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檢視中國債務陷阱
Examining the Chinese Debt-Trap Diplomacy

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Chapter 1: Introduction

As China's economic influence continues to grow, its lending practices to developing countries have come under scrutiny. The concept of "debt-trap diplomacy", whereby China extends excessive loans to countries in exchange for political or economic concessions, has become a topic of heated debate (Chellaney, 2017).

From the political science aspect, an analysis from the Belfer Center for Science and International Affairs states that China utilizes the debt-trap diplomacy as a technique to achieve strategic objectives, such as projecting power across South Asian trading routes, undermining regional opposition to its South China Sea claims, and supporting its naval efforts to break out into the Pacific (Parker and Chefitz, 2018). On the opposite side of the spectrum, critics of this phenomenon argue that claims of the debt-trap diplomacy are often exaggerated or based on incomplete information. For example, Brautigam (2020) argues that the debt-trap is based on a flawed understanding of Chinese lending practices and the histories of the target countries, and that China is not strategically pursuing the debt-trap diplomacy on developing countries.

Does the excessive debt to China cause potential impact on the economy of low income developing countries (LIDC), especially those in the Belt and Road Initiatives (BLI)? Not many empirical studies examine this issue. On one hand, the lack of transparency in the Chinese lending system, whereby loan terms, conditions and collateral requirements are not always disclosed to the borrowers, makes it difficult for economists to fully grasp the magnitude of the issue. On the other hand, the decision for a country to continue honoring debt obligations is typically an optimal decision in the absence of enforcement, hence it is not obvious whether the country is genuinely in good standings, or is it that the country would actually be better off if it were to default, but is somehow forced not to due to other enforcement, in this case might be the political leverage from China. As a result, recent studies on whether debt-trap diplomacy is a myth have primarily been conducted normatively in the field of political science, rather than a positive economics analysis (See, e.g., Himmer and Rod, 2023; Chen, 2020).

In this thesis, I aim to shed light on whether a country fell into the debt-trap by using the sovereign debt model proposed by Na et al. (2018) and the graphical approach presented by Hinrichsen (2021), to provide insights into the sustainability of the debt of borrowing countries. The concept of addressing the issue through the above methods follows Ho (2023). By calibrating the model for a particular country, a set of tradable-output levels which would cause the country to default could be obtained, given its current debt level. The approach of Hinrichsen (2021) then allows us to present the default set graphically, with each data point on the space representing a debt-output pair for a specific year. This visual representation allows for an examination of whether the country has been in the default zone but has managed to avoid default due to other enforcement mechanisms.

In particular, the default sets for Sri Lanka and Pakistan are examined in this thesis. Sri Lanka is mentioned in the origin of the debt-trap diplomacy narrative (Chellaney, 2017), and Pakistan, being the centerpiece of the China Pakistan Economic Corridor (CPEC), is also discussed frequently in the literature (Hurley et al., 2019).

To my best knowledge, there are so far no empirical study on the issue of debt-trap diplomacy based on an economic model with microfoundation. My thesis contributes to the literature of debt-trap diplomacy by conducting the first empirical result based on a sovereign debt model calibrated with the data of receiving countries under debate, to investigate whether the loans provided by China indeed push the countries to the brink of default, and report counterfactual results of China not lending the considerable amount of loans to the countries of the BRI.

The remaining chapters are organized as follows: In Chapter 2, a comprehensive literature review is conducted on the topics of Debt-trap diplomacy and sovereign default models. Chapter 3 briefly describes the characteristics of external debts to China, as well as the current situation of Sri Lanka and Pakistan. Chapter 4 outlines the specific sovereign debt model that will be applied in this study. Chapter 5 presents the calibration of the model and provides the corresponding empirical results. Finally, Chapter 6 concludes the thesis.

Chapter 2: Literature Review

2.1 Debt-Trap Diplomacy

Whether the debt-trap diplomacy is just a conspiracy used as an instrument for the western countries to justify its political strategy, or is it in fact causing stress on the receiving counties intentionally, is continuously under great debates and defenses.

The term “debt-trap diplomacy” is first coined in Chellaney (2017), which states that the infrastructures supported financially by the China government in Sri Lanka is burdensome and causing Sri Lanka, as well as other small and poor countries, to endure the unsustainable loans, forcing them to cede strategic leverage to China. From a political aspect, researchers from the Belfer Center for Science and International Affairs propose that China may pursue three main strategic objectives using this approach (Parker and Chefitz, 2018). These objectives include: expanding its “String of Pearls” to address the “Malacca Dilemma” and extend its influence along crucial South Asian trade routes, destabilizing and fragmenting the regional coalition led by the United States that challenges China’s claims in the South China Sea, and facilitating the People’s Liberation Army Navy (PLAN) in advancing beyond the ”Second Island Chain” and into the open waters of the Pacific Ocean.[1] This is considered as the reshaping of “soft” infrastructure that China is hoping to enhance (Hillman, 2018).

[1] The Strait of Malacca, located between Malaysia and Indonesia, is one of the busiest and most critical shipping routes in the world, connecting the Indian Ocean and the Pacific Ocean. The “Malacca dilemma” refers to the strategic vulnerability faced by China due to its heavy dependence on the Strait of Malacca for maritime trade and energy imports (Parker and Chefitz, 2018).

A common criticism to the loans from China is that the terms and conditions are typically not transparent compared to other creditors in order to encourage their dependency to China (Tillerson, 2018). For example, an analysis on the contract of 100 China's long foreign lending suggests that the terms of these agreements feature unusual confidentiality clauses, seek advantages over other creditors through collateral arrangements, and potentially grant influence over debtors' policies (Gelpern et al., 2022). The authors' analysis reveals that the contracts signed with Chinese state-owned entities after 2014 in their study frequently contain extensive clauses to maintain confidentiality, which impose broad obligations on the debtor to keep contract details and related information undisclosed. Furthermore, about 30% of these agreements require the borrowing country to establish a dedicated bank account, typically controlled by the lender, as a form of collateral for debt repayment. These revenue accounts, uncommon in sovereign lending, restrict the borrowing country's authority over its own finances. Additionally, compared to other types of debt contracts, Chinese agreements more commonly include cross-default clauses, allowing lenders to demand immediate repayment (called an *acceleration*) if the borrower defaults on other lenders. These contract characteristics, along with their creative design to manage credit risks and enforcement hurdles, portray China as a powerful and commercially astute lender to developing nations.

There are several studies regarding the estimation of debt vulnerability caused by China's loan. Hurley, Morris and Portelance (2019) study the debt sustainability in BRI countries by examining their dept-to-GDP ratio versus the share of China's debt. Following the threshold of 50-60 percent rising debt-to-GDP ratio constructed by Chudik et al. (2015), they identify eight countries that are particularly risky.[2] Bandiera and Tsiropoulos (2020) examine the impact of investment and infrastructure projects under the BRI on the debt-to-GDP ratios of recipient countries. The authors analyze the growth effects of BRI investment and estimates the potential increase in debt vulnerabilities for certain countries

[2] These countries are Djibouti, Kyrgyzstan, Laos, the Maldives, Mongolia, Montenegro, Pakistan, and Tajikistan

through a model-based growth projection. The findings suggest that approximately 28% of BRI investment recipient countries, consisting of 7 low-income developing countries and 5 emerging markets, are expected to face increased debt vulnerability in the medium term, while 37% of countries, including 5 low-income developing countries and 6 emerging markets, may experience a rise in their debt-to-GDP ratio due to BRI investment and financing in the long term, with 8 of them being vulnerable to changes in financing costs.

In stark contract, critics of the “debt-trap diplomacy” narrative often argue that the benefit of China’s lending on the receiving country is neglected, and state that the concerns are often exaggerated. Eom, Brautigam and Benabdallah (2018) argue at the year 2018, Chinese loans did not play a significant role in causing debt distress in Africa. They identify 17 African countries that were low-income and under high risk of debt distress, and find that for less than half (8) of the countries, the levels of debt to China were relatively small compared to their total external debt, and that the debt distress were caused primarily by other conflicts in the nation; six other countries had higher loans from China, but so were to other official creditors; in only three countries, Chinese loans were the main contributions to the debt distress. Brautigam (2020) indicates that debtor countries have voluntarily accepted Chinese loans and report positive experiences, suggesting that concerns over Chinese infrastructure funding are exaggerated, as many view China as an appealing economic model and development partner. In the case of Sri Lanka, Brautigam (2020) also argues that the project of Hambantota Port was the concept of former President Mattala Rajapaksa, and that Chinese Banks have shown willingness to assist their restructuring of existing loans. The Rhodium Group reviews 40 cases of China’s external debt renegotiations, and finds that not only are asset seizures a rare occurrence, but China is limited in the leverage in negotiation due to external events such as change in leadership (Kratz, Feng and Wright, 2019).

Notably, governments of the indebted countries often defends their own decision of loan. The Minister of Finance of the Trinidad and Tobago, for instance, argued that choosing a loan without the need for retrenchment, currency devaluation, or other adverse measures, especially when the interest rates are similar, is an obvious and favorable choice.[3]

2.2 Sovereign Debt Model

Sovereign debt models under the Eaton-Gersovitz framework has been widely used to analyze default decision drivens by reputation and sanction(Eaton and Gersovitz, 1981). Several important features are included in the framework recently to replicate important stylized facts that were not captured under the benchmark model of Eaton-Gersovitz. For instance, Aguiar and Gopinath (2006) and Arellano (2008) take into consideration the risk premium in the market price of debt. As a result, country spread, defined by the difference between the country's net interest rate and the world risk-free interest rate, can be accounted for quantitatively. Na et al. (2018) further investigate the role of government's optimal policy under wage rigidity in a decentralized economy to study the *Twin Ds* phenomenon. They find that being able to freely set the optimal taxation rate and devaluation rate improves the welfare of a country by reducing unemployment, which provides further incentive for a country to default. Mendoza and Yue (2012) endogenize output cost by combining sovereign default and business cycle, which provides microfoundation for the ad-hoc assumption of output costs during default episodes in Arellano (2008). The firm in the economy faces a working capital constraint, which amplifies the effect caused by a shock in the total factor productivity. Chatterjee and Eyigungor (2012) consider the fact that sovereign bond have long and short maturities, and the authors extend the model such that the maturity is realized with a probability. Seniority of a debt is another popular topic in recent literatures. Seniority is the priority of a debt contract among all debts. Empirical study by Schlegl, Trebesch and Wright (2019) shows that private creditors are often paid first, compared to bilateral official creditors, and multilateral institutions (IMF and World Bank) are indeed senior creditors. Chatterjee and Eyigungor (2015) propose a

[3] Loop News "Imbert: Choosing between IMF, Chinese loan a 'no-brainer,"'June 15, 2021

model to analytically track the seniority of debts; Ho and Ritschl (2023) extend the Eaton-Gersovitz model to include both senior and junior debts, representing either commercial debts or war reparations. The assignment of seniority for the different debts can then be switched, which the authors refer to as "seniority reversal."

Another important stylized fact regarding sovereign defaults is that these episodes are in general partial defaults, contrary to the assumption that the country resets its debt levels after a temporary exclusion from the international financial market (See Sturzenegger and Zettelmeyer, 2008; Cruces and Trebesch, 2013). The fraction that a country chooses to default is called *haircuts*, and is usually determined through debt renegotiations. Yue (2010) combines the benchmark Eaton-Gersovitz model with a Nash bargaining game model to simultaneously explain default and debt negotiation. The author proves that the bargaining power has a great impact on debt recovery rate and bond spread. Bi (2008) incorporates the Eaton-Gersovitz model with a stochastic bargaining model to explain why delays in debt negotiation can actually be beneficial. However, empirical data shows that debt does not necessarily reduce following a restructuring; debt during the episode in fact increases first, creating a hump-shaped dynamic. Arellano et al. (2023) first emphasize this stylized fact, and proposes a model and accounting framework that explains the phenomenon.

A rich literature examines the default episodes using calibration on Argentina (Arellano, 2008; Schmitt-Grohé and Uribe, 2016; Mendoza and Yue, 2012; Na et al., 2018). Hinrichsen (2021) examines the effect of war reparations on countries' default set using data from France in the 1870s, Germany in the 1930s, and Finland in the 1940s. Ho and Ritschl (2023) investigate transfer protection in the Dawes Plan by calibrating the German economy during the 1920s.

Chapter 3: Debt to China

International debt to China lacks transparency since their official lending is predominantly undertaken by state-owned entities, and unlike other major economies, the Chinese government does not report or publish any data on its official international lending or outstanding overseas debt claims (Horn, Reinhart and Trebesch, 2021).

Horn, Reinhart and Trebesch (2021) combines a variety of sources to construct a consensus database of Chinese official loans. Their database spans from 1949, the establishment of the People's Republic of China, to 2017. It contains a granular dataset of 2151 loans and 2824 grants with information such as the creditor agent, borrower type, commitment, maturity, etc. It also provides an aggregate panel data of the external debt to China for each country. To have a more precise data of the China debt issue, the database constructed by Horn, Reinhart and Trebesch (2021) is utilized as the primary source of Chinese debt data. This database is combined with data from other creditors obtained from the International Debt Statistics (IDS) from World Bank.

The top 30 countries with the largest debts to China's official creditors are displayed in Figure 1a. Notably, Russia owes China over \$70 billion, while Pakistan's debt amounts to \$27 billion, both topping the list. Brazil and Venezuela are among the top 10 countries with the highest debt to China in Latin America. Contrary to what many people believe, African countries have not borrowed much from China. However, if we consider the ratio of Chinese debt to GDP in Figure 1b, some African countries appear to be highly indebted to China. Djibouti, for instance, has an alarming ratio of 68.5% of its GDP consisting of Chinese debt, while Tonga, Niger, and Zambia have ratios exceeding 10%. This result is in line with the description in Eom et al. (2018).

A main finding in Horn, Reinhart and Trebesch (2021) is that China had become the world's largest creditor to developing countries after 2013, surpassing the amount of World Bank. Figure 2 shows the change of the total amount of debt from different main creditors, including China, World Bank[1], IMF, and the aggregation of all countries in the Paris Club. The debt amount started to rise rapidly after 2000, when the China government launched the "Go Out Policy" in 1999. In 2017, the debt to China in global had reached $355 billion, while the debt to World Bank was $300 billion.

The main focus on my thesis is to evaluate the debt sustainability for a country after it receives the considerable amount of loans from China. Among all countries, Sri Lanka and Pakistan are often under the discussion regarding the debt-trap issue. From the geostrategic aspect, Sri Lanka could serve as the military base for China (Chellaney, 2017), and Pakistan allows China to better connect with the Arabian Sea (Hurley et al., 2019). A preliminary view of the change in debt to China for the two countries in Figure 3 also provides some idea of how China is accumulating an increasing amount of debt to the two country. I briefly introduce the debt-trap diplomacy narrative regarding the two countries in the following sections.

3.1 Sri Lanka

In the original article where the terminology "Debt-trap Diplomacy" was coined, Chellaney (2017) specifically mentioned the predicament faced by the Sri Lankan government. He argues that the Chinese government supported large infrastructure projects in Sri Lanka and provided heavy loans to their government, and as the project eventually failed to repay the debt, the country is then ensnared in the concessions to China. Figure 3a shows the change in composition of the creditors to Sri Lanka.

China started to provide loans to Sri Lanka in 2005, and during 2006 to 2008, the

[1] Including the International Development Association (IDA) and the International Bank for Reconstruction and Development (IBRD)

debt amount to China remains to be around $1 billion, which is roughly 2.9% of GDP. Starting from 2009, debt to China has increased to $3 billion, and it reached $7.5 billion in 2014, accounting for 9.5% of GDP. The loans are primarily composed of constructions of infrastructures, such as the Hambantota International Port and the Mattala Rajapaksa International Airport.

The issue of the Hambantota port is regarded as a typical example of a debt-trap (Moramudali, 2020). The construction of the port was initiated in 2007 and entrusted to the state-owned Chinese companies — China Harbour Engineering Company and Sinohydro Corporation. The first phase started in 2008, where China lent $307 million at a yearly interest rate of 6.3% through the Chinese Exim Bank[2]. The second phase started in 2012 shortly after the completion of the first phase, which transfers the Hambantota port into a container port.

China's involvement in Sri Lanka's infrastructure development was facilitated by President Mahinda Rajapaksa, during which China became Sri Lanka's leading investor and lender. This gave China significant diplomatic leverage over Sri Lanka (Chellaney, 2017). However, when Rajapaksa was unexpectedly defeated in the early 2015 election by Maithripala Sirisena, who campaigned on the promise to extricate Sri Lanka from the Chinese debt trap, work on major Chinese projects was suspended.

However, Sri Lanka's government was already on the brink of default, and Sirisena eventually acquiesced to a series of Chinese demands in 2017[3], including the sale of an 70% stake in the Hambantota port to China Merchants Port (CM Port) and a 99-year lease to China.

Notably, as argued by Moramudali (2019), the lease did not write off the loans obtained to construct Hambantota port. The proceeds from the lease were used to boost

[2] From AidData Project ID #33409, "China Exim Bank provides $306.7 million buyers credit loan for Phase I of Hambantota"

[3] This narrative origins from Chellaney (2017). Brautigam (2020), however, provides a different narrative. She mentioned that "*The proceeds were used to increase Sri Lanka's US dollar reserves in 2017-18 with a view to the repayment of maturing international sovereign bonds …Therefore, the sale of Hambantota was originally a fire sale designed to raise money to deal with larger debt problems.*"

the country's dollar reserves in 2017-18, especially in preparation for the large amount of external debt that needed to be serviced when international sovereign bonds matured in early 2019. This means that the lease is not a debt-equity-swap, as common narratives elaborated (Moramudali, 2020).

The Mattala Rajapaksa International Airport appears to have the same problem as the Hambantota Port. The airport is launched in 2009, and an $181 million loan with 2% interest rate was provided by the Chinese Exim Bank. The airport opened in 2013, but according to the Civil Aviation Authority of Sri Lanka, only 21,000 passengers were served at the airport, and was criticized as "the world's emptiest international airport(Shepard, 2016).

Sri Lanka eventually declared a suspension on payment on most foreign debt from April 12, 2022. According to the International Debt Statistics (IDS), debt stock of Sri Lanka in 2021 has already reached at least $56 billion, yielding a 63% debt-to-GDP ratio, under which debt to China is at least $7.22 billion[4]. Whether Sri Lanka was indeed already under the extreme "brink of default" during 2015 is a major gap in the literature of sovereign default that has not yet been investigated.

3.2 Pakistan

Similar to Sri Lanka, Pakistan has also experienced a significant surge in its debt to China, while simultaneously maintaining a relatively high debt level with other official creditors. As depicted in Figure 3b, the composition of creditors to Pakistan has undergone notable changes. Beginning in 2013, the loans from China escalated from $4 billion to $13 billion in 2014, and further rose to $27.6 billion in 2017, more than doubling the amount in 2014.

Pakistan is the centerpiece of the China Pakistan Economic Corridor (CPEC), a 3000 km corridor that connects China with the Arabian Sea. CPEC serves as an important net-

[4] The debt statistics reported in the International Debt Statistics (IDS) is underestimated as under the Debt Reporting System (DRS), state-owned commercial banks are not included. However, according to the definition of Chinese official lendings adopted in Horn, Reinhart and Trebesch (2021), this term should be included. As a result, the debt level in IDS might be underestimated.

work as it reduces the passage for China's energy import from the Middle Eastern countries (Wikipedia contributors, 2023). In order to accomodate the needs for CPEC, China has launched enormous amounts of infrastructure in Pakistan after 2015. These items include a deep water port, road and rail lines, and most importantly, energy sector projects. Massive amount of commercial loans are provided to Pakistan in 2016, with most of them focusing on power projects. These projects include: \$2.7 billion on the Gwadar-Nawabshah LNG terminal and pipeline project; \$1.26 billion on the Karot Hydropower Project; \$1.28 billion on the Matiari to Lahore Transmission Line; \$1.55 billion on the Pakistan Port Qasim Power Project; and \$0.75 billion on the Qasim Datang Power Station (Horn, Reinhart and Trebesch, 2021). Up to 2018, the estimation of the total value of projects under the CPEC is \$62 billion, out of which around \$33 billion is allocated for energy projects (Hurley, Morris and Portelance, 2019). China is expected to finance about 80% of this amount. The private investments for energy projects in Pakistan will be financed by the Exim Bank of China at an interest rate of 5-6%. Private Independent Power Producers (IPP) will be responsible for constructing the energy projects under CPEC, instead of the governments of China or Pakistan. In turn, the government of Pakistan will be legally bound to buy electricity from these companies at rates that were agreed upon before. However, despite this significant investment, some projects have already been cancelled, such as three major road projects that were cancelled at the end of 2017(Hurley, Morris and Portelance, 2019).

Gwadar Port, located in the western corner of Pakistan, is another prominent component of the CPEC. It serves as a strategic trade shortcut, connecting the Middle East and Europe while bypassing the Malacca Straits. The corridor originates from Kashgar in China's Xinjian province, passes through the Gate of Khunjerab, traverses Pakistan's capital city Islamabad, and extends all the way to Karachi, where Pakistan meets the Arabic sea. At the terminus of the corridor lies Gwadar, which provides access to the Oman Gulf. Recognizing its pivotal location, China has planned to expand the port and create additional berths. In November 2015, the China Overseas Port Holding Company secured a 43-year lease for Gwadar, enabling port expansion and the establishment of a ”Special Economic Zone,” similar to Sri Lanka’s situation (Ranade, 2017). The level of debt resulting from this project, however, remains uncertain. A governor of the Central Bank of Pakistan once admitted that he lacked precise information regarding the composition of the $46 billion investment, stating, “I don’t know out of the $46 billion how much is debt, how much is equity, and how much is in kind (Small, 2020).”

Consequently, the total debt in Pakistan is at least $130 billion, which yields about 37.5% debt-to-GDP ratio[5]. The sudden increase of debt to China draws the attention of researchers and journalists. For example, a report from the Financial Time titled “Pakistan is on the brink” states that Pakistan is following Sri Lanka into default. Given the recent frequent analogy drawn between Pakistan and Sri Lanka, it is essential to analyze Pakistan from the perspective of the sovereign default model.

[5] As with the reason of Sri Lanka, debt data retrieved from International Debt Statistics is underestimated since it does not include state-owned commercial banks.

Chapter 4: Analytic Model

So far, most discussions on China's debt trap have been limited to narrative statistics. To objectively assess this issue, we adopt the up-to-date sovereign debt default model in the literature as a tool for our empirical analysis. Na et al. (2018) proposes a model to study the Argentine economy; Hinrichsen (2021) uses the model to study the enforcement of sovereign debt under war reparations. The model in my thesis strictly follows Na et al. (2018) and Hinrichsen (2021).

International debt often lacks enforcement, and governments hold the decision of whether to repay the debt or default, based on the comparison of future values (Eaton and Gersovitz, 1981). Therefore, default can be considered an optimal policy for a country that faces unsustainable debt levels. By defaulting, the country avoids the burden of paying interest on the debt, but it also faces the consequence of being excluded from the international credit market for a period of time. As a result, the country would have to rely solely on its own financial resources until it regains access to international credit markets. Moreover, studies have pointed out that sovereign debt defaults are often accompanied by a devaluation of the currency; Reinhart (2002) refers to this phenomenon as "Twin Ds." Empirical analysis by Na et al. (2018) further observes that the devaluation rate often decreases after the time of default, suggesting that the Twin Ds phenomenon is the joint result of an optimal policy. They proposed a model that incorporates two key frictions: limited commitment to repay external debts and downward nominal wage rigidity. It is a decentralized version of the Eaton-Gersovitz sovereign debt model. The model predicts that default will occur only after a series of increasingly negative output shocks. Prior to default, domestic absorption experiences a severe contraction, which leads to a decline in demand for labor. Nevertheless, since the nominal wage has a downward rigidity that limits the adjustment, real wages fail to adjust downward, causing involuntary unemployment to occur. To prevent this situation, the government's optimal policy is to devalue the domestic currency, thereby reducing the real value of wages. Therefore, default episodes are usually accompanied by significant currency devaluations in both the model and the data (Na et al., 2018).

Therefore, for the sovereign debt model, I closely follow Na et al. (2018) to examine the set of conditions under which default is the optimal decision. The calibrated model will then serve as a benchmark metric that allows us to investigate whether China has potentially trapped heavily indebted poor counties into default, using the approach proposed by Hinrichsen (2021).

4.1 Households

The model assumes that the economy is populated by a large number of representative households who maximize their expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t), \tag{1}$$

where $\beta \in (0, 1)$ denotes the discount factor, and c_t represents the consumption good, which is composed of tradable consumption c_t^T and nontradable consumption c_t^N. Assume that c_t follows an aggregate technology

$$c_t = A(c_t^T, c_t^N), \tag{2}$$

where A is an increasing, concave, and linearly homogeneous function that captures characteristics such as the ratio or elasticity of substitution between tradable and nontradable consumption. The period utility function $U(c_t)$ follows the standard assumption, which is a strictly increasing and strictly concave function.

Assume that the household only has access to the one-period and non-state-contingent bond. The household spends on consumption of tradable and nontradable goods, along with their debt which comes due in the current period. Its resources consist of labor incomes, dividend incomes, lump-sum transfers from the government, and incomes from borrowing from foreign lenders. The household is also endowed with tradable goods, which follow a stochastic process. The budget constraint of the representative household is then

$$P_t^T c_t^T + P_t^N c_t^N + P_t^T d_t = P_t^T \tilde{y}_t^T + W_t h_t + (1-\tau_t^d) P_t^T q_t^d d_{t+1} + F_t + \Phi_t, \tag{3}$$

where $P_t^T (P_t^N)$ denotes the nominal price of tradable (nontradable) goods, d_t the bond denominated in tradable goods which is due in period t, q_t the price of debt to be repaid at $t+1$, $\tilde{y}_t^T$ the endowment of tradable goods to the household, W_t the nominal wage, h_t the hours worked, τ_t^d the tax on debt, F_t a lump-sum transfer from the government, and finally Φ_t the nominal profits from owning firms. The household's working hour is bounded by an upper limit

$$h_t \leq \bar{h}, \tag{4}$$

and it takes the working hour h_t as given.

Further, denote the relative price of nontradable in terms of tradable goods as $p_t \equiv \frac{P_t^N}{P_t^T}$, we have the following budget constraint,

$$c_t^T + p_t c_t^N + d_t = \tilde{y}_t^T + w_t h_t + (1-\tau_t^d) q_t^d d_{t+1} + f_t + \phi_t, \tag{5}$$

where $w_t = \frac{W_t}{P_t^T}$, $f_t = \frac{F_t}{P_t^T}$, $\phi_t = \frac{\Phi_t}{P_t^T}$ are the variables expressed in price of tradable goods. The household's problem is to choose $\{c_t, c_t^T, c_t^N, d_{t+1}\}$ such that its utility (1) is maximized subject to the budget constraints (2) – (5) and the no-Ponzi-game debt limit.

The Lagrangian for the household is

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U(A(c_t^T, c_t^N)) + \lambda_t \left[\tilde{y}_t^T + w_t h_t + (1-\tau_t^d) q_t^d d_{t+1} + f_t + \phi_t - c_t^T - p_t c_t^N - d_t \right] \right\}.$$

The first order equations are the following:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_t^T} &= A_1(c_t^T, c_t^N) U'(c_t) - \lambda_t = 0 \\
\frac{\partial \mathcal{L}}{\partial c_t^N} &= A_2(c_t^T, c_t^N) U'(c_t) - \lambda_t p_t = 0 \\
\frac{\partial \mathcal{L}}{\partial d_{t+1}} &= (1-\tau_t^d) q_t^d \lambda_t - E_t \lambda_{t+1} = 0,
\end{aligned}$$

where λ_t is the Lagrange multiplier. $A_1(\cdot,\cdot) = \frac{\partial A}{\partial c_t^T}$ and $A_2(\cdot,\cdot) = \frac{\partial A}{\partial c_t^N}$ is respectively the first derivative of the aggregation function with respect to tradable and nontradable consumption. The first order conditions can be concluded as

$$p_t = \frac{A_2(c_t^T, c_t^N)}{A_1(c_t^t, c_t^N)} \tag{6a}$$

$$\lambda_t = U'(c_t) A_1(c_t^T, c_t^N) \tag{6b}$$

$$(1-\tau_t^d) q_t^d \lambda_t = \beta E_t \lambda_{t+1}. \tag{6c}$$

4.2 Firms

Perfectly competitive firms produce nontradable goods y_t^N according to the production technology

$$y_t^N = F(h_t), \tag{7}$$

where F is strictly increasing and strictly concave. Each firm maximizes its profit by choosing the amount of labor. Profit is given by

$$\Phi_t(h_t) = P_t^N F(h_t) - W_t h_t, \tag{8}$$

and the optimal labor demand is then

$$P_t^N F'(h_t) = W_t.$$

Dividing both side by the price of tradable goods, and define $w_t \equiv \frac{W_t}{P_t^T}$ as the real wage in terms of tradable goods, the first order condition can be written as

$$p_t F'(h_t) = w_t. \tag{9}$$

4.3 Downward Nominal Wage Rigidity

The key assumption in Schmitt-Grohé and Uribe (2016) and Na et al. (2018) is the downward nominal wage rigidity. As the wage is unable to be adjusted to a lower level, involuntary unemployment is inevitable, hence the government has the incentive to allow devaluation. The model imposes a lower bound to the growth rate of nominal wage

$$W_t \geq \gamma W_{t-1}, \qquad \gamma > 0. \tag{10}$$

This implies that the growth rate $\frac{W_t - W_{t-1}}{W_{t-1}} \geq \gamma - 1$. When this inequality is unbinding ($W_t > \gamma W_{t-1}$), the economy is fully employed ($h_t = \bar{h}$). However, if the condition binds, the economy might have unemployment ($h_t < \bar{h}$). This relationship can be written as the following equation

$$(\bar{h} - h_t)(W_t - \gamma W_{t-1}) = 0. \tag{11}$$

4.4 Government

We assume here that, under the lack of enforcement in the international credit market, the government has the option to benevolently free up domestic balance sheet by choosing to default or not. Denote I_t as the indicator of whether the government chooses to honor its debt in period t. If the government repays in this period ($I_t = 1$), the country will be able to borrow in the following period, hence $d_{t+1} > 0$. However, if the government chooses to default ($I_t = 0$), then the country will enter the status of financial autarky and is unable to have any sovereign debt in the next period, hence $d_{t+1} = 0$. The above scenario can be written as a slackness condition

$$(1 - I_t)d_{t+1} = 0. \tag{12}$$

To model the duration of financial exclusion, assume that once the country is in bad standing in the international credit market, it can regain reputation and access to financial markets with probability $\theta \in [0, 1)$, and remain in bad standing with probability $1 - \theta$. This implies that the country has an average exclusion duration of $\frac{1}{\theta}$ periods[1].

Assume that the government distributes the proceeds from the debt tax to households as a lump-sum payment. If the government honors the debt, it repays d_t, but if the government decides to default, it will not make any payments to foreign lenders, and instead will return any payments made by households directly to them. The budget constraint for the government can then be expressed as

$$f_t = \tau_t^d q_t^d d_{t+1} + (1 - I_t)d_t, \tag{13}$$

where $f_t \equiv \frac{F_t}{P_t^T}$ is the lump-sum transfer in terms of tradable goods. Right-hand side of the equation states that the transfer to households will include d_t when $I_t = 0$, which is when the country decides to default. Nevertheless, the transfer of debt tax will be zero after default since $d_{t+1} = 0$ when $I_t = 1$, according to Equation (12).

[1] The expected exclusion period $= \sum_{t=1}^{\infty} t\theta(1-\theta)^{t-1} = \theta \sum_{t=1}^{\infty} t(1-\theta)^{t-1} = \frac{1}{\theta}$.

4.5 Foreign Lenders

The behavior of foreign lenders is not explicitly modeled in this framework, but as all rational agents, the expected marginal benefit of lending to the domestic country must be equivalent to the opportunity cost of funds. Let r^* represent the opportunity cost for the foreign lenders; this could be the world interest rate. Since q_t is the price of debt that repays one unit of d_{t+1} tomorrow, the return on the debt is $\frac{1}{q_t}$. The lenders take the risk of default into consideration, therefore, the expected return will actually be lower. Assume that foreign lenders are risk neutral and don't require risk premium, this gives

$$\frac{\Pr(I_{t+1} = 1 \mid I_t = 1)}{q_t} = 1 + r^*. \tag{14}$$

Equivalently, the equation can be written as

$$I_t \left[q_t - \frac{E_t I_{t+1}}{1 + r^*} \right] = 0.$$

4.6 Competitive Equilibrium

Under equilibrium, the households' consumption equals the production of firms

$$c_t^N = y_t^N. \tag{15}$$

The tradable goods are purely endowed exogenously under an AR(1) process

$$\ln\left(y_t^T\right) = \rho \ln\left(y_{t-1}^T\right) + \mu_t, \tag{16}$$

where $\mu_t \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma_\mu^2)$ is an i.i.d. shock, and $|\rho| \in [0, 1)$ is the autocorrelation parameter. When the country decides to default, it is in bad standing, hence it faces an output loss defined by $L(y_t^T)$. The loss function is non-negative and increasing in the tradable goods.

The endowment of tradable goods to the household is then

$$\tilde{y}_t^T = \begin{cases} y_t^T - L(y_t^T) & \text{if } I_t = 0 \\ y_t^T & \text{otherwise} \end{cases} \tag{17}$$

When the country defaults ($I_t = 0$), the endowment decreases.

Price of debt offered by foreign lenders q_t should be equal to the price of the domestic debt q_t^d, but only during the good standing

$$I_t(q_t^d - q_t) = 0. \tag{18}$$

The market clearing condition can be established by combining various equations, including the household budget constraint (3) and (4), the firm's production function (7) and profit equation (8), the government's constraint on debt (12) and lump-sum return (13), and conditions from (15), (17), and (18). Eventually, the clearing condition for tradable goods is

$$c_t^T = y_t^T - (1 - I_t)L(y_t^T) + I_t(q_t d_{t+1} - d_t) \tag{19}$$

Assume that the law of one price applies to tradable goods. The foreign currency price of tradable goods is denoted as P_t^{T*}, while the nominal exchange rate is represented by $\mathcal{E}_t$. The law of one price states that the price of tradable goods in the domestic currency is equal to the foreign currency price multiplied by the nominal exchange rate.

$$P_t^T = P_t^{T*}\mathcal{E}_t$$

This implies that the price of a tradable good should be the same in both domestic and foreign currency terms in an efficient market. Without loss of generosity, the foreign-currency price of the tradable goods is normalized to 1 ($P_t^{T*} = 1$), hence the nominal price for tradable goods can be expressed as the nominal exchange rate

$$P_t^T = \mathcal{E}_t. \tag{20}$$

For convenience, also define the devaluation rate of domestic currency as

$$\epsilon_t \equiv \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}} = \frac{P_t^T}{P_{t-1}^T}. \tag{21}$$

The conditions are now sufficient to define a competitive equilibrium.

Definition 1 (Competitive Equilibrium in Na et al. (2018)**).** A competitive equilibrium is a set of stochastic process $\left\{c_t^T, h_t, w_t, d_{t+1}, \lambda_t, q_t, q_t^d\right\}$ satisfying

$$c_t^T = y_t^T - (1 - I_t)L(y_t^T) + I_t(q_t d_{t+1} - d_t), \tag{22}$$

$$(1 - I_t)d_{t+1} = 0, \tag{23}$$

$$\lambda_t = U'(A(c_t^T, F(h_t)))A_1(c_t^T, c_t^N), \tag{24}$$

$$(1 - \tau_t^d)q_t^d \lambda_t = \beta E_t \lambda_{t+1}, \tag{25}$$

$$I_t(q_t^d - q_t) = 0, \tag{26}$$

$$\frac{A_2(c_t^T, F(h_t))}{A_1(c_t^t, F(h_t))} = \frac{w_t}{F'(h_t)}, \tag{27}$$

$$w_t \geq \gamma \frac{w_{t-1}}{\epsilon_t}, \tag{28}$$

$$h_t \leq \bar{h}, \tag{29}$$

$$\left(h_t - \bar{h}\right)\left(w_t - \gamma \frac{w_{t-1}}{\epsilon_t}\right) = 0, \tag{30}$$

$$I_t \left[q_t - \frac{E_t I_{t+1}}{1 + r^*}\right] = 0, \tag{31}$$

given processes $\left\{y_t^T, \epsilon_t, \tau_t^d . I_t\right\}$ and initial conditions w_{-1} and d_0.

As proven by Na et al. (2018), if the government is able to set the devaluation rate and the tax on debt freely, then the stochastic process of the variables $\left\{c_t^T, h_t, d_{t+1}, q_t\right\}$ can be determined by the process of $\{y_t^T, I_t\}$ and the initial debt level d_0.

As discussed previously, the decision of I_t is an optimal policy for the government due to lack of commitment to repay in the international credit market. Furthermore, the default decision of the government in the next period $t+1$ is also affected by the current decision. To see this argument, first notice that the default decision in $t+1$ is determined by the state variables $\{y_{t+1}^T, d_{t+1}\}$. However, d_{t+1} is determined in period t, which means that the government in period t understands that it is able to affect the default decision in $t+1$ via the choice of d_{t+1}. As y_{t+1}^T follows a first-order Markov process, the expected value of y_{t+1}^T is a function of y_t^T, hence the expected value for the default decision on period t is actually a function of y^T and d_{t+1}. Recall that the price for the debt q_t is related to the probability of default in the next period, according to Equation (14), it can be expressed in the contemporary variables

$$q_t = q(y_t^T, d_{t+1}). \tag{32}$$

On the one hand, this provides us the economic intuition that the government internalizes the fact that its choice of debt in the next period can affect the price of the debt. On the other hand, this allows us to clarify the dependencies of variables in the value function.

4.7 Default Decision

Following the standard Eaton-Gersovitz framework, this model considers the following three value functions: value of continuing to repay the debt v^c, value of being in good standing v^g, and value of being in bad standing v^b.

Under the period of being in good financial standing, the value for the government to continue repaying the debt is the maximum value of the utility gained by the households this period, plus the discounted value of being in a good financial standing, subject to the households' budget constraints. Formally,

$$
\begin{aligned}
v^c(y_t^T, d_t) = \max_{\{c_t^T, h_t, d_{t+1}\}} \quad & \left\{U\left(A\left(c_t^T, F(h_t)\right)\right) + \beta E_t v^g\left(y_{t+1}^T, d_{t+1}\right)\right\} \\
\text{s.t} \quad & c_t^T + d_t = y_t^T + q(y_t^T, d_{t+1})d_{t+1} \\
& h_t \leq \bar{h}.
\end{aligned}
\tag{33}
$$

Where the first constraint is obtained by setting $I_t = 1$ in Equation (19), and the second is the constraint on working hour.

If the country is in bad standing, the consumption on tradable goods experiences a loss. The government has probability θ of regaining access to international financial markets, and probability $1-\theta$ of continuing in bad standing. During the period in bad standing, the country obtains no international borrowing, hence, the state variable for debt is excluded. Formally,

$$
\begin{aligned}
v^b(y_t^T) = \max_{\{h_t\}} \quad & \left\{U\left(A\left(y_t^T - L(y_t^T), F(h_t)\right)\right) + \beta E_t\left[\theta v^g\left(y_{t+1}^T, 0\right) + (1-\theta)v^b\left(y_{t+1}^T\right)\right]\right\} \\
\text{s.t} \quad & h_t \leq \bar{h}.
\end{aligned}
\tag{34}
$$

The tradable consumption $c_t^T = y_t^T - L(y_t^T)$ again follows Equation (19) by setting $I_t = 0$, and is substituted explicitly into the value function.

If the country is in good standing, the government has the freedom to choose which is best for the country: to continue or to default. The decision is made by comparing the value functions of the two scenarios, given the current output shock for tradable goods and the current level of debt

$$
v^g(y_t^T, d_t) = \max\left\{v^c(y_t^T, d_t), v^b(y_t^T)\right\}. \tag{35}
$$

Define the default set $D(d_t)$ as the set of tradable-output levels y_t^T examined by the government in period t, in which the government's optimal respond is to default. Formally,

$$D(d_t) = \left\{ y_t^T : v^b(y_t^T) > v^c(y_t^T, d_t) \right\}. \tag{36}$$

In other words, given a current debt level d_t, if the government observes that y_t^T is inside $D(d_t)$, it chooses to default.

Under rational expectations, the foreign lenders recognize the default set, hence the price for debt is determined by Equation (14), given by

$$q(y_t^T, d_{t+1}) = \frac{1 - \Pr\left\{ y_{t+1}^T \in D(d_{t+1}) \mid y_t^T \right\}}{1 + r^*}. \tag{37}$$

Note that the price of debt enters the value function of continuing, $v^c(y_t^T, d_t)$.

It is obvious that the optimal labor supply is $h_t = \bar{h}$ since all functions, F, A, U, are monotonic, which implies that under the freedom to choose the devaluation rate and the tax on debt, the government can ensure full employment. Denote $w^f(c_t^T)$ the equilibrium wage function under full employment given the consumption of tradable goods. Combining Equation (9) and the Euler equation in (6a) and impose the optimal policy $h_t = \bar{h}$, we have

$$w_t = w^f(c_t^T) \equiv \frac{A_2(c_t^T, F(\bar{h}))}{A_1(c_t^T, F(\bar{h}))} F'(\bar{h}). \tag{38}$$

Knowing that the wage has downward nominal rigidity, the government sets the devaluation rate accordingly. The downward rigidity (11) states that

$$\gamma \leq \frac{W_t}{W_{t-1}} = \frac{w_t}{w_{t-1}} \frac{P_t^T}{P_{t-1}^T} = \epsilon \frac{w_t}{w_{t-1}},$$

where the second equal sign comes from Equation (21). Substitute the wage under full employment, we get

$$\epsilon_t \geq \gamma \frac{w_{t-1}}{w^f(c_t^T)}. \tag{39}$$

This is the family of optimal devaluation policies. Following Na et al. (2018) and Hinrichsen (2021), we assume that the government chooses the minimal devaluation target that stabilizes nominal wages, that is, $\epsilon_t = \gamma \frac{w_{t-1}}{w^f(c_t^T)}$.

Chapter 5: Empirical Results

Typically, a model under the Eaton-Gersovitz framework does not have an analytical solution. Therefore, the optimal default set defined by Equation (36), as well as the value functions and the policy functions, must be obtained numerically via the technique of value function iteration. This requires the assignment of functional forms as well as structural parameters that matches the economy. I follow the functional forms and the calibration approach introduced in Na et al. (2018) and Hinrichsen (2021).

5.1 Calibration

5.1.1 Functional Forms

Following Na et al. (2018), the time unit is assumed to be one quarter, and the periodic utility function is assumed to be the constant relative risk aversion (CRRA) type

$$U(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma}, \tag{40}$$

where σ is the inverse of elasticity of intertemporal substitution of the consumption. The aggregator function for tradable and non-tradable consumption takes the constant elasticity of substitution (CES) form

$$c_t = A(c_t^T, c_t^N) = \left[a \left(c_t^T\right)^{1-\frac{1}{\xi}} + (1-a) \left(c_t^N\right)^{1-\frac{1}{\xi}} \right]^{\frac{1}{1-\frac{1}{\xi}}}. \tag{41}$$

The CES aggregator states that the share of tradable consumption is $a \in [0, 1]$, and the elasticity of substitution between the tradable and non-tradable consumption is ξ. Moreover, following the literature, to make the consumption of tradable goods c_t^T and the external debt d_t independent of the outputs in the nontradable sector in the equilibrium, assume that the inter- and intratemporal elasticity of substitution is equivalent (See Uribe and Schmitt-Grohé, 2017, Chapter 9.5). That is,

$$\xi = \frac{1}{\sigma}. \tag{42}$$

The production technology for the nontradable goods follows a simple form

$$y_t^N = F(h_t) = h_t^{\alpha}. \tag{43}$$

The loss-function in Equation (17) is positive and increasing with y_t^T, and following Chatterjee and Eyigungor (2012), I adopt the quadratic form with two parameters

$$L(y_t^T) = \max\left\{0, \delta_1 y_t^T + \delta_2 \left(y_t^T\right)^2\right\}. \tag{44}$$

This is also adopted in Na et al. (2018). In this setting, if we set $\delta_1 < 0$ and $\delta_2 > 0$, the output-loss increases as y_t^T increases, indicating that the more a country is endowed, the more it loses during default.

5.1.2 Calibration of Sri Lanka

The model is calibrated to Sri Lanka before 2008, when the Chinese government started to provide the increasing amount of loans. As described in Chapter 3, China started to provide loans to Sri Lanka in 2005, and during 2006 to 2008, the debt amount to China remains to be around \$1 billion, which is roughly 2.9% of GDP. Starting from 2009, debt to China has increased to \$3 billion, and it reached \$7.5 billion in 2014, accounting for

9.5% of GDP (see Figure 3a).

I proxy the output process of Equation (16) by the annual detrended log-real-tradable-GDP of Sri Lanka from 1980 to 2021. I follow Na et al. (2018) and calculate real tradable GDP by summing the real GDP in agriculture, forestry, fishing and industry[1]. The cyclical component of the output is obtained by filtering the time series with an HP-filter with smoothing parameter λ set to 100. Estimation of the AR(1) on the cyclical component thus yields $\rho = 0.9114$ and $\sigma_u = 0.0180$, which yields an unconditional standard deviation of 4.37% for the output[2]. Figure 4 presents the decomposition of the log-real-GDP.

The global risk-free world interest rate r^* is set to match the U.S. 3-month treasury bill rate during 1990 to 2007[3], which is roughly 4% annually, or 1% for one quarter. This is in line with Chatterjee and Eyigungor (2012) and Na et al. (2018). The probability of reentry is difficult to assess due to the lack of data. As a result, following Chatterjee and Eyigungor (2012) and Hinrichsen (2021), I set the probability of reentry to 0.0385, which implies that the country will be in default on average for about 6.5 years.

The labor share is set as $\alpha = 0.65$ based on the calibration in Jegajeevan (2016), which matches the estimation of labor share in Duma (2007). The share of tradable consumption is approximated by the share of tradable output in total output, as suggested in Uribe and Schmitt-Grohé (2017). Calculating the mean of tradable-to-GDP ratio over 1980 to 2021, I set the value as $a = 0.36$. The elasticity of substitution between tradable and nontradable goods ξ is set as 0.5 following Uribe and Schmitt-Grohé (2017), which is close to the cross-country estimation of 0.44 by Stockman and Tesar (1995). According to the assumption in

[1] Industry includes: mining, manufacturing, construction, electricity, water, and gas. In the original specification in Na et al. (2018), only mining and manufacturing is included. The case of Pakistan, however, suggested that energy related sector is a crucial component traded with China. Therefore a broader "industry" sector is included in my thesis.

[2] Since the AR(1) estimation is conducted on annual data, the estimated coefficients must be quarterized. Specifically, $\rho = 1 - \frac{1-\hat{\rho}}{4}$, and $\sigma_u = \frac{\hat{\sigma}}{\sqrt{4}}$, where $\hat{\rho}$ and $\hat{\sigma}$ are the estimated parameters for the AR(1) via OLS. The unconditional standard deviation is evaluated by $\frac{\sigma_u}{\sqrt{1-\rho^2}}$

[3] Quarterly averaged data retrieved from FRED. The span of 1990 to 2007 is set to match the periods of the Great Moderation, when the business cycle fluctuation is reduced (Hakkio, 2013). The average of 3-month treasury bill rate over 1990 to 2007 is 4.10%.

Equation (42), $\sigma = 1/\xi = 2$. The calibration on the two parameters ξ and σ is inline with most real-business-cycles (Uribe and Schmitt-Grohé, 2017; Na et al., 2018). Nominal wage rigidity is set as $\gamma = 1.109$ based on the empirical estimation of downward wage rigidity in 2014 by Matschke and Nie (2022)[4].

Following Na et al. (2018), the rest of the parameters $(\beta, \delta_1, \delta_2)$, which is respectively the subjective discount factor and the two parameters for the loss function, is chosen to match three equilibrium outcomes[5]: (i) the average debt-to-tradable-GDP ratio in periods of good standing is 175% per quarter; (ii) the frequency of default is 2.6 times per century; and (iii) the average output loss is 7% per year conditional on being in financial autarky. The following justifies this choice of targets.

(i) The average debt-to-tradable-GDP ratio to be targeted is motivated by the fact that the average annual debt-to-tradable-GDP ratio of Sri Lanka in the data is about 118%. The value is calculated by averaging the nominal external-debt-to-GDP ratio over 2001 to 2008[6]. Multiplying this by an average of 37% haircut[7] implies that about 44% of the debt is unsecured annually[8]. Since we are dealing with a model with quarterly period, this results in the 175% debt-to-tradable-GDP ratio targeted during calibration.

(ii) Sri Lanka suspended all debt payment in April 2022, but else there is no other default episode within the periods of calibration. Note that Sri Lanka interacted with Paris Club in 2005 for a moratorium due to the tsunami of December 2004, but this event

[4] Matschke and Nie (2022) estimates downward wage rigidity by measuring the annualized average gross real wage growth during an unemployment cycle for each country.

[5] In particular, I use the surrogate optimization solver in Matlab to search for the optimal values for the three parameters. Essentially, VFI must be proceeded for each triplet of the parameters to obtain the targeting equilibrium outcomes. Details are mentioned in section 5.2.

[6] Data source: International Debt Statistics. The period of year is chosen to be 8 years before China's increasing support of loans. The time span of 8 years is inline with that in Uribe and Schmitt-Grohé (2017).

[7] This is the average sovereign haircut between 1970 and 2010 (Cruces and Trebesch, 2013). The haircut data for the current Sri Lanka sovereign default is not available since it is still under restructuring.

[8] In the model, we assume that the country defaults on 100% of the debt, hence this approach is necessary to handle the case of a haircut.

is not counted as a default according to global rating systems such as S&P Global (Kraemer et al., 2022). Due to this uncertainty of determining a default episode, I set the default frequency to be 2.6 as the benchmark target, which is the frequency of Argentina following Na et al. (2018). Attempts on counting default episodes based on a different definition will be conducted as a robustness check in later section.

(iii) Na et al. (2018) adopts a growth accounting approach proposed by Zarazaga (2012) to calculate the output loss associated with the default. Applying this method to Sri Lanka, however, suggests that the output increases along with default, indicating that the method is not suitable in this case. See Appendix A for more detail. I therefore follow Na et al. (2018) and set the output loss to be 7%, matching the case of Argentina.

Table 2 summarizes the calibrated parameters and their sources, and Table 5 summarizes the fitness.

5.1.3 Calibration of Pakistan

The calibration strategy for Pakistan is similar to that of Sri Lanka. The parameters for the output process is obtained from the cyclical component of the HP-filter on the annual log-real-GDP for Pakistan from 1980 to 2021, which yields $\rho = 0.8518$ and $\sigma_u = 0.0116$ (see Figure 4). The risk-free interest rate remains to be 4% annually, hence r =1%. Pakistan defaulted on January 1999, completed its debt restructuring on December 1999 (Kraemer et al., 2022), but gained partial reaccess (flows > 0) in 2004, and full reaccess (flow > 1% of GDP) in 2006 (Trebesch, 2011, Table 5.6). The model adopted in my thesis does not distinguish between partial or full reaccess, hence the reentry period is set as the first year Pakistan gain positive flow of debt. Accordingly, the reentry period is set to 6 years (24 quarters), and $\theta = 0.0417$.

The labor share is set as 0.4 to match the capital share in real GDP, following Rehman et al. (2020). The share of tradable consumption is calibrated according to the tradable-to-GDP ratio over 1980 to 2021, which gives $a = 0.33$. The intratemporal elasticity of substitution of consumption $\xi = 0.5$ and $\sigma = 2$, following the same justification for Sri Lanka (Rehman et al., 2020; Uribe and Schmitt-Grohé, 2017). Nominal wage rigidity is set as $\gamma = 1.048$ based on the empirical estimation of downward wage rigidity in 2014 by Matschke and Nie (2022).

Finally, the triplet $(\beta, \delta_1, \delta_2)$ is also chosen to match the three equilibrium results: (i) the average debt-to-traded-GDP ratio in periods of good standing is 44% per quarter; (ii) the frequency of default is 2.6 times per century; and (iii) the average output loss is 7% per year conditional on being in financial autarky.

(i) The average debt-to-GDP-ratio between 2006 and 2013[9] is 30% according to the International Debt Statistics. Multiplying it with an average 37% haircut ratio[10] estimated in Cruces and Trebesch (2013) yields an 11.1% of annual unsecured debt, which gives an 44.4% unsecured debt-to-GDP ratio quarterly.

(ii) Default record according to Kraemer et al. (2022) and Uribe and Schmitt-Grohé (2017) shows that in Pakistan there is only one default episode in 1999. However, Pakistan sought debt relief from Paris Club several times during the 1970s and 1980s (Kundi, 2016), indicating that if we only consider the event of 1999, we might underestimate the default frequency[11]. Following the same logic in Sri Lanka, since determining a default episode is not straightforward in this case, I set the default frequency to be 2.6 times per century as the benchmark target, following Na et al. (2018). A robustness check with a different approach of determining default episodes will be provided in later section.

[9] Following the same logic as with Sri Lanka, I choose an 8-years window before the increasing amount of loans from China in 2013.

[10] Cruces and Trebesch (2013) estimates that the haircut ratio for the 1999 default of Pakistan is about 15%. This yields an 18% unsecured debt-to-GDP ratio to be targeted, which is too low for the model. I therefore adopt the average haircut ratio instead.

[11] The Government of Pakistan (2008) reports a comprehensive list of debt restructuring events in Pakistan. In May 1972, Pakistan interacted with the Paris Club for a short-term debt relief of \$234 million following the separation of East Pakistan. In June 1974, an arrangement with member countries of Aid-to-Pakistan

(iii) The capital-output ratio rose from 1.36 to 1.4 after the default episode, which suggests that the approach of Zarazaga (2012) is also not suitable in this case. See Appendix A for more details[12]. As a result, I again follow Na et al. (2018) and set the target average output loss to be 7%.

Table 3 summarizes the calibrated parameters for Pakistan, and Table 5 summarizes the fitness.

5.2 Numerical Computation

The approximated equilibrium is obtained by conducting value function iteration over an $n_y \times n_d$ discretized and equally spaced state space, where $n_y = 200$ is the number of grids for the output process and $n_d = 200$ is the number of grids for the debt (Na et al., 2018). Denote $[\underline{y}^T, \overline{y}^T]$ as the lower and upper bound of output grid. Following Uribe and Schmitt-Grohé (2017), this is set as $[-4.2\sigma_u, 4.2\sigma_u]$. Also following the authors, since the average debt levels for both countries do not exceed 150%, the upper bound for debt is set as 1.5, therefore the debt range for the value function iteration is $[\underline{d}, \overline{d}] = [0, 1.5]$.

I follow Schmitt-Grohé and Uribe (2016) and discretize the AR(1) process for output by constructing a transition probability matrix over the grids of the output. A time series of 10 million observations was generated based on Equation (16). Each observation is then assigned to the nearest grid point among the 200 discrete values of $\ln y^T$. The discretized series is analyzed to calculate the probabilities of transitioning from one discrete state to another in consecutive periods. To obtain the transition probability matrix, a 200×200

Consortium agreed to provide another debt relief of $650 million. By 1981, political crisis worsen the debt service, hence a debt relief was granted by the Consortium. The nuclear test in 1998 by Pakistan led to international sanction and embargo, causing Pakistan to fail its debt service and had to seek debt restructuring with the Paris Club. The followed debt treatments (in 1999 and 2001) from the Paris Club were under the "Houston term", which differs from the previous debt relief in a sense that it granted a smaller concessional interest rate and a longer grace periods (Siddiqui et al., 2001).

[12] In the calculation, I first consider the period of partial reaccess to the credit market in 2004 as the end of autarky, following Trebesch (2011). The attempt to calculate output loss by defining the emergence of default as full reaccess to financial market in 2006 instead of partial reaccess in 2004 is examined, but it yields an output loss of 0.25%, which is too subtle compared to cross-country studies of output loss during default (Borensztein and Panizza, 2009).

matrix is initialized with zeros. For each pair of consecutive observations, the corresponding element in the matrix is incremented by 1. After considering all the observations, the matrix is normalized by dividing each row by the sum of its elements. This results in the estimated transition probability matrix, which effectively captures the covariance matrices of order 0 and 1 (Uribe and Schmitt-Grohé, 2017).

Value function iteration is then conducted for a given set of calibrated parameters, which is composed of the set of fixed parameters $(\rho, \sigma_u, r^*, \theta, \alpha, a, \xi, \gamma)$ and the tuple $(\beta, \delta_1, \delta_2)$. The default dynamic and moments can then be conducted once the value function converges. Following Uribe and Schmitt-Grohé (2017) and Na et al. (2018), a simulation of the model based on the policy function is conducted sequentially[13] for 1.1 million iterations, of which the first 0.1 million periods are discarded.

A critical procedure of the calibration is to select the tuple $(\beta, \delta_1, \delta_2)$ such that it matches the three targets described in section 5.1, which are the default frequency, debt-to-GDP ratio, and output loss. In particular, the tuple is chosen to minimize the Euclidean distance between the target value and the corresponding moments generated by the dynamic simulation[14]. Since the value function iteration and dynamic simulation are time-consuming processes, I use the surrogate optimization algorithm in MATLAB as the searching algorithm, as it is suitable for minimizing time-consuming objective functions.

5.3 Default Set

As described in Section 4.7 and Appendix B, the default decision is jointly decided by the current debt level and its output for tradable goods during the period. Under a certain

[13] The main randomness comes from the AR(1) output. Once y_{t+1}^T is determined, along with determination of the debt level for next period d_{t+1}, we have the state (y_{t+1}^T, d_{t+1}) for $t+1$, which is a grid-point. Values for the rest of the endogenous variables such as c_{t+1}, I_{t+1} can then be obtained by extracting the corresponding value from the state grid-point on the policy function.

[14] Default frequency is evaluated by taking the mean of $\{I_t\}_{t \in T}$ and multiply it by 100×4 quarters per century. Debt-to-GDP ratio is calculated by taking the mean of $\{d_t / y_t^T\}_{t \in \text{Good T}}$, which is the average debt-to-tradable-output ratio condition on being in good standing. Output loss is evaluated by taking the mean of $\{(y_t^T - \tilde{y}_t^T)/y_t^T\}_{t \in \text{Bad T}}$, which is the average output difference between the endowment and the actual received output, divided by the endowment, over periods of bad standings.

debt stock d_t, the less the tradable output y_t^T it is endowed, the lower the benefit a country receives when it chooses to repay its debt. Ultimately, if the output falls below a certain threshold value, it becomes optimal for the country to default. Hinrichsen (2021) compares the theoretical scenarios of defaulting with the actual data with a graphical approach. He maps the default set evaluated via value function iteration into the state space (y_t^T, d_t), with the output as the horizontal axis and the debt level as the vertical axis. A grid on the space is colored in gray if the corresponding debt-output pair is *not* in the default set. Figure 7 demonstrates the default set for Sri Lanka and Pakistan. The debt level in the default set plot differs between the two countries due to the difference in the calibration and the targeted equilibrium moments. As shown in the figure and proven in Appendix B, the upper limit of a default set (the intersection of the white and gray areas given a certain debt level) increases as the debt level increases, indicating that it requires a higher endowment of tradable output in order to incentivize the country to continue to fulfill its obligations to repay[15].

The actual data is then plotted on the default set for comparison. The output level corresponds to the cyclical component of the per capita tradable-GDP obtained by the HP-filter for the respective year. The corresponding debt level is calculated by multiplying the debt-to-nominal-tradable-GDP ratio by the haircut ratio and 4 quarters. This calculation is necessary because the targeted debt level during calibration specifically refers to the quarterly unsecured portion of the debt, and thus the data should undergo the same calculation to accurately represent the unsecured debt component.

Furthermore, the debt data used in this analysis consists of two components: the debt stock excluding China obtained from the International Debt Statistics (IDS), and the debt stock specifically for China sourced from the database created by Horn, Reinhart and

[15] In the default set depicted in Hinrichsen (2021), there is a discontinuity in the upper bound of the default set. This indicates that when the output level drops to a certain threshold, the country chooses to default, regardless of the current debt level. This is contradicted to the analytical conclusion proved in Proposition 3 in Appendix B. During a private correspondence, the author acknowledged this mistake in his doctoral thesis, but he has already corrected it in his latest book.

Trebesch (2021). It is worth noting that Chinese government loans channeled through state-owned enterprises are typically not reported in the World Bank Debt Reporting System (DRS), which is the basis for IDS. This omission leads to an underestimation of the actual debt burden faced by the country. The additional loans provided by state-owned commercial banks, referred to as "hidden debts" by Horn, Reinhart and Trebesch (2021), are included in their database. Consequently, in order to capture the total debt burden accurately, the total debt stock reported in IDS is adjusted by excluding the debt to China from IDS and adding the debt to China obtained from the database by Horn, Reinhart and Trebesch (2021).

5.3.1 Default Decision for Sri Lanka

Figure 8a plots the debt-output pair for Sri Lanka from 2007 to 2017. Each point represents the actual data pair for the year, adjusted by the haircut ratio. Recall that China initiated its constructions in infrastructures such as the Hambantota International Port in 2007, and Mattala Rajapaksa International Airport in 2009. The selection of this time span allows us to examine the change in debt burden for Sri Lanka comprehensively in a chronological way.

As shown in the figure, at the initial phase of China's investment in 2007 and 2008, Sri Lanka is still under its safe zone of not defaulting, despite that it experienced a little drop in the output. Sri Lanka first landed into the default set during 2009, as debt to China reached \$3 billion. During 2010 and 2011, Sri Lanka ended its 25-years civil war, and experienced an unprecedented output growth. Agriculture sector in 2010 grew for 7%, and industry sector grew for 8%. This rapid increasing in GDP largely lowered the debt-to-tradable-GDP ratio, both in 2010 and 2011, and hence Sri Lanka was able to avoid the risk of default and stayed under the non-default set. However, as GDP growth in tradable good slowed down to a rate of 5%, the debt to China increased by 6%, which slowly pushed Sri Lanka to the brink of default. Eventually, in 2013, as total debt reached \$41 billion, in which China accounted for 13%, Sri Lanka was again entering the default set. The status continued until 2016, with 2014 being an ambiguous exception.

In this sense, debt to China indeed boosted Sri Lanka into the predicament of debt unsustainability, or in other words, pushed Sri Lanka into a debt trap. It is worth mentioning that in 2015, as President Maithripala Sirisena defeated former President Mahinda Rajapaksa in the election, he promised to extricate Sri Lanka from the debt burden from China. However, according to Figure 8a, Sri Lanka was too indebted during 2015 and 2016, and it would be difficult for the new government to take actions besides undergoing debt restructuring. Eventually, in 2017, the infamous event of Sri Lanka leasing the Hambantota International Port for 99 years and selling 70% of the stake to China Merchants Port stroke the headline. This action, however, did not write off the loans for the construction of the infrastructures, as it is used as a raise in money to repay debt to other official creditors (Brautigam, 2020; Moramudali, 2019). Regardless of the narrative of whether the lease is involuntary or not, along with the outstanding output level in 2017, Sri Lanka was back to the non-default set after the lease. Observe that all the cyclical components of the tradable goods (the x coordinate) are above 1 after 2011. This indicates that Sri Lanka is under the default set even when the output is mostly above the steady state level of tradable outputs.

An intriguing question arises: What if the debt to China is not included, holding other conditions fixed? Hinrichsen (2021) compares the debt for countries in war reparations with and without indemnities, and here I adopt the same approach. Figure 8b shows the comparison of the data. In the figure, the solid black dots represent data points including debts owed to China, while the gray dots represent data points excluding debts from China. It is obvious that, *ceteris paribus*, all years spanning from 2009 to 2017 are not under the default set. This is, indeed, an inexact comparison. First, the debt level given in the model is endogenous, and we do not distinguish between China and other lenders. It is possible that Sri Lanka sought for other creditors for loans if the infrastructures were planned voluntarily, as mentioned in Brautigam (2020). In this case the debt level in the absence of China can be underestimated. Second, the impact on GDP would be uncertain if China had not been involved in the construction of the infrastructure projects. For instance, Bandiera and Tsiropoulos (2020) estimates from a long-term growth model that in 2020, the additional growth from investment on the Belt and Road Initiative (BRI) is about 0.08%. Estimations of the extra growth due to investments during 2009 to 2017 are crucial in order to examine the counterfactual debt burden in the absence of China's intervention. If investments in China indeed contributed to the GDP growth during the periods, the debt-to-tradable-GDP ratio excluding China is also underestimated.

5.3.2 Default Decision for Pakistan

Figure 9a illustrates the debt-output pair for Pakistan from 2013 to 2017. The starting period is chosen to be two years before the massive amount of projects on the Chinese infrastructure networks was launched for the China-Pakistan Economic Corridor (CPEC) in 2015.

Pakistan is not under the default set during 2013, and is relatively safe from being in the default status, as even an 8% contraction in the output (if y_t^T = 0.92) is not making Pakistan willing to default. Debt level started to increase intensively, and up to 2015, debt to China had reach about $15 billion, surpassing the amount of debt to World Bank, which in that time was about $13 billion. Since then, China has become the largest creditor to Pakistan (see Figure 3b). As a result, debt-to-tradable GDP increased, and both data points in 2014 and 2015 located above 2013 in Figure 6b. In 2016, 6 additional major loans on infrastructure constructions mainly on the energy sector was launched, which summed up to $8.6 billion, and hence in 2016 Pakistan was around the brink of default. Details about the projects are described in Section 3.2. Eventually in 2017, with 3 more major loans in power project initialized, the total debt stock is up to $108 billion, and compared with the nominal tradable GDP of $136 billion, the debt-to-tradable-GDP is 80%, which corresponds to about 120% unsecured debt per quarter. This pushed Pakistan into the default set, and made Pakistan strangled in the debt trap.

Similar to Sri Lanka, the following attempts to illustrate the economy of Pakistan excluding China. Figure 9b illustrates the debt-output relationship when excluding China's interventions. Throughout the entire time span, the debt-to-tradable-GDP ratio remains well below the default threshold. Notably, in 2017, the ratio closely resembles that of 2013. However, it is important to reiterate that this result does not consider the potential increase in debt owed to other creditors or the benefits derived from investments related to the BRI. Pakistan has been suffering from energy shortage since the 1990s. During 2010 to 2013, the former Pakistan Prime Minister Yousuf Raza Gilani launched an "energy policy" that aimed to resolve the power crisis, with measures including banning neon signs, closing street markets early, and introducing of 13 independent power producers (IPP)[16]. While the construction of large-scale power plants, including nuclear plants, could have directly increased energy supply, the Pakistani government primarily focused on energy conservation policies. Financial restrictions may have been the main concern for the government that prevented them from initializing large constructions, especially after the tragic natural disasters such as the 2010 floods in Pakistan[17]. Hence, it is reasonable to argue that without the loans and projects initiated by China, Pakistan would not have undertaken these energy projects, and its debt would not have accumulated dramatically from other official creditors.

Another aspect of uncertainty when assessing the counterfactual debt-to-tradable-GDP ratio is the impact on GDP following China's investment in infrastructure. According to estimates from Bandiera and Tsiropoulos (2020), using a long-term growth model, additional investment from the BRI accounted for 1.93% of GDP, resulting in a 0.41% increase in GDP compared to the baseline model in 2020. This effect is five times larger than that

[16] BBC "Pakistan's PM announces energy policy to tackle crisis," April 22, 2010

[17] Asian Development Bank and World Bank jointly estimated the total cost of the damage at approximately \$10.1 billion, and the reconstruction costs at least \$6.8 billion (Asian Development Bank and World Bank, 2011).

observed in Sri Lanka. Although estimations for the period between 2013 and 2017 have not been conducted, a preliminary observation can be made from Figure 6b. The nominal tradable GDP demonstrates increasing growth after 2013, despite the total debt level increases at a faster pace. Since this observation is not sufficient for causal inference, further examination of the infrastructure's contribution to GDP is a topic worthy of future research.

5.4 Robustness Check

The value function iteration of the model depends on the persistency and volatility of the AR(1) process for the tradable output, namely (ρ, σ_u). The parameters are obtained with the cyclical component after conducting the HP-filter. In addition, the x-coordinate of the data points plotted on the default set is also obtained with the cyclical component, representing the deviation from the trend. One may concern that the method of detrending might yield a different conclusion. In Na et al. (2018) and Hinrichsen (2021), the output process is detrended using the log-quadratic filter[18]. Generally, the volatility for the cyclical component obtained with log-quadratic filter is higher than that of HP-filter (Uribe and Schmitt-Grohé, 2017).

Figure 5 illustrates the decomposition of the (per capita) tradable output using the log-quadratic filter. It yields $(\rho, \sigma_u) =$ (0.9325, 0.0266) for Sri Lanka and $(\rho, \sigma_u) =$ (0.9239, 0.0174) for Pakistan. Compared to the unconditional standard deviation of 4.37% for Sri Lanka's per capita tradable output using the HP-filter, the log-quadratic filter yields 7.38%; similarly, the unconditional standard deviation for Pakistan's per capita tradable output using the HP-filter is 2.21%, while the log-quadratic filter yields 4.55%. See Table 4 for the comparison.

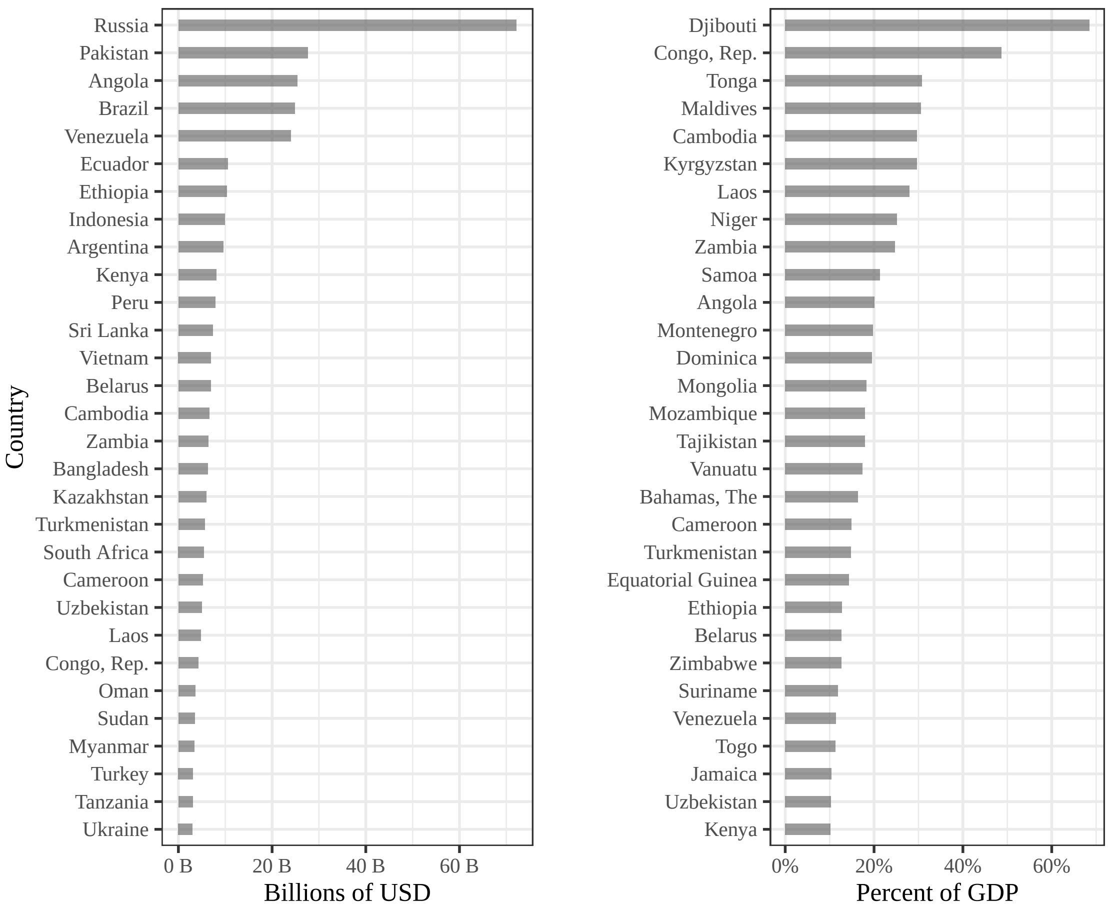
The default set and the corresponding debt-output pairs when the output process is

[18] In specific, assume that the real GDP can be expressed by the cyclical component and trend (secular) component $y_t = y_t^s + y_t^c$. The components are then estimated by running the OLS: $y_t = a + bt + ct^2 + \epsilon_t$, then set $y_t^c = \epsilon_t$ and $y_t^s = a + bt + ct^2$ (Uribe and Schmitt-Grohé, 2017).

obtained via log-quadratic filter is shown in Figure 10. Interestingly, the number of years when Sri Lanka was within the default set reduces, as depicted in Figure 10a. In contrast to the results obtained with the HP-filter, the year 2009 is now considered safe, albeit on the verge of default. The remaining two years within the default set are 2015 and 2016, coinciding with the period when the newly elected president expressed his determination to alleviate Sri Lanka's burden of debt to China. These findings regarding the predicament faced by the new government remains robust when using the log-quadratic filter. As for the case of Pakistan, the year 2017 is no longer under the default set, contrary to the result obtained with the HP-filter. Nevertheless, the trend of the debt-to-tradable-ratio kept increasing, and during the year 2017, the output deviated towards the positive direction ($y_t^T > 1$). If the output shock were to contract just by a mere 1% (at $y_t^T = 0.99$), Pakistan would be falling into the default set.

Chapter 6: Conclusion and Discussion

Figure 1: Debt to China Statistic by Country in 2017

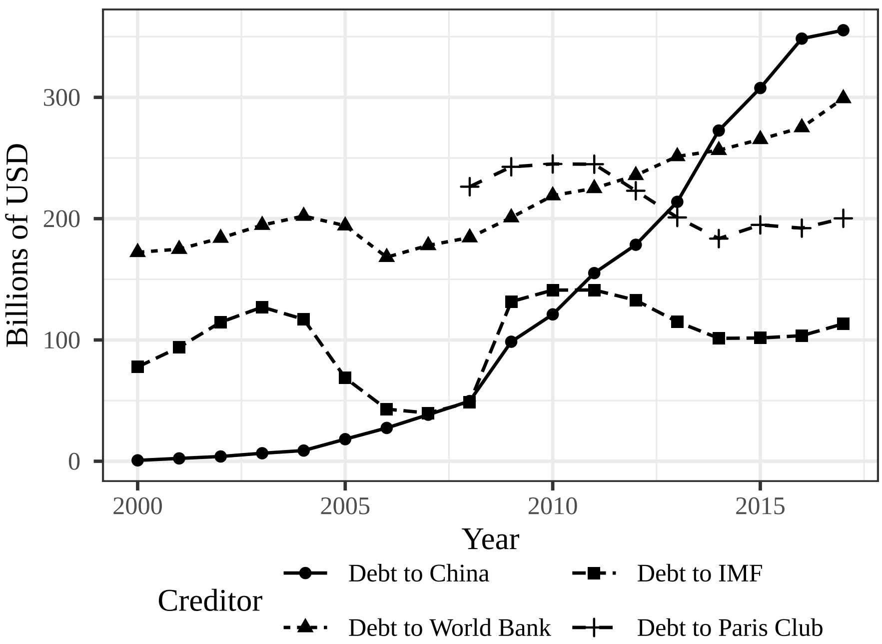


(a) Top 30 Debtor by Total Debt in USD

(b) Top 30 Debtor by Dept-to-GDP Ratio

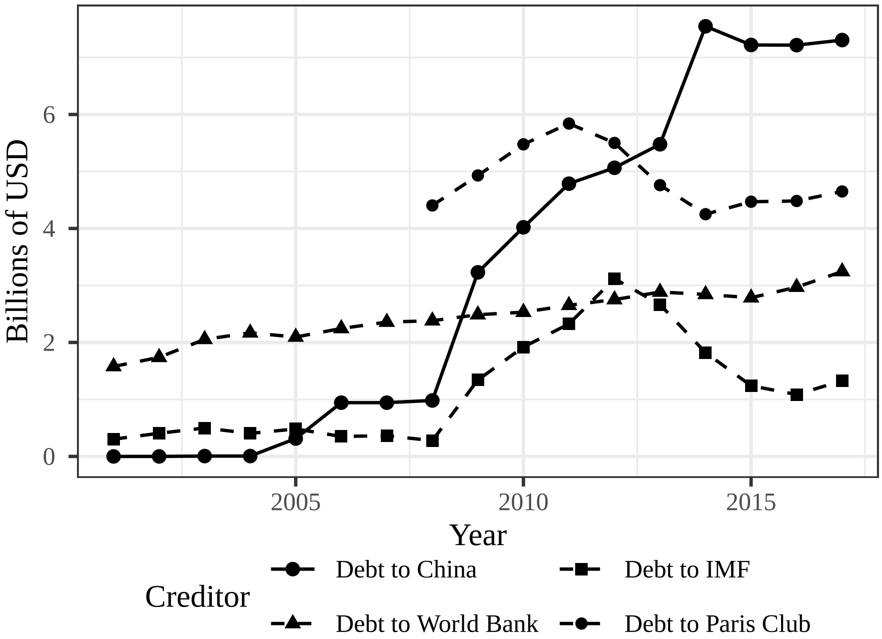
Source: Horn, Reinhart and Trebesch (2021) database
Note: The figure on the left presents the top 30 countries in amount of total external debt to China in 2017. The figure on the right compares by the China-debt-to-GDP ratio.

Figure 2: Change of Aggregate Public Debt for Different Official Creditors

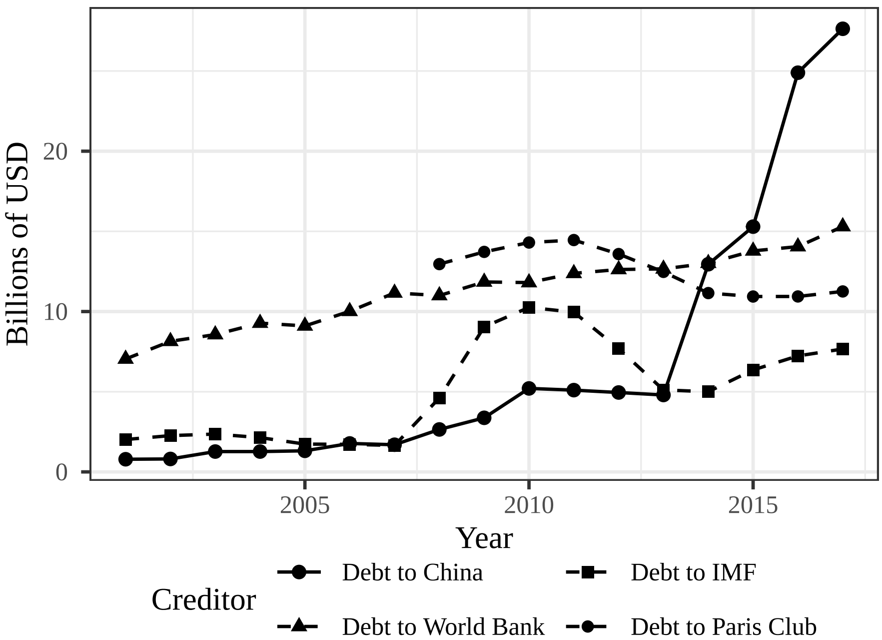


Source: Horn, Reinhart and Trebesch (2021) database
Note: The figure shows the change in the aggregate external public debt that the developing countries owed to different official creditors. These include China, World Bank (excluding China), IMF, and all 22 Paris Club governments. It is obvious that China had become the largest official creditors in the world according to the estimation of Horn, Reinhart and Trebesch (2021).

Figure 3: Debt to Main Creditors



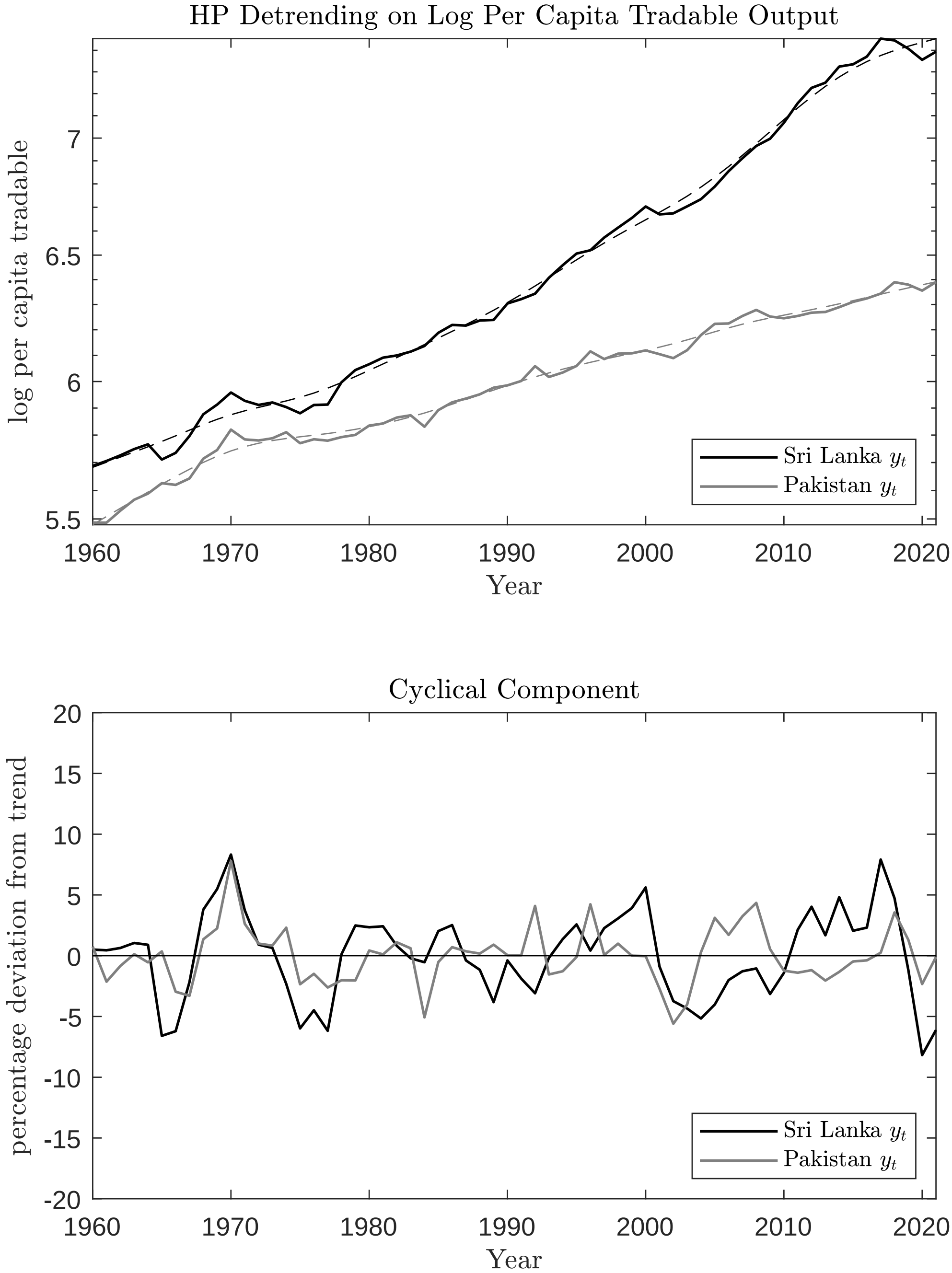
(a) Sri Lanka



(b) Pakistan

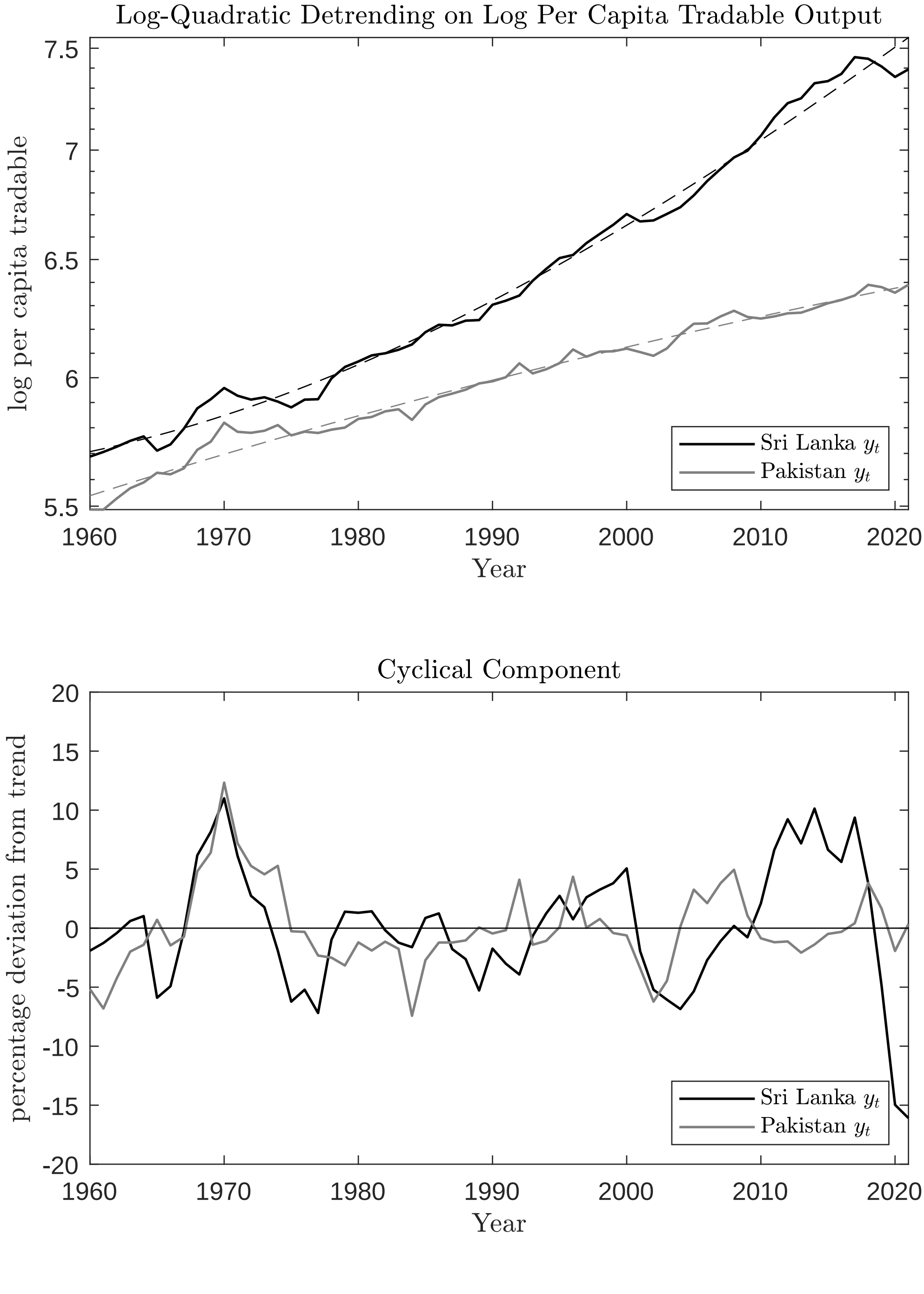
Source: Horn, Reinhart and Trebesch (2021) database
Note: The figure shows the change in the external public debt that Sri Lanka and Pakistan owed to different official creditors. These include China, World Bank (excluding China), IMF, and all 22 Paris Club governments.

Figure 4: Decomposition of log-real-tradable-GDP for Sri Lanka and Pakistan Using HP-Filter



Source: World Bank national accounts data.
Note: The cyclical component in the right is obtained by the HP-filter with smoothing parameter $\lambda = 100$ for the log-real-GDP in the left. The quarterized AR(1) estimation for Sri Lanka yields $(\rho, \sigma_u) = (0.9114, 0.0180)$, and for Pakistan yields $(\rho, \sigma_u) = (0.8518, 0.0116)$

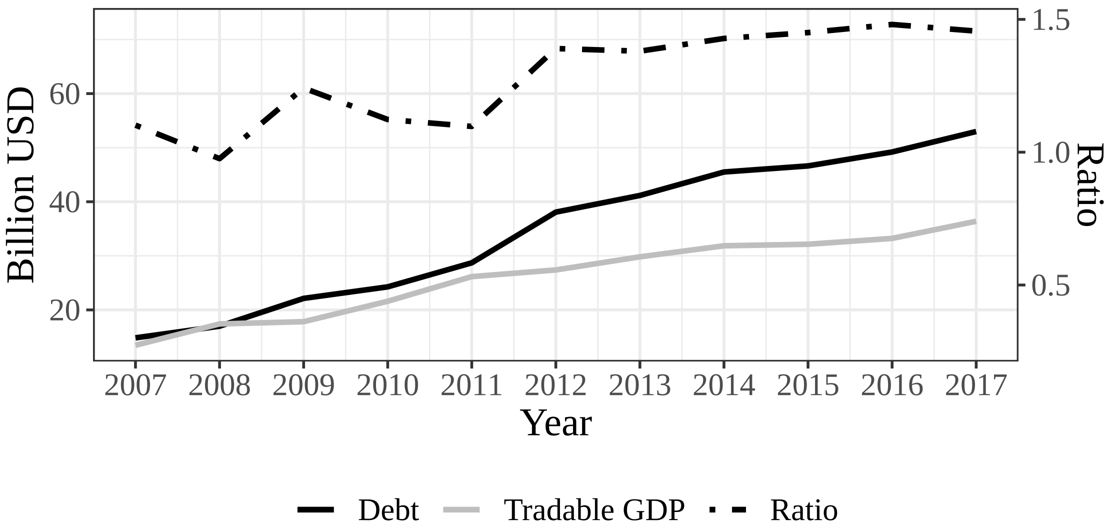
Figure 5: Decomposition of log-real-tradable-GDP for Sri Lanka and Pakistan Using Log-quadratic Filter



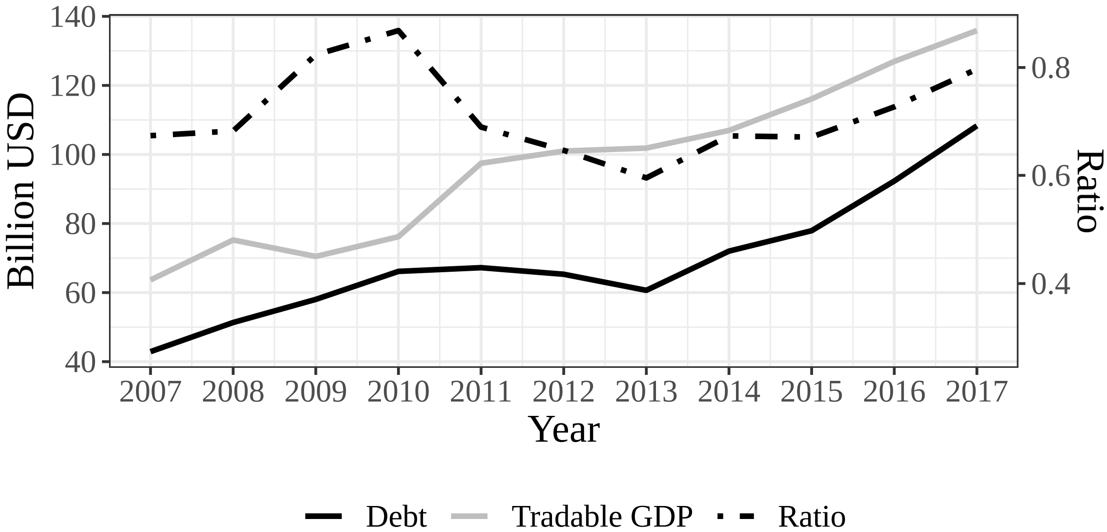
Source: World Bank national accounts data.

Note: The cyclical component in the right is obtained by the log-quadratic filter for the log-real-GDP in the left. The quarterized AR(1) estimation for Sri Lanka yields $(\rho, \sigma_u) = (0.9325, 0.0266)$, and for Pakistan yields $(\rho, \sigma_u) = (0.9239, 0.0174)$

Figure 6: Debt Stock, Tradable GDP, and the Ratio, 2007 – 2017



(a) Sri Lanka

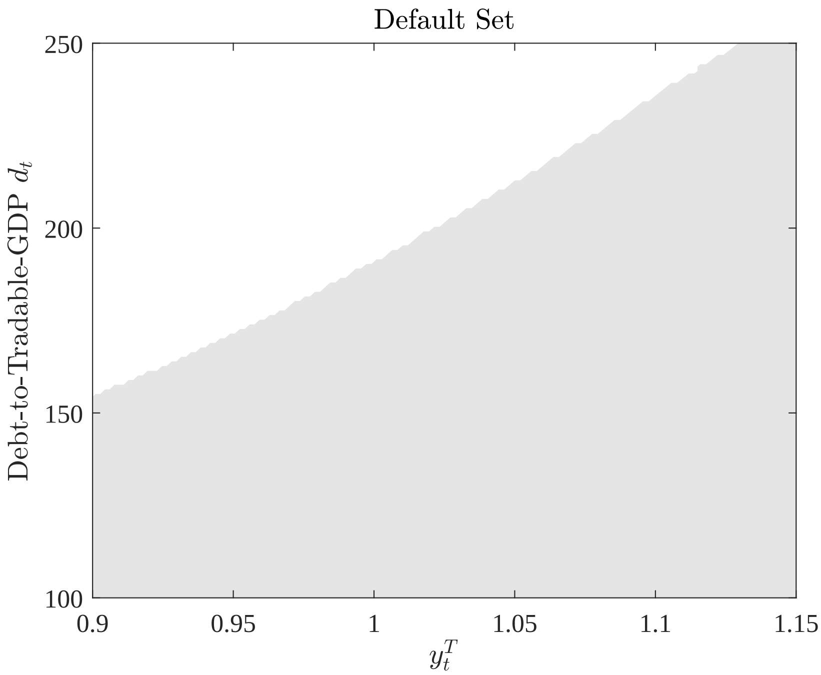


(b) Pakistan

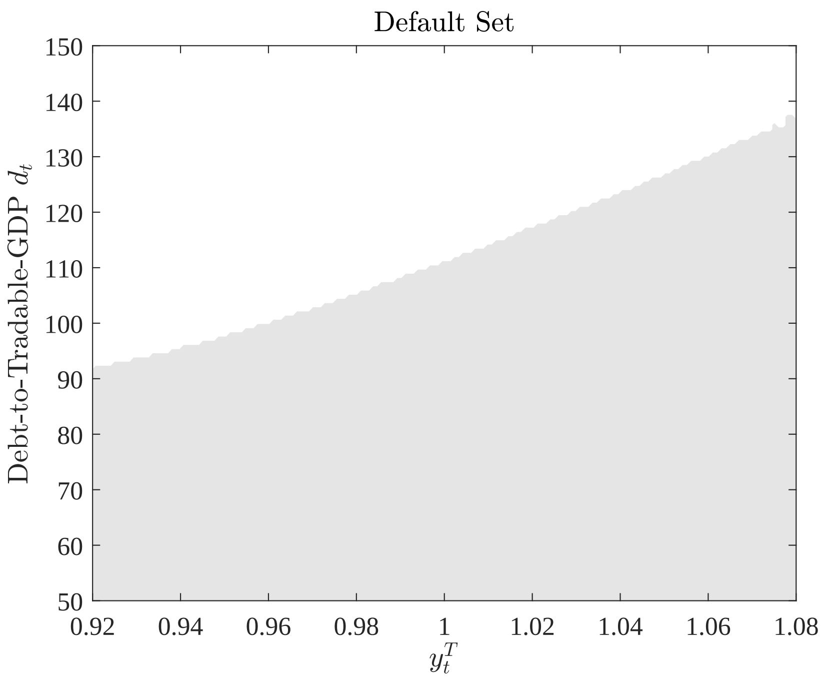
Source: Debt data from IDS Database and Horn, Reinhart and Trebesch (2021) Database. GDP data from World Bank.

Note: The figure shows the change of total debt level versus the nominal GDP for tradable goods, defined as the sum of agriculture, forestry, fishing, and industry. The left vertical axis represents the values of debts and nominal tradable-GDP in billion USD, depicted by solid lines. The right vertical axis corresponds to the ratio shown as a dashed black line, reflecting the ratio of debt-to-tradable-GDP.

Figure 7: Graphical Representation of Default Set



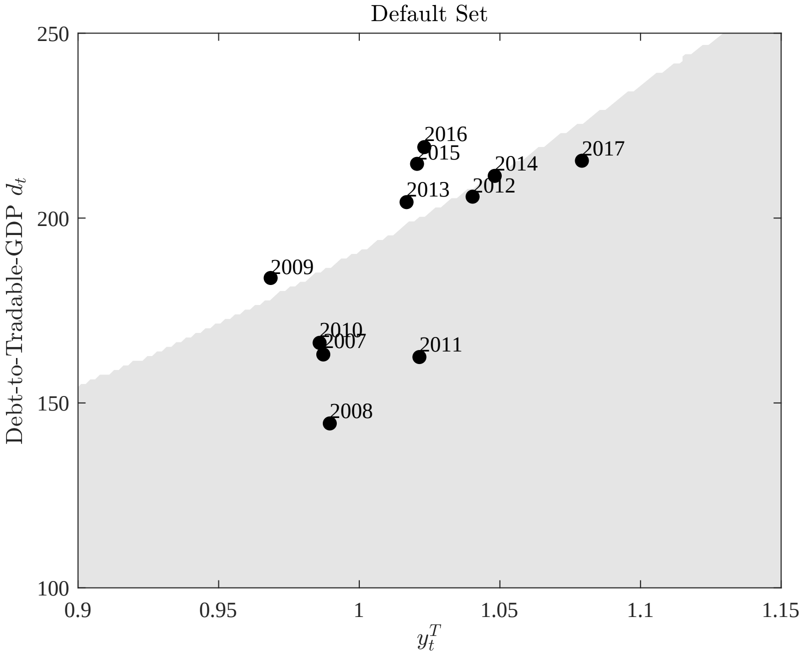
(a) Sri Lanka



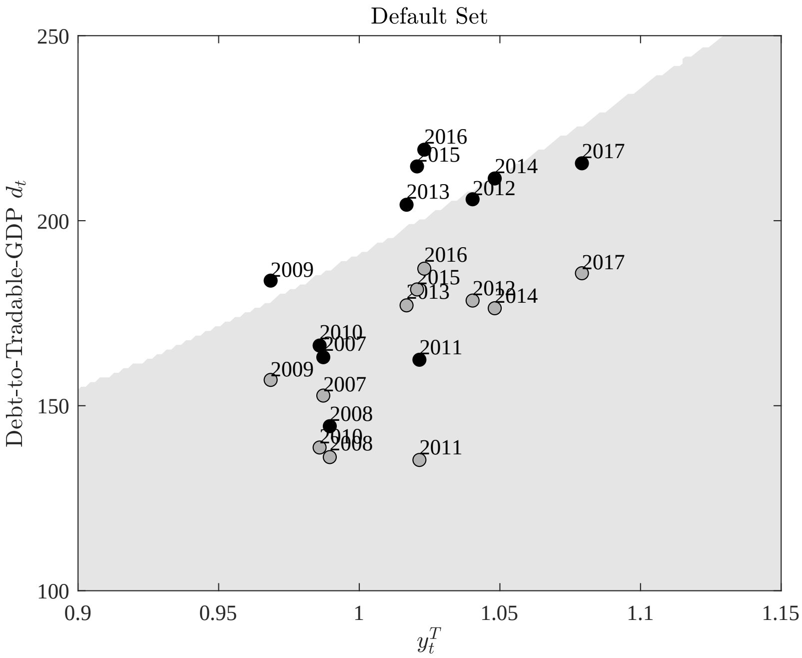
(b) Pakistan

Note: The default set derived via value function iteration is plotted on the state space. The horizontal axis represents the tradable output level, and the vertical axis represents the debt stock. The gray area represents the grid point where the country chooses not to default, and the white area is the grid point where default is the optimal action for the country.

Figure 8: Mapping Actual Data onto Analytical Model for Sri Lanka, 2007 – 2017



(a) Debt to All Creditors

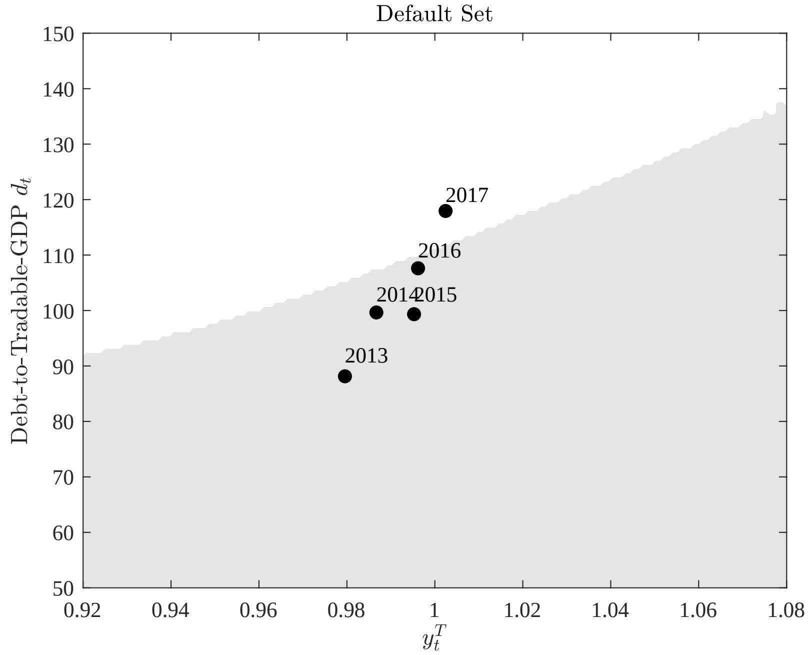


(b) Excluding Debt to China

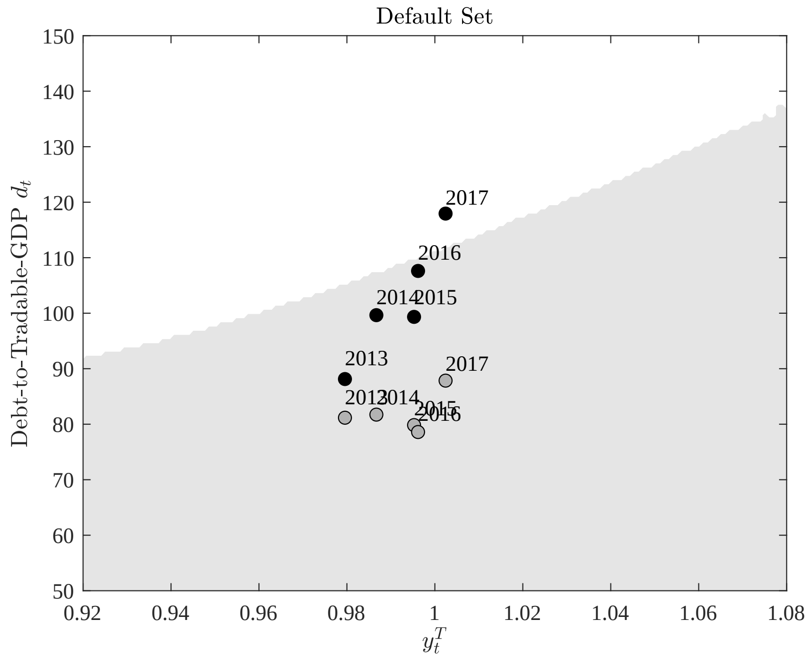
Source: Debt data from IDS Database and Horn, Reinhart and Trebesch (2021) Database. GDP data from World Bank

Note: The gray region represents the non-default set of Sri Lanka, and the white region represents the default set. Black dots in (a) and (b) represents the output-debt pair in the real data, and the gray dots in (b) represents the output-debt pair that excludes the debts from China. Debt here corresponds to the unsecured debt amount per quarter.

Figure 9: Mapping Actual Data onto Analytical Model for Pakistan, 2013 – 2017



(a) Debt to All Creditors

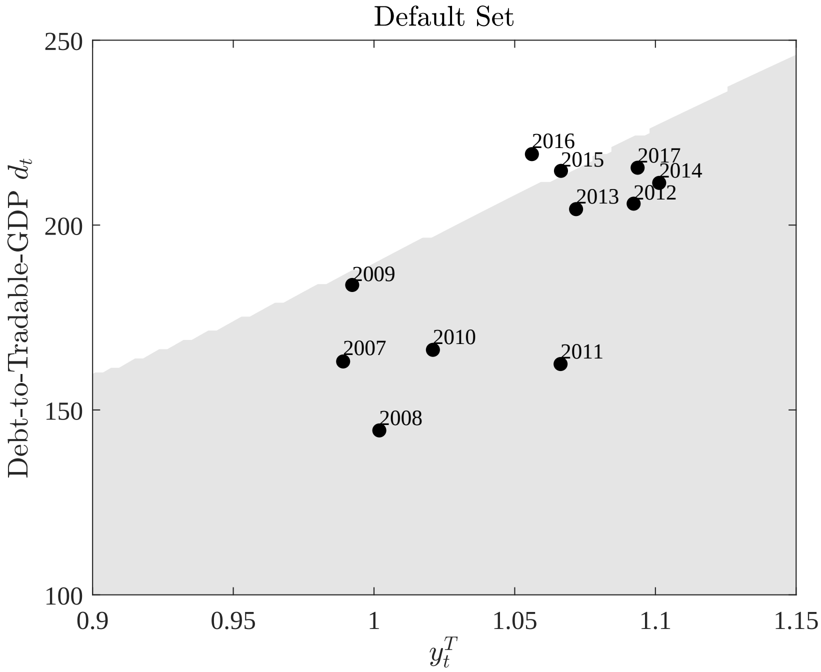


(b) Excluding Debt to China

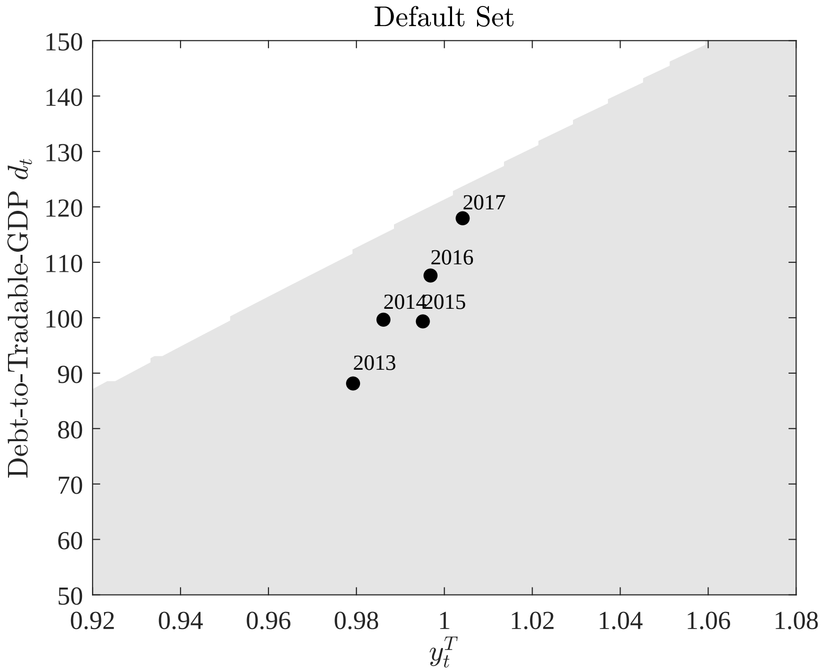
Source: Debt data from IDS Database and Horn, Reinhart and Trebesch (2021) Database. GDP data from World Bank

Note: The gray region represents the non-default set of Pakistan, and the white region represents the default set. Black dots in (a) and (b) represents the output-debt pair in the real data, and the gray dots in (b) represents the output-debt pair that excludes the debts from China. Debt here corresponds to the unsecured debt amount per quarter.

Figure 10: Default Set and Mapped Data Using Log-quadratic Filter



(a) Sri Lanka



(b) Pakistan

Source: Debt data from IDS Database and Horn, Reinhart and Trebesch (2021) Database. GDP data from World Bank
Note: The figures demonstrates the default set as well as the data points when the cyclical component of the output process is obtained via a log-quadratic filter. The gray region represents the non-default set of Pakistan, and the white region represents the default set. Black dots represent the output-debt pair in the real data.

Table 1: Data Source

Description	Source	Code/Method
Real GDP on Agriculture	World Bank	NV.AGR.TOTL.KD
Real GDP on Industry	World Bank	NV.IND.TOTL.KD
Real Tradable GDP		Sum of the above two data
Real GDP	World Bank	NY.GDP.MKTP.KD
Nominal GDP on Agriculture	World Bank	NV.AGR.TOTL.CD
Nominal GDP on Industry	World Bank	NV.IND.TOTL.CD
Nominal Tradable GDP		Sum of the above two data
Nominal GDP	World Bank	NY.GDP.MKTP.CD
Population	World Bank	SP.POP.TOTL
Nominal Trade Balance	World Bank	BN.GSR.MRCH.CD
External Debt to All Creditors	International Debt Statistics	
External Debt to China (part)	International Debt Statistics	
External Debt to China	Horn, Reinhart and Trebecsh (2021)	

Note: Real GDP refers to using data in constant 2015 U.S. dollar. The sector name “agriculture, forestry, and fishing” is simplified as “Agriculture” in the table. “Industry” consists of mining, manufacturing, construction, electricity, water, and gas. External debt to China has two sources: International Debt Statistics and Horn, Reinhart and Trebesch (2021). In the empirical result throughout the thesis, the main data for the debt to China is from Horn, Reinhart and Trebesch (2021) as it includes loans from state-owned commercial banks as well. Data for the debt to China reported in International Debt Statistics does not contain this category, hence is marked “part” in the table.

Table 2: Calibration for Sri Lanka

Parameter	Description	Value	Source
ρ	Autocorrelation of output	0.9114	Estimation of AR(1) on GDP
σ_u	Standard deviation of output	0.0180	Estimation of AR(1) on GDP
r^*	Risk-free rate	0.01	U.S. 3-month treasury bill rate
θ	Probability of reentry	0.0385	Chatterjee and Eyigungor (2012)
α	Labor share in non-tradable goods sector	0.65	Jegajeevan (2016)
a	Share of tradable consumption	0.35	Share of tradable goods in GPD
ξ	Intratemporal elasticity of substitution of consumption	0.5	Na et al. (2018)
σ	Inverse of intertemperal elasticity of substitution of consumption	2	$1/\xi$
γ	Downward wage rigidity	1.109	Matschke and Nie (2022)
β	Discount factor	0.6919	Estimated
δ_1	Coefficient of the linear term in loss function	-0.4391	Estimated
δ_2	Coefficient of the quadratic term in loss function	0.5530	Estimated
$\bar{h}$	Labor endowment	1	Normalized to 1

Note: The time unit is one quarter. AR(1) is performed on annual tradable GDP data but quarterized following the approach of Hinrichsen (2021). “Estimated” means that the coefficient is obtained by matching certain equilibrium conditions, described in detail in Section 5.1.

Table 3: Calibration for Pakistan

Parameter	Description	Value	Source
ρ	Autocorrelation of output	0.8518	Estimation of AR(1) on GDP
σ_u	Standard deviation of output	0.0116	Estimation of AR(1) on GDP
r^*	Risk-free rate	0.01	3 month treasury bill rate
θ	Probability of reentry	0.0417	Trebesch (2011)
α	Labor share in non-tradable goods sector	0.4	Rehman et al. (2020)
a	Share of tradable consumption	0.33	Share of tradable goods in GDP
ξ	Intratemporal elasticity of substitution of consumption	0.5	Na et al. (2018)
σ	Inverse of intertemperal elasticity of substitution of consumption	2	$1/\xi$
γ	Downward wage rigidity	1.048	Matschke and Nie (2022)
β	Discount factor	0.6252	Estimated
δ_1	Coefficient of the linear term in loss function	-0.5148	Estimated
δ_2	Coefficient of the quadratic term in loss function	0.5789	Estimated
$\bar{h}$	Labor endowment	1	Normalized to 1

Note: The time unit is one quarter. AR(1) is performed on annual tradable GDP data but quarterized following the approach of Hinrichsen (2021). “Estimated” means that the coefficient is obtained by matching certain equilibrium conditions, described in detail in Section 5.1.

Table 4: AR(1) Estimation Result with Different Filtering Methods

Filtering	Sri Lanka			Pakistan		
	ρ	σ	Unconditional std	ρ	σ	Unconditional std
HP	0.9114	0.0180	4.37%	0.8518	0.0116	2.21%
Log-Q	0.9325	0.0266	7.38%	0.9239	0.0174	4.55%

Note: "HP" refers to HP-filter with $\lambda = 100$; "Log-Q" refers to the log-quadratic detrending. ρ and σ_u are the parameters obtained by fitting the AR(1) process of the cyclical component $y_t^c = \rho y_{t-1}^c + u_t$, where $u_t \sim \mathcal{N}(0, \sigma_u^2)$. "Unconditional std" stands for the corresponding unconditional standard deviation of the AR(1) process, evaluated by $\frac{\sigma_u}{\sqrt{1-\rho^2}}$.

Table 5: Calibrated Parameters Under Different Filtering Methods

Filtering	Sri Lanka			Pakistan		
	β	δ_1	δ_2	β	δ_1	δ_2
HP	0.6919	-0.4391	0.5530	0.6252	-0.5148	0.5789
Log-Q	0.6320	-0.2878	0.4248	0.8627	-0.4167	0.4973
	d/y^T	freq	L	d/y^T	freq	L
Target	*1.75*	*2.6*	*0.07*	*1.02*	*2.6*	*0.07*
HP	1.73	1.26	0.102	1.02	1.26	0.057
Log-Q	1.70	1.8	0.122	1.00	1.06	0.067

Note: "HP" refers to HP-filter with $\lambda = 100$; "Log-Q" refers to the log-quadratic detrending. The upper part of the table shows the estimated parameters $(\beta, \delta_1, \delta_2)$ that are tuned to match three equilibrium conditions: debt-to-tradable-GDP ratio (d/y^T), frequency of default (freq), and the output loss L. The lower part of the table shows the fitness of the estimation by comparing the targeted moments with the model-generated moments.

Bibliography

Aguiar, Mark and Gita Gopinath (2006) “Defaultable Debt, Interest Rates and the Current Account,” *Journal of International Economics*, 69 (1), 64–83, Matlab code for programs attached.

Arellano, Cristina (2008) “Default Risk and Income Fluctuations in Emerging Economies,” *American Economic Review*, 98 (3), 690–712.

Arellano, Cristina, Xavier Mateos-Planas, and José-Víctor Ríos-Rull (2023) “Partial default,” *Journal of Political Economy*, 131 (6), 000–000.

Asian Development Bank and World Bank (2011) “2011 PAKISTAN FLOODS: Preliminary Damage and Needs Assessment,” Report.

Bandiera, Luca and Vasileios Tsiropoulos (2020) “A Framework to Assess Debt Sustainability under the Belt and Road Initiative,” *Journal of Development Economics*, 146, 102495.

Bi, Ran (2008) “Beneficial delays in debt restructuring negotiations.”

Borensztein, Eduardo and Ugo Panizza (2009) “The Costs of Sovereign Default,” *IMF Staff Papers*, 56 (4), 683–741.

Brautigam, Deborah (2020) “A Critical Look at Chinese ‘DTD’: The Rise of a Meme,” *Area Development and Policy*, 5 (1), 1–14.

Chatterjee, Satyajit and Burcu Eyigungor (2012) “Maturity, Indebtedness, and Default Risk,” *American Economic Review*, 102 (6), 2674–99.

——— (2015) “A seniority arrangement for sovereign debt,” *American Economic Review*, 105 (12), 3740–3765.

Chellaney, Brahma (2017) “China’s Debt-Trap Diplomacy,” Project Syndicate, January.

Chen, Wanjing Kelly (2020) “Sovereign Debt in the Making: Financial Entanglements and Labor Politics Along the Belt and Road in Laos,” *Economic Geography*, 96 (4), 295–314.

Chudik, Alexander, Kamiar Mohaddes, M. Hashem Pesaran, and Mehdi Raissi (2015) “Is There a Debt-threshold Effect on Output Growth?,” Cambridge Working Papers in Economics 1520, Faculty of Economics, University of Cambridge.

Cruces, Juan J. and Christoph Trebesch (2013) “Sovereign Defaults: The Price of Haircuts,” *American Economic Journal: Macroeconomics*, 5 (3), 85–117.

Duma, Nombulelo (2007) “Sri Lanka’s Sources of Growth.”

Eaton, Jonathan and Mark Gersovitz (1981) “Debt with Potential Repudiation: Theoretical and Empirical Analysis,” *The Review of Economic Studies*, 48 (2), 289–309.

Eom, Janet, Deborah Brautigam, and Lina Benabdallah (2018) “The Path Ahead: The 7th Forum on China-Africa Cooperation,” SAIS-CARI Briefing Papers 01/2018, Johns Hopkins University, School of Advanced International Studies (SAIS), China Africa Research Initiative (CARI).

Gelpern, Anna, Sebastian Horn, Scott Morris, Brad Parks, and Christoph Trebesch (2022) “How China Lends: A Rare Look into 100 Debt Contracts with Foreign Governments,” *Economic Policy*, eiac054.

Government of Pakistan (2008) "Manual on Foreign Debt Management," Economic Affairs Division Islamabad, Dec.

Hakkio, Craig S (2013) "The Great Moderation," *Federal Reserve History, November*, 22.

Hillman, Jonathan (2018) "China's Belt and Road Initiative: Five Years Later," Center for Strategic and International Studies, January.

Himmer, Michal and Zdeněk Rod (2023) "Chinese Debt Trap Diplomacy: Reality or Myth?" *Journal of the Indian Ocean Region*.

Hinrichsen, Simon (2021) "Essays on War Reparations and Sovereign Debt: Two Hundred Years of War Debts and Default, from the Napoleonic Wars to Iraq," Ph.D. thesis, London School of Economics and Political Science.

Ho, Tai-kuang (2023) "China's Debt Trap: Reality and Illusion," Mimeo.

Ho, Tai-kuang and Albrecht Ritschl (2023) "Seniority Reversals and Endogenous Sudden Stops: Some Transfer Problem Dynamics," *Working Paper*.

Horn, Sebastian, Carmen M. Reinhart, and Christoph Trebesch (2021) "China's Overseas Lending," *Journal of International Economics*, 133, 103539.

Hurley, John, Scott Morris, and Gailyn Portelance (2019) "Examining the Debt Implications of the Belt and Road Initiative from a Policy Perspective," *Journal of Infrastructure, Policy and Development*, 3 (1), 139–175.

Jegajeevan, Sujeetha (2016) "Understanding Sri Lankan Business Cycles Through an Estimated DSGE Model," in *Conference Proceedings, 9th International Research Conference (2016), Central Bank of Sri Lanka, Colombo*.

Kraemer, Nick W, Evan M Gunter, and Vincent R Conti (2022) "Default, Transition, and Recovery: 2021 Annual Global Sovereign Default and Rating Transition Study," May.

Kratz, Agatha, Allen Feng, and Logan Wright (2019) “New Data on the “Debt Trap” Question,” Rhodium Group, April.

Kundi, Mansoor Akbar (2016) “Paris Club, International Debt Regime and Pakistan,” *Pakistan Horizon*, 69 (4), 15–25.

Matschke, Johannes and Jun Nie (2022) “Downward Wage Rigidities and Recession Dynamics in Advanced and Emerging Economies,” *Federal Reserve Bank of Kansas City Working Paper* (22-10).

Mendoza, Enrique G. and Vivian Z. Yue (2012) “A General Equilibrium Model of Sovereign Default and Business Cycles,” *The Quarterly Journal of Economics*, 127 (2), 889–946.

Moramudali, Umesh (2019) “Is Sri Lanka Really a Victim of China’s ‘Debt Trap’,” *The Diplomat*, 14.

——— (2020) “The Hambantota Port Deal: Myths and Realities,” *The diplomat*, 1, 2020.

Na, Seunghoon, Stephanie Schmitt-Grohé, Martín Uribe, and Vivian Yue (2018) “The Twin Ds: Optimal Default and Devaluation,” *American Economic Review*, 108 (7), 1773–1819.

Parker, Sam and Gabrielle Chefitz (2018) “Debtbook Diplomacy,” *Belfer Center for Science and International Affairs*.

Ranade, Jayadeva (2017) “Does the China-Pakistan Economic Corridor Really Help Pakistan?” *Liberal Stud.*, 2, 33.

Rehman, Muhammad, Sajawal Khan, Zafar Hayat, and Faruk Balli (2020) “A Small Open Economy DSGE Model with Workers’ Remittances,” *Journal of Economic Studies*, 47 (6), 1339–1361.

Reinhart, Carmen M. (2002) "Default, Currency Crises, and Sovereign Credit Ratings," *The World Bank Economic Review*, 16 (2), 151–170.

Schlegl, Matthias, Christoph Trebesch, and Mark LJ Wright (2019) "The seniority structure of sovereign debt,"Technical report, National Bureau of Economic Research.

Schmitt-Grohé, Stephanie and Martín Uribe (2016) "Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment," *Journal of Political Economy*, 124 (5), 1466 – 1514.

Shepard, Wade (2016) "The Story Behind the World's Emptiest International Airport," Forbes, May.

Siddiqui, Rizwana, Rehana Siddiqui, and Aqdas Ali Kazmi (2001) "Determinants of Debt Rescheduling in Pakistan [with Comments]," *The Pakistan Development Review*, 40 (4), 689–704.

Small, Andrew (2020) "Returning to the Shadows: China, Pakistan, and the Fate of CPEC."

Stockman, Alan C. and Linda L. Tesar (1995) "Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements," *The American Economic Review*, 85 (1), 168–185.

Sturzenegger, Federico and Jeromin Zettelmeyer (2008) "Haircuts: estimating investor losses in sovereign debt restructurings, 1998–2005," *Journal of international Money and Finance*, 27 (5), 780–805.

Tillerson, Rex (2018) "US-Africa relations: A new framework," *US Department of State. March*, 6.

Trebesch, Christoph (2011) "Sovereign Default and Crisis Resolution," Ph.D. thesis.

Uribe, Martín and Stephanie Schmitt-Grohé (2017) *Open Economy Macroeconomics*: Princeton University Press.

Wikipedia contributors (2023) "China-Pakistan Economic Corridor — Wikipedia, The Free Encyclopedia," [Online; accessed 12-May-2023].

Yue, Vivian Z (2010) "Sovereign default and debt renegotiation," *Journal of international Economics*, 80 (2), 176–187.

Zarazaga, Carlos E. (2012) "Default and Lost Opportunities: A Message from Argentina for Euro-zone Countries," *Economic Letter*, 7 (may).

Appendix A: Output Loss

The following demonstrates the calculation of output loss associated with the default using the growth accounting approach proposed by Zarazaga (2012).

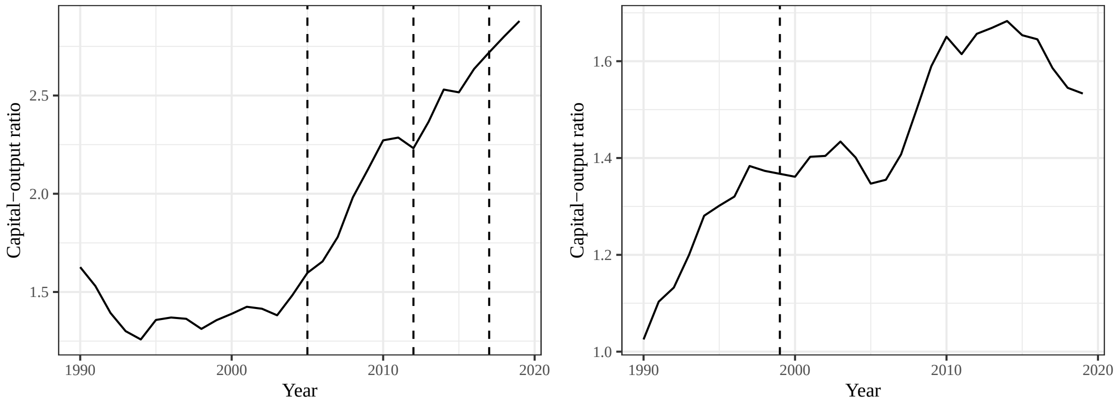
Following Zarazaga (2012), assume that the production function follows the form $y_t = h_t^{\alpha} k_t^{1-\alpha}$ where y_t denotes output, k_t denotes physical capital, and h_t denotes employment. This implies that by the relationship $\frac{y_t}{h_t} = \left(\frac{k_t}{y_t}\right)^{\frac{1-\alpha}{\alpha}}$. If the capital-output ratio before the default episode $\kappa_b = \frac{k_b}{y_b}$ falls to $\kappa_a = \frac{k_a}{y_a}$ after the default episode, the output per worker would be $\Delta = \left[\left(\frac{\kappa_a}{\kappa_b}\right)^{\frac{1-\alpha}{\alpha}} - 1\right] \times 100$ percent higher. If we ascribe all the observed decrease in capital to the sovereign default, we conclude that the output loss is on average $\frac{\Delta}{2}$% per worker during the period. Note that $\left(\frac{\kappa_a}{\kappa_b}\right)^{\frac{1-\alpha}{\alpha}} - 1 > 0$ if and only if $\kappa_a > \kappa_b$. This implies that there is an output loss only if the capital-output-ratio decreases. As argued in Zarazaga (2012), the output loss is associated with a trough in the capital-output-ratio.

Using data from the Penn World Table, the annually capital-output ratio is calculated by dividing the capital stock at current PPPs (variable *cn*) by the output-side real GDP at current PPPs (variable *cgdpo*), both in million 2017 U.S. dollar.

In the case of Sri Lanka, capital-output ratio associated with the three default episodes (2005, 2012, 2017) recorded in the BoC-BoE Sovereign Default Database all increase, even for the consecutive years, as shown in Figure 11a. It is therefore unreasonable to attribute all the variations of the output to the sovereign default episodes.

For the case of Pakistan's biggest default in 1999, according to previous calibration, $\alpha = 0.4$, which implies that by the relationship $\frac{y_t}{h_t} = \left(\frac{k_t}{y_t}\right)^{\frac{3}{2}}$. Pakistan gained partial reaccess (debt flow > 0) and emerged from financial autarky in 2004 (Trebesch, 2011),

Figure 11: Capital-Output Ratio, 1990 to 2020



(a) Sri Lanka (b) Pakistan

Source: Penn World Table
Note: The solid line represents the capital-output ratio for Sri Lanka and Pakistan. Default episode examined is plotted in vertical dashed lines.

therefore the growth accounting will be conducted within 1999 to 2004. The capital-output-ratio was about 1.36 in 1999. It rose to about 1.43 in 2003, and slowly fell to 1.4 in 2004, as shown in Figure 11b. Following the exact same logic, since $\frac{k_t}{y_t}$ rose from 1.36 to 1.4 between 1999 and 2004, the output per worker actually increased, which is contradicting our assumption of an output loss. Alternatively, if we consider the year of full reaccess as the end of default (debt flow > 1% GDP), which is in 2006 (Trebesch, 2011), the calculation yields that the output per worker would have been $\left[\left(\frac{1.36}{1.355}\right)^{\frac{3}{2}} - 1\right] \times 100 = 0.5\%$ higher, which gives an average output loss of 0.25%. The value is too low compared to the average output loss of 5.5% according to cross-country studies (Uribe and Schmitt-Grohé, 2017; Borensztein and Panizza, 2009), which indicates that it is also unreasonable to ascribe all the effect on output to capital. Hence, we conclude that the estimation of output cost of the default for Pakistan following Zarazaga (2012) is also not applicable.

Appendix B: Properties of the Default Set

In Chapter 4, I introduce the default set given by the decentralized Eaton-Gersovitz model by Na et al. (2018). It is worth examining some properties of the default set, as it justifies visually the empirical results of the thesis. Despite the fact that the default set can only be established through numerical computation instead of an analytical expression, some properties of the default set can still be obtained without knowing the exact form of the set.

Recall that the default set is defined as

$$D(d_t) = \{y_t^T : v^b(y_t^T) > v^c(y_t^T, d_t)\},$$

which is the set of output levels within which the country is best to default given a debt level d_t. I derive the following three properties regarding the default set under the model specifications of Na et al. (2018):

1. If the default set is not empty, then the trade balance deficit is less than the output loss. That is, $q(y_t^T, d_{t+1})d_{t+1} - d_t < -L(y_t^T)$.
2. If $y_1 \in D(d_t)$ and $\underline{y} \leq y_2 \leq y_1$, then $y_2 \in D(d_t)$
3. The default set $D(d_t)$ is an interval $[\underline{y}, y^*(d_t)]$, where $y^*(d_t)$ is increasing in d_t.

Here, $\underline{y}$ denotes the lower bound on the endowment level during numerical computaiton.

Similar properties are proved in Arellano (2008) and Uribe and Schmitt-Grohé (2017) for the centralized version of Eaton-Gersovitz model.

Proposition 1. If $D(d_t) \neq \emptyset$, then $q(y_t^T, d_{t+1})d_{t+1} - d_t < -L(y_t^T)$ for all d_{t+1}.

Proof. The proof is by contradiction. Suppose that $q(y_t^T, \tilde{d}_{t+1})\tilde{d}_{t+1} - d_t > -L(y_t^T)$ for some $\tilde{d}_t$, according to the definition of $v^c(d_t, y_t^T)$,

$$
\begin{aligned}
v^c(d_t, y_t^T) &= \max_{d_{t+1}, h_t} \left\{ U\left(A(y_t^T + q_t(y_t^T, d_{t+1})d_{t+1} - d_t, F(h_t))\right) + \beta E_t v^g(y_{t+1}^T, d_{t+1}) \right\} \\
&\geq U\left(A(y_t^T + q_t(y_t^T, \tilde{d}_{t+1})\tilde{d}_{t+1} - d, \bar{h})\right) + \beta E_t v^g(y_{t+1}^T, \tilde{d}_{t+1}) \\
&\geq U\left(A(y_t^T - L(y_t^T), \bar{h})\right) + \beta E_t v^b(y_{t+1}^T) \\
&\equiv v^b(y_t^T).
\end{aligned}
$$

For third line, the first term holds due to the fact that both the utility function and the aggregation function is strictly increasing and concave, and the second term holds due to the definition of $v^g = \max\{v^b, v^c\}$.

This result, however, yields a contradiction since if $v^c(d_t, y_t^T) \geq v^b(y_t^T)$ for all possible endowments of tradable output, then we have $D(d_t) = \emptyset$ by definition. Therefore, we conclude that if under the certain debt level, the default set is not empty, then we must have $q(y_t^T, d_{t+1})d_{t+1} - d_t < -L(y_t^T)$ for all d_{t+1}. $\square$

Proposition 2. If $y_1 \in D(d_t)$ and $\underline{y} \leq y_2 \leq y_1$, then $y_2 \in D(d_t)$.

Proof. Consider the difference between the value function under bad standings $v^b(y_t^T)$ and the value function of continuing to repay its debt $v^c(y_t^T, d_t)$ as $\Delta(y_t^T, d_t) \equiv v^b(y_t^T) - v^c(y_t^T, d_t)$. By definition, any tradable output in the default set $y_t^T \in D(d_t)$ satisfies $\Delta(y_t^T, d_t) > 0$.

Consider the first derivative of the difference function $\Delta_{y,d} = \frac{\partial \Delta}{\partial y_t^T} = v_y^b(y) - v_y^c(y, d)$.

Recall that

$$v^c(y,d) = \max_{\{d'\}} \quad \{U\left(A\left(y + q(y,d')d' - d, F(1)\right)\right) + \beta E_t v^g\left(y', d'\right)\}$$
$$v^b(y) = U\left(A\left(y - L(y), F(1)\right)\right) + \beta E_t\left[\theta v^g\left(y', 0\right) + (1-\theta)v^b\left(y'\right)\right].$$

The notation for tradable output y_t^T is simplified as y and y_{t+1}^T as y', and similarly d_t is simplified as d and d_{t+1} as d'. Also, from previous discussion we know that the optimal working hours is $h_t^* = \bar{h}$ when the government chooses to devaluate during default as its optimal policy, which is normalized to unity for simplicity. Applying the envelope theorem on $v^c(y,d)$,

$$\begin{aligned} v_y^c \equiv \frac{\partial v^c}{\partial y} &= \frac{\partial}{\partial y} U\Big[A\Big(y + q(y,d')d' - d, F(1)\Big)\Big] \\ &= \Big(1 + q_y(y,d')d'\Big) A_1(c_c, F(1)) U'\Big(A(c_c, F(1))\Big), \end{aligned}$$

where $q_y(y,d') \equiv \frac{\partial q}{\partial y}$, $c_c \equiv y + q(y,d')d' - d$. This is easily derived by the chain rule. As for $v^b(y)$,

$$v_y^b \equiv \frac{\partial v^b}{\partial y} = \Big[1 - L'(y)\Big] A_1(c_b, F(1)) U'\Big(A(c_b, F(1))\Big),$$

where $L' \equiv \frac{\partial L}{\partial y}$ and $c_b \equiv y - L(y)$ For the sake of simplicity, the second parameter for the aggregation function $A(\cdot,\cdot)$ and its derivative $A_1(\cdot,\cdot)$ will be simplified by showing only the first argument since the second argument is always $F(1)$.

Accordingly, the difference function

$$
\begin{aligned}
\Delta_y =& v_y^b(y) - v_y^c(y, d) \\
=& \Big[1 - L'(y)\Big] A_1(c_b) U'\Big(A(c_b)\Big) - \Big(1 + q_y(y, d')d'\Big) A_1(c_c) U'\Big(A(c_c)\Big) \\
=& A_1(c_b) U'\Big(A(c_b)\Big) - A_1(c_c) U'\Big(A(c_c)\Big) \\
& - L'(y) A_1(c_b) U'(A(c_b)) - q_y(y, d')d' A_1(c_c) U'(c_c).
\end{aligned} \tag{45}
$$

Note that $A(\cdot, \cdot)$ and $U(\cdot)$ are both concave and increasing by assumption. This implies that if $c_1 < c_2$, then (i) $A(c_1) < A(c_2)$, (ii) $A_1(c_1) > A_1(c_2) > 0$, and (iii) $U'(c_1) > U'(c_2) > 0$ Together, it implies that $U'(A(c_1)) > U'(A(c_2))$. Furthermore, since $\frac{A_1(c_1)}{A_1(c_2)} > 1$ and $\frac{U'(A(c_1))}{U'(A(c_2))} > 1$, we have

$$
\frac{A_1(c_1) U'(A(c_1))}{A_1(c_2) U'(A(c_2))} > 1 \implies A_1(c_1) U'(A(c_1)) > A_1(c_2) U'(A(c_2)). \tag{46}
$$

The first two terms in Equation (45) resembles this relationship. Since

$$
c_b \equiv y - L(y) > y + q(y, d')d' - d \equiv c_c
$$

according to Proposition 1, by Equation (46)

$$
A_1(c_b) U'\Big(A(c_b)\Big) - A_1(c_c) U'\Big(A(c_c)\Big) < 0.
$$

The third term in Equation (45) is negative since the loss function is assumed to be nonnegative and nondecreasing (Na et al., 2018), hence $L'(y) > 0$. The marginal price of debt offered by foreign lenders $q_y(y, d')$ is positive since a better condition of output today y yields a higher output tomorrow y' due to the AR(1) nature of output, which in turn decreases the probability of default tomorrow. As a result, the price of bond increases.

Overall, we have

$$\Delta_y(y, d) < 0$$

if the default set is not empty. That is, $v^b(y) - v^c(y, d)$ is a decreasing function of y.

When default is an optimal policy under the tuple (y_1, d), which means that $y_1 \in D(d)$, then by definition $v^b(y_1) > v^c(y_1, d)$. For any given $y_2 \leq y_1$, since $v^b(y) - v^c(y, d)$ is decreasing in y, we also have $v^b(y_2) > v^c(y_2, d)$, hence $y_2 \in D(d)$. $\square$

Proposition 3. The default set $D(d_t)$ is an interval $\big[\underline{y}, y^{T*}(d_t)\big)$, where $y^{T*}(d_t)$ is increasing in d_t.

Proof. An output is in the default set if $v^b(y_t^T) - v^c(y_t^T, d_t) > 0$. It is trivial that as the output goes to infinity, the country has no incentive to default hence $v^c(\infty, d_t) > v^b(\infty)$. By the intermediate value theorem, it is obvious that there exist some y^{T*} such that $\Delta(y^{T*}, d_t) = v^b(y^{T*}) - v^c(y^{T*}, d_t) = 0$, where $y^{T*} = y^{T*}(d_t)$ is the upper bound of default set that depends on the current debt level. Since $\Delta_y(y_t^T, d_t) = v_y^b(y_t^T) - v_y^c(y_t^T, d_t) < 0$ when $D(d_t) \neq \emptyset$, all values such that $y_t^T < y_t^{T*}$ has $\Delta(y_t^T, d_t) < 0$. This proves that the default set is an interval[1].

Taking the total derivative of the upper limit with respect to the debt level using the equation $\Delta(y^{T*}(d_t), d_t) = 0$, we get

$$\frac{\mathrm{d}y^{T*}(d_t)}{\mathrm{d}d_t} = -\frac{\frac{\partial \Delta}{\partial d_t}}{\frac{\partial \Delta}{\partial y^T}} = -\frac{-v_d^c(y^{T*}(d_t), d_t)}{v_y^b(y^{T*}) - v_y^c(y^{T*}, d_t)}.$$

We know that $v_y^b(y^{T*}) - v_y^c(y^{T*}, d_t j) < 0$. Applying the envelope theorem to $v^c(y_t^{T*}, d_t)$, we get

$$\frac{\partial v^c}{\partial d_t} = -A_1\Big[y_t^{T*} + q_t(y_t^{T*}, d_{t+1})d_{t+1} - d_t\Big]U'\Big[A(y_t^{T*} + q_t(y_t^{T*}, d_{t+1})d_{t+1} - d_t)\Big] < 0.$$

[1] The lower bound of the interval is the lowest level of endowment $\underline{y}$

Eventually,

$$\frac{\mathrm{d}y^{T*}(d_t)}{\mathrm{d}d_t} > 0. \tag{47}$$

This result implies that as the debt level increases (decreases), the upper bound of the default set should be strictly increasing (decreasing). $\square$

These results match those of Arellano (2008) and Uribe and Schmitt-Grohé (2017). As discussed by Na et al. (2018), when optimal devaluation and taxation policies are implemented, the equilibrium allocation in the economy aligns with that of the centralized Eaton-Gersovitz model. Therefore, it is not surprising that the default set in the decentralized economy exhibits similar behavior to that observed in a traditional centralized economy.