \bar{b}(k'')$ for π close to 0 from the continuity of the functions in (18). ||

A2. Proof of Proposition 2

Consider first the case where $K_0 = k^\pi$. Either the maximum of $V_g^T(B_0)$ is achieved for some finite T or it is not. If it is not, then

$$V_g^\infty(B_0) = \lim_{T \rightarrow \infty} V_g^T(B_0) > V_g^T(B_0)$$

for all finite T , and never running down the debt is optimal. (Notice that $B_0 \leq B^s(\pi)$ implies that at the optimal T , $V_g^T(B_0) \geq V_g^d(B_0)$.)

We now argue that as B_0 increases above $\bar{b}(k'')$, it can pass through critical points where the optimal T increases by one period. For $\bar{b}(k'') < B_0 \leq \bar{b}(k'')/(1 - \pi)$, it is optimal to set $T = 1$, because the yield from selling $\bar{b}(k'')$, $\beta \bar{b}(k'')$, is greater than βB_1 for any $\bar{b}(k'') < B_1 < B_0$. As we increase B_0 , we can pass through a critical point where the optimal T increases to $T = 2$. It cannot increase by more, because the optimal government policy is to steadily decrease B if it is to decrease at all. Therefore, there must be a region of values of B_0 where it is optimal to set $T = 2$ in between the regions where it is optimal to set $T = 1$ and where it is optimal to set $T = 3$. As we increase B_0 even further, we can increase T , but we can never decrease it. To see why the optimal T can

never decrease, observe that for $\bar{b}(k^n) < B_0 < \bar{b}(k^n)/(1 - \pi)$,

$$V_g^1(B_0) > V_g^2(B_0) > \dots > V_g^\infty(B_0).$$

By setting $T = 1$ for B_0 in this region, the government can avoid crises without sacrificing government spending, and every period that it delays is costly. Differentiating our formula for $V_g^T(B_0)$, we obtain

$$0 > \frac{d}{dB_0} V_g^\infty(B_0) > \dots > \frac{d}{dB_0} V_g^2(B_0) > \frac{d}{dB_0} V_g^1(B_0).$$

Therefore, if $V^T(B) \leq V^{T^*}(B)$ for $T^* > T$, then for any $B^* > B$, $V^T(B^*) < V^{T^*}(B^*)$. Hence, if it is optimal to reduce the debt from B_0 to $\bar{b}(k^n)$ in T periods, then for higher B_0 , it cannot be optimal to reduce the debt in $T - 1$ periods.

As π tends to 0, the equilibria tend to those of the zero-probability crisis equilibria. For π close enough to 0, it is easy to show that there are necessarily regions of B_0 with the full range of possibilities $T = 1, 2, \dots, \infty$. Let $V_g(B_0, T) = V_g^T(B_0)$ and $g(B_0, T) = g^T(B_0)$, where we think of T as a continuous variable. We can differentiate our formula for V_g^T with respect to T . If we can find that for fixed $B_0 > \bar{b}(k^n)$ and for π small enough, $\partial V_g(B_0, T)/\partial T > 0$ for all T , then we know that even for discrete T , increasing T yields a higher expected payoff for the government. Differentiating $V_g(B_0, T)$, we obtain

$$\begin{aligned} \frac{\partial}{\partial T} V_g(B_0, T) = & \hat{\beta}^{T-1} \{ (1 - \pi)(\log \hat{\beta}) v'(g(B_0, T)) (\beta \bar{b}(k^n) - \hat{\beta} B_0) / (1 - \hat{\beta}^T)^2 \\ & - (\log \hat{\beta}) [(1 - \pi)(c^\pi(k^\pi) + v(g(B_0, T))) + \pi V_g^d] / (1 - \hat{\beta}) \\ & + (\log \hat{\beta}) [(1 - \pi)(c^n(k^n) + v(g(B_0, T))) + \pi V_g^d] \\ & + \hat{\beta} (\log \hat{\beta}) [c^n(k^n) + v(\theta f(k^n) - (1 - \beta)\bar{b}(k^n))] / (1 - \beta) \}. \end{aligned}$$

Fortunately, this formula is easy to interpret. As T tends to ∞ , $\partial V_g(B_0, T)/\partial T$ tends to 0, because $V_g(B_0, T)$ tends to $V_g^\infty(B_0)$. Even so, we are concerned with the sign of $\partial V_g(B_0, T)/\partial T$. The benefit of increasing T is that the government can maintain a higher level of government spending, and this benefit is captured by the first term in the formula above. Notice that as π tends to 0, this benefit remains positive once we factor out $\hat{\beta}^{T-1}$. The cost of increasing T is that the government risks crises for more periods, and this cost is captured by the last three terms in the formula above. Notice that as π tends to 0, this cost goes to zero, even after we factor out $\hat{\beta}^{T-1}$.

Now fix a $B_0 > \bar{b}(k^n)$ and a π for which $\partial V_g(B_0, T)/\partial T > 0$ for all T . The optimal government policy is to set spending equal to $g^\infty(B_0)$ and maintain debt at B_0 . Our previous arguments now imply that for any T , there exists some initial B , $\bar{b}(k^n) < B < B_0$, for which the optimal government policy is to run its debt down to $\bar{b}(k^n)$ in T periods. We know that for $\bar{b}(k) < B \leq \bar{b}(k^n)/(1 - \pi)$, it is optimal to run down the debt in one period. We also know that for $B = B_0 > \bar{b}(k^n)/(1 - \pi)$, it is optimal to never run down the debt. Somewhere between $\bar{b}(k^n)/(1 - \pi)$ and B_0 , all other intermediate possibilities must exist.

To rule out the possibility of there being a sudden jump from a finite T being optimal to it being optimal to maintain the debt level constant, suppose to the contrary that such a jump does occur. Then, at the debt level B where this jump occurs, we know that $V_g^T(B) = V_g^\infty(B)$, but $V_g^{T^*}(B) < V_g^T(B)$ for all $T^* > T$. Furthermore, $V_g^\infty(B^*) > V_g^T(B^*)$ for all $B^* > B$. The continuity of V_g^T and V_g^{T+1} implies that we can choose $\tilde{B} > B$ so that $V_g^T(\tilde{B}) > V_g^{T+1}(\tilde{B})$. Since

$$0 > \frac{d}{dB} V_g^{T+1}(B) > \frac{d}{dB} V_g^T(B),$$

$V_g^T(B) > V_g^{T+1}(B)$ for all $B \leq \tilde{B}$. Since $V_g^T(\tilde{B}) \rightarrow V_g^\infty(\tilde{B})$ as $T \rightarrow \infty$, however, we know that there exists a $\tilde{T} > T + 1$ sufficiently large so that $V_g^{\tilde{T}}(\tilde{B}) > V_g^T(\tilde{B})$. Consequently, if we restrict the government's choices to the set $1, 2, \dots, \tilde{T}$, we know that at $B_0 = \tilde{B}$ it would choose to run down its debt in \tilde{T} periods. Our previous arguments now imply that there has to be a region where $B < \tilde{B}$ and where it is optimal to run down the debt in $T + 1$ periods, in particular where $V_g^{T+1}(B) > V_g^T(B)$. This contradiction rules out the possibility of a sudden jump.

For the case when $K_0 \neq k^\pi$, a similar variational argument implies that under the optimal policy, g is constant during the transition to $\bar{b}(k^n)$. Furthermore, if instead of (K_0, B_0) as its state, the government has $(k^n, B_0 + \theta(f(k^n) - f(K_0)))$, where $B_0 + \theta(f(k^n) - f(K_0)) \leq B^s(\pi)$ so that the participation constraint does not bind, the government's problem is unchanged, except that private consumption is different in period 0. Hence, the solution is unchanged. Notice that B_1 is weakly less than $B_1 + \theta(f(k^n) - f(K_0))$, but this does not imply that $B_1 \leq B_0$ if $K_0 < k^\pi$. ||

A3. Proof of Proposition 3

The strategy of the proof is to first go through the case where $\pi = 0$, and where crises are unanticipated, then extend the analysis to general values of π .

We first consider the situation where the participation constraint does not bind in period 0 or period 1. We argue that in this case the equilibrium is stationary after period 1. Consequently, if the participation constraint does not bind in period 1, it does not bind afterward.

To determine optimal government behaviour when the participation constraint does not bind, we can solve variational problem (13) to show that $g_t = g_{t+1}$. We can now characterize equilibrium outcomes when the participation constraint does not bind: The initial capital stock is given by K_0 , but is thereafter set at k^n . Consumption is determined by the capital stock according to the function $c^n(k)$. Government consumption is constant at

$$g^n(B_0, K_0) = \theta f(K_0) - (1 - \beta)B_0 + \beta \theta (f(k^n) - f(K_0)),$$

and government debt becomes constant after one period at

$$B^n(B_0, K_0) = B_0 + \theta (f(k^n) - f(K_0)).$$

The government's payoff from not defaulting in period 0 is

$$V_g^n(s_0, B^n(B_0, K_0), \beta) = c^n(K_0) + v(g^n(B_0, K_0)) + \beta [c^n(k^n) + v(g^n(B_0, K_0))]/(1 - \beta),$$

where $s_0 = (B_0, K_0, 1, \cdot)$. Thereafter it is

$$V_g^n(s, B^n(K_0, B_0), \beta) = [c^n(k^n) + v(g^n(K_0, B_0))]/(1 - \beta),$$

where $s = (B^n(B_0, K_0), k^n, 1, \cdot)$. If both of these payoffs satisfy the partition constraint, then this is the equilibrium outcome.

When the participation constraint is violated in the proposed equilibrium described above, there are two possibilities: either the government can choose to default, or it can sell less new debt to reduce its incentive to default and thereby to satisfy the participation constraint.

To show that the participation constraint binds for at most two periods, we consider the case where $K = k^n$ since we know that $K_1 = k^n$ if the government never defaults. We argue that there is a unique $B^s > 0$ such that the stationary debt sequence B^s satisfies the participation constraint with equality and that any stationary debt sequence $B < B^s$ satisfies the participation constraint strictly. Furthermore, if $B_1 > B^s$ satisfies the participation constraint, then $B_2 < B^s$. Consequently, once $K = k^n$, it takes at most one period for the government to adopt the stationary debt policy described previously.

To show that there is a unique $B^s > 0$ such that $B_1 = B_2 = B^s$ satisfies the participation constraint with equality, let us write this constraint as $(V_g^n - V_g^d)(B_1, B_2) \geq 0$ where

$$\begin{aligned} (V_g^n - V_g^d)(B_1, B_2) &= c^n(k^n) + v(\theta f(k^n) - B_1 + \beta B_2) \\ &\quad + \beta [c^n(k^n) + v(\theta f(k^n) - (1 - \beta)B_2)]/(1 - \beta) \\ &\quad - c^d(k^n) - v(\theta \alpha f(k^d) + \beta B_2) \\ &\quad + \beta [c^d(k^d) + v(\theta \alpha f(k^d))]/(1 - \beta). \end{aligned} \tag{A1}$$

Notice that $(V_g^n - V_g^d)(0, 0) > 0$ and that $(V_g^n - V_g^d)(B, B) \rightarrow -\infty$ as $B \rightarrow \theta f(k^n)/(1 - \beta)$. Furthermore, differentiating $V_g^n - V_g^d$ yields

$$\frac{\partial}{\partial B_1} (V_g^n - V_g^d)(B, B) + \frac{\partial}{\partial B_2} (V_g^n - V_g^d)(B, B) < 0.$$

Consequently, since $(V_g^n - V_g^d)$ is continuous in (B_1, B_2) , there is a unique B^s , $0 < B^s < \theta f(k^n)/(1 - \beta)$, such that $(V_g^n - V_g^d)(B^s, B^s) = 0$ and $(V_g^n - V_g^d)(B, B) < 0$ for all $B < B^s$, while $(V_g^n - V_g^d)(B, B) > 0$ for all $B > B^s$.

To show that, if the government does not default and, if $B_1 > B^s$, then $B_2 < B^s$, we differentiate $V_g^n - V_g^d$ with respect to B_1 and B_2 separately. Since

$$\frac{\partial}{\partial B_2} (V_g^n - V_g^d)(B_1, B_2) < 0,$$

for all $B_2 \geq B_1$, if $B_2 = B_1$ violates the participation constraints, that is, if $(V_g^n - V_g^d)(B_1, B_2) < 0$, then the government cannot satisfy this constraint by trying to sell even more new debt B_2 . Instead, the government can try to

sell less new debt, and $B_2 < B_1$ if any B_2 that satisfies the participation constraint exists. To see this, we apply the implicit function theorem to $(V_g^n - V_g^d)(B_1, B_2) = 0$ at $B_1 = B_2 = B^s$ to calculate $dB_2(B_1)/dB_1$

$$\frac{d}{dB_1} B_2(B^s) = -\frac{\partial}{\partial B_1} (V_g^n - V_g^d)(B^s, B^s) \Big/ \frac{\partial}{\partial B_2} (V_g^n - V_g^d)(B^s, B^s) < 0.$$

This says that setting B_1 above B^s requires us to decrease B_2 below B^s to satisfy the participation constraint. For B_1 high enough, however, there is no value of B_2 for which the government satisfies the participation constraints and it is optimal to default.

We now argue that there is a continuous increasing function $\bar{B}(K_0)$ such that for all $B_0 \leq \bar{B}(K_0)$, it is optimal for the government to never default, and for all $B_0 > \bar{B}(K_0)$, it is optimal for the government to default in period 0. We need to consider both the participation constraint in period 1 (A1) and that in period 0, $(V_g^n - V_g^d)(B_0, B_1, B_2, K_0) \geq 0$ where

$$\begin{aligned} (V_g^n - V_g^d)(B_0, B_1, B_2, K_0) = & c''(K_0) + v(\theta f(K_0) - B_0 + \beta B_1) \\ & + \beta[c''(k'') + v(\theta f(k'') - B_1 + \beta B_2)] \\ & + \beta^2[c''(k'') + v(\theta f(k'') - (1 - \beta)B_2)]/(1 - \beta) \\ & - c^d(K_0) - v(\theta \alpha f(K_0) + \beta B_1) \\ & - \beta[c^d(k^d) + v(\theta \alpha f(k^d))]/(1 - \beta). \end{aligned} \quad (A2)$$

We first argue that there is a continuous function $B_1(B_2)$ such that the participation constraint in period 1 is satisfied with equality. A similar argument implies that there is a continuous function $B_0(B_1, B_2, K_0)$ such that the participation constraint in period 0 is satisfied with equality. We then let

$$\begin{aligned} \bar{B}(K_0) = & \max B_0(B_1, B_2, K_0), \\ & \text{subject to} \\ & 0 \leq B_1 \leq B_1(B_2), \\ & 0 \leq B_2 \leq B^s. \end{aligned} \quad (A3)$$

The constraint $B_1 \leq B_1(B_2)$ binds if and only if the participation constraint binds in period 1 when $B_0 = \bar{B}(K_0)$.

Differentiating $V_g^n - V_g^d$ in (A1), we obtain

$$\frac{\partial}{\partial B_1} (V_g^n - V_g^d)(B_1, B_2) < 0.$$

Furthermore, $(V_g^n - V_g^d)(0, B_2) > 0$ and $(V_g^n - V_g^d)(B_1, B_2) \rightarrow -\infty$ as $B_1 \rightarrow \theta f(k'') + \beta B_2$. Consequently, for any B_2 there is a unique $B_1(B_2)$ for which the participation constraint holds with equality and $(V_g^n - V_g^d)(B_1, B_2) > 0$ for all $B_1 < B_1(B_2)$ and $(V_g^n - V_g^d)(B_1, B_2) < 0$ for all $B_1 > B_1(B_2)$. Since $(V_g^n - V_g^d)/\partial B_1 \neq 0$, the Implicit Function Theorem implies that $B_1(B_2)$ is continuous (in fact, continuously differentiable).

We now repeat the same argument for the function $V_g^n - V_g^d$ in (A2): Since

$$\frac{\partial}{\partial B_0} (V_g^n - V_g^d)(B_0, B_1, B_2, K_0) < 0$$

and $(V_g^n - V_g^d)(0, B_1, B_2, K_0) > 0$, while $(V_g^n - V_g^d)(B_0, B_1, B_2, K_0) \rightarrow -\infty$ as $B_0 \rightarrow \theta f(K_0) + \beta B_1$, there is a continuous function $B_0(B_1, B_2, K_0)$ for which the participation constraint in period 0 is satisfied with equality. Furthermore, we can use the Implicit Function Theorem to show that $B_0(B_1, B_2, K_0)$ is increasing in K_0

$$\frac{\partial}{\partial K_0} B_0(B_1, B_2, K_0) = \frac{-\partial(V_g^n - V_g^d)(B_0, B_1, B_2, K_0)/\partial K_0}{\partial(V_g^n - V_g^d)(B_0, B_1, B_2, K_0)/\partial B_0} > 0$$

for all (B_0, B_1, B_2, K_0) such that $(V_g^n - V_g^d)(B_0, B_1, B_2, K_0) = 0$.

Consider now the problem (A3). Since $B_0(B_1, B_2, K_0)$ is continuous in (B_1, B_2) , it achieves a maximum on the compact constraint set. Furthermore, since $B_0(B_1, B_2, K_0)$ is also jointly continuous in (B_1, B_2) and K_0 , this maximum varies continuously with K_0 .

Showing that $\bar{B}(K_0)$ is increasing in K_0 would seem to be a simple matter of applying the Implicit Function Theorem. Unfortunately, unless we are willing to make assumptions on the second derivatives of v like nonincreasing absolute risk aversion, we cannot ensure that the (B_1, B_2) that solve (A3) vary continuously with K_0 . Suppose, however, that $B_0(B_1, B_2, K_0) = \bar{B}(K_0)$ and that $\tilde{K}_0 > K_0$. Fixing (B_1, B_2) , we know that $B_0(B_1, B_2, \tilde{K}_0) > \bar{B}(K_0)$. Consequently, as we allow (B_1, B_2) to vary, we know that $\bar{B}(\tilde{K}_0) \geq B_0(B_1, B_2, \tilde{K}_0) > \bar{B}(K_0)$.

We turn now to the case of a general value for the probability of a crisis, $0 < \pi \leq 1$. The characterization of the equilibrium works similarly to that above. In the case where $\pi = 0$, the stationary debt policy characterizes optimal government behaviour and, implicitly, equilibrium outcomes when the participation constraint does not bind. In the crisis equilibrium, $T(B_0)$ and $V_g^T(B_0)$ characterize optimal government behaviour and, implicitly, equilibrium outcomes when the participation constraint does not bind.

When the participation constraint does bind, we can use the identical logic as in the case where $\pi = 0$ to argue that, if $K = k^\pi$, then the equilibrium adjusts to that characterized by $T(B)$ and $V_g^T(B)$ in at most one period; in particular, if $B_1 > B^s(\pi)$, then $B_2 < B^s(\pi)$ and the government runs down its debt in $T(B_2)$ periods starting in the period after $K = k^\pi$. If $K_0 = k^\pi$, this is period 1, but if $K_0 \neq k^\pi$ and if the participation constraint binds in period 1, it is period 2. We need to also allow for the possibility that $K = k^n$ if the government needs to lower either B_1 or B_2 so much to satisfy the participation constraints in period 0 or period 1 so that B_1 or B_2 is less than or equal to $\bar{b}(k^n)$. Otherwise, the proof follows the identical logic as in the case where $\pi = 0$. The notation involved in writing out the expressions for $V_g^n - V_g^n$ analogous to those in (A1) and (A2) is straightforward, but tedious. We omit it here. ||

A4. Proof of Proposition 4

Consider varying α and letting $B(\alpha)$ be such that

$$W_p(B(\alpha)) = \bar{W}(\alpha).$$

In the extreme case, as $\alpha \rightarrow 0$, we know that $B(\alpha) \rightarrow f(k^n)/(1 - \beta)$. Consequently, for α sufficiently small, $B(\alpha) > f(k^n)$ and $W_{nl}(B(\alpha)) = -\infty$. This implies that

$$W_p(B(\alpha)) = \bar{W}(\alpha) > W_{nl}(B(\alpha)). \quad ||$$

A5. Proof of Proposition 5

If, in the event of a crisis, the government chooses not to default, then the worst possible outcome will be for it to have to repay all of the debt coming due each period. In this case, the no-lending condition becomes

$$\begin{aligned} (1 - \beta^N)[c^n(k^n) + v(\theta f(k^n) - B_N)]/(1 - \beta) + \beta^N[c^n(k^n) + v(\theta f(k^n))]/(1 - \beta) \\ \geq c^d(k^n) + v(\theta \alpha f(k^n)) + \beta[c^d(k^d) + v(\theta \alpha f(k^d))]/(1 - \beta). \end{aligned} \quad (\text{A4})$$

Recall that, with one period bonds, the maximum stationary debt level B^s is characterized by the equation

$$\begin{aligned} [c^n(k^n) + v(\theta f(k^n) - (1 - \beta)B^s)]/(1 - \beta) \\ = c^d(k^n) + v(\theta \alpha f(k^n) + \beta B^s) + \beta[c^d(k^d) + v(\theta \alpha f(k^d))]/(1 - \beta). \end{aligned} \quad (\text{A5})$$

Notice that, as $N \rightarrow \infty$, $B_N \rightarrow 0$ and $B_N \rightarrow (1 - \beta)B$. Consequently, as $N \rightarrow \infty$, the left-hand side of (A5) tends to

$$[c^n(k^n) + v(\theta f(k^n) - (1 - \beta)B)]/(1 - \beta), \quad (\text{A6})$$

which is greater than or equal to the left-hand side of (A6) for all B in the interval $\bar{b}(k^n) < B \leq B^s$. The right-hand side of (A5) is always strictly less than the right-hand side of (A6), however. Consequently, there exists a finite maturity level N' such that, for all maturity levels $N \geq N'$, (A5) is satisfied for all B in the interval $\bar{b}(k^n) < B \leq B^s$. ||

A6. Proof of Proposition 6

First fix γ and let β vary. Notice that, as $\beta \rightarrow 1$, $\bar{W}(\beta) \rightarrow -\infty$. Consider the level of debt $B(\beta)$ such that

$$W_{nl}(B(\beta)) = \bar{W}(\beta).$$

As $\beta \rightarrow 1$, $B(\beta) \rightarrow \theta f(k^n)$ and $W_p(B(\beta)) \rightarrow \infty$. Consequently,

$$W_p(B(\beta)) > \bar{W} \geq W_{nl}(B(\beta)).$$

Hence, for a fixed $\beta < 1$ for which this inequality is satisfied, there is a nonempty interval of debt levels $B > B(\beta)$ in the crisis zone.

Now fix β and let γ vary. Since $\gamma > 0$, we can divide W_p , W_{nl} , and \bar{W} by γ . Note that W_p/γ and W_{nl}/γ are independent of γ , while \bar{W}/γ can be made arbitrarily negative (since $\bar{W} < 0$) as γ goes to zero. Since W_{nl} tends to $-\infty$ for a lower value of B than does W_p , by choosing γ sufficiently close to zero, we can ensure that a nonempty crisis zone exists. (See Figure 1.) ||

Acknowledgements. We are grateful to seminar participants at numerous institutions and conferences, V. V. Chari, Ed Green, Patrick Kehoe, Lee Ohanian, Hyun Song Shin, Carlos Vegh, and three anonymous referees. The research of the second author has been supported by grant SBR-9618370 from the National Science Foundation. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

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