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Maturity, Indebtedness, and Default Risk[†]

By Satyajit Chatterjee and Burcu Eyigungor*

We advance quantitative-theoretic models of sovereign debt by proving the existence of a downward sloping equilibrium price function for long-term debt and implementing a novel method to accurately compute it. We show that incorporating long-term debt allows the model to match Argentina's average external debt-to-output ratio, average spread on external debt, the standard deviation of spreads, and simultaneously improve upon the model's ability to account for Argentina's other cyclical facts. We also investigated the welfare properties of maturity length and showed that if the possibility of self-fulfilling rollover crises is taken into account, long-term debt is superior to short-term debt. (JEL E23, E32, F34, O11, O19)

Until recently, the existing literature on debt and default—both the consumer debt and the sovereign debt parts—has considered only one-period debt. In reality, both consumers and countries can and do borrow for more than one period. In this paper, we present a new approach to incorporating long-term debt in equilibrium models of unsecured debt and default in the style of Eaton and Gersovitz (1981).

We make four contributions. First, we show that there exists an equilibrium price function for unsecured long-term debt with the property that the supply curve for credit is rising in the interest rate. Thus, a key implication of the Eaton-Gersovitz framework is shown to carry over to the case of long-term debt.[1]

Second, we contribute to the quantitative-theoretic literature on emerging market business cycles by providing a more complete accounting of the facts. Specifically, we show that our model can easily account for the observed high debt-to-output ratio, the average spread, as well as the volatility of spreads in emerging markets without compromising the model's ability to account for emerging market business

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[1] The models of consumer bankruptcy analyzed in Chatterjee et al. (2007) and others bear a strong resemblance to the Eaton-Gersovitz model. The model and the solution procedure developed in this paper are thus relevant to the consumer bankruptcy literature as well.

cycle facts.[2] We show that the role of long-term debt is critical in this accounting in that a model with one-period debt cannot match the three key first and second moments without generating excessive cyclical volatility in consumption and the trade balance and low correlations of these quantities with output.[3]

Third, we investigate the optimal maturity length. Although long-term debt is a better hedge against low realizations of output, the fact that the sovereign cannot commit to limit its future borrowing makes long-term debt more expensive—the so-called "debt dilution problem." In our calibrated model, the cost imposed by the debt dilution problem turns out to be the dominant effect and welfare is decreasing in maturity. However, we show that this result is reversed when we modify our model to allow for a small probability of a self-fulfilling rollover crisis. A sovereign that issues long-term debt is less vulnerable to a rollover crisis than one that issues short-term debt.[4] Indeed, we show that this additional source of shock does not affect the properties of the baseline long-term debt model but affects the properties of the one-period debt model significantly. Specifically, if debt is short-term, the sovereign constrains its borrowing in order to reduce the likelihood of rollover crises. In our calibrated model, this scaling back of debt makes one-period debt inferior to long-term debt.

Finally, we present a novel approach to accurately computing models with unsecured long-term debt and default. Our approach relies on the presence of a low-variance i.i.d. output shock drawn from a continuous cumulative distribution function (CDF). As we explain later in the paper, continuity of the CDF is key to avoiding a lack of convergence and the i.i.d. nature of the shock is key to developing an algorithm that can solve for the equilibrium for very tight convergence criterion.

The paper is organized as follows. Section I provides a brief literature review. Section II introduces the sovereign debt environment. Section III gives the main theoretical results and explains the computational challenge involved in solving this class of models and how this challenge can be met. Section IV presents all the quantitative results when the model is calibrated to Argentina's experience in the 1990s. The two online appendices contain results of a more technical nature, including the proofs of the main propositions.

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[2] As documented in Neumeyer and Perri (2005), open emerging market economies display a high cyclical volatility of consumption and a countercyclical trade balance. Aguiar and Gopinath (2006) and Arellano (2008) showed that the Eaton-Gersovitz framework, with its rising supply curve for credit, can quantitatively account for these patterns.

[3] We suspect that any endowment model that (i) has only one-period (i.e., quarterly) debt; (ii) matches a high level of debt (70 percent of quarterly output on average, as in our case); and (iii) has a spread volatility as high as in the data will tend to imply a counterfactually high consumption volatility. The reason is that in such a model a large fraction of consumption must be financed by issuance of new debt and it would take counterfactually small shifts in the Eaton-Gersovitz supply curve for credit to generate consumption volatility of the magnitude we see in the data. To have a model consistent with realistic debt-to-output ratios and observed volatilities of spreads and consumption we must recognize that only a small portion of consumption is financed by new issuance of debt. This is precisely what a model with long-term debt permits.

[4] A self-fulfilling rollover crisis is default resulting from a coordination failure, wherein if all lenders continue to lend the sovereign will continue to repay but if each lender suspects that other lenders will not extend new loans and therefore refuse to lend in anticipation of a default then the sovereign defaults. Cole and Kehoe (2000) provide a theoretical demonstration that long-term debt can reduce the probability of rollover crisis relative to short-term debt.

I. Literature Review

There is a related literature on long-term sovereign debt. Hatchondo and Martinez (2009) introduce consols with geometrically declining coupon payments. They show that having long-duration debt considerably improves the cyclical volatility of spreads relative to a one-period debt model.[5] Nevertheless, the standard deviation of spreads in their model is very low, at most 0.33 percent, compared to the data, which they report to be 2.51 percent.

Bi (2008) examines maturity choice in a model of one- and two-period debt with renegotiation on defaulted debt and shows that when default is likely in the near future, it is relatively attractive for the sovereign to borrow short-term. Thus, the maturity structure of debt shortens when approaching a crisis. Arellano and Ramanarayanan (forthcoming) examine maturity choice using the long-duration debt model of Hatchondo and Martinez. The focus of their paper is on the cyclical properties of the term spread and the duration of debt (maturity).

There have been a number of recent additions to the quantitative sovereign debt literature that extend the Eaton-Gersovitz framework in important directions while maintaining the assumption that debt is one-period. Bi (2008), D'Erasmo (2011), Benjamin and Wright (2009), and Yue (2010) explicitly model the debt renegotiation process that follows sovereign default. Cuadra and Sapriza (2008) examine the role of political uncertainty in affecting the level and volatility of sovereign spreads. Mendoza and Yue (2012) endogenize the costs of default by combining a production model featuring foreign working capital loans (as in Neumeyer and Perri 2005 and Uribe and Yue 2006) with the Eaton-Gersovitz framework. Several of these extensions were motivated by the desire to generate debt-to-output ratios that come closer to the high levels observed for emerging markets. However, with one exception, these studies do not come close to generating the high debt-to-output ratios we see in the data.[6]

II. Environment

A. *Preferences and Endowments*

Time is discrete and denoted $t \in \{0, 1, 2, \ldots\}$. The sovereign receives a strictly positive endowment x_t each period. The stochastic evolution of x_t is governed by the following process:

$$(1) \qquad\qquad\qquad x_t = y_t + m_t.$$

[5] They depart from the Eaton-Gersovitz framework in assuming that sovereigns do not lose access to international capital markets upon default but simply suffer a one-period proportional output loss (up to 50 percent in the worst case).

[6] Arellano (2008) obtained a mean debt-to-output ratio of 6 percent; Bi (2008) 21.2 percent; Aguiar and Gopinath (2006) 19 percent; Yue (2010) 10.1 percent; Cuadra and Sapriza (2008) 6.9 percent; and Mendoza and Yue (2009) 23.1 percent. Benjamin and Wright (2009) are able to generate a debt-to-output ratio of 65 percent, but they do not compare the level and the standard deviation of spreads in their model to that in the data.

Here, $m_t \in M = [-\overline{m}, \overline{m}]$ is a transitory income shock drawn independently each period from a mean zero probability distribution with continuous CDF $G(m)$, and y_t is a persistent income shock that follows a finite-state Markov chain with state space $Y \subset \mathbb{R}_{++}$ and transition law $\Pr\{y_{t+1} = y' \mid y_t = y\} = F(y, y') > 0, y$ and $y' \in Y$. As noted in the introduction, the i.i.d shock m is included to make robust computation of the model possible. In the quantitative analysis to follow, the endowment process (1) is estimated assuming a very small variance for m.

The sovereign maximizes expected utility over consumption sequences, where the utility from any given sequence c_t is given by

$$(2) \qquad \sum_{t=0}^{\infty} \beta^t u(c_t), \ 0 < \beta < 1.$$

The momentary utility function $u(\cdot) : [0, \infty) \to \mathbb{R}$ is continuous, strictly increasing, strictly concave, and bounded above by the quantity U.

B. Option to Default and the Market Arrangement

The sovereign can borrow in the international credit market and has the option to default on a loan. Default is costly in several ways. First, upon default, the sovereign loses access to the international credit market—cannot borrow or save in the period of default—and remains in financial autarky for some random number of periods. Specifically, following the period of default, the sovereign is let back into the international credit market with probability $0 < \xi < 1$. Second, during its sojourn in financial autarky, the sovereign loses some amount $\phi(y) > 0$ of the persistent component of output y. Third, the sovereign's transitory component of income drops to $-\overline{m}$ in the period of default.[7] We assume that $y - \phi(y) - \overline{m} > 0$ (which ensures that $y - \phi(y) + m > 0$ for all $(y, m) \in (Y \times M)$) and that $y - \phi(y)$ is increasing in y.[8]

We analyze long-term debt contracts that mature probabilistically. Specifically, each unit of outstanding debt matures next period with probability λ. If the unit does not mature, which happens with probability $1 - \lambda$, it gives out a coupon payment z. Note that, going forward, a unit bond of type (z, λ) issued $k \geq 1$ periods in the past has exactly the same payoff structure as another (z, λ) unit bond issued $k' > k$ periods in the past. This means that we need to keep track of the *total* number of (z, λ) bonds only. This cuts down on the number of state variables.[9] In what follows we

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[7] This technical assumption is made for the purpose of speeding up computation. It is not important that m take the lowest value possible. As we verify in the sensitivity analysis section, setting $m = 0$ (the mean value of the transitory shock) works just as well.

[8] In this paper, a function $f(x)$ is *increasing (decreasing)* in x if $x' > x$ implies $f(x') \geq (\leq)f(x)$ and *is strictly increasing (strictly decreasing)* in x if $f(x') > (<)f(x)$.

[9] If bonds mature deterministically in T periods, the sovereign's state vector will contain the vector $(b_0, b_1, b_2, \ldots, b_{T-1})$, where b_τ is the quantity of bonds due for repayment τ periods in the future. Thus one has to keep track of at least T state variables, each of which can take many values. Hatchondo and Martinez (2009) use a similar trick of rendering outstanding obligations "memoryless." In their setup, all bonds last forever (consols) but each pays a geometrically declining sequence of coupon payments. Thus, a bond issued in the current period promises to pay the sequence $\{1, \delta, \delta^2, \delta^3, \ldots\}$. This payoff structure is the same as that of a unit random maturity bond with $\lambda = 1 - \delta$ and $z = 1$. Our specification has the advantage that it allows separate targeting of maturity length and coupon payments.

assume that unit bonds are infinitesimally small—meaning that if b unit bonds of type (z, λ) are outstanding at the start of next period, the issuer's coupon obligations next period will be $z \cdot (1 - \lambda)b$ with certainty and the issuer's payment-of-principal obligations will be λb with certainty.

We assume that there is a single type of bond (z, λ) available in this economy. We assume that lenders are risk-neutral and that the market for sovereign debt is competitive. The unit price of a bond of size b is given by $q(y, b)$. The price of a unit bond does not depend on the transitory shock m because knowledge of current period m does not help predict either m or y in the future and, therefore, does not inform the likelihood of future default. We assume that the sovereign can choose the size of its debt from a finite set $B = \{b_I, b_{I-1}, \ldots b_2, b_1, 0\}$, where $b_I < b_{I-1} < \cdots < b_2 < b_1 < 0$.[10] As is customary in this literature, we will view debt as negative assets.

C. Decision Problem

Consider the decision problem of a sovereign with $b \in B$ of type (z, λ) bonds outstanding and endowments (y, m). Denote the sovereign's lifetime utility conditional on repayment by the function $V(y, m, b) : Y \times M \times B \to \mathbb{R}$, its lifetime utility conditional on being excluded from international credit markets by the function $X(y, m) : Y \times M \to \mathbb{R}$, and its unconditional (optimal) lifetime utility by the function $W(y, m, b) : Y \times M \times B \to \mathbb{R}$.

Then,

$$(3) \quad X(y, m) = u(c) + \beta\{[1 - \xi] E_{(y'm')|y} X(y', m') + \xi E_{(y'm')|y} W(y', m', 0)\}$$

subject to

$$c = y - \phi(y) + m.$$

The sovereign's lifetime utility under financial autarchy reflects the fact that it loses $\phi(y)$ of its output and can expect to be let back into the international credit market next period with probability ξ.

And,

$$(4) \quad V(y, m, b) = \max_{b' \in B} u(c) + \beta E_{(y'm')|y} W(y', m', b')$$

subject to

$$c = y + m + [\lambda + (1 - \lambda)z]b - q(y, b')[b' - (1 - \lambda)b].$$

The above assumes that the budget set under repayment is nonempty, meaning there is at least one choice of b' that leads to nonnegative consumption. But it is possible that (y, b, m) is such that all choices of b' lead to negative consumption. In this case, repayment is simply not an option, and the sovereign must default.

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[10]For simplicity, we do not allow the sovereign to accumulate assets. In our application, the no accumulation constraint is never binding in the simulations.

Finally,

$$(5) \qquad W(y,m,b) \;=\; \max\{V(y,m,b), X(y, \,-\,\bar{m})\}.$$

Since W determines both X and V (via equations (3) and (4), respectively) equation (11) defines a Bellman equation in W.

We assume that if the sovereign is indifferent between repayment and default, it repays. Hence, the sovereign defaults if and only if $X(y, \,-\,\bar{m}) > V(y,m,b)$. This decision problem implies a default decision rule $d(y,m,b)$ (where $d = 1$ is default and $d = 0$ is repayment) and, in the region where repayment is feasible, a debt decision rule $a(y,m,b)$. We assume that if the sovereign is indifferent between two distinct bs, it chooses the larger one (i.e., chooses a lower debt level over a higher one).

D. *Equilibrium*

The world one-period risk-free rate r_f is taken as exogenous. Given a competitive market in sovereign debt, the unit price of a bond of size b, $q(y,b')$, must be consistent with zero profits adjusting for the probability of default. That is

$$(6) \quad q(y,b') = E_{(y'm')|y}\left[[1 - d(y',m',b')] \frac{\lambda + [1 - \lambda][z + q(y',a(y',m',b'))]}{1 + r_f} \right].$$

In the event of default, the creditors get nothing. In the event of repayment, the creditors get λ, which is the fraction of a unit bond that matures next period, and on the remaining fraction, $(1 - \lambda)$, the creditors get the coupon payment z. In addition, the fraction that remains outstanding has some value that depends on the persistent component of the sovereign's endowment next period and on the sovereign's debt next period. Since the right-hand side of the equation depends directly and indirectly (through the decision rules d and a) on the function $q(y,b')$, (6) can be viewed as the functional equation: $q(y,b') = (Hq)(y,b')$, where H is the operator defined by the right-hand side of equation (6).

III. Theory and Computational Algorithm

A. *Theory*

This section states results regarding how default and borrowing decisions change with regard to variations in b. These results are then used to establish the main theoretical result of this paper, namely, that an equilibrium pricing function exists with the property that price is increasing in b' (proofs of all assertions are available in the online Appendix).

PROPOSITION 1: $d(y,m,b)$ *is decreasing in* b.

For one-period debt, Proposition 1 implies that the equilibrium pricing function is increasing in b', or, equivalently, that more credit is supplied at higher interest rates.

In the one-period case $z = 0$ and $\lambda = 1$ and the equilibrium pricing equation (6) reduces to $q(y,b') = E_{(y'm')|y}[1 - d(y',m',b')]/[1 + r_f]$. Since the right-hand side of this equality is increasing in b' by virtue of Proposition 1, $q(y,b')$ is increasing in b'. But if $\lambda < 1$ (average maturity is greater than one period) the price function also depends on $a(y,m,b)$. The behavior of $q(y,b')$ with respect to b' now depends on how $a(y,m,b)$ varies with b. But the behavior of $a(y,m,b)$ with respect to b depends, in turn, on how $q(y,b')$ varies with b'. Proposition 2 states that if the price function is increasing in b', the debt decision rule is increasing in b. Proposition 3 then establishes that there always exists a solution to the pricing equation (6) that is increasing in b'.

PROPOSITION 2: *If $q(y,b')$ is increasing in b' then $a(y,m,b)$ is increasing in b.*

PROPOSITION 3: *There exists an equilibrium price function $q^*(y,b')$ that is increasing in b'.*

The existence of an equilibrium price function requires the presence of the random variable m with continuous CDF. With this addition, the operator H is a continuous function of $q(y,b')$. The reason why m is needed to make H continuous is given in the next section.

B. Computation of the Equilibrium Price Function

In this section, we explain why computing the equilibrium price function can be challenging and how this challenge is met in our paper. The solution procedure is to iterate on (6) until convergence. More precisely, let k denote the iteration number and let $Z^k(y,b') = E_{(y',m'|y)}W^k(y',m',b')$ be the expected lifetime utility conditional on y and b'. Let $d(y,m,b;q^k,Z^k)$ and $a(y,m,b;q^k,Z^k)$ be the default and debt decision rules when the price function is $q^k(y,b')$ and the expected lifetime utility is $Z^k(y,b')$. Let $H[q^k,Z^k]$ denote the expectation on the right-hand side of (6) given $d(y,m,b;q^k,Z^k)$ and $a(y,m,b;q^k,Z^k)$ and let ζ be a "relaxation parameter." Then,

$$(7) \qquad q^{k+1}(y,b') = (1 - \zeta)H[q^k,Z^k](y,b') + \zeta q^k(y,b'),$$

$$(8) \qquad Z^{k+1}(y,b') = E_{(m',y')|y}\max\left\{X(y',-\bar{m};Z^k), V(y',m',b';q^k,Z^k)\right\},$$

and the iteration is continued until both $\max_{y,b'}|H(q^k,Z^k)(y,b') - q^k(y,b')|$ and $\max_{y,b'}|Z^{k+1}(y,b') - Z^k(y,b')|$ are sufficiently close to zero.

For the iterations to converge, a solution to (6) must exist. But if both y and b are discrete and m is identically zero, there is no assurance that (6) will have a solution because the operator H need not be continuous in the function $q(y,b')$. For instance, if for some $q(y,b')$ function and some $(y,b,0)$, the sovereign is indifferent between default and repayment, an infinitesimal change in the function $q(y,b')$ will cause a switch in behavior and, therefore, a discrete change in the expectation on the right-hand side of (6). Similarly, even if the sovereign strictly prefers to repay, it may be indifferent between two different choices of debt. Once again, an infinitesimal change in the $q(y,b')$ function can result in a discrete change in behavior and a
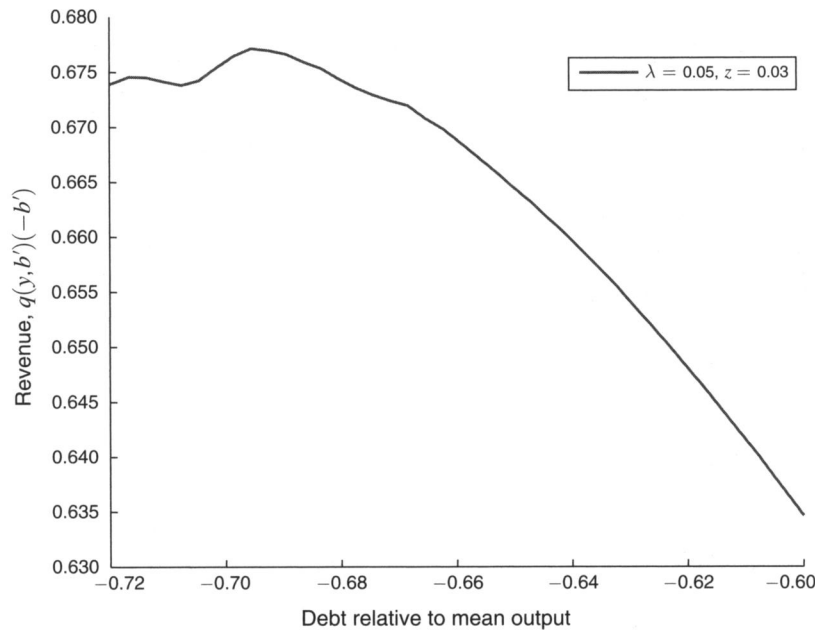
FIGURE 1. REVENUE FROM LONG-TERM DEBT $(b = 0)$

discrete change in the expectations term.[11] Note that the scope for getting jumps due to indifference is much greater for long-term debt than short-term debt because of the fact that $a(y, m, 0)$ appears in the pricing equation. This additional complication of long-term debt cannot be attenuated by making the grid on B fine because the points of indifference can be far apart on the grid. This is because the budget set under repayment is typically not convex.

Figure 1 shows a portion of $q(y, b')[b' - (1 - \lambda)b]$ function for the case in which $b = 0$ and $m = 0$ from our quantitative model presented later in the paper. Observe the kink and the ensuing depression in the upward sloping portion of the function. Figure 2 displays the variation in total lifetime utility from different choices of b' for $b = 0$ and $m = 0$.

Observe the many nonconcave segments in this function. These nonconcavities imply that, given (y, b) and q, the sovereign may be indifferent between two widely separated values of b', which will make the right-hand side of (6), and therefore the operator H, discontinuous in $q(y, b')$.

In the online Appendix we document that solving the model without the m shock leads to poor convergence outcomes for the equilibrium pricing function. The points marked A (the global maximum) and B (a local maximum) in Figure 2 illustrate what goes wrong. When the variance of m is set to 0, it often happens that a point like A is the optimal choice for some iteration k, but when that choice is incorporated in the price function, a point like B becomes the optimum choice for some

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[11] Exact indifference never happens in computations but changes in $q(y, b')$ from one iteration to the next are not infinitesimal either. The point is that if two options are near-indifferent, very small changes in $q^k(y, b')$ can cause a discrete change in behavior and, therefore, a discrete change in $q^{k+1}(y, b')$.
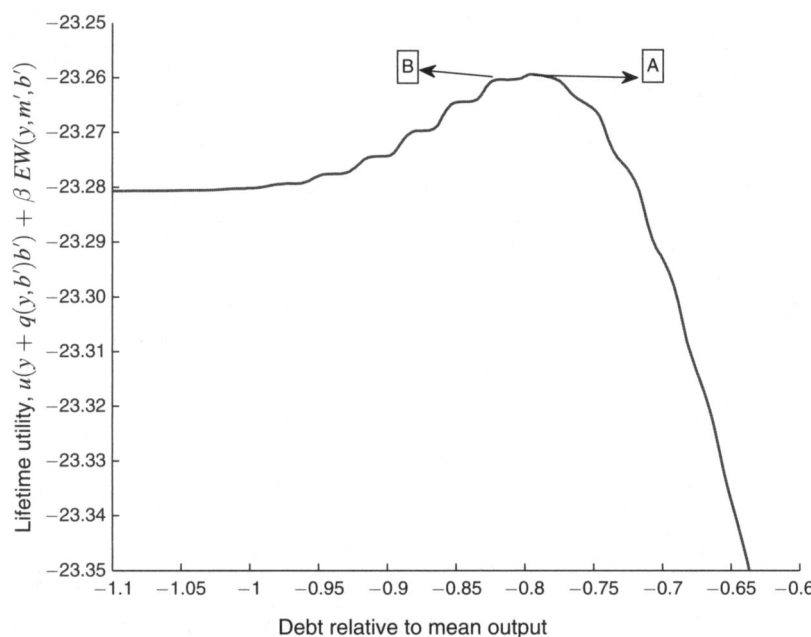
FIGURE 2. VARIATION OVER b' IN TOTAL LIFETIME UTILITY (GIVEN $m = 0, b = 0$)

iteration $k' > k$; and when that choice is incorporated in the price function, the point A reemerges as the new optimum for some iteration $k'' > k'$. Thus, the asset decision rule meanders back and forth and (7) fails to converge. We suspect that this lack of convergence occurs because there is, in fact, no solution to (6).[12]

We know from general equilibrium theory that nonexistence resulting from nonconvexities can be avoided by allowing agents to randomize over decisions. Introducing m is like introducing randomization: There is now a *probability* that an action d or b' is chosen given (y, b) and $q(y, b')$ and this probability changes continuously with the function $q(y, b')$. The nonconvexity of the budget set and non-concavity of the value function continue to imply that the decision rule $a(y, m, b; q)$ is a discontinuous function of $q(y, b')$. But as long as the points of discontinuity are finite in number, infinitesimal changes in q will *not* cause jumps in the expectation term (which is now an integral over y' *and* the continuous variable m') since each jump point has probability zero. In this way, a continuous CDF for m ensures the continuity of the right-hand side of the functional equation (6) with respect to $q(y, b')$ and the existence of an equilibrium.[13]

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[12] We consider an algorithm to have failed to converge if the maximum relative error between two successive iterates of the price function does not fall below 10^{-5} within 3,000 iterations. If the lack of convergence is due to nonexistence of an equilibrium then no matter how long we allow the program to run, it will not converge. Given this, some stopping rule is needed and we chose 3,000 (we have confirmed that the algorithm without the i.i.d. shock does not converge for higher bounds as well).

[13] An alternative strategy to prove existence is to work with decision correspondences (as opposed to decision rules). In this approach, if the decision-maker is indifferent between two (or more) actions then each action is taken with some probability that is determined in equilibrium. The proof of existence of an equilibrium relies on the Kakutani Fixed Point Theorem for compact- and convex-valued correspondences. While this approach solves the existence issue, it does not appear to be computationally tractable. In particular, computing mixed strategies when the "support points" of the mixed strategy are not known in advance—and the choice set is very large—seems to be a challenging task.

However, the introduction of m brings its own computational issues. Since m is a *continuous* variable and nonconvexities make $a(y,m,b;q^k,Z^k)$ potentially "discontinuous" in m, it is not obvious how this decision rule is to be computed.[14] This is where the assumption that m is i.i.d. plays an important role—it allows us to establish that $d(y,m,b)$ and $a(y,m,b)$ are monotone with respect to m. We have:

PROPOSITION 4: $a(y,m,b)$ *is increasing in* m *and* $d(y,m,b)$ *is decreasing in* m.

The task of computing the decision rules with respect to m thus boils down to (i) locating the value of m at which $d(y,m,b;q^k,Z^k)$ switches from 1 to 0 and (ii) the values of m at which the $a(y,m,b;q^k,Z^k)$ switches from one debt level to a lower debt level.[15] Note that it is not known in advance which lower value of debt the sovereign switches to as m increases; in particular it need *not* be the next lower debt level on the grid. However, an algorithm exists, described in the online Appendix, that can recover $a(y,m,b;q^k,Z^k)$ near exactly.[16] Once behavior with respect to m is known, $\max\{X^k(y,-\overline{m}), V^k(y,m,b;q^k,Z^k)\} = W^k(y,m,b;q^k,Z^k)$ can be integrated with respect to m accurately using the integration method also described in the online Appendix. In this way, both value functions and the price function can be computed for very tight convergence criterion.

Another issue is that the iteration (7) may fail to converge if the variance of m is too small. As q changes, the thresholds for m change. If the variance of m is very small, any given change in thresholds will result in a large change in the choice probabilities. Setting ζ very close to 1 can counteract this sensitivity (by making the change between $q^{k+1}(y,b')$ and $q^k(y,b')$ itself very small) but at the expense of making the number of iterations needed to achieve convergence much larger than our upper bound of 3,000. Thus, to achieve convergence σ_m must not be too small.[17] More generally, there is a trade-off between σ_m and ζ with regard to achieving convergence: the lower σ_m, the higher ζ must be to achieve convergence (an example of this trade-off is given in the online Appendix).

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[14] Since $a(y,m,b;q^k,Z^k)$ takes values in a discrete set, by discontinuous we mean that small changes in m can lead to large jumps in the optimal debt level. See footnote 16 for more clarification.

[15] The behavior of $d(y,m,b)$ with respect to m is easy to characterize also because of the assumption that the act of default resets m to $-\overline{m}$. This assumption makes the payoff from default independent of m. If the level of transitory income were to remain unaffected by the act of default, the payoff from default, X, would also depend positively on m. As shown in Chatterjee et al. (2007), this would result in the default set being characterized by *two* thresholds, m^L and m^U, with default occurring when $m \in (m^U, m^L)$. Since the role of the transitory shock in this paper is to ensure convergence of (7), it is computationally efficient to eliminate the dependence of X on m so that only one default threshold needs to be computed.

[16] For each (y,b) and q, the algorithm recovers $\{-\overline{m} < m^{K-1} < m^{K-2} < \cdots < m^1 < \overline{m}\}$ and $\{b'^K < b'^{K-1} < \cdots < b'^1\}$ such that b'^K is chosen for $m \in [-\overline{m}, m^{K-1})$, b'^{K-1} is chosen for $m \in [m^{K-1}, m^{K-2}), \ldots$, b'^1 is chosen for $m \in (m^1, \overline{m}]$ ($K = 1$ means the same debt level b'^1 is chosen for all $m \in M$). Note that $K - i$ need not be adjacent to $K - (i+1)$ on the grid. Note also that it is not possible to apply these methods to a continuous y because y is not i.i.d and, therefore, the debt decision rule and the default decision rule are nonmonotonic in y. For instance, if current y is above its mean, the price of debt is low, and the sovereign has an incentive to issue more debt. On the other hand, an above mean y implies that the country will be poorer in the future, which gives the sovereign an incentive to save more (i.e., borrow less). These two effects pull in opposite directions and lead to nonmonotonic behavior of b' with respect to y. In the absence of monotonicity, it is unclear if an algorithm can be devised to locate the values of y at which there is a switch in debt levels or default decision.

[17] Although we cannot prove that there is a unique equilibrium, we have not found instances of multiple equilibria. We do know, theoretically, that given the price vector $q(y,b')$, and a tie-breaking rule in case of indifference, the decision rules $d(y,m,b)$ and $a(y,m,b)$ are unique (see the online Appendix for a proof).

The final computational point is whether there are advantages to solving long-duration debt models assuming that y and b are discrete as opposed to continuous. We believe there are two advantages. First, if either y or b (or both) is a continuous variable, $q(y, b')$ is infinite dimensional and it is much harder to establish the existence of a solution to $q(y, b') = (Hq)(y, b')$. If a solution is not guaranteed, and a computational algorithm fails to converge, it is not possible to tell if this failure results from a lack of a solution or from a defective algorithm. Second, with continuous y and/or b', any computation scheme must involve interpolating value functions and the price function. For the interpolations to be justified, the functions must either be differentiable (see, for instance, Theorems 6.7.3 and 6.9.1 in Judd 1998) or be Lipschitz continuous with a *known* Lipschitz bound. But for this class of models neither the value functions nor the price function are differentiable everywhere and discovering a Lipschitz bound for $q(y, b')$ appears to be a challenging task.[18]

IV. Maturity, Indebtedness, and Spreads: The Argentine Case

A. Calibration

We apply the framework developed in the previous sections to Argentina. The main contribution is to show that long-duration bonds, besides being a closer fit with reality, can help account for the average level of spreads, the volatility of spreads, and the average level of debt in Argentina without generating counterfactual implications regarding Argentina's business cycle facts. Thus, introducing long-duration bonds into the Eaton-Gersovitz model significantly improves its quantitative performance. We focus on the eight year period between 1993:I and 2001:IV during which Argentina was on a fixed exchange rate vis-à-vis the dollar and was borrowing in international credit markets via marketable bonds.[19]

For the quantitative work we make the following specific functional form or distributional assumptions:

- Endowment processes:

$$\ln y_t = \rho \ln y_{t-1} + \epsilon_t, \quad \text{where} \quad 0 < \rho < 1 \quad \text{and} \quad \epsilon_t \sim N(0, \sigma_\epsilon^2)$$

$$m_t \sim \text{trunc } N(0, \sigma_m^2) \quad \text{with points of truncation} \quad -\bar{m} \quad \text{and} \quad \bar{m}$$

- Utility function: $u(c) = c^{1-\gamma}/(1 - \gamma)$.

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[18] How well smoothness-based interpolation techniques work in practice is an open research question. Hatchondo, Martinez, and Sapriza (2010) show that for the one-period debt model, interpolation techniques can deliver accurate results in the sense that interpolation methods give the same answer as the discrete state space method with a very fine grid for the model described in Arellano (2008). They apply a variant of their method to their long-duration bond model in Hatchondo and Martinez (2009) but they do not compare how well their method performs in solving the pricing equation relative to the discrete state space method. Also, it is not known if interpolation methods work well for the empirically relevant parameter space. In Arellano (2008), Hatchondo, Martinez, and Sapriza (2010), as well as in Hatchondo and Martinez (2009), the level and volatility of spreads, as well as the level of debt, are quite low relative to the data.

[19] This is also the time period analyzed in Arellano (2008).

- The loss in the persistent component of output in the event of default or exclusion:

$$\phi(y) \; = \; \max\{0, d_0 y \; + \; d_1 y^2\}, d_1 \; \geq \; 0.$$

The specification for $\phi(y)$ allows for a variety of cost functions. If $d_0 > 0$ and $d_1 = 0$, the cost is proportional to output; if $d_0 = 0$ and $d_1 > 0$, the cost rises more than proportionately with output; if $d_0 < 0$ and $d_1 > 0$, the cost is 0 for $0 \leq y \leq -d_0/d_1$ and rises more than proportionately with output for $y > -d_0/d_1$. This last case resembles the cost function in Arellano (2008).[20] The reasons for choosing this flexible form are discussed in the findings section.[21] With these assumptions, the numerical specification of the model requires giving values to 11 parameters. These are (i) three endowment process parameters, σ_m, ρ and σ_ϵ^2; (ii) two preference parameters, β and γ; (iii) two parameters describing the bond, the maturity parameter λ, and the coupon payment z; (iv) two default output loss parameters, d_0 and d_1; (v) the probability of reentry following default, ξ; and (vi) the risk-free rate r_f.

The parameters of the endowment process are estimated on linearly detrended quarterly real GDP data for the period 1980:I–2001:IV.[22] As noted earlier, convergence of (7) requires that the standard deviation of the i.i.d. shock m be not too low. Experimentation shows that $\sigma_m = 0.003$ is a good lower bound for our purposes—meaning that convergence is achieved within 3,000 iterations for a wide range of parameter values. Thus, the endowment process is estimated assuming that $\sigma_m = 0.003$. The estimated value of ρ and σ_ϵ are 0.948503 and 0.027092, respectively.[23] In the computations, we approximate the y process by a 200-state Markov chain and set $\bar{m} = 2\sigma_m = 0.006$.[24] Of the preference parameters, the value of γ is set equal to 2, which is the standard value used in this literature.

The parameters describing the bond were determined to match the maturity and coupon information for Argentina reported in Broner, Lorenzoni, and Schmukler (2007). The median maturity of Argentine bonds is 20 quarters so $\lambda = 1/20 = 0.05$. We set $z = 0.03$, corresponding to an annual coupon rate of 12 percent. In the data, the value-weighted average coupon rate is about 11 percent.[25]

[20] In Arellano, $\phi(y) = \max\{0, y - \bar{y}\}$. Thus, cost is 0 for $0 \leq y \leq \bar{y}$ and rises linearly at rate 1 beyond $\bar{y}$. Thus, default costs as a proportion of y, namely, $(1 - \bar{y}/y)$, increase strongly with y.

[21] With this specification, the cost can exceed y for large y. This situation never arises in our application but could be formally ruled out by setting $\phi(y) = \min\{y, \max\{0, d_0 y + d_1 y^2\}\}$.

[22] The quarterly data series on real GDP, real aggregate consumer expenditure, real exports, real imports, and the (nominal) interest rate on Argentine sovereign debt is taken from Neumeyer and Perri (2005). All the quantity variables were deseasonalized using the multiplicative X-12 routine in EViews.

[23] If the process is estimated without the transitory shock, the estimates of ρ and σ_ϵ are 0.930139 (0.038395) and 0.027209 (0.001577), respectively, where the values in parenthesis are standard errors. Note that the values of ρ and σ_ϵ used in the calibration are well within one standard deviation of these AR1 estimates and statistically indistinguishable from them. Note also that adding m to the AR1 equation is equivalent to assuming that log GDP is measured with some noise. Since the standard deviation of log GDP in the sample is 0.076107, setting $\sigma_m = 0.003$ implies that σ_m^2 is 0.16 percent of the variance of log GDP. This is small compared with the standard deviation of measurement errors assumed in estimation of DSGE models (see, for instance, Ireland 2004; see Del Negro and Schorfheide 2010) for a discussion of this practice).

[24] The value of $\bar{m}$ is small enough that the requirement $y - \max\{0, d_0 y + d_1 y^2\} - \bar{m} > 0$ is satisfied for all values of y in the Markov chain and for all d_0 and d_1 used in the computations.

[25] We chose 12 percent because with an annual risk-free rate of 4 percent and an average spread of around 8 percent, a bond with coupon of 12 percent will trade roughly at par. So, whether the debt is recorded at face value (which is the accounting practice) or at market prices (which is economically more sensible) will not matter for the calibration of the model.

We set $\xi = 0.0385$, which gives an average period of exclusion of 26 quarters, or 6.5 years. One measure of the exclusion period is the time it took to reach settlement on the defaulted debt. Beim and Calomiris (2000, Table A) report that for the 1982 Argentine default, settlement did not occur until 1993. For the 2001 default, Argentina reached settlement with a majority of its creditors in 2005. Benjamin and Wright (2009, Figure 15) also report Argentina as being in a state of default between 1982 and 1993 and between 2001 and 2005. By these measures, the average exclusion period for Argentina is 7.5 years. Gelos, Sahay, and Sandleris (2011) measure exclusion as the years between default and the date of the next issuance of public and publicly guaranteed bonds or syndicated loans. By this measure, exclusion following the 1982 default lasted only four years (Table A7). They do not report the exclusion period for the 2001 default. In our calibration, we give somewhat more weight to the settlement-date measures of the exclusion period and set the average exclusion period to 6.5 years.

The risk-free rate, r_f, was set at 0.01, which is roughly the real rate of return on a 3-month (one quarter) US Treasury bill.

The three remaining parameters β, d_0, and d_1 are selected to match (i) an average external debt-to-output ratio of 0.7, which is 70 percent of the average external debt-to-output ratio for Argentina over the period 1993:I–2001:IV; (ii) the average default spread over the same period of 0.0815; and (iii) the standard deviation of the spread of 0.0443.[26] We seek to match only a portion of debt because we do not model repayment. In reality, sovereign debt that goes into default eventually pays off something. In Argentina's case, the repayment on debt defaulted on in 2001 has been around 30 cents to the dollar. Thus, we treat only 70 cents out of each dollar of debt as the truly unsecured portion of the debt. But, as part of our sensitivity analysis, we also examine the case in which we fully match average external debt-to-output ratio.

Finally, we need to specify the model analogs of the external debt-to-output ratio and spreads. In the GDF database, the external commitments of a country are reported on a cash-accounting basis, which means that commitments are recorded at their face value, i.e., they are recorded as the undiscounted sum of future promised payments of principal.[27] The coupon payments agreed to do not figure directly in this accounting because they are not viewed as obligations until they are past due. Given this valuation principle, the model analog of debt as reported in the data is simply b, and the external debt-to-output ratio is simply b/y.[28] The default spread in the model is calculated as in the data. We compute an internal rate of return $r(y, b')$ which makes the present discounted value of the promised sequence of future payments on a unit bond

[26]Debt is total long-term public and publicly guaranteed external debt outstanding and disbursed and owed to private and official creditors at the end of each year, as reported in the World Bank's Global Development Finance Database (GDF) (series DT.DOD.DPPG.CD). The average debt-to-output ratio is the average ratio of debt to GNP measured at a quarterly rate. The spread was calculated as the difference between the interest rate data reported in Neumeyer and Perri (2005) (which is the same as the EMBI data) and the 3-month T-bill rate. The T-bill rate series used is the TB3MS series available at http://research.stlouisfed.org/fred2/categories/116. Both the interest rate data and the T-bill rate are reported in annualized terms.

[27]See "Coverage and Accounting Rules" in Section 3 of the World Bank Statistical Manual on External Debt (also available at http://go.worldbank.org/6FB4093970).

[28]The reason for this is that each bond can be viewed as a combination of unit bonds with varying maturities. For instance, a measure λ of unit bonds is due next period, a measure $(1 - \lambda)\lambda$ is due in two periods, ..., a measure $(1 - \lambda)^{j-1}\lambda$ is due in j periods, and so on. Since each of these obligations has a face value of 1, each would be recorded as a unit obligation. Thus, the total obligation is simply $\sum_{j=1} \lambda(1 - \lambda)^{j-1} = 1$.

TABLE 1—PARAMETERS SELECTED DIRECTLY

Parameter	Description	Value
γ	risk aversion	2
$\bar{m}$	bound on m	0.006
σ_m	standard deviation of m	0.003
σ_ϵ	standard deviation of ϵ	0.027092
ρ	autocorrelation	0.948503
ξ	probability of reentry	0.0385
r_f	risk-free return	0.01
λ	reciprocal of average maturity	0.05
z	coupon payments	0.03

TABLE 2—PARAMETERS SELECTED BY MATCHING MOMENTS

Parameter	Description	Value
β	discount factor	0.95402
d_0	default cost parameter	−0.18819
d_1	default cost parameter	0.24558

TABLE 3—RESULTS AND COMPARISON

Statistics	Data	Baseline	Arellano Table 4, p.706
Average spread	0.0815	0.0815	0.0358
SD of spread	0.0443	0.0443	0.0636
Debt-to-Y ratio	1.00	0.70	0.06

equal to the unit price, that is, $q(y,b') = [\lambda + (1 - \lambda)z]/[\lambda + r(y,b')]$. The difference between $(1 + r(y,b'))^4 - 1$ and $(1 + r_f)^4 - 1$ is the annualized default spread in the model.[29]

The parameter selections are summarized in the following two tables. Table 1 lists the values of the parameters that are selected directly without solving for the equilibrium of the model. Table 2 lists the parameter values that are selected by solving the equilibrium of the model and choosing the parameters so as to make the model moments come as close as possible to the three data moments mentioned above.

B. Findings

The results of the moment matching exercise are reported in Table 3. The first column of numbers shows the data for Argentina. The second column reports the moments in the model. All model moments are (sample) averages calculated by simulating the economy over many periods but always discarding the first 20 periods after reentry following each default.[30] Evidently, the matching exercise is fully

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[29] If there is no possibility of default, the unit price would be a constant $\bar{q}$ such that $\bar{q} = [\lambda + (1 - \lambda) \times (z + \bar{q})]/[1 + r_f]$, which implies $\bar{q} = [\lambda + (1 - \lambda)z]/[\lambda + r_f]$. Since $q(y,b) \le \bar{q}$, it follows that $r(y,b') \ge r_f$. Furthermore, the higher is the probability of default, the lower is $q(y,b')$ and higher is $r(y,b')$.

[30] We do this because the model economy reenters capital markets without any debt, whereas Argentina emerged from each default/restructuring episode with debt. By ignoring the first five years following reentry, we ignore years with counterfactually low debt in our model.
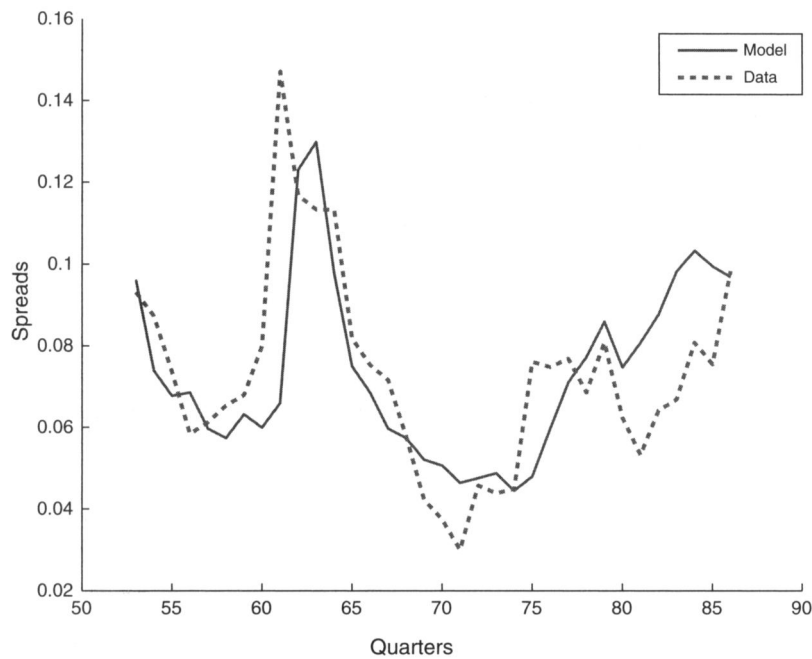
FIGURE 3. SIMULATED SPREADS FOR MODEL AND DATA (1993:I–2001:III)

successful.[31] Figure 3 shows the path of the model-simulated spreads for 1993–2001 if the initial level of debt is chosen to exactly match the spread in 1993:I. The close correspondence between the model-implied spreads and the data is striking.

For comparison purposes, the last column reports the corresponding model statistics of Arellano (2008) one-quarter debt model. Although Arellano did not target these statistics, the fact remains that there are significant deviations between her model and the data: The debt-to-output ratio is very low, the average spread is about 50 percent lower and the volatility of spreads is about 44 percent higher.

Table 4 reports some key cyclical properties of Argentine data and corresponding model moments.[32] Since we did not target these moments, the results are informative about the performance of our model.

The first column of numbers is the data for Argentina. Several features of the data stand out. First, the relative volatility of consumption is about the same as output— in stark contrast to small, open, developed economies. Second, the trade balance is countercyclical, which is also in contrast to small, open, developed economies. Third, spreads on sovereign debt are countercyclical and Argentina displayed a high propensity to default during the 1975–2001 period.[33] The following column reports

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[31] For the record, the average debt-to-output ratio in the baseline model when debt is measured at its market value is 0.703. So, it is only slightly higher than its face value. The reason is that the average interest rate on debt, 0.0292 percent per quarter, is only slightly larger than the $0.0285 (= (1 − \lambda)z = 0.95 \times 0.03)$ coupon payment on each unit of debt.

[32] Second moments for consumption and output were computed using logged and linearly detrended series. Since net exports (NX) can be negative, it was expressed as a proportion of output and then linearly detrended. The spread series was also linearly detrended, although the trend component is negligible.

[33] The frequency of default is the number of default episodes as a fraction of the number of years Argentina was in good standing with international creditors in the 27 years between 1975 and 2001. Argentina defaulted in August

TABLE 4—CYCLICAL PROPERTIES, DATA, AND MODELS

Variable	Data 1993:I–2001:IV	Baseline	1-period debt	Arellano Table 4, p.706
$\sigma(c)/\sigma(y)^a$	1.09	1.11	1.59	1.10
$\sigma(NX/y)/\sigma(y)$	0.17	0.20	1.06	0.26
corr$(c, y)^a$	0.98	0.99	0.73	0.97
corr$(NX/y, y)$	−0.88	−0.44	−0.16	−0.25
corr$(r − r_f, y)$	−0.79	−0.65	−0.55	−0.29
Average debt service^c	0.053	0.055	0.699	0.056
Default freqency^b	0.125	0.068	0.073	0.030

Notes: ^{a}Sample period is 1980:I–2001:IV; ^{b}Sample period is 1975–2001; ^{c}Principal and interest payments as a fraction of output.

the same statistics for the model. The model gets the qualitative patterns of the data right: Model consumption and trade balance have about the right level of volatility relative to output, and the trade balance and spreads are countercyclical while consumption is highly procyclical. The forces in the model that lead to these patterns are the ones emphasized in Aguiar and Gopinath (2006) and Arellano (2008).[34] The average probability of default in the model is lower than the observed frequency of default; but default is a rare event and it is hard to estimate its frequency accurately from relatively short data series.

The third data column reports results if debt is assumed to be one period and (β, d_0, d_1) are chosen to match the same three statistics as in the baseline model. The parameter vector that achieves this match is $(0.67, −0.46, 0.57)$. Aside from the implausibility of such a low value of β, the match generates serious anomalies with respect to business cycle statistics. The relative volatility of consumption and the trade balance is now much higher than in the data and the correlations of consumption, the trade balance, and spreads with output are much lower. Incorporating long-term debt moves virtually every model moment in Table 4 closer to the data, a clear indicator of the superior performance of the long-term debt model.

Why does the one-period debt model imply such a high volatility of consumption? The reason is simple: If there are b dollars of debt outstanding, the debt service obligation is b and the sovereign must refinance *all* of b at the new price $q(y, b)$ to maintain its debt level. Thus, changes in $q(y, b)$ will tend to imply large changes in consumption and the trade balance because b is large relative to output. In contrast, with long-term debt, the debt service obligation is only $[\lambda + (1 − \lambda)z]b$ and the sovereign can maintain its debt level by refinancing the much smaller quantity λb at the new price $q(y, b)$. Thus, for a given volatility of spreads, the long-term debt model can match the average level of debt without creating a counterfactually high volatility of consumption and the trade balance.

1982 and regained access in March 1993. We assume that Argentina was in default for 11 years. Thus it defaulted twice in a 16-year period of good standing. We chose 1975 as the start date because that is when Argentina began accumulating significant amounts of debt. If we start in 1946 and use the "years in restructuring" reported in Beim and Calomiris (2000, Table A), Argentina would show three defaults in a 35-year period of good standing. This would give a default frequency of 0.086. If we start in 1800 and use Beim and Calomiris again, the default frequency would drop to 0.03. But it is questionable if our model is the right framework to address such a long sweep of history.

[34]When output is below trend, the probability of default on new loans rises. If this rise is sharp enough, it is optimal for the sovereign to *reduce* debt rather than to increase it (which is what would be optimal holding interest rates constant). Thus, there is a tendency for consumption to decline more than the decline in output and for the trade balance to improve with a fall in output.

The last column reports the results for the benchmark model in Arellano (2008, Table 4). Even ignoring the frequency of default, the model with long-term debt comes substantially closer to accounting for the cyclical moments. In Arellano's model, the (negative) correlations between net exports and output, and spreads and output are, on average, about 33 percent as large as in the data. In contrast, in the model with long-term debt these correlations, respectively, are 50 percent and 82 percent as large as in the data.

C. Model Mechanics

Role of the Default Cost Function.—Since the calibrated values d_0 and d_1 are negative and positive, respectively, our specification shares the feature that Arellano (2008) introduced in her specification of default costs, namely, that the default cost as a proportion of output declines with output and becomes zero for low enough output levels.

It is now well-understood that this structure of default punishment is important in generating higher default rates, whether the default cost function is endogenous or exogenous (see, for instance, the discussion in Mendoza and Yue 2012). The key is the asymmetry in default costs: The country is punished much more severely for default when income is high than when income is low. The severe punishment for default in high-income periods implies that investors do not expect the sovereign to default in the near future (given the persistence in output) when output is high. This results in low spreads, and the (impatient) sovereign borrows aggressively. But when output declines, the punishment from default declines as well. This raises the likelihood of default and spreads rise. The high spreads make debt servicing more onerous, and, eventually, if income stays low, the sovereign defaults. Without the asymmetry, it is impossible to generate a significantly positive default frequency without making the sovereign very impatient.[35]

What appears not to have been appreciated in the literature is that the structure of default costs is also important for the *volatility* of spreads. For Arellano's specification, the default cost as a proportion of output is $1 - \bar{y} \cdot y^{-1}$ (where $\bar{y}$ is the level of output below which costs are zero), which is very sensitive to changes in y. Consequently, the probability of default is correspondingly sensitive to fluctuations in y and so is the spread (recall that Arellano's model predicted a higher volatility of spread than the data). With our specification, we can match both the level *and* the volatility of spreads. The larger d_1 is, the more volatile the spreads are likely to be. This intuition is verified in Table 5 which shows the results of varying d_1 while choosing d_0 and β to match the targets for average debt-to-output ratio and the average spreads. Notice that the volatility of spreads rises with d_1.

The accompanying changes in β and d_0 are informative about the economics of the model and are worth commenting on. The higher d_1 is, the more sensitive spreads are to variation in output and the easier it is for the model to achieve a higher

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[35] For instance, with a proportional default cost, the cost does not vary much with the level of output and spreads remain relatively high over a wide range of output and debt levels. Consequently, the sovereign rarely borrows enough to enter into regions where the probability of default is measurably positive (unless the sovereign is very impatient).

TABLE 5—ROLE OF DEFAULT COST PARAMETERS

d_1	d_0	β	$\sigma(r - r_f)$	avg. $(r - r_f)$	avg. b/y
0.15	−0.098	0.93696	0.0264	0.0815	0.70
0.25 (baseline)	−0.188	0.95402	0.0443	0.0815	0.70
0.35	−0.288	0.96195	0.0577	0.0815	0.70

TABLE 6—ROLE OF LONG-TERM DEBT

Moment	Data	Baseline	Baseline with $\lambda = 1$
Average $(r - r_f)$	0.0815	0.0815	0.0026
$\sigma(r - r_f)$	0.0443	0.0443	0.0037
Average b/y	1	0.70	0.81
$\sigma(c)/\sigma(y)$	1.09	1.11	1.14
$\sigma(NX/y)/\sigma(y)$	0.17	0.20	0.34
$corr(c,y)$	0.98	0.99	0.95
$corr(NX/y, y)$	−0.88	−0.44	−0.24
$corr(r - r_f, y)$	−0.79	−0.65	−0.42
Debt service	0.053	0.055	0.812
Default frequency	0.075	0.068	0.002

frequency of default. Since higher default frequency and high spreads are easier to achieve with a higher d_1, the sovereign needs to be *more* patient in order for it to willingly hold the level of debt that implies the observed probability of default. This explains why the value of β rises with d_1. We also see that d_0 falls as d_1 rises. This is because an increase in d_1 shifts up the default cost function which expands the maximum amount of debt the sovereign can carry without defaulting. As long as β is sufficiently less than $1/(1 + r_f)$, the sovereign will gravitate to this maximum and that will increase the average level of debt. To keep the average debt level constant, the overall default punishment should remain roughly constant. Thus, d_0 falls to counterbalance the increase in d_1.

The Role of Long-Term Debt.—In this section, we explain the role of long-term debt in our model. One way to understand its role is to compute the equilibrium of the baseline model with short-term debt, i.e., all parameter values are held fixed at their baseline values but the sovereign is permitted to issue only one-period debt. The results of this exercise are shown in the last column in Table 6. The equilibrium has stark differences. The average spread, the volatility of spreads, and the default frequency are minuscule compared with the long-term bond case.

This raises the question as to why increasing the maturity length beyond one period induces the sovereign to willingly extend its borrowing into the region where the probability of default is significantly positive. The answer lies in the differing incentives to issue additional debt in the two cases. Treating b' as a continuous variable, the marginal gain from borrowing is given by

$$(9) \qquad \left(-q(y, b') - \frac{\partial q(y, b')}{\partial b'} [b' - (1 - \lambda) b] \right) u'(c).$$

When the sovereign issues an extra unit of debt, it gets revenue from that extra unit but faces a decrease in the price of the bond, which decreases the revenue on
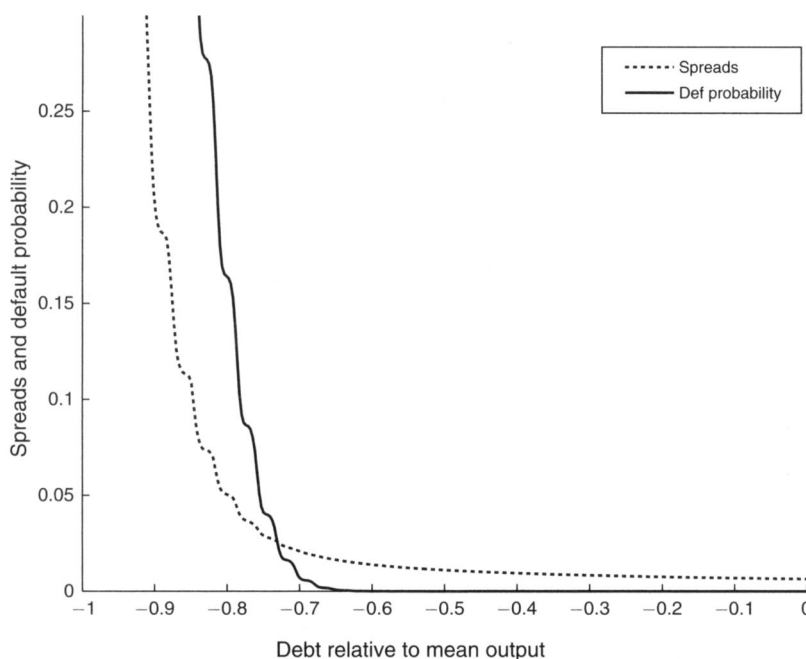
FIGURE 4. CURRENT SPREADS AND NEXT PERIOD DEFAULT PROBABILITY FOR LONG-TERM BONDS WHEN $y = $ MEAN(y)

all bonds being currently issued. In the case of short-term debt, the decrease in price applies to the entire stock of debt b', whereas with long-term debt it applies to $[b' - (1 - \lambda)b]$. Thus, the sovereign faces a much greater disincentive to borrow when default probabilities become positive in the short-term case.

In addition, $|\partial q(y, b')/\partial b'|$ is larger for short-term debt in the region where default probabilities are positive, as shown in Figures 4 and 5, which plot how spreads and the default probability vary with debt in the two cases. The reason why spreads rise faster with short-term debt is that servicing one-period debt becomes onerous more quickly than servicing long-term debt. Although debt levels are not perfectly comparable across the two cases (they involve different future obligations), the fact that spreads rise faster for short-term bonds is another reason the sovereign is less willing to extend borrowing into the region where default probability is positive when debt is short-term.[36]

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[36]It is worth noting that spreads on long-term debt start out positive and rise even though the probability of default next period may be zero. Even when the sovereign borrows a very small amount in the current period (so the default probability for next period is 0), lenders understand that the sovereign's optimal decision *next* period is to take on a significant amount of debt. And since $q(y, b')$ is decreasing in b', lenders rationally expect to suffer a capital loss on the nonmaturing portion of the debt. This depresses the current price of debt and leads to a positive spread from the start. And, initially, the spread rises with debt simply because the $a(y', m', b')$ is increasing in b' (Proposition 3), and the expected capital loss is increasing. This shows that it is not necessary to invoke risk-aversion on the part of lenders to account for gaps between spreads and default probabilities. With long-term debt, a gap can arise (and vary) because of the dynamics of debt accumulation. A gap can also arise if there is repayment on defaulted debt, which we have ruled out.
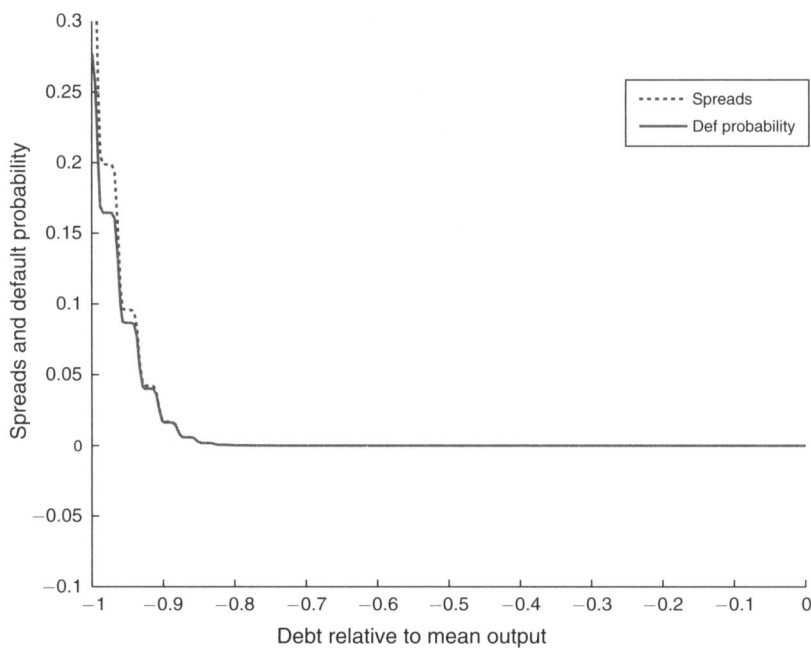
FIGURE 5. CURRENT SPREADS AND NEXT PERIOD DEFAULT PROBABILITY FOR SHORT-TERM BONDS WHEN $y = \text{MEAN}(y)$

TABLE 7—WELFARE COMPARISON ACROSS MATURITY LENGTH

(Quarters, λ)	Cert. eqv. cons	Avg. spread	Avg. b/y	Def. freq.
$(1,1)$	1.0175	0.0026	0.81	0.0024
$(2,0.5)$	1.0174	0.0049	0.81	0.0047
$(4,0.25)$	1.0169	0.0102	0.79	0.0096
$(6,0.167)$	1.0161	0.0166	0.76	0.0156
$(8,0.125)$	1.0150	0.0241	0.74	0.0224
$(10,0.1)$	1.0139	0.0327	0.73	0.0298
$(12,0.083)$	1.0129	0.0420	0.71	0.0375
$(14,0.071)$	1.0118	0.0519	0.70	0.0455
$(16,0.063)$	1.0108	0.0619	0.70	0.0534
$(18,0.056)$	1.0099	0.0719	0.70	0.0608
$(20,0.05)$	1.0092	0.0815	0.70	0.0675

D. *The Welfare Cost of Debt Dilution*

In this section, we examine the welfare effects in the baseline model of moving from one-period debt ($\lambda = 1$) to long-term debt ($\lambda = 0.05$). We assume that the b and m are both zero and compute $\Sigma_y V_\lambda(y,0,0)\Pi(y)$, where $\Pi(y)$ is the invariant distribution of the Markov chain for y. Rather than report utilities, we report the value of c that makes $c^{1-\sigma}/[(1 - \beta)(1 - \sigma)]$ equal to $\Sigma_y V_\lambda(y,0,0)\Pi(y)$ (the flow certainty equivalent consumption). The results are given in Table 7.

Welfare is highest for short-term debt and declines monotonically as λ falls toward 0.05. Thus, the sovereign is best off issuing short-term (one-quarter) debt. The difference in consumption equivalent in going from 20-quarter maturity to one-quarter maturity is 0.81 percent, which is significant by the standards of welfare comparisons.

Why is short-term debt better than long-term debt? If the sovereign can commit to not default, the (implicit) interest rate on short-term and long-term debt would

be the risk-free rate and maturity length would make no difference to welfare or consumption. Evidently, the risk of default makes a difference. But the reason for the difference is subtle. It turns out that if lenders insist that the sovereign compensate them for declines in the market value of outstanding debt and, conversely, the sovereign insists that the lenders compensate the sovereign for improvements in the value of outstanding debt, long-term debt becomes equivalent to short-term debt *even in the presence of the default risk*. This equivalence result can be understood as follows. With this arrangement, the value of repayment is (ignoring the m shock which is not important for this demonstration):

$$V(y_-,y,b) = \max_{b'}\Big\{u\big(y + [\lambda + (1 - \lambda)(z + q(y_-,b))]b - q(y,b')b'\big)$$
$$+ \beta E_{y'|y}\big[\max\{V(y,y',b'),X(y')\}\big]\Big\}.$$

where y_- is the realization of income in the previous period. Observe that the portion of the bond that does not mature in the current period pays off the coupon z and its *last period* market value as opposed to its *current* market value. This is equivalent to the sovereign transferring $[(q(y_-,b) - q(y,b')](1 - \lambda)b$ to the lenders each period (the sovereign pays if this quantity is negative and receives if it is positive). The value of default is: $X(y) = u(y - \phi(y)) + \beta E_{y'|y}\{(1 - \xi)X(y') + \xi W(y,y',0)\}$ and $W(y_-,y,b) = \max\{V(y_-,y,b),X(y)\}$. Denote the decision rules by $d(y_-,y,b)$ and $a(y_-,y,b)$. Then, the equilibrium price of a unit bond is given by

$$(10) \quad q(y,b') = [\lambda + (1 - \lambda)(z + q(y,b'))]\left(\frac{E_{y'|y}(1 - d(y,y',b'))}{1 + r}\right).$$

We now do a change of variables based on the guess that the default decision depends only on y and the *total* obligation of the sovereign at the start of each period, which is $[\lambda + (1 - \lambda)(z + q(y_{-1},b))]b = A$. Let $d(y,A)$ be the default decision rule and notice that multiplying both sides of (10) by b' gives $q(y,b')b' = [\lambda + (1 - \lambda)(z + q(y,b'))]b'(E_{y'|y}(1 - d(y,y',b'))/(1 + r))$. Observe that $q(y,b')b'$ can be written as $(E_{y'|y}(1 - d(A',y'))/(1 + r))A' = \tilde{q}(y,A')A'$. Thus, we can rewrite the value of repayment as $V(y,A) = \max_{A'}\{u(y + A - \tilde{q}(y,A')A') + \beta E_{y'|y}\max\{V(y',A'),X(y')\}\}$, which is equivalent to a one-period debt problem. Thus, freezing the value of future outstanding debt at its current market value makes long-term and short-term debt equivalent.

If the future value of outstanding debt is fixed at its value at issue, the sovereign cannot dilute the future value of outstanding debt by issuing more debt in the future. This arrangement, therefore, solves the debt dilution problem and reduces the interest rate on debt. On the other hand, the future market value of debt can also change due to changes in y and these *exogenous* fluctuations in the market value of outstanding debt lead to corresponding fluctuations in disposable income of the risk-averse sovereign.[37] For our calibration, the welfare-reducing effect of a more

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[37] When the market value falls, the sovereign makes a payment to lenders and when the market value rises the sovereign receives a payment from lenders.

volatile disposable income is dominated by the welfare-enhancing effect of lower borrowing costs, making short-term debt better than long-term debt.[38]

E. Rollover Crises and the Superiority of Long-Term Debt

The results of the previous section lead to the awkward conclusion that even though long-term debt improves model performance, in the model itself the sovereign would prefer to issue one-period debt. In this section we extend the baseline model to allow for a small probability of a rollover crisis (self-fulfilling default), and show that this small additional source of shocks makes long-term debt better than short-term debt without affecting the superior performance of the long-term debt model emphasized earlier.

This extension is motivated by the following two observations. First, if the sovereign issues short-term debt in our model, it issues a large amount of it—on average 81 percent of mean output. Thus, the sovereign rolls over a very large fraction of current consumption each period, on average. Second, many observers have noted that a large volume of short-term debt exposes a borrower to the possibility of a "run equilibrium" wherein lenders' refusal to roll over maturing debt can force the borrower into default, thereby justifying the lenders' refusal to lend. Cole and Kehoe (2000) provide a theoretical foundation for this view in the context of sovereign borrowing. Importantly for our purposes, they also show that the "run equilibrium" is less likely if the sovereign issues long-term debt. Even with a large stock of long-term debt, the *maturing* portion of debt can be small, so lenders' refusal to roll over is of little consequence to the borrower. Knowing this, lenders do not run and runs fail to be an equilibrium outcome.

To proceed, consider the following static coordination game played by the sovereign and a single new lender at the start of any period in which the sovereign has some outstanding debt and, conditional on meeting its current obligations, desires to issue new loans. The columns give the strategies of the sovereign and the rows give the strategies of the lender. If the lender makes the new loan (L) and the sovereign repays its existing debt (R), the sovereign receives the payoff from repaying the loan and borrowing, denoted $V^+(y, m, b)$, and the lender earns a net return of 0 (i.e, the lender earns the risk-free return—in expectation—which is also the opportunity cost of its funds). If the lender lends and the sovereign defaults (D), we assume that the new loan is returned to the lender without it earning any interest—hence the (discounted) loss of interest earnings $(r_f/(1 + r_f))\Delta$, where Δ is the amount of new lending.[39] If the lender does not lend (N) and the sovereign repays but cannot borrow, the sovereign receives $V^-(y, m, b) \leq V^+(y, m, b)$ and the lender earns 0. Finally,

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[38] In an earlier version of the paper, we showed that if we give the sovereign a choice between short-term and long-term debt each period, and we match the same three statistics as in the baseline model, the sovereign always chooses to issue short-term debt in every state. These results are available in the working paper version of this paper. Note, however, that to solve this model in reasonable time we employed a much coarser grid for Y and b than in the baseline model.

[39] In Cole and Kehoe, the game is sequential with the sovereign deciding on default after lenders have made their lending decisions. If we had followed Cole and Kehoe, the payoff from default conditional on having received new loans would take into account the additional consumption afforded by the new loan. And, if lenders lend and the sovereign defaults lenders would lose not only the interest but the entire loan as well.

if the lender does not lend and the sovereign defaults, the payoffs are 0 and $X(y, -\bar{m})$ for the lender and sovereign, respectively.

	R	D
L	$0, V^+(y,m,b)$	$-(r_f/1 + r_f)\Delta, X(y, -\bar{m})$
N	$0, V^-(y,m,b)$	$0, X(y, -\bar{m})$

Under the tie-breaking rule that if the sovereign is indifferent between repaying and defaulting it always repays, and if the lender is indifferent between lending and not lending it always lends, this game has the following set of Nash equilibria, depending on the value of $X(y, -\bar{m})$. When $X(y, -\bar{m}) \leq V^-(y,m,b) \leq V^+(y,m,b)$, the unique equilibrium is (L,R); if $V^-(y,m,b) \leq V^+(y,m,b) < X(y, -\bar{m})$, the unique equilibrium is (N,D); and if $V^-(y,m,b) < X(y, -m) \leq V^+(y,m,b)$, both (L,R) and (N,D) are equilibria of the game. In this case, we assume that the equilibrium selected depends on the realization of a sunspot variable, denoted ω. If $\omega = 0$, the (L,R) equilibrium is selected and if $\omega = 1$, the (N,D) equilibrium is selected. The latter case corresponds to a self-fulfilling "rollover crisis": The lender refuses to lend because it believes that the sovereign will default and the sovereign defaults because it believes that the lender will refuse to lend.[40]

In what follows, we modify the model presented in earlier sections in light of this game. First, we need to be precise about the values $V^+(y,m,b)$ and $V^-(y,m,b)$ and $X(y, -\bar{m})$. Let $W(y,m,b,\omega)$ denote the lifetime utility of the sovereign, which now depends on the sunspot variable ω in addition to the other state variables. Then,

$$V^+(y,m,b) = \max_{b' \in B} u(c) + \beta E_{(y'm')|y}[(1 - \pi)W(y',m',b',0) + \pi W(y',m',b',1)]$$

subject to

$$c = y + m + [\lambda + (1 - \lambda)z]b - q(y,b')[b' - (1 - \lambda)b],$$

where we assume that ω is i.i.d. and takes the value 1 with probability π. If there is no b' for which consumption is nonnegative, then we set $V^+(y,m,b)$ to $-\infty$. And

$$V^-(y,m,b) = \max_{b' \in B} u(c) + \beta E_{(y'm')|y}[(1 - \pi)W(y',m',b',0) + \pi W(y',m',b',1)]$$

subject to

$$c = y + m + [\lambda + (1 - \lambda)z]b - q(y,b')[b' - (1 - \lambda)b]$$

$$\text{and} \quad b' \geq (1 - \lambda)b.$$

Again, if there is no b' for which consumption is nonnegative, we set $V^-(y,m,b)$ to $-\infty$. Evidently, $V^-(y,m,b) \leq V^+(y,m,b)$. If the sovereign desires to issue new

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[40] In Cole and Kehoe (2000), the game is played between the sovereign and many lenders acting independently. The multiplicity of lenders makes the coordination failure implicit in a rollover crisis more plausible. For simplicity, we assume that a coordination failure may occur between the sovereign and a single lender.

loans (conditional on meeting its current obligations), $V^-(y,m,b) < V^+(y,m,b)$. The value under exclusion has the same structure as in the rest of the paper, specifically, $X(y,m)$ solves $X(y,m) = u(y - \phi(y) + m) + \beta\{[1 - \xi]E_{(y'm'\omega')|y}X(y',m') + \xi E_{(y'm'\omega')|y}W(y',m',0,\omega')\}$ which then pins down the value under default, $X(y,-\overline{m})$.

The functional equation that determines $W(y,m,b,\omega)$ is given by

$$(11) \quad W(y,m,b,\omega)$$

$$= \begin{cases} V^+(y,m,b) & \text{if } X(y,-\overline{m}) \leq V^-(y,m,b) \quad \text{and} \quad \omega \in \{0,1\} \\[2mm] X(y,-\overline{m}) & \text{if } V^+(y,m,b) < X(y,-\overline{m}) \quad \text{and} \quad \omega \in \{0,1\} \\[2mm] V^+(y,m,b) & \text{if } V^-(y,m,b) < X(y,-\overline{m}) \leq V^+(y,m,b) \quad \text{and} \quad \omega = 0 \\[2mm] X(y,-\overline{m}) & \text{if } V^-(y,m,b) < X(y,-\overline{m}) \leq V^+(y,m,b) \quad \text{and} \quad \omega = 1. \end{cases}$$

To see why (11) holds, observe that when $X(y,-\overline{m}) \leq V^-(y,m,b)$ the unique equilibrium of the game is (L,R). Therefore, regardless of the value of ω, the (equilibrium) lifetime utility of the sovereign is $V^+(y,m,b)$. Similarly, when $V^+(y,m,b) < X(y,-\overline{m})$, the unique equilibrium of the game is (N,D) and the lifetime utility of the sovereign is $X(y,-\overline{m})$, regardless of the value of ω. When $V^-(y,m,b) < X(y,-\overline{m}) \leq V^+(y,m,b)$, the equilibrium of the game depends on the value of ω. If $\omega = 0$, the equilibrium is (L,R) and lifetime utility is $V^+(y,m,b)$, but if $\omega = 1$ the equilibrium is (N,D) and lifetime utility is $X(y,-\overline{m})$. Correspondingly, the default decision rule, denoted $d(y,m,b,\omega)$, takes the value 1 in those cases where lifetime utility is $X(y,-\overline{m})$ and 0 otherwise; the asset decision rule conditional on repayment, denoted $a(y,m,b,\omega)$, is one that solves (11). Finally, the pricing equation now solves the functional equation

$$q(y,b') = E_{(y'm'\omega')|y}\left[[1 - d(y',m',b',\omega')]\frac{\lambda + [1 - \lambda][z + q(y',a(y',m',b',\omega'))]}{1 + r_f}\right].$$

Before we turn to the quantitative results, we note that the model analyzed in the rest of the paper corresponds to the case where $\omega = 0$ with probability 1.[41] Table 8 reports the results for long-term and short-term debt for values of sunspot probabilities ranging from 0 to 10 percent. For long-term debt (panel A), the rising sunspot probability leaves the average spread, the average b/y and the default frequency essentially unchanged, although it does marginally lower welfare (the drops are in the order of 10^{-5} percent). The reason the sunspot probability has so little additional effect is that the gap between $V^+(y,m,b)$ and $V^-(y,m,b)$ is positive only when the sovereign wishes to increase its debt to more than $0.95b$. Generally, the gap will be positive if borrowing costs are low, which happens when output is high. During

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[41] In this case, the "rollover crisis" equilibrium is never played and the sovereign defaults only if $V^+(y,m,b) < X(y,-\overline{m})$. Note also that if $V^-(y,m,b) = V^+(y,m,b)$ (which happens if the sovereign does not wish to issue new loans or if the budget sets are empty) then the bottom two branches of $W(y,m,b,\omega)$ are not relevant and, once again, the sovereign defaults only if $V^+(y,m,b) < X(y,-\overline{m})$.

TABLE 8—MATURITY LENGTH AND THE EFFECTS OF SUNSPOT PROBABILITY

$\Pr[\omega = 1]$	Cert. eqv. cons	Avg. spread	Avg. b/y	Def. prob.
Panel A. Maturity length 5 years ($\lambda = 0.05$)				
0.00	1.0092	0.0815	0.70	0.0676
0.005	1.0092	0.0815	0.70	0.0675
0.01	1.0092	0.0815	0.70	0.0674
0.02	1.0092	0.0815	0.70	0.0675
0.05	1.0092	0.0815	0.70	0.0677
0.1	1.0091	0.0816	0.70	0.0676
Panel B. Maturity length 1 quarter ($\lambda = 1$)				
0.00	1.0175	0.0026	0.81	0.0024
0.005	1.0108	0.0240	0.71	0.0223
0.01	1.0079	0.0066	0.43	0.0062
0.02	1.0074	0.0029	0.40	0.0027
0.05	1.0071	0.0024	0.39	0.0021
0.1	1.0069	0.0022	0.38	0.0021

such times the default cost is high and $X(y, -\bar{m})$ is low. Thus, the conditions for a sunspot-driven default (which require that $V^-(y,m,b) < X(y,-\bar{m}) \leq V^+(y,m,b)$) rarely occur and the randomness introduced by ω is of little consequence.

In contrast, if the sovereign is carrying a large amount of one-period debt, the gap between $V^+(y,m,b)$ and $V^-(y,m,b)$ will be very large since fully paying down a large amount of debt is likely to be very costly. Thus, it is much more likely that the conditions for a sunspot-driven default will be satisfied. Thus, the extraneous uncertainty matters more for short-term debt. The effects can be seen in panel B of Table 8. When the probability of a sunspot is half a percent, the sovereign reduces its average level of short-term debt to about 70 percent of output.[42] As the sunspot probability is raised to 1 percent, the sovereign responds by cutting debt back to 43 percent of output, on average. The reason for the strong response is twofold. Most importantly, with a lot of short-term debt outstanding the sovereign is vulnerable to a rollover crisis whenever $\omega = 1$. Furthermore, default triggered by $\omega = 1$ tends to be more costly than default triggered by poor fundamentals because the sunspot shock is independent of fundamentals and can happen when output is high (recall that in our model the cost of default as a proportion of output is increasing in output). In sum, even though $\omega = 1$ with low probability, its occurrence is sufficiently painful for the sovereign that it wants to avoid any crisis that might ensue. It does so by limiting its borrowing and thereby eliminating the bad "run" equilibrium. Importantly, the scaling back of debt reduces welfare, and for a sunspot probability of 1 percent or higher, the sovereign is better off issuing long-term debt than short-term debt.

These results provide one explanation for why emerging market economies choose to issue longer-term debt despite the fact that the debt dilution problem makes longer-term debt expensive. The benefit of longer-term debt is that it allows the sovereign to credibly commit to service the debt in the event of a sudden stop in lending even when the level of outstanding debt is large relative to output. This commitment in turn reduces the likelihood of such crises. If the sovereign is impatient, it may

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[42]Even with this reduced level of debt, spreads rise to 2.40 percent and default frequency to 2.23 percent. The strong response of default frequency to a small probability of a sunspot occurs because $V^+(y,m,b) - X(y, -\bar{m})$ also declines with a rise in spreads so that the likelihood of default when $\omega = 0$ also goes up.

prefer to issue longer-term debt and borrow a large amount rather than issue cheaper short-term debt but constrain its borrowing so as to limit the likelihood of rollover crises. It is worth noting that even when the probability of a crisis is 10 percent, the gain in welfare from issuing long-term debt is only 0.002 percent of consumption. This is small but in line with the common finding that welfare costs of fluctuations tend to be small (Lucas 1987).

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