# Abstract

~~Debates~~ surrounding whether the excessive loans provided by China under the Belt and Road Initiative lead to high indebtedness and eventual "debt traps" ~~in~~ recipient countries ~~remain ongoing~~. This thesis ~~aims to~~ empirically examine this ~~question~~ through the calibration of a sovereign debt model. Specifically, the study focuses on two strategically important countries — Sri Lanka and Pakistan. The research findings validate the notion that these two countries indeed fell into the default set once they received substantial loan amounts~~. Based on the results, two categories of debt traps are proposed, with Sri Lanka and Pakistan falling into distinct categories~~. This categorization offers an objective assessment and presentation method within the literature on debt-trap diplomacy.

**Keywords:** Debt-Trap Diplomacy, Belt and Road Initiative, Sovereign Debt, Optimal Default Decision

# Appendix A: Output Loss

The following demonstrates the calculation of output loss associated with the default using the growth accounting approach proposed by Zarazaga (2012).

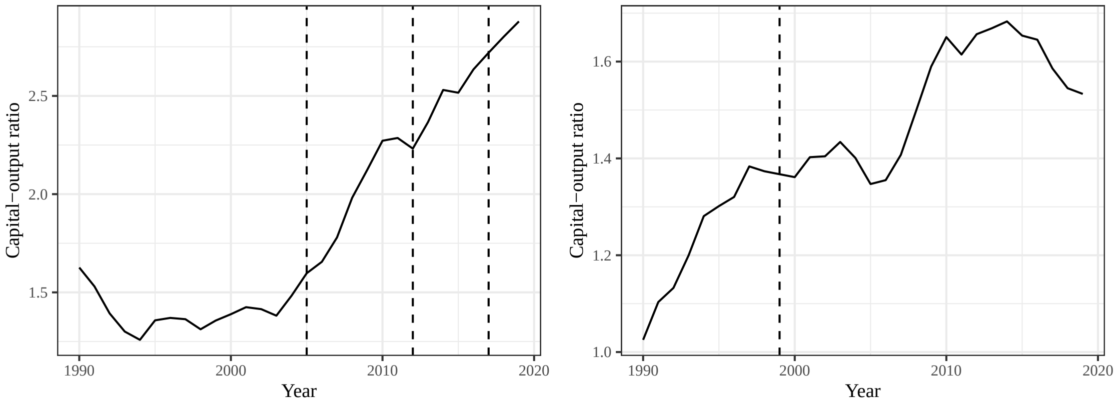
Following Zarazaga (2012), assume that the production function follows the form $y_t = h_t^{\alpha} k_t^{1-\alpha}$ where $y_t$ denotes output, $k_t$ denotes physical capital, and $h_t$ denotes employment. This implies ~~that~~ by the relationship $\frac{y_t}{h_t} = \left(\frac{k_t}{y_t}\right)^{\frac{1-\alpha}{\alpha}}$. If the capital-output ratio before the default episode $\kappa_b = \frac{k_b}{y_b}$ falls to $\kappa_a = \frac{k_a}{y_a}$ after the default episode, the output per worker would be $\Delta = \left[\left(\frac{\kappa_a}{\kappa_b}\right)^{\frac{1-\alpha}{\alpha}} - 1\right] \times 100$ percent higher. If one ascribes all the observed decrease in capital to the sovereign default, one conclude~~s~~ that the output loss is on average $\frac{\Delta}{2}$% per worker during the period. Note that $\left(\frac{\kappa_a}{\kappa_b}\right)^{\frac{1-\alpha}{\alpha}} - 1 > 0$ if and only if $\kappa_a > \kappa_b$. This implies ~~that there is~~ an output loss only if the capital-output-ratio decreases. As argued in Zarazaga (2012), the output loss is associated with a trough in the capital-output-ratio.

Using data from the Penn World Table, the annual~~ly~~ capital-output ratio is calculated by dividing the capital stock at current PPPs (variable *cn*) by the output-side real GDP at current PPPs (variable *cgdpo*), both in ~~million~~ 2017 ~~U.S.~~ dollar.

In the case of Sri Lanka, capital-output ratio associated with the three default episodes (2005, 2012, 2017) recorded in the BoC-BoE Sovereign Default Database all increase, even for ~~the~~ consecutive years, as shown in Figure 13a. It is therefore unreasonable to attribute all the variations of the output to the sovereign default episodes.

For the case of Pakistan's biggest default in 1999, according to previous calibration, $\alpha = 0.4$, which implies ~~that~~ by the relationship $\frac{y_t}{h_t} = \left(\frac{k_t}{y_t}\right)^{\frac{3}{2}}$. Pakistan gained partial reaccess (debt flow > 0) and emerged from financial autarky in 2004 (Trebesch, 2011)~~;~~ therefore the growth accounting will be conducted within 1999 to 2004. The capital-output-ratio was ~~about~~ 1.36 in 1999. It rose to ~~about~~ 1.43 in 2003~~,~~ and slowly fell to

Figure 13: Capital-Output Ratio, 1990 to 2020




(a) Sri Lanka

(b) Pakistan

Source: Penn World Table
Note: The solid line represents the capital-output ratio for Sri Lanka and Pakistan. ~~Default~~ episode examined is plotted in vertical dashed lines.

1.4 in 2004, as shown in Figure 13b. Following the exact same logic, since $\frac{k_t}{y_t}$ rose from 1.36 to 1.4 between 1999 and 2004, the output per worker actually increased, which ~~is~~ contradict~~ing~~ our assumption of an output loss. Alternatively, considering the year of full reaccess as the end of default (debt flow > 1% GDP), which is in 2006 (Trebesch, 2011), the calculation yields that the output per worker would have been $\left[\left(\frac{1.36}{1.355}\right)^{\frac{3}{2}} - 1\right] \times 100 =$ 0.5% higher, which gives an average output loss of 0.25%. The value is too low compared to the average output loss of 5.5% according to cross-country studies (Uribe and Schmitt-Grohé, 2017; Borensztein and Panizza, 2009), ~~which indicates~~ that it is also unreasonable to ascribe all the effect on output to capital. Hence, I conclude that the estimation of output cost of the default for Pakistan following Zarazaga (2012) is also not applicable.

# Appendix B: Properties of the Default Set

In Chapter 4, I introduce the default set given by the decentralized Eaton-Gersovitz model ~~by~~ Na et al. (2018). It is worth examining some properties of the default set, as ~~it~~ justifies visually the empirical results of the thesis. Despite the fact that the default set can only be established through numerical computation instead of an analytical expression, some properties of the default set can still be obtained without knowing ~~the~~ exact form ~~of the set~~.

Recall that the default set is defined as:

$$D(d_t) = \{y_t^T : v^b(y_t^T) > v^c(y_t^T, d_t)\},$$

which is the set of output levels within which the country is best to default given a debt level $d_t$. I derive the following three properties regarding the default set under the model specifications of Na et al. (2018)~~:~~

1. If the default set is not empty, then the trade balance deficit is less than the output loss. ~~That is~~, $q(y_t^T, d_{t+1})d_{t+1} - d_t < -L(y_t^T)$.

2. If $y_1 \in D(d_t)$ and $\underline{y} \le y_2 \le y_1$, then $y_2 \in D(d_t)$

3. The default set $D(d_t)$ is an interval $[\underline{y}, y^*(d_t)]$, where $y^*(d_t)$ is increasing in $d_t$.

Here, $\underline{y}$ denotes the lower bound on the endowment level during numerical computa~~iton~~. Similar properties are proved in Arellano (2008) and Uribe and Schmitt-Grohé (2017) for the centralized version of Eaton-Gersovitz model.

**Proposition 1.** If $D(d_t) \neq \emptyset$, then $q(y_t^T, d_{t+1})d_{t+1} - d_t < -L(y_t^T)$ for all $d_{t+1}$.

*Proof.* The proof is by contradiction. Suppose that $q(y_t^T, \tilde{d}_{t+1})\tilde{d}_{t+1} - d_t > -L(y_t^T)$ for some $\tilde{d}_t$, according to the definition of $v^c(d_t, y_t^T)$:

$$
\begin{aligned}
v^c(d_t, y_t^T) &= \max_{d_{t+1}, h_t} \left\{U\left(A(y_t^T + q_t(y_t^T, d_{t+1})d_{t+1} - d_t, F(h_t))\right) + \beta E_t v^g(y_{t+1}^T, d_{t+1})\right\} \\
&\geq U\left(A(y_t^T + q_t(y_t^T, \tilde{d}_{t+1})\tilde{d}_{t+1} - d, \bar{h})\right) + \beta E_t v^g(y_{t+1}^T, \tilde{d}_{t+1}) \\
&\geq U\left(A(y_t^T - L(y_t^T), \bar{h})\right) + \beta E_t v^b(y_{t+1}^T) \\
&\equiv v^b(y_t^T).
\end{aligned}
$$

For third line, the first term holds due to the fact that both the utility function and the aggregation function ~~is~~ strictly increasing and concave, and the second term holds due to the definition of $v^g = \max\{v^b, v^c\}$.

This result, however, yields a contradiction since if $v^c(d_t, y_t^T) \geq v^b(y_t^T)$ for all possible endowments of tradable output, then $D(d_t) = \emptyset$ by definition. This conclude that if under the certain debt level, the default set is not empty, then it must be true that $q(y_t^T, d_{t+1})d_{t+1} - d_t < -L(y_t^T)$ for all $d_{t+1}$. $\square$

**Proposition 2.** If $y_1 \in D(d_t)$ and $\underline{y} \leq y_2 \leq y_1$, then $y_2 \in D(d_t)$.

*Proof.* Consider the difference between the value function under bad standing~~s~~ $v^b(y_t^T)$ and the value function of continuing to repay ~~its~~ debt $v^c(y_t^T, d_t)$ as $\Delta(y_t^T, d_t) \equiv v^b(y_t^T) - v^c(y_t^T, d_t)$. By definition, any tradable output in the default set $y_t^T \in D(d_t)$ satisfies $\Delta(y_t^T, d_t) > 0$.

Consider the first derivative of the difference function $\Delta_y(y, d) = \frac{\partial \Delta}{\partial y_t^T} = v_y^b(y) - v_y^c(y, d)$. Recall that:

$$
\begin{aligned}
v^c(y, d) &= \max_{\{d'\}} \quad \{U\left(A\left(y + q(y, d')d' - d, F(1)\right)\right) + \beta E_t v^g\left(y', d'\right)\} \\
v^b(y) &= U\left(A\left(y - L(y), F(1)\right)\right) + \beta E_t\left[\theta v^g\left(y', 0\right) + (1-\theta)v^b\left(y'\right)\right].
\end{aligned}
$$

The notation for tradable output $y_t^T$ is simplified as $y$ and $y_{t+1}^T$ as $y'$, and similarly $d_t$ is simplified as $d$ and $d_{t+1}$ as $d'$. ~~Also,~~ previous discussion states that the optimal working hours ~~is~~ $h_t^* = \bar{h}$ when the government chooses to devaluate during default as its optimal policy, which is normalized to unity for simplicity. Applying the envelope theorem on $v^c(y,d)$:

$$
\begin{aligned}
v_y^c \equiv \frac{\partial v^c}{\partial y} &= \frac{\partial}{\partial y} U\Big[A\Big(y + q(y,d')d' - d, F(1)\Big)\Big] \\
&= \Big(1 + q_y(y,d')d'\Big) A_1(c_c, F(1)) U'\Big(A(c_c, F(1))\Big),
\end{aligned}
$$

where $q_y(y,d') \equiv \frac{\partial q}{\partial y}$, $c_c \equiv y + q(y,d')d' - d$. This is easily derived by the chain rule. As for $v^b(y)$:

$$
v_y^b \equiv \frac{\partial v^b}{\partial y} = \Big[1 - L'(y)\Big] A_1(c_b, F(1)) U'\Big(A(c_b, F(1))\Big),
$$

where $L' \equiv \frac{\partial L}{\partial y}$ and $c_b \equiv y - L(y)$ For the sake of simplicity, the second parameter for the aggregation function $A(\cdot,\cdot)$ and its derivative $A_1(\cdot,\cdot)$ will be simplified by showing only the first argument since the second argument is always $F(1)$.

~~Accordingly, the~~ difference function:

$$
\begin{aligned}
\Delta_y =& v_y^b(y) - v_y^c(y,d) \\
=& \Big[1 - L'(y)\Big] A_1(c_b) U'\Big(A(c_b)\Big) - \Big(1 + q_y(y,d')d'\Big) A_1(c_c) U'\Big(A(c_c)\Big) \\
=& A_1(c_b) U'\Big(A(c_b)\Big) - A_1(c_c) U'\Big(A(c_c)\Big) \\
& - L'(y) A_1(c_b) U'(A(c_b)) - q_y(y,d')d' A_1(c_c) U'(c_c).
\end{aligned} \tag{45}
$$

Note that $A(\cdot,\cdot)$ and $U(\cdot)$ are both concave and increasing by assumption. This implies that if $c_1 < c_2$, then (i) $A(c_1) < A(c_2)$, (ii) $A_1(c_1) > A_1(c_2) > 0$, and (iii) $U'(c_1) > U'(c_2) > 0$ Together, it ~~implies~~ that $U'(A(c_1)) > U'(A(c_2))$. Furthermore, since $\frac{A_1(c_1)}{A_1(c_2)} > 1$ and $\frac{U'(A(c_1))}{U'(A(c_2))} > 1$, this results in:

$$
\frac{A_1(c_1) U'(A(c_1))}{A_1(c_2) U'(A(c_2))} > 1 \implies A_1(c_1) U'(A(c_1)) > A_1(c_2) U'(A(c_2)). \tag{46}
$$

The first two terms in Equation (45) resemble~~s~~ this relationship. Since:

$$c_b \equiv y - L(y) > y + q(y, d')d' - d \equiv c_c$$

according to Proposition 1, by Equation (46)

$$A_1(c_b)U'\Big(A(c_b)\Big) - A_1(c_c)U'\Big(A(c_c)\Big) < 0.$$

The third term in Equation (45) is negative since the loss function is assumed to be non-negative and nondecreasing (Na et al., 2018), and hence $L'(y) > 0$. The marginal price of debt offered by foreign lenders $q_y(y, d')$ is positive since a better condition of output today $y$ yields a higher output tomorrow $y'$ due to the AR(1) nature of output, which in turn decreases the probability of default tomorrow. As a result, the price of bond increases. Overall, this gives:

$$\Delta_y(y, d) < 0$$

if the default set is not empty. ~~That is~~, $v^b(y) - v^c(y, d)$ is a decreasing function of $y$.

When default is an optimal policy under the tuple $(y_1, d)$, which means that $y_1 \in D(d)$, then by definition $v^b(y_1) > v^c(y_1, d)$. For any given $y_2 \leq y_1$, since $v^b(y) - v^c(y, d)$ is decreasing in $y$, it is clear that $v^b(y_2) > v^c(y_2, d)$, and hence $y_2 \in D(d)$. $\square$

**Proposition 3.** The default set $D(d_t)$ is an interval $\big[\underline{y}, y^{T*}(d_t)\big)$, where $y^{T*}(d_t)$ is increasing in $d_t$.

*Proof.* An output is in the default set if $v^b(y_t^T) - v^c(y_t^T, d_t) > 0$. It is trivial that as the output goes to infinity, the country has no incentive to default hence $v^c(\infty, d_t) > v^b(\infty)$. By the intermediate value theorem, it is obvious that there exist some $y^{T*}$ such that $\Delta(y^{T*}, d_t) = v^b(y^{T*}) - v^c(y^{T*}, d_t) = 0$, where $y^{T*} = y^{T*}(d_t)$ is the upper bound of default set that depends on the current debt level. Since $\Delta_y(y_t^T, d_t) = v_y^b(y_t^T) - v_y^c(y_t^T, d_t) < 0$ when $D(d_t) \neq \emptyset$, all values such that $y_t^T < y_t^{T*}$ ~~has~~ $\Delta(y_t^T, d_t) < 0$. This proves that the default set is an interval.[1]

[1] The lower bound of the interval is the lowest level of endowment $\underline{y}$

Taking the total derivative of the upper limit with respect to the debt level using the equation $\Delta(y^{T*}(d_t), d_t) = 0$ yields:

$$\frac{\mathrm{d}y^{T*}(d_t)}{\mathrm{d}d_t} = -\frac{\frac{\partial \Delta}{\partial d_t}}{\frac{\partial \Delta}{\partial y^T}} = -\frac{-v_d^c(y^{T*}(d_t), d_t)}{v_y^b(y^{T*}) - v_y^c(y^{T*}, d_t)}.$$

~~Previous~~ proposition shows that $v_y^b(y^{T*}) - v_y^c(y^{T*}, d_t j) < 0$. Applying the envelope theorem to $v^c(y_t^{T*}, d_t)$ gives:

$$\frac{\partial v^c}{\partial d_t} = -A_1\Big[y_t^{T*} + q_t(y_t^{T*}, d_{t+1})d_{t+1} - d_t\Big]U'\Big[A(y_t^{T*} + q_t(y_t^{T*}, d_{t+1})d_{t+1} - d_t)\Big] < 0.$$

Eventually,

$$\frac{\mathrm{d}y^{T*}(d_t)}{\mathrm{d}d_t} > 0. \tag{47}$$

This result implies that as the debt level increases (decreases), the upper bound of the default set should be strictly increasing (decreasing). □

These results match those of Arellano (2008) and Uribe and Schmitt-Grohé (2017). As discussed by Na et al. (2018), when optimal devaluation and taxation policies are implemented, the equilibrium allocation in the economy aligns with that of the centralized Eaton-Gersovitz model. Therefore, it is not surprising that the default set in the decentralized economy exhibits similar behavior to that observed in a traditional centralized economy.