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Default Risk and Income Fluctuations in Emerging Economies

By Cristina Arellano*

Recent sovereign defaults are accompanied by interest rate spikes and deep recessions. This paper develops a small open economy model to study default risk and its interaction with output and foreign debt. Default probabilities and interest rates depend on incentives for repayment. Default is more likely in recessions because this is when it is more costly for a risk averse borrower to repay noncontingent debt. The model closely matches business cycles in Argentina predicting high volatility of interest rates, higher volatility of consumption relative to output, and negative correlations of output with interest rates and the trade balance. (*JEL* E21, E23, E32, E43, F34, O11, O19)

Emerging markets tend to have volatile business cycles and experience economic crises more frequently than developed economies. Recent evidence suggests that this may be related to cyclical changes in the access to international credit. In particular, emerging market economies face volatile and highly countercyclical interest rates, usually attributed to countercyclical default risk.[1] Figure 1 illustrates these correlations by plotting aggregate consumption, output, and interest rate spreads for Argentina.[2] In December 2001, Argentina defaulted on its international debt and fell into a deep economic crisis. During the crisis, consumption and output collapsed, interest rates increased, and the trade balance experienced a sharp reversal.[3] This evidence indicates that a priority for theoretical work in emerging market macroeconomics is understanding markets for international credit, and in particular the joint analysis of default risk, interest rates, and aggregate fluctuations.

This paper develops a stochastic general equilibrium model with endogenous default risk. The model studies the relation between default events, interest rates, and output, shedding light on potential mechanisms generating the comovements described above. The terms of international loans are endogenous to domestic fundamentals and depend on incentives to default. The paper extends the approach developed by Jonathan Eaton and Mark Gersovitz (1981) in their seminal study on international lending, and analyzes how endogenous default probabilities and fluctuations in output are related. In a quantitative exercise the model is applied to analyze the default experience of Argentina. The model can predict the recent default and can account well for the business cycle statistics in Argentina.

* Department of Economics, University of Minnesota, 1035 Heller Hall, 271–19th Avenue South, Minneapolis, MN 55455, and Federal Reserve Bank of Minneapolis (e-mail: arellano@econ.umn.edu). I would like to thank Patrick Kehoe, Tim Kehoe, John Coleman, Adam Szeidl, Martin Uribe, three anonymous referees, and the editor for many useful suggestions. I especially thank Enrique Mendoza and Jonathan Heathcote for all the advice and guidance. I thank the National Science Foundation for financial support under grant 0551334. The views expressed herein are those of the author and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

[1] Pablo A. Neumeyer and Fabrizio Perri (2005) and Martin Uribe and Vivian Yue (2006) document the countercyclicality of country interest rates for Argentina, Brazil, Ecuador, Mexico, Peru, Philippines, and South Africa.

[2] The figure plots quarterly series for: linearly detrended GDP and aggregate consumption, and the interest rate spread defined as the difference of the Emerging Markets Bond Index (EMBI) yield and the yield of a five-year US bond. See Section IV for details on data and sources.

[3] The dynamics of interest rates, consumption, output, and the trade balance around the 1999 Russian default and 1999 Ecuadorean default are similar to those experienced in Argentina.

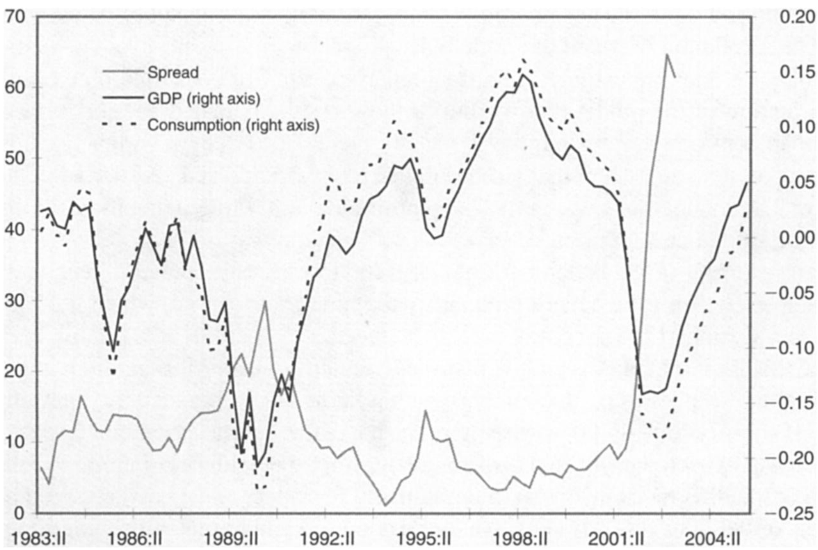

FIGURE 1. ARGENTINA'S DEFAULT

The model in this paper accounts for the empirical regularities in emerging markets as an equilibrium outcome of the interaction between risk neutral creditors and a risk averse borrower that has the option to default. The borrower is a benevolent government of a small open economy which trades bonds with foreign creditors. Bond contracts reflect default probabilities that are endogenous to the borrower's incentives to default. Thus, the equilibrium interest rate the economy faces is linked to default. Default entails temporary exclusion from international financial markets and direct output costs. Default happens along the equilibrium because the asset structure is incomplete, since it includes only bonds that pay a noncontingent face value. Asset incompleteness is necessary in this framework to study time-varying default premia due to equilibrium default. With noncontingent assets, risk neutral competitive lenders are willing to offer debt contracts that in some states will result in default by charging a higher premium on these loans. In addition to more closely reflecting the actual terms of international financial markets where foreign debt is largely contracted at noncontingent interest rates, this market structure has the potential to deliver countercyclical default risk, since repayment of noncontingent, nonnegotiable loans in low-output, low-consumption times is more costly than repayment in boom times.

In the first part of the paper, a simpler version of the model with i.i.d. shocks and only exclusion costs from default is considered in order to characterize analytically the equilibrium properties of credit markets. It is shown that default occurs in recessions, and also when the borrower cannot roll over the current debt. This result contrasts with standard participation constraint models that have a complete set of assets, which have the feature that default incentives are higher in good times. The key intuition for why asset market incompleteness reverses the relation between default and output is that after a prolonged recession, debt holdings can grow so much that the economy experiences net capital outflows. These capital outflows are more costly for a risk averse borrower in times of low shocks, making default more attractive in recessions.

In the quantitative part of the paper, the general model is calibrated to Argentina to study its recent default episode. A successful calibration of the historical default probability requires a flexible specification that makes the output costs of default disproportionately larger in booms. The model replicates well the business cycles statistics in Argentina. It can account for the high volatility of interest rates, the negative correlations of output and consumption with interest rates, the negative correlation between the trade balance and output, the positive correlation between the trade balance and interest rates, and the higher volatility of consumption relative to output. The main feature of the model that facilitates these results is that, with persistent shocks, the terms of bond contracts are much more stringent in recessions than in booms because of default risk. Thus recessions are accompanied by higher interest rates and smaller trade deficits than booms are. The model can also predict Argentina's default while generating the high interest rates and collapse in consumption observed.

The main anomaly of the benchmark model is the low average spread it generates. Risk neutral pricing closely links the default probability to the average spread, which is at odds with the data. The last section of the paper documents the necessary features of a pricing kernel that can rationalize the disconnect between low historical default probabilities and high average spreads in emerging markets bonds. If the lenders' pricing kernel—i.e., the lenders' intertemporal marginal rate of substitution—is sufficiently high during default events, then bond prices will reflect not only a lower expected payoff but also compensation for default risk premia. We illustrate that within the model this mechanism can quantitatively reproduce the empirical spread if the lender's pricing kernel is sufficiently sensitive to the domestic conditions of the emerging country.

The paper is related to several studies that have looked at the relation between interest rates and business cycles. Neumeyer and Perri (2005) model the effect that exogenous interest rate fluctuations have on business cycles and find that interest rate shocks can account for 50 percent of the volatility of output in Argentina. Uribe and Yue (2006) construct an empirical VAR to uncover the relationship between country interest rates and output, and then estimate this relationship with a theoretical model. They find that country spreads explain 12 percent of movements in output, and that output explains 12 percent of movements in country interest rates. These papers, however, do not model endogenous country spreads responding to probabilities of default in international loans.

The debt contractual arrangement in this paper is related to the optimal contract arrangements in the presence of commitment problems, such as the analyses by Timothy Kehoe and David Levine (1993), Narayana Kocherlakota (1996), and Fernando Alvarez and Urban Jermann (2000). These studies assume, however, that a complete set of contingent assets is available, and they search for allocations that are efficient, subject to a lack of enforceability. While it is useful to characterize allocations under the constrained efficiency benchmark, this market structure may not be useful as a framework for understanding actual emerging markets. First, default, defined as a breach of contract, never arises in equilibrium so that default premia are never observed. Second, default incentives in this class of models are typically higher in periods of high output, which is when efficiency dictates loan repayment. These features put these models at odds with the empirical evidence regarding default risk in emerging markets where bond yields are countercyclical and where debt prices largely reflect the risk of default. This paper delivers the correct empirical prediction because it assumes an incomplete set of assets, as in William Zame (1993), where default occurs with a positive probability. In this regard, the paper is closely related to the analysis on unsecured consumer credit with the risk of default by Satyajit Chatterjee et al. (2007), which models equilibrium default in an incomplete markets setting.

Recent quantitative models of sovereign debt build on the framework of this paper and address other very important features in emerging markets. In contemporaneous work, Mark Aguiar and Gita Gopinath (2006) take a more serious look at the process for output in emerging countries and find that shocks to the trend are important in these economies. With permanent shocks, more debt is demanded in booms because a high output today predicts a high growth rate in the future. Thus, in their model trend shocks are the rationale for the positive relation between the trade balance and spreads. Regarding renegotiation procedures, this paper assumes that the defaulted debt is never paid back, but most of the sovereign defaults are resolved through settlements with creditors. Yue (2006) precisely studies the role of renegotiation after default and finds that the bargaining power of the lender and borrower can affect substantially the terms of contracts and interest rates. Political economy factors are generally considered important determinants of interest rate spreads and are studied in Gabriel Cuadra and Horacio Sapriza (2006), who find that greater political uncertainty increases the frequency of default events in emerging countries.

The focus in this paper is on understanding the interaction among the level and volatility of output, sovereign default, and interest rate spreads in an environment of incomplete markets. Results match the empirical facts that default incentives are higher when the economy has large debt positions and is in a recession. The paper is organized as follows: Section I presents the theoretical model, Section II characterizes the equilibrium, Section III assesses the quantitative implications of the model in explaining the data, and Section IV concludes.

I. The Model Economy

Consider a small open economy that receives a stochastic stream of income. The government of the economy trades bonds with risk neutral competitive foreign creditors. Debt contracts are not enforceable and the government can choose to default on its debt at any time. If the government defaults, it is assumed to be temporarily excluded from international intertemporal trading and to incur direct output costs. The price of each bond available to the government reflects the likelihood of default events, such that creditors break even in expected value.

Households are identical and risk averse, and have preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{1}$$

where $0 < \beta < 1$ is the discount factor, c is consumption, and $u(\cdot)$ is increasing and strictly concave. Households receive a stochastic stream of a tradable good y. The output shock is assumed to have a compact support and to be a Markov process with a transition function $f(y', y)$. Households also receive a transfer of goods from the government in a lump sum fashion.

The government is benevolent and its objective is to maximize the utility of households. The government has access to the international financial markets, where it can buy one-period discount bonds B' at price $q(B', y)$. The government also decides whether to repay or default on its debt. The bond price function $q(B', y)$ is endogenous to the government's incentives to default, and depends on the size of the bond B' and on the aggregate shock y, because default probabilities depend on both. A purchase of a discount bond with a positive value for B' means that the government has entered into a contract where it saves $q(B', y)B'$ units of period t goods to receive $B' \geq 0$ units of goods the next period. A purchase of a discount bond with negative face value for B' means that the government has entered into a contract where it receives $-q(B', y)B'$ units of period t goods and promises to deliver, conditional on not declaring default, B' units of goods the following period. The government rebates back to households all the proceedings from its international credit operations in a lump sum fashion.

When the government chooses to repay its debts, the resource constraint for the small open economy is the following:

$$c = y + B - q(B', y)B'. \tag{2}$$

Given that the government is benevolent, it effectively uses international borrowing to smooth consumption and alter its time path. The idiosyncratic income uncertainty induced by y cannot, however, be insured away with the set of bonds available, which pay a time and state invariant amount. Thus, asset markets in this model are incomplete, not only because of the endogenous default risk, but also because of the set of assets available.

Driven by recent emerging market default episodes, we model the costs from default as consisting of two components: exclusion from international financial markets and direct output costs.[4] We take a simple specification in modeling the value of default such that it replicates the fact that recent sovereign defaults are accompanied by a temporary loss of access to international borrowing and by low aggregate output. Specifically, if the government defaults, we assume that current debts are erased from the government's budget constraint and that saving or borrowing is not allowed. The government will remain in financial autarky for a stochastic number of periods and will reenter financial markets with an exogenous probability. Default also entails direct costs such that output is lower during the periods the government is in autarky.

When the government chooses to default, consumption equals output:

$$c = y^{def}, \tag{3}$$

where $y^{def} = h(y) \leq y$, and $h(y)$ is an increasing function.

Foreign creditors have access to an international credit market in which they can borrow or lend as much as needed at a constant international interest rate $r > 0$. They have perfect information regarding the economy's endowment process and can observe the level of income every period. Creditors are assumed to price defaultable bonds in a risk neutral manner such that in every bond contract offered they break even in expected value. In particular, every period lenders choose loans B' to maximize expected profits ϕ, taking prices as given:

$$\phi = qB' - \frac{(1-\delta)}{1+r}B', \tag{4}$$

where δ is the probability of default.

For positive levels of foreign asset holdings, $B' \geq 0$, the probability of default is zero, and thus the price of a discounted bond is equal to the opportunity cost for creditors. For negative asset holdings, $B' < 0$, the equilibrium price accounts for the risk of default creditors face, such that the price of a discount bond equals to the risk-adjusted opportunity cost.[5] This requires that bond prices satisfy

$$q = \frac{(1-\delta)}{1+r}. \tag{5}$$

The probability of default δ is endogenous to the model and depends on the government incentives to repay debt. Since $0 \leq \delta \leq 1$, the zero profit requirement implies that bond prices q lie in the closed interval $[0,(1+r)^{-1}]$. We define the country gross interest rate as the inverse of the discount bond price, $1 + r^c = 1/q$, and the country spread as the difference between the country interest rate and the risk-free rate $r^c - r$.

The timing of decisions within each period is as follows. The government starts with initial assets B, observes the income shock y, and decides whether to repay its debt obligations or

[4] Daniel Cohen and Jeffrey Sachs (1986) and Harold Cole and Timothy Kehoe (2000) also model sovereign defaults as having negative implications on output.

[5] Risk adjustment in this framework is not due to compensation for risk aversion, as lenders are risk neutral. It reflects the risk neutral compensation for a lower expected payoff.

default. If the government decides to repay, then taking as given the bond price schedule $q(B',y)$, the government chooses B' subject to the resource constraint. Then, creditors taking q as given choose B'. Finally, consumption c takes place.

II. Recursive Equilibrium

We define a recursive equilibrium in which the government does not have commitment and in which the government, foreign creditors, and households act sequentially. Given aggregate states $s = (B,y)$, the policy functions for the government B', the price function for bonds q, and the policy functions for the consumers c determine the equilibrium.

Households simply consume their endowment plus the transfers from the government's foreign credit operations. Foreign creditors are risk neutral and lend the amount of debt demanded by the government as long as the gross return on the bond equals $(1 + r)$. Given loan size B' and income state y, the bond price satisfies

$$q(B',y) = \frac{(1 - \delta(B',y))}{1 + r}. \tag{6}$$

The government observes the income shock y and, given initial foreign assets B, chooses whether to repay or default. If the government chooses to repay its debt obligations and remain in the contract, then it chooses the new level of foreign assets B'. The government understands that the price of new borrowing $q(B',y)$ depends on the states y and on its choice of B'.

Define $v^o(B,y)$ as the value function for the government that has the option to default and that starts the current period with assets B and endowment y. The government decides whether to default or repay its debts to maximize the welfare of households. Note that the default option can be optimal only when the government has debt (i.e., negative assets).

Given the option to default, $v^o(B,y)$ satisfies

$$v^o(B,y) = \max_{\{c,d\}} \{v^c(B,y),\, v^d(y)\}, \tag{7}$$

where $v^c(B,y)$ is the value associated with not defaulting and staying in the contract and $v^d(y)$ is the value associated with default.

When the government defaults, the economy is in temporary financial autarky and income falls and equals consumption. The value of default is given by the following:

$$v^d(y) = u(y^{def}) + \beta \int_{y'} [\theta v^o(0,y') + (1 - \theta) v^d(y')] f(y',y)\, dy', \tag{8}$$

where θ is the probability that the economy will regain access to international credit markets.

As we document below, after recent default episodes, countries experienced contractions in economic activity and lacked access to international borrowing. Our specification for the value of default in the model economy encompasses these two elements exogenously. However, a large literature has studied how both can arise endogenously as an equilibrium outcome from a relation between a lender and a borrower who lacks commitment. Regarding exclusion costs, reputation models of sovereign debt have studied extensively how positive sovereign borrowing can be sustained when exclusion from financial markets is the optimal trigger punishment lenders impose on a borrower in default. For example, Mark Wright (2002) studies how a country's concern for its reputation can work to enforce repayment because lenders have incentives to tacitly collude in punishing a country in default, even if they are making zero profits.[6] Regarding output costs, Cole and Patrick Kehoe (1997) present a model where sovereign default damages other relations outside the credit market, generating additional welfare losses for the borrower. Moreover, within the context of this model, Yue (2006) studies the renegotiation process after default as an endogenous outcome of a game between the lender and borrower.

When the government chooses to remain in the credit relation, the value conditional on not defaulting is the following:

$$v^c(B,y) = \max_{(B')} \left\{ u(y - q(B',y)B' + B) + \beta \int_{y'} v^o(B',y')\, f(y',y)\, dy' \right\}. \tag{9}$$

The government decides on optimal policies B' to maximize utility. The decision to remain in the credit contract and not default is a period-by-period decision. The expected value from next period onward incorporates the fact that the government could choose to default in the future. The government also faces a lower bound on debt, $B' \geq -Z$, which prevents Ponzi schemes but is otherwise not binding in equilibrium.

The government default policy can be characterized by default sets and repayment sets. Let $A(B)$ be the set of y's for which repayment is optimal when assets are B, such that

$$A(B) = \{y \in Y : v^c(B,y) \geq v^d(y)\},$$

and let $D(B) = \widetilde{A}(B)$ be the set of y's for which default is optimal for a level of assets B:

$$D(B) = \{y \in Y : v^c(B,y) < v^d(y)\}. \tag{10}$$

Now that we have developed the problem for each of the agents in the economy, the equilibrium is defined. Let $s = \{B, y\}$ be the aggregate states for the economy.

DEFINITION 1: *The recursive equilibrium for this economy is defined as a set of policy functions for (i) consumption $c(s)$; (ii) government's asset holdings $B'(s)$, repayment sets $A(B)$, and default sets $D(B)$; and (iii) the price function for bonds $q(B',y)$ such that:*

1. *Taking as given the government policies, households' consumption $c(s)$ satisfies the resource constraint.*

2. *Taking as given the bond price function $q(B',y)$, the government's policy functions $B'(s)$, repayment sets $A(B)$, and default sets $D(B)$ satisfy the government optimization problem.*

3. *Bonds prices $q(B',y)$ reflect the government's default probabilities and are consistent with creditors' expected zero profits.*

The equilibrium bond price function $q(B',y)$ has to be consistent with the government's optimization and with expected zero profits for lenders, such that the price correctly assesses the

[6] A large number of other papers have studied alternative mechanisms to solve the Jeremy Bulow and Kenneth Rogoff (1989) paradox, which states that if the government has an enforcement technology of its own such that it can save at the same interest rate after defaulting, no international borrowing can be sustained in equilibrium because default will happen with probability one. Kenneth Kletzer and Brian Wright (2000) show that by introducing lack of commitment from the side of lenders, positive borrowing can be supported in equilibrium. Manuel Amador (2003) shows that political economy considerations, with a short-sighted government that faces political shocks, can also address this paradox.

probability of default of the government. Default probabilities $\delta(B',y)$ and default sets $D(B')$ are then related in the following way:

$$\delta(B',y) = \int_{D(B')} f(y',y)\ dy'. \tag{11}$$

When default sets are empty, $D(B') = \emptyset$, equilibrium default probabilities $\delta(B',y)$ are equal to zero because, with assets B', the government never chooses to default for all realizations of the endowment shocks. When $D(B') = Y$, default probabilities $\delta(B',y)$ are equal to one. More generally, default sets are shrinking in assets, as the following proposition shows.

PROPOSITION 1 (Default sets are shrinking in assets): *For all $B^1 \leq B^2$, if default is optimal for B^2 in some states y, then default will be optimal for B^1 for the same states y, that is, $D(B^2) \subseteq D(B^1)$.*

PROOF:
See Appendix.

This result is proven in Chatterjee et al. (2007) and in Eaton and Gersovitz (1981). The result follows from the property that the value of staying in the contract is increasing in B and that the value of default is independent of B. As assets decrease, the value of the contract monotonically decreases while the value of default is constant. Thus, if default is preferred in a given state y for some level of assets B, the value of the contract is less than the value of default. As assets decrease, the value of the contract will be even lower than before and so default will continue to be preferred.

Since stochastic shocks are assumed to have a bounded support, there exists a level of assets that is low enough, such that default sets equal the entire endowment set. On the other hand, given that default can be preferable only when assets are negative (i.e., when the government is holding debts), there exists a level of assets $\bar{B} \leq 0$, such that default sets are empty.[7] These two properties of default sets can be summarized as follows.

DEFINITION 2: *Denote as $\underline{B}$ the upper bound of assets for which the default set constitutes the entire set, and let $\bar{B}$ be the lower bound of assets for which default sets are empty, where $\underline{B} \leq \bar{B} \leq 0$ due to Proposition 1:*

$$\underline{B} = \sup\{B : D(B) = Y\},$$

$$\bar{B} = \inf\{B :\ D(B) = \emptyset\}.$$

Condition (11) implies that the equilibrium price function $q(B',y)$ is increasing in B' such that a low discount price for a large loan compensates lenders for a possible default. Bond prices are also contingent on the endowment shock because the probability distribution from which shocks are drawn the next period depends on today's shock. Since the risk of default varies with the level of debt and depends on the stochastic structure of shocks, competitive risk-neutral pricing requires that the equilibrium bond price be a function of both B' and y.

[7] Harold Zhang (1997) introduced $\bar{B}$ as the no-default debt limit in his work on participation constraints under incomplete markets.

A. *Case of i.i.d. Shocks*

This section characterizes the bond price function and the default decision for the case of i.i.d. endowment shocks. Here, equilibrium bond prices $q(B')$ are independent of the shock realization because today's shock gives no information on the likelihood of tomorrow's shock, and therefore of a default event. We assume that $h(y) = y$, no output loss in autarky, and $\theta = 0$, financial autarky is permanent after default.

PROPOSITION 2: *If, for some B, the default set is non-empty $D(B) \neq \emptyset$, then there are no contracts available $\{q(B'), B'\}$ such that the economy can experience capital inflows, $B - q(B')B' > 0$.*

PROOF:
See Appendix.

Default arises only when the borrower does not have access to a contract that lets him roll over the current debt due. If the borrower could roll over the current debt, then he would simply consume more today and default tomorrow on a higher debt. In particular, given that from tomorrow onward the borrower under the contract has the option to default, if default is chosen today, then it must be that today's period utility is lower under the contract than under default. But given that debt contracts are chosen to maximize the contract value, it must be that today consumption under the contract is less than the endowment for all contracts available.

PROPOSITION 3: *Default incentives are stronger the lower the endowment. For all $y_1 \leq y_2$, if $y_2 \in D(B)$, then $y_1 \in D(B)$.*

PROOF:
See Appendix.

This result comes from the property that utility is increasing and concave in consumption and that under no default the economy experiences net capital outflows due to Proposition 2. The idea is that net repayment is more costly when income is low due to concavity, making default a more likely choice. In low-income times, the contracts available are not useful insurance instruments for a highly indebted borrower because none can increase consumption relative to income. Thus, the asset the borrower is giving up is not very valuable and default may be preferable in recessions.

Endowment shocks have generally two opposing effects on default incentives. When output is high, the value of default is relatively high, increasing default incentives. But, at the same time, the value of repayment is high, which decreases default incentives. With an incomplete set of assets and i.i.d. shocks, the latter effect dominates and thus default is more likely the lower income. This result contrasts with the participation constraint models that have a complete set of contingent assets. These models have the feature that default incentives are higher in times of good shocks and capital outflows in recessions are never part of the contract (see the textbook treatment of such an economy in Lars Ljungqvist and Thomas Sargent 2000).

Due to Proposition 3, for $B < \bar{B}$, it is immediate that default sets can be characterized by a closed interval, where only the upper bound is a function of assets $[\underline{y}, y^*(B))$. The default boundary $y^*(B)$ divides the $\{y, B\}$ space into the repayment and default regions and is decreasing in assets due to Proposition 1. At the boundary, the value of the contract equals the value of default: $v^d(y^*(B)) = v^c(B, y^*(B))$ for $B \in (\underline{B}, \bar{B})$. The equilibrium price $q(B')$ is, in turn, a function of the

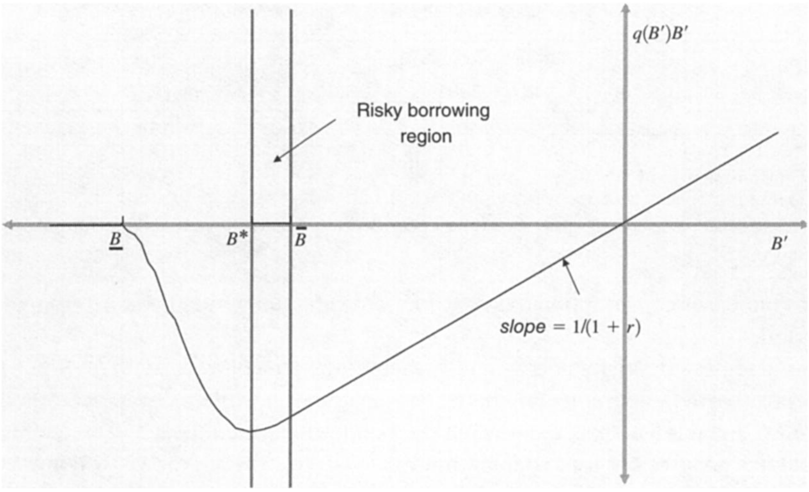

FIGURE 2. TOTAL RESOURCES BORROWED

default boundary and the distribution of shocks such that $q(B') = [1/(1 + r)][1 - F(y^*(B'))]$, where F is the cumulative probability distribution of shocks.

Equilibrium bond prices determine the borrower's budget set in every state y and B. In particular, each contract $\{q(B'), B'\}$ changes consumption today by the product $q(B')B'$, and the entire set of contracts available to the borrower is characterized by

$$q(B')B' = \frac{1}{1+r}[1 - F(y^*(B'))]B' \tag{12}$$

over the space B'. With i.i.d. shocks, the set of contracts available to the borrower is exactly the same every period for all income y states.[8]

Budget sets are bounded from above by $\Psi = \min_{B'}\left([1/(1 + r)][1 - F(y^*(B'))]B'\right)$ because bond prices go to zero as debt increases. The bond contract that generates the maximum increase in consumption is $\Psi \equiv q(B^*)B^*$. Figure 2 plots the set of contracts for a parameterized example and illustrates this endogenous borrowing limit at B^*.[9] Borrowing limits imply that the borrower faces a limited set of feasible consumption levels each period and that in some low-income, low-wealth state, although the borrower would like to increase his consumption further, he does not have access to such a loan contract and is, in turn, constrained.

The figure shows the total resources borrowed that are available for consumption, $q(B')B'$, under various asset choices. For all assets $B' \geq \overline{B}$, bond prices are the risk-free rate; for assets $B' \leq \underline{B}$, bond prices are zero and thus these contracts give zero resources to the borrower. For intermediate asset levels, $B' \in (\underline{B}, \overline{B})$, bond prices are increasing in the level of assets because $y^*(B')$ is decreasing in this range, but $q(B')B'$ is first decreasing and then increasing in B'. Figure 2 illustrates the endogenous "Laffer Curve" for borrowing that the model generates. The borrower would never choose optimally a bond contract with $B < B^*$ because he can find an alternative

[8] With persistent shocks, which are analyzed in the next section, the set of contracts available depends on today's state y.
[9] The figure is plotted for the case of i.i.d. Gaussian shocks, $h(y) = y$ and $\theta = 0$.

TABLE 1—BUSINESS CYCLE STATISTICS FOR ARGENTINA

	Default episode x: Q1–2002	std(x)	corr(x,y)	corr(x,r^c)
Interest rates spread	28.60	5.58	−0.88	
Trade balance	9.90	1.75	−0.64	0.70
Consumption	−16.01	8.59	0.98	−0.89
Output	−14.21	7.78		−0.88

contract that increases consumption today by the same amount while incurring a smaller liability for next period.

The relevant region for "risky borrowing" is then limited to contracts with $B' \in (B^*, \bar{B})$ because these carry positive default premia and increase consumption while incurring the smallest liability. Uncertainty in endowments smooths out the bond price function $q(B')$, extending the range of B' that carries positive but finite default premia to $(\underline{B}, \bar{B})$.[10] However, risky contracts that will be chosen in equilibrium correspond only to $B' \in (B^*, \bar{B})$ due to the endogenous Laffer Curve. Thus, for the region $B' \in (B^*, \bar{B})$ to be non-empty, the bond price function needs to decrease slowly enough such that lower asset levels are associated with larger capital inflows.[11]

Regarding the comovement between interest rates and income, the model generates a negative relation, even with i.i.d. shocks. The reason is that more debt is demanded in recessions, as in Mark Huggett (1993), which implies that although the bond price function is independent of the shock, recessions are associated with high interest rates. This produces a counterfactual feature, however: recessions are correlated with trade deficits. The following section analyzes the relation between interest rates, debt dynamics, and output for a persistent income process. Here, the negative relation between output and interest rates remains, while the empirically correct negative relation between trade balances and output emerges due to the state-dependent debt contracts offered.

III. Quantitative Analysis

A. *Data*

In December 2001, in one of the largest defaults in history, Argentina defaulted on \$100 billion of its external government debt, which represented 37 percent of its 2001 GDP. It also experienced a severe economic crisis, with output decreasing about 14 percent at the time of the default. This section documents this default event and the business cycle features of the Argentinean economy.

The data in Table 1 are quarterly real series, seasonally adjusted, and are taken from the Ministry of Finance (MECON). The business cycle statistics include all the data available up to the default episode, the last quarter of 2001. Output and consumption data are log and filtered with a linear trend; the series start in 1980. The trade balance data are reported as a percentage of output and the series start in 1993. The interest rate series are the EMBI for Argentina and are taken from the dataset in Neumeyer and Perri (2005) and MECON. The interest rate series

[10] In a deterministic model of borrowing with a varying but perfectly forecastable endowments sequence, the bond price function will jump from $1/(1+r)$ to zero at a threshold $B \leq 0$. In this case, default will not arise in equilibrium because default events can be perfectly forecasted.

[11] Although Figure 2 presents an example with a non-empty risky borrowing region, we find that, for some parameterizations, the default boundary and the price function become very steep and this region disappears.

TABLE 2—BUSINESS CYCLE STATISTICS FOR OTHER DEFAULTERS

Ecuador	Default episode x: Q3–1999	std(x)	corr(x,y)	corr(x,r^c)
Interest rates spread	47.58	5.44	−0.63	
Trade balance	10.96	4.47	−0.39	0.05
Consumption	−7.14	2.78	0.92	−0.53
Output	−6.46	2.53		−0.63
Russia	**Default episode x: Q4–1999**	**std(x)**	**corr(x,y)**	**corr(x,r^c)**
Interest rates spread	30.43	17.50	−0.70	
Trade balance	12.40	5.40	−0.17	0.86
Consumption	−17.20	7.08	0.79	−0.80
Output	−12.60	11.80		−0.70

start in the third quarter of 1983.[12] The interest rate spread is the difference between the interest rate for Argentina and the yield of the five-year US treasury bond.[13] The second column of Table 1 reports the standard deviations of all variables, and the third and fourth columns report correlations of each variable with output and interest rate spreads. The first column presents the deviations from trend of the variables in the first quarter of 2002, the default period.[14]

Output and consumption are negatively correlated with interest rate spreads. These negative relations are much stronger in the default episode because during the crisis output plummeted and spreads skyrocketed. Consumption is also more volatile than output, and the trade balance is countercyclical and positively correlated with spreads. Interest rate spreads in Argentina are high and volatile. The mean spread in Argentina from 1983 to 2001 is 10.25 percent. In addition, all variables experienced very dramatic deviations at the time of the default.

Table 2 presents statistics for business cycles and default events in two additional defaulter countries: Ecuador and Russia. The data are series taken from International Financial Statistics (IFS) and the Central Bank of Ecuador and are treated in similar fashion as for Argentina. The interest rate spread series are also their respective EMBI spreads. Both countries experienced a sovereign default in 1999, along with a deep recession.[15] In Ecuador and Russia, the time series properties for interest rates, output, and the trade balance are similar to the Argentinean case. The high volatility of interest rate spreads, together with the countercyclicality of interest rates and the trade balance, appear to be regularities for recent data in emerging countries.

B. *Calibration and Functional Forms*

The model is solved numerically to evaluate its quantitative predictions regarding the occurrence of default events, the business cycle properties of interest rates, consumption and the trade balance, and the real dynamics observed in emerging markets in times of default and crises.

The quantitative implementation of the model requires a flexible specification for default costs that increases the set of risky loans available, so that high default probabilities can be calibrated.

[12] Statistics for the trade balance and the interest rate spread are reported as percentages.

[13] The EMBI for Argentina is an index composed of Argentina's dollar bonds that are mostly long maturity. Thus, to calculate spreads, we use a long maturity US bond.

[14] The linear trend for the statistics in the default episode is computed with series covering the period up to 2005: II.

[15] More generally, David Miller, Michael Tomz, and Wright (2006) document that in the last century defaults generally occur during periods of low output.

Without direct output costs after default, the range of risky borrowing is very small and the equilibrium set of risky loans is limited, as Figure 2 illustrates. Thus, we assume that default entails some direct output cost of the following form:

$$h(y) = \left\{ \begin{array}{lll} \hat{y} & if & y > \hat{y} \\ y & if & y \leq \hat{y} \end{array} \right\}. \tag{13}$$

The asymmetric default output costs make the value of autarky a less sensitive function of the shock, which is key for extending sufficiently the range of B' that carry positive but finite default premia, $(\underline{B}, \overline{B})$. All else equal, a large set $(\underline{B}, \overline{B})$ increases the set of risky loans that can be attractive in equilibrium for borrowers $(B^*, \overline{B})$, giving the quantitative model the possibility to deliver the historical default probabilities.[16]

Moreover, output contractions after default of the form in (13) can be rationalized under two assumptions that are consistent with empirical observations during recent sovereign defaults: first, that sovereign default disrupts the functioning of the financial private sector and diminishes the aggregate credit available in the economy; and, second, that private credit is an essential input for production. The idea is that prior to default, given that private financial markets function well, credit can be adjusted according to shocks, and thus output covaries closely with the productivity shocks. After default, however, private credit is constrained, and thus output cannot be large, even under a good shock, because an essential input is scarce.[17]

Decline in credit and output contractions are features of recent sovereign defaults. Eduardo Borensztein, Eduardo Levy-Yeyati, and Ugo Panizza, for the Inter-American Development Bank (2007), document that the sovereign defaults of the last two decades have been accompanied by substantial decreases in private credit. For the case of Argentina, private credit was dramatically lower during the default period relative to the proceeding period: the cumulative private domestic credit during the 13 quarters when Argentina was in default (December 2001 to March 2004) was 454 billion real US dollars, or 53 percent of that during the 13 quarters prior to default, 855 billion real US dollars.[18] Using a comprehensive firm-level dataset for Ecuador, Arellano and Katya Kartashova (2007) find that during the 1999 sovereign default, which featured 24 percent reduction in private credit, firms with the largest dependency on credit decrease their output disproportionately and account for a large fraction of the output collapse.[19, 20]

In this paper, we assume this reduced form specification for default costs that is consistent with empirical observations, and use it to calibrate the historical default probability for Argentina. The discipline then is on how the model performs in terms of spread fluctuations and comovements, given an empirical default probability.

[16] Compare, for example, the set $(\underline{B}, \overline{B})$ arising when the default value is the value of permanent autarky and no output costs $v^d(y)$ to a new set $(\underline{B}^1, \overline{B}^1)$ arising when the default value is a constant corresponding to the autarky value of the lowest shock $v^d(\underline{y})$. The reason the new set is larger is that $\underline{B}^1 < \underline{B}$ because $v^c(\underline{B}^1, \overline{y}) = v^d(\underline{y}) < v^d(\overline{y}) = v^c(\underline{B}, \overline{y})$ and $v^c(B, y)$ is increasing in B.

[17] See Enrique Mendoza and Yue (2007) for a comprehensive model that formalizes a related idea.

[18] See Guido Sandleris (2006) for a model where sovereign defaults affect the availability of credit to the private sector. Jean Tirole (2003) also presents a model where international private lending is distorted by government interventions.

[19] The authors find that firms with short-term debt-to-asset ratios in the top 50 percentile in 1998 account for 80 percent of the aggregate sales decline of 19 percent in 1999. The disproportional decrease in sales for highly indebted firms is maintained even after controlling for firm-specific fixed effects in a panel regression.

[20] The output implications of financial constraints have been studied extensively in works such as Ben Bernanke and Mark Gertler (1989) and Nobuhiro Kyotaki and John Moore (1997). See also Mendoza (2006) for a quantitative exploration of the 1995 Mexican recession based on financial constraints.

TABLE 3—PARAMETERS

Risk-free interest rate	$r = 1.7\%$	US 5-year bond quarterly yield
Risk aversion	$\sigma = 2$	
Stochastic structure	$\rho = 0.945$, $\eta = 0.025$	Argentina's GDP
Calibration	Values	Target statistics
Discount factor	$\beta = 0.953$	3% default probability
Probability of reentry	$\theta = 0.282$	Trade balance volatility 1.75
Output costs	$\hat{y} = 0.969\, E(y)$	5.53% debt service to GDP

The following utility function is used in the numerical simulations:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}.$$

The risk aversion coefficient σ is set to two, which is a common value used in real business cycle studies. The risk-free interest rate r is set to 1.7 percent, which is the average quarterly interest rate of a five-year US treasury bond during this time period. The stochastic process for output is estimated from the series of Argentina's GDP. It is assumed to be a log-normal AR(1) process, $\log(y_t) = \rho \log(y_{t-1}) + \varepsilon_t^y$, with $E[\varepsilon^y] = 0$ and $E[\varepsilon^2] = \eta_y^2$. The estimated values are $\rho = 0.945$ and $\eta = 0.025$. The shock is then discretized into a 21-state Markov chain, using a quadrature-based procedure (George Tauchen and Robert Hussey 1991).

The time preference parameter β, the probability of reentering financial markets after default θ, and the default costs threshold $\hat{y}$ are calibrated to match the following moments of the Argentinean economy: a default probability of 3 percent, an average debt service–to-GDP ratio of 5.53 percent, and the standard deviation of the trade balance. The Argentinean government defaulted on its foreign debt three times in the last 100 years, which provides this rough estimate for a default probability.[21] The average debt service–to-GDP ratio in Argentina was obtained from the World Bank for 1980–2001. Table 3 summarizes the parameter values.

The calibrated probability to reenter financial markets of 0.282 is consistent with the estimates of Gaston Gelos, Ratna Sahay, and Sandleris (2004), who find that during the default episodes of the 1990s, economies were excluded from the credit markets for only a short period of time. The calibrated output costs are also consistent with the empirical observation that Argentina's output was below trend for 85 percent of the time while in state of default (December 2001 to March 2004) before the country renegotiated its debt.[22]

C. *Simulation Results*

This section first analyzes policy functions for the calibrated model and then examines its quantitative performance in comparison with the data.

Figure 3 shows the bond price schedule and the equilibrium interest rate faced by the borrower in the model, as a function of assets B (reported as ratio of mean output) for two income shocks that are 5 percent above and below trend. The left panel of Figure 3 plots the price schedule,

[21] David Beim and Charles Calomiris (2001) report two episodes of sovereign default in Argentina's foreign debt for 1900–2001: one in 1956 when Argentina defaulted on its suppliers' credits in the post-Peron budget crisis, and another in 1982 when it defaulted on its foreign bank loans in the midst of another budget crisis. In 2001, Argentina defaulted a third time on their foreign debt.

[22] For the case of the sovereign defaults in Russia and Ecuador, aggregate GDP was below trend for 100 percent of the time before each country renegotiated its debt.

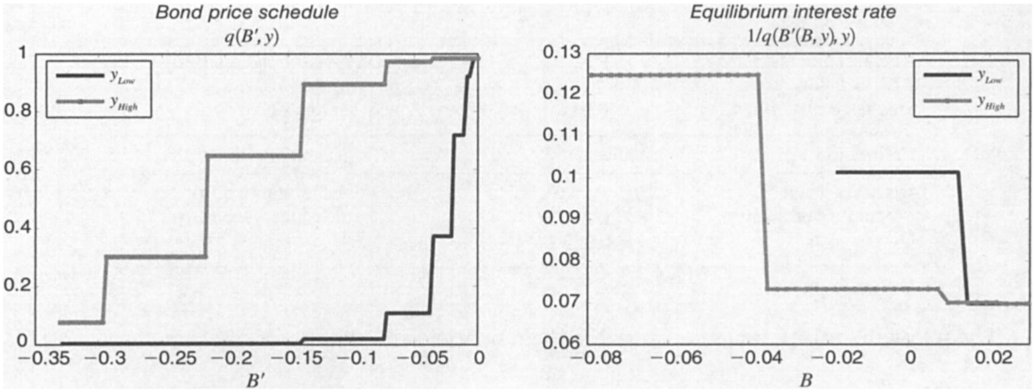

Figure 3. Bond Prices and Assets

which determines the set of contracts $\{q(B',y),B'\}$ the borrower can choose from every period. Bond prices are an increasing function of assets, making larger levels of debt carry higher interest rates. Importantly, booms are associated with more lenient financial contracts, as the interest rate charged for every loan size is lower during booms. In fact, the model delivers countercyclical borrowing constraints with booms having much looser borrowing limits than recessions: $B^*(y_{High}) < B^*(y_{Low})$. The reason is that default is preferable mostly during recessions, and shocks are persistent. Thus, a low shock today predicts that tomorrow the shock will likely be low again, and this is when the borrower defaults even for a small amount of debt. The endogenous countercyclical interest rate schedule due to default is the essential mechanism for the model to match the data in emerging markets.

The right panel of the figure shows the actual annual interest rate $1/q(B',y)$ the economy pays along the equilibrium path in state $\{B,y\}$ given its choice of borrowing $B'(B,y)$. If assets relative to output are above -0.02, in recessions the borrower chooses relatively higher levels of debt and thus faces higher interest rates. However, if initial assets are smaller (larger debt) then in recessions the borrower defaults while in booms he chooses to borrow risky.

The borrower of the model has essentially two instruments to affect his time path of consumption: borrowing and default. The use of debt is twofold: First, debt is used to smooth income fluctuations relative to the mean level of income and mean debt, as in standard incomplete market models (Mendoza 1991). Second, given that β is lower than the inverse of the risk-free interest rate, debt can be used to tilt the consumption profile toward the present. In standard models with incomplete assets and a noncontingent borrowing constraint, this second effect is reflected simply by a lower mean in asset holdings in the limiting distribution.[23] In this default model, however, the financial contracts available are state dependent, and thus front loading consumption is easier in high-income shocks when debt is in fact cheaper and borrowing limits are loose.

The left panel of Figure 4 presents the savings policy function $B'(B,y)$ conditional on not defaulting as a function of assets B for a high and a low y shock. Savings B' and assets B are reported as a percentage of mean output, and the two y shocks are 5 percent above and below trend. When wealth is large ($B > 0.1$), the economy saves less in recessions than in booms, as in standard models (Huggett 1993). When wealth is small and negative, however, the economy borrows more in booms than in recessions because of the countercyclical interest rate schedules.

[23] In fact, in standard incomplete markets models with a noncontingent borrowing constraint, it is a requirement that $\beta(1 + r) < 1$ in order to have $u'(c_t)$ converging to a random variable, and thus to have a limiting distribution of assets with a finite mean.

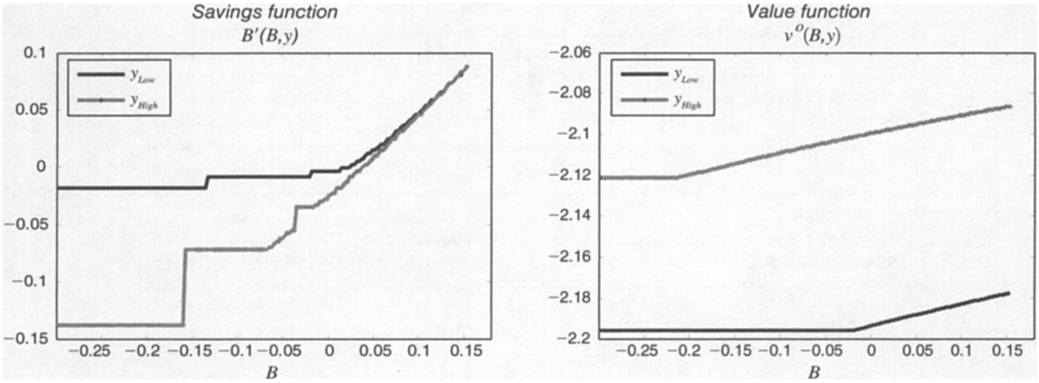

Figure 4. Savings and Value Functions

When wealth is small the borrower would like to borrow heavily during bad shocks, but it cannot because such financial contracts are not available. In fact in recessions the borrower is often at the constraint.

The second policy the borrower has is whether to default. The right panel of Figure 4 shows the value of the option to default or repay, $v^o(B,y)$, as a function of assets B for a high and a low y shock. For a given output realization, default is chosen for all levels of assets below a threshold—when the outside option is better than the option of staying in the contract. In the figure, default is chosen for assets less than -2 percent of mean output when y is 5 percent below trend, and for assets less than -21 percent of mean output when y is 5 percent above trend. The particular thresholds are somewhat mechanical given the assumed reduced form of the default value. However, if one compares the thresholds of assets for each output realization below which default is chosen, the model delivers defaults for larger assets levels when output is lower. Thus, for a given level of assets, having the option to default reduces the spread in lifetime utility across shocks and completes markets, as in Zame (1993). In fact, the asymmetric costs from default amplifies the role of default as a policy for completing markets.

An interesting feature of the model that matches the data is that larger capital outflows (i.e., $y - c$) can occur in recessions because here is when interest rates are high and borrowing is constrained. For example, when debt is 2 percent of output, the consumption-output ratio when the shock is 5 percent above trend is 1.04, whereas when the shock is 5 percent below trend this ratio is 0.99. This result is similar to that of Andrew Atkeson (1991), where he shows that in an insurance model of debt that features moral hazard and unenforceability of debt contracts, the optimal debt contract will feature capital outflows in recessions. Here, the result is driven by the incompleteness of assets and the endogenous cyclical borrowing constraints that arise due to default risk.

We now turn to discuss the quantitative predictions of the model in terms of matching the data. As Table 4 shows, the model matches well the business cycle statistics in Argentina. To make the model business cycle statistics comparable to the data, we choose the observations prior to default events from the limiting distribution of assets. In particular, we simulate the model over time, find 100 default events, extract the 74 observations before the default event, and report mean statistics from these 100 samples.[24] The time series in the model are treated in an equal fashion as in the data.

[24] We choose 74 observations prior to a default event to mimic the period length between 1983:III and 2001:IV in Argentina, which constitutes the period between default events.

TABLE 4—BUSINESS CYCLE STATISTICS IN THE BENCHMARK MODEL

	Default episodes	std(x)	corr(x, y)	corr(x, r^c)
Interest rates spread	24.32	6.36	−0.29	
Trade balance	−0.01	1.50	−0.25	0.43
Consumption	−9.47	6.38	0.97	−0.36
Output	−9.60	5.81		−0.29
Other statistics				
Mean debt (percent output)	5.95	Mean spread		3.58
Default probability	3.00	Output deviation in default		−8.13

In terms of the calibrated parameters, the model approximately matches the probability of default, the volatility of the trade balance, and the ratio of debt to GDP. In the model, low β, low θ, and low $\hat{y}$ all tend to increase the mean debt level. As illustrated in Aguiar and Gopinath (2006), however, exclusion costs alone, which are parameterized by θ, are not enough to quantitatively sustain large levels of borrowing because the welfare costs of fluctuations are small, as in Lucas (1987).

The model matches the data in that it simultaneously delivers a higher volatility of consumption relative to income, countercyclical interest rates, and a countercyclical trade balance. Matching these three moments is surprising given that this is an insurance model of debt. However, the cyclical borrowing schedules provide a mechanism for generating these features. Consumption in recessions is close to output because borrowing is very expensive and the borrower is constrained. In booms, however, debt is cheap and is used to tilt the consumption profile, especially when wealth is low. Thus, in good times the trade balance is negative, spreads are low, and consumption is higher than output, making consumption more volatile than output, on average.[25] State-contingent financial contracts that are harsher in recessions provide a unified rationale for the fluctuations of consumption and the trade balance in emerging markets. This mechanism can potentially complement that in Aguiar and Gopinath (2007), where consumption and trade balance fluctuations can also be understood as an optimal response to shocks that are permanent even under perfect financial markets.

The model matches the volatility of interest rate spreads in Argentina. Varying default probabilities seem to be the driving force for the spread volatility, as an average default probability calibrated to 3 percent is enough to account well for it. Time-varying default probabilities alone cannot, however, account for the level of spreads. The model generates a mean annual spread of 3.58 percent, which is smaller than the mean spread in Argentina of 10.25 percent. The reason for this anomaly is the one-to-one mapping from default probabilities to spreads due to risk neutral pricing. Yet, as documented in Fernando Broner, Guido Lorenzoni, and Sergio Schmukler (2005), excess returns are an important component of interest rate spreads. Below, we experiment with how variations in the pricing kernel can address this anomaly.[26]

Table 4 also reports mean percentage deviations for the statistics in the model during the period prior to the default event. In periods of default, the model economy experiences significant collapses in consumption and output, and high interest rate spreads, as in Argentina. However,

[25] Persistence in shocks is essential for the model to generate these facts. When shocks are i.i.d., the bond price schedule is independent of the shock and the model behaves similarly to standard income fluctuation models with incomplete markets delivering lower volatility of consumption relative to income and a procyclical trade balance.

[26] The fact that default probabilities do not account for all the spread in bonds is a well-known puzzle in the finance literature on corporate defaultable bonds (Jing-Zhi Huang and Ming Huang 2003).

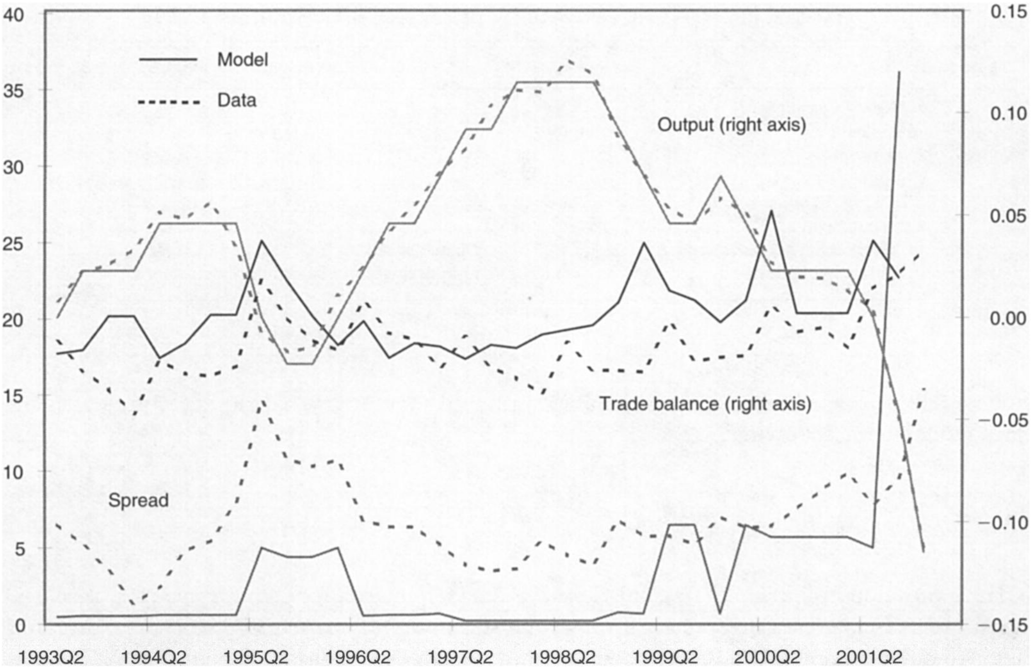

FIGURE 5. ARGENTINA AND MODEL TIME SERIES

the model underestimates the massive collapse and misses the reversal in the trade balance observed. Finally, the mean output deviation during the periods when the economy is in default and excluded from financial markets is −8.13 percent in the model, which matches closely the mean deviation from trend of Argentinean output of −7.3 percent while in a state of default.

The model can predict the recent default in Argentina. We feed into the model the time series of Argentina's GDP starting in 1993, and the model predicts a default in the fourth quarter of 2001, the period when the Argentinean government defaulted. Figure 5 plots the time series of output, trade balance, and interest rate spreads in the data and in the model. The model predicts the higher spreads experienced in Argentina in the periods between 1995–1996 and 2000–2001. It underestimates the relatively high spreads between 1996 and 1999 because income is very high and the probability of default is close to zero. But, overall, the model does well at tracing the spread dynamics in Argentina. The dynamics of the trade balance are traced less well by the model, but it predicts the trade balance surpluses during 1995–1996 and 2001.[27]

D. *Risk Averse Pricing*

The main anomaly of the benchmark model is the low average interest rate spread it generates with a default probability calibrated to the historical average. Risk neutral pricing establishes a tight link between default probabilities and spreads which is at odds with the data. This section introduces an example where default risk premium is the additional component in the spread of defaultable bonds. We model directly the lenders' stochastic discount factor m as a stochastic

[27] If we feed in shocks starting in 1983, the model predicts an additional default event in the third quarter of 1989 because GDP in Argentina was 20 percent below trend in this period. Standard & Poors actually states 1989 contained an additional default event in Argentina.

TABLE 5—BUSINESS CYCLE STATISTICS WITH RISK AVERSE PRICING KERNEL

	Default episodes	std(x)	corr(x, y)	corr(x, r^c)
Interest rates spread	53.69	10.65	−0.22	
Trade balance	−0.69	2.89	−0.15	0.17
Consumption	−8.11	7.17	0.91	−0.24
Output	−8.37	5.90		−0.22
Other statistics				
Mean debt (percent output)	5.95	Mean spread		10.40
Default probability	3.00	Output deviation in default		−7.21

process that prices default risk. In particular, we modify the pricing equation (5) in the benchmark model to the following:

$$q(B', y) = \int_{A(B')} m(y') f(y', y)\ dy'. \tag{14}$$

Time variation in the lender's pricing kernel—lender's intertemporal marginal rate of substitution—affects interest rate spreads through the sensitivity of the lender's stochastic discount factor to default events. If defaults occur when the lender's stochastic discount factor is high, defaultable loans will carry a premium higher than the probability of default. The idea is that lenders will require a default risk premium to compensate for the fact that the low default payoff happens when their stochastic discount factor is high. Moreover, the extent to which this co-variation generates larger spreads depends on the volatility of the lenders' pricing kernel.

To make this specification comparable to the benchmark model, we assume that m is an i.i.d. random variable with a constant mean equal to the inverse of the risk-free rate and with an innovation correlated with the small open economy's income. In particular, we assume m follows this process: $m_{t+1} = 1/(1 + r) - \lambda \varepsilon^y_{t+1}$ such that $E(m) = 1/(1 + r)$ and $\text{var}(m) = \lambda^2 \eta^2_{\varepsilon^y}$. For $\lambda > 0$, the correlation between the endowment process (in logs) and the lenders' stochastic discount factor is $-(1 - \rho)$.

The parameters λ and β are calibrated in this example such that the model reproduces the average spread and the historical default probability. We maintain all other parameters equal to the benchmark model. The calibrated values are $\beta = 0.882$ and $\lambda = 24$. Table 5 presents the business cycle statistics for this case. As the table shows, this parameterization breaks the link between the average spread and the default probability, bringing the model closer to the data. In terms of business cycles, this parameterization delivers similar statistics as the benchmark model but overestimates the volatility of the trade balance and spreads.

These results show that default risk premium can potentially rationalize the large difference between historical default probabilities and spreads if lenders have a sufficiently high stochastic discount factor in default states. The large sensitivity (parameterized by λ) of the lenders' pricing kernel required is equivalent to a high degree of risk aversion in the lenders' marginal rate of substitution such that the compensation for risk is large.[28] The relation between defaults and the lenders' stochastic discount factor could be rationalized in a model where lenders are specialists in emerging market assets and have portfolios with returns affected by particular default events.

[28] This finding relates to the vast literature on asset pricing that documents that high risk aversion is needed for models to generate the large stock excess returns observed in the data.

A precise modeling of these issues is important, and Sandra Lizarazo's (2006) work is a step in this direction.

IV. Conclusion

This paper models endogenous default risk in a stochastic dynamic framework of a small open economy that features incomplete markets. The paper presents a model where interest rates respond to output fluctuations through endogenous time-varying default probabilities. In the first part, the paper studies analytically the relationship between default and output in an environment of incomplete assets, and establishes that incomplete markets deliver default events in recessions. Second, it explores quantitatively the predictions of the model in explaining the real dynamics observed during the 2001 Argentinean default. The model predicts the recent default and can match well multiple features of the data, such as the volatility of interest rates, the high volatility of consumption relative to income, the negative correlation between output and interest rates, and the negative correlation between the trade balance and output.

Even though this paper provides a framework to study sovereign defaults and fluctuations in country spreads, our understanding of international interest rates in emerging markets is still at a very early stage. The growing literature on quantitative models of sovereign defaultable debt is studying such other important issues as: alternative borrowing motives and bailouts (Aguiar and Gopinath 2006), renegotiation with creditors (Yue 2006), default risk premium (Lizarazo 2006), political economy considerations (Cuadra and Sapriza 2006), risk sharing implications (Yan Bai and Jing Zhang 2005), and optimal maturity structure (Arellano and Ramanarayanan 2007). Given the significant costs for emerging markets associated with default and high and volatile interest rates, the further study of these issues seems of special value.

Appendix 1

PROPOSITION 1: *For all $B^1 \leq B^2$, if default is optimal for B^2, in some states y, then default will be optimal for B^1 for the same states y, that is, $D(B^2) \subseteq D(B^1)$.*

This result is similar to Eaton and Gersovitz (1981) and Chatterjee et al. (2007).

For all $\{y\} \in D(B^2)$, $u(y) + \beta E(\theta v^o(0,y') + (1-\theta)v^d(y')) > u(y + B^2 - q(B',y)B') + \beta E v^o(B',y')$. Since $y + B^2 - q(B',y)B' > y + B^1 - q(B',y)B'$ for all B', $u(y + B^2 - q(B',y)B') + \beta E v^o(B',y') > u(y + B^1 - q(B',y)B') + \beta E v^o(B',y')$. Thus, the value of the contract under no default is increasing in foreign asset holdings. Hence, $u(y) + \beta E(\theta v^o(0,y') + (1-\theta)v^d(y')) > u(y + B^1 - qB') + \beta E v^o(B',y')$, which implies that $\{y\} \in D(B^1)$.

PROPOSITION 2: *If, for some B, the default set is nonempty $D(B) \neq \emptyset$, then there are no contracts available $\{q(B'),B'\}$ such that the economy can experience capital inflows, $B - q(B')B' > 0$.*

This is a proof by contradiction. Suppose there are contracts $\{q(B'),B'\}$ available to the economy such that $B - q(B')B' > 0$, but that the government chooses under the contract utility some $\hat{B}$ to maximize utility such that $B - q(\hat{B})\hat{B} < 0$, and then finds default to be the optimal option because $u(y) + \beta E v^d(y') > u(y + B - q(\hat{B})\hat{B}) + \beta E v^o(\hat{B},y')$.

Now note that under all contracts $\{q(B'),B'\}$ that deliver $B - q(B')B' > 0$, staying in the contract is always preferable to default because $Ev^o(B',y') \geq Ev^d(y')$, and $u(y+B - q(B')B') > u(y)$. This implies that $\hat{B}$ cannot be the maximizing level of assets and then default be optimal, because it is a contradiction.

Thus, if $D(B) \neq \emptyset$, given that B' is chosen to maximize the value of the contract, then it must be that not only $B - q(B')B' < 0$ but also it $\nexists$ a contract available $\{q(B'),B'\}$ such that $B - q(B')B' > 0$.

PROPOSITION 3: *Default incentives are stronger the lower the endowment. For all* $y_1 \leq y_2$, *if* $y_2 \in D(B)$, *then* $y_1 \in D(B)$.

If $y_2 \in D(B)$, then by definition $u(y_2) + \beta E v^d(y') > u(y_2 + B - q(B')B') + \beta E v^o(B',y')$. If

$$\text{(A1)} \qquad u(y_2 + B - q(B^2)B^2) + \beta E v^o(B^2,y') - \{u(y_1 + B - q(B^1)B^1) + \beta E v^o(B^1,y')\}$$
$$> u(y_2) + \beta E v^d(y') - \{u(y_1) + \beta E v^d(y')\},$$

then $y_2 \in D(B)$ implies $y_1 \in D(B)$. Now, it is necessary to show that expression (A1) holds.

Given that shocks are i.i.d., the right side of equation (A1) simplifies to $[u(y_2)] - [u(y_1)]$ and, because of utility maximization,

$$u(y_2 + B - q(B^2)B^2) + \beta E v^o(B^2,y') \geq u(y_2 + B - q(B^1)B^1) + \beta E v^o(B^1,y').$$

Thus, if

$$\text{(A2)} \qquad u(y_2 + B - q(B^1)B^1) + \beta E v^o(B^1,y') - \{u(y_1 + B - q(B^1)B^1) + \beta E v^o(B^1,y')\}$$
$$> \{u(y_2) - u(y_1)\}$$

holds, then through transitivity expression (A1) holds.

Simplifying (A2):

$$u(y_2 + B - q(B^1)B^1) - u(y_1 + B - q(B^1)B^1) > u(y_2) - u(y_1).$$

Due to Proposition 2, if $y_2 \in D(B)$ then $B - q(B')B' < 0$ for all available $\{q(B'),B'\}$, thus $B - q(B^1)B^1 < 0$. Hence, given that utility is increasing and strictly concave, then (A2) holds, which implies that $y_1 \in D(B)$.

Appendix 2: Computational Algorithm

The following algorithm is used to solve the model:

1. Start with some guess for the parameters to be calibrated: β, θ, and $\hat{y}$ and a discretized state space for assets consisting of a grid of 200 points equally spaced.

2. Start with a guess for the bond price schedule such that $q^0(B,y) = 1/(1 + r)$ for all B' and y.

3. Given the bond price schedule, solve the optimal policy functions for consumption $c(B,y)$, asset holdings $B'(B,y)$, repayment sets $A(B)$, and default sets $D(B)$ via value function iteration. For each iteration of the value function, we need to compute the value of default which is endogenous because it depends on the value of the contract at $B = 0$. We iterate on the value function until convergence for a given q^0.

4. Using default sets and repayment sets, compute new bond price schedule $q^1(B,y)$ such that lenders break even and compare it to the bond price schedule of the previous iteration: $q^0(B,y)$. If a convergence criterion is met, $\max\{q^0(B,y) - q^1(B,y)\} < \varepsilon$, then move to the next step. Otherwise, update the price using a Gauss-Seidel algorithm and go back to step 3.

5. Compute business cycles statistics from 100 samples of data containing a default. If the model business cycles match the data we stop; otherwise we adjust parameters and grid, and go to step 2.

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