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Examining the Chinese Debt-Trap Diplomacy

陳家威

Chia-Wei Chen

指導教授：何泰寬博士

Advisor: Tai-kuang Ho, Ph.D.

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Chapter 1: Introduction

As China's economic influence continues to grow, its lending practices to developing countries have come under scrutiny. The concept of "debt-trap diplomacy", whereby China extends excessive loans to countries in exchange for political or economic concessions, has become a topic of heated debate (Chellaney, 2017).

From the political science aspect, an analysis from the Belfer Center for Science and International Affairs states that China utilizes the debt-trap diplomacy as a technique to achieve strategic objectives, such as projecting power across South Asian trading routes, undermining regional opposition to its South China Sea claims, and supporting its naval efforts to break out into the Pacific (Parker and Chefitz, 2018). On the opposite side of the spectrum, critics of this phenomenon argue that claims of the debt-trap diplomacy are often exaggerated or based on incomplete information. For example, Brautigam (2020) argues that the debt-trap is based on a flawed understanding of Chinese lending practices and the histories of the target countries, and that China is not strategically pursuing the debt-trap diplomacy on developing countries.

Does the excessive debt to China cause potential impact on the economy of low income developing countries (LIDC), especially those in the Belt and Road Initiatives (BLI)? Not many empirical studies examine this issue. On one hand, the lack of transparency in the Chinese lending system, whereby loan terms, conditions and collateral requirements are not always disclosed to the borrowers, makes it difficult for economists to fully grasp the magnitude of the issue. On the other hand, the decision for a country to continue honoring debt obligations is typically an optimal decision in the absence of enforcement, hence it

is not obvious whether the country is genuinely in good standings, or is it that the country would actually be better off if it were to default, but is somehow forced not to due to other enforcement, in this case might be the political leverage from China. As a result, recent studies on whether debt-trap diplomacy is a myth have primarily been conducted normatively in the field of political science, rather than a positive economics analysis (See, e.g., Himmer and Rod, 2023; Chen, 2020).

In this thesis, I aim to shed light on whether a country fell into the debt-trap by using the sovereign debt model proposed by Na et al. (2018) and the graphical approach presented by Hinrichsen (2021), to provide insights into the sustainability of the debt of borrowing countries. The concept of addressing the issue through the above methods follows Ho (2023). By calibrating the model for a particular country, a set of tradable-output levels which would cause the country to default could be obtained, given its current debt level. The approach of Hinrichsen (2021) then allows us to present the default set graphically, with each data point on the space representing a debt-output pair for a specific year. This visual representation allows for an examination of whether the country has been in the default zone but has managed to avoid default due to other enforcement mechanisms.

In particular, the default sets for Sri Lanka and Pakistan are examined in this thesis. Sri Lanka is mentioned in the origin of the debt-trap diplomacy narrative (Chellaney, 2017), and Pakistan, being the centerpiece of the China Pakistan Economic Corridor (CPEC), is also discussed frequently in the literature (Hurley et al., 2019).

To my best knowledge, there are so far no empirical study on the issue of debt-trap diplomacy based on an economic model with microfoundation. My thesis contributes to the literature of debt-trap diplomacy by conducting the first empirical result based on a sovereign debt model calibrated with the data of receiving countries under debate, to investigate whether the loans provided by China indeed push the countries to the brink of default, and report counterfactual results of China not lending the considerable amount of loans to the countries of the BRI.

The remaining chapters are organized as follows: In Chapter 2, a comprehensive literature review is conducted on the topics of Debt-trap diplomacy and sovereign default models. Chapter 3 briefly describes the characteristics of external debts to China, as well as the current situation of Sri Lanka and Pakistan. Chapter 4 outlines the specific sovereign debt model that will be applied in this study. Chapter 5 presents the calibration of the model and provides the corresponding empirical results. Finally, Chapter 6 concludes the thesis.

Chapter 2: Literature Review

Debt-Trap Diplomacy

Whether the debt-trap diplomacy is just a conspiracy used as an instrument for the western countries to justify its political strategy, or is it in fact causing stress on the receiving countries intentionally, is continuously under great debates and defenses.

The term “Debt-trap Diplomacy” is first coined in Chellaney (2017), which states that the infrastructures supported financially by the China government in Sri Lanka is burdensome and causing small and poor countries to endure the unsustainable loans, forcing them to cede strategic leverage to China. From a political aspect, researchers from the Belfer Center for Science and International Affairs propose that China may pursue three main strategic objectives using this approach (Parker and Chefitz, 2018). These objectives include: expanding its “String of Pearls” to address the “Malacca Dilemma¹” and extend its influence along crucial South Asian trade routes, destabilizing and fragmenting the regional coalition led by the United States that challenges China’s claims in the South China Sea, and facilitating the People’s Liberation Army Navy (PLAN) in advancing beyond the “Second Island Chain” and into the open waters of the Pacific Ocean. This is considered as the reshaping of “soft” infrastructure that China is hoping to enhance (Hillman, 2018).

A common criticism to the loans from China is that the terms and conditions are typi-

¹ The Strait of Malacca, located between Malaysia and Indonesia, is one of the busiest and most critical shipping routes in the world, connecting the Indian Ocean and the Pacific Ocean. The “Malacca dilemma” refers to the strategic vulnerability faced by China due to its heavy dependence on the Strait of Malacca for maritime trade and energy imports (Parker and Chefitz, 2018).

cally not transparent compared to other creditors in order to encourage their dependency to China (Tillerson, 2018). For example, an analysis on the contract of 100 China's long foreign lending suggests that the terms of these agreements feature unusual confidentiality clauses, seek advantages over other creditors through collateral arrangements, and potentially grant influence over debtors' policies (Gelpern et al., 2022). The authors' analysis reveals that the contracts signed with Chinese state-owned entities after 2014 in their study frequently contain extensive clauses to maintain confidentiality, which impose broad obligations on the debtor to keep contract details and related information undisclosed. Furthermore, about 30% of these agreements require the borrowing country to establish a dedicated bank account, typically controlled by the lender, as a form of collateral for debt repayment. These revenue accounts, uncommon in sovereign lending, restrict the borrowing country's authority over its own finances. Additionally, compared to other types of debt contracts, Chinese agreements more commonly include cross-default clauses, allowing lenders to demand immediate repayment (called an *acceleration*) if the borrower defaults on other lenders. These contract characteristics, along with their creative design to manage credit risks and enforcement hurdles, portray China as a powerful and commercially astute lender to developing nations.

There are several studies regarding the estimation of debt vulnerability on caused by China's loan. Hurley, Morris and Portelance (2019) studies the debt sustainability in BRI countries by examining their debt-to-GDP ratio versus the share of China's debt. Following the threshold of 50-60 percent rising debt-to-GDP ratio constructed by Chudik et al. (2015), they identify eight countries² that are particularly risky. Bandiera and Tsiropoulos (2020) examines the impact of investment and infrastructure projects under the BRI on the debt-to-GDP ratios of recipient countries. It analyzes the growth effects of BRI investment and estimates the potential increase in debt vulnerabilities for certain countries through a model-based growth projection. The findings suggest that approximately 28%

² These countries are Djibouti, Kyrgyzstan, Laos, the Maldives, Mongolia, Montenegro, Pakistan, and Tajikistan

of BRI investment recipient countries, consisting of 7 low-income developing countries and 5 emerging markets, are expected to face increased debt vulnerability in the medium term, while 37% of countries, including 5 low-income developing countries and 6 emerging markets, may experience a rise in their debt-to-GDP ratio due to BRI investment and financing in the long term, with 8 of them being vulnerable to changes in financing costs.

In stark contrast, critics of the debt-trap diplomacy narrative often argue the benefit of China's lending on the receiving country, and state that the concerns are often exaggerated. Eom, Brautigam and Benabdallah (2018) argues at the time (2018) that Chinese loans did not play a significant role in causing debt distress in Africa. They identify 17 African countries that were low-income and under high risk of debt distress, and find that for less than half (8) of the countries, the levels of debt to China were relatively small compared to their total external debt, and that the debt distress were caused primarily by other conflicts in the nation; six other countries had higher loans from China, but so were to other official creditors; in only three countries, Chinese loans were the main contributions to the debt distress. Brautigam (2020) indicates that debtor countries have voluntarily accepted Chinese loans and report positive experiences, suggesting that concerns over Chinese infrastructure funding are exaggerated, as many view China as an appealing economic model and development partner. In the case of Sri Lanka, Brautigam (2020) also argues that the project of Hambantota Port was the concept of former President Mattala Rajapaksa, and that Chinese Banks have shown willingness to assist their restructuring of existing loans. The Rhodium Group reviews 40 cases of China's external debt renegotiations, and finds that not only are asset seizures a rare occurrence, but China is limited in the leverage in negotiation due to external events such as change in leadership (Kratz, Feng and Wright, 2019).

Notably, governments of the indebted countries often defends their own decision of loan. The Minister of Finance of the Trinidad and Tobago, for instance, argued that choosing a loan without the need for retrenchment, currency devaluation, or other adverse mea-

tures, especially when the interest rates are similar, is an obvious and favorable choice³.

Sovereign Debt Model

Sovereign debt models under the Eaton-Gersovitz framework has been widely used to analyze default (Eaton and Gersovitz, 1981). Developed by Aguiar and Gopinath (2006) and Arellano (2008), several important features are included in the framework recently. For instance, Schmitt-Grohé and Uribe (2016) extends the model by implementing the nominal wage rigidity into the model, and Na et al. (2018) further investigates the role of government's optimal policy under wage rigidity in a decentralized economy to study the Twin Ds phenomenon. They find that being able to freely set the optimal taxation rate and devaluation rate improves the welfare of a country by reducing unemployment, which provides further incentive for a country to default. Arellano et al. (2023) further considers the fact that contrary to the assumption of defaulting completely to all creditors and resetting all its debts after a default episode, as in standard Eaton-Gersovitz models, the data shows that emerging countries usually partially default, and that defaults are accompanied by haircuts but not reductions in debt. The authors proposed a theoretical model to deal with this gap in the sovereign default literature. In the output models mentioned above, output cost is set exogenously with an ad hoc loss function; Mendoza and Yue (2012) endogenizes output cost by combining sovereign default and business cycle.

A rich literature examines the default episodes using calibration on Argentina (Arellano, 2008; Schmitt-Grohé and Uribe, 2016; Mendoza and Yue, 2012; Na et al., 2018). Hinrichsen (2021) examines the effect of war reparations on countries' default set using data from France in the 1870s, Germany in the 1930s, and Finland in the 1940s. Ho and Ritschl (2023) investigate transfer protection in the Dawes Plan by calibrating the German economy during the 1920s.

³ Loop News “Imbert: Choosing between IMF, Chinese loan a ‘no-brainer,’” June 15, 2021

Chapter 3: Debt to China

International debt to China lacks transparency since their official lending is predominantly undertaken by state-owned entities, and unlike other major economies, the Chinese government does not report or publish any data on its official international lending or outstanding overseas debt claims (Horn, Reinhart and Trebesch, 2021).

Horn, Reinhart and Trebesch (2021) combines a variety of sources to construct a consensus database of Chinese official loans. Their database spans from 1949, the establishment of the People's Republic of China, to 2017. It contains a granular dataset of 2151 loans and 2824 grants with information such as the creditor agent, borrower type, commitment, maturity, etc. It also provides an aggregate panel data of the external debt to China for each country. To have a more precise data of the China debt issue, the database constructed by Horn, Reinhart and Trebesch (2021) is utilized as the primary source of Chinese debt data. This database is combined with data from other creditors obtained from the International Debt Statistics (IDS) from World Bank.

The top 30 countries with the largest debts to China's official creditors are displayed in Figure 1a. Notably, Russia owes China over \$70 billion, while Pakistan's debt amounts to \$27 billion, both topping the list. Brazil and Venezuela are among the top 10 countries with the highest debt to China in Latin America. Contrary to what many people believe, African countries have not borrowed much from China. However, if we consider the ratio of Chinese debt to GDP in Figure 1b, some African countries appear to be highly indebted to China. Djibouti, for instance, has an alarming ratio of 68.5% of its GDP consisting of Chinese debt, while Tonga, Niger, and Zambia have ratios exceeding 10%. This result is

in line with the description in Eom et al. (2018).

A main finding in Horn, Reinhart and Trebesch (2021) is that China had become the world's largest creditor to developing countries after 2013, surpassing the amount of World Bank. Figure 2 shows the change of the total amount of debt from different main creditors, including China, World Bank¹, IMF, and the aggregation of all countries in the Paris Club. The debt amount started to rise rapidly after 2000, when the China government launched the "Go Out Policy" in 1999. In 2017, the debt to China in global had reached \$355 billion, while the debt to World Bank was \$300 billion.

The main focus on my thesis is to evaluate the debt sustainability for a country after it receives the considerable amount of loans from China. Among all countries, Sri Lanka and Pakistan are often under the discussion regarding the debt-trap issue. From the geostrategic aspect, Sri Lanka could serve as the military base for China (Chellaney, 2017), and Pakistan allows China to better connect with the Arabian Sea (Hurley et al., 2019). A preliminary view of the change in debt to China for the two countries in Figure 3 also provides some idea of how China is accumulating an increasing amount of debt to the two country. I briefly introduce the debt-trap diplomacy narrative regarding the two countries in the following sections.

Sri Lanka

In the original article where the terminology "Debt-trap Diplomacy" was coined, Chellaney (2017) specifically mentioned the predicament faced by the Sri Lankan government. He argues that the Chinese government supported large infrastructure projects in Sri Lanka and provided heavy loans to their government, and as the project eventually failed to repay the debt, the country is then ensnared in the concessions to China. Figure 3a shows the change in composition of the creditors to Sri Lanka.

The issue of the Hambantota port is regarded as a typical example of a debt-trap for

¹ Including the International Development Association (IDA) and the International Bank for Reconstruction and Development (IBRD)

some scholars (Moramudali, 2020). The construction of the port was initiated in 2007 and entrusted to the state-owned Chinese companies — China Harbour Engineering Company and Sinohydro Corporation. The project was valued at \$361 million, of which Exim Bank financed 85% at a yearly interest rate of 6.3%².

China's involvement in Sri Lanka's infrastructure development was facilitated by President Mahinda Rajapaksa, during which China became Sri Lanka's leading investor and lender. This gave China significant diplomatic leverage over Sri Lanka (Chellaney, 2017). However, when Rajapaksa was unexpectedly defeated in the early 2015 election by Maithripala Sirisena, who campaigned on the promise to extricate Sri Lanka from the Chinese debt trap, work on major Chinese projects was suspended.

However, Sri Lanka's government was already on the brink of default, and Sirisena eventually acquiesced to a series of Chinese demands in 2017³, including the sale of a 70% stake in the Hambantota port to China Merchants Port (CM Port) and a 99-year lease. Notably, as argued by Moramudali (2019), the lease did not write off the loans obtained to construct Hambantota port. The proceeds from the lease were used to boost the country's dollar reserves in 2017-18, especially in preparation for the large amount of external debt that needed to be serviced when international sovereign bonds matured in early 2019. This means that the lease is not a debt-equity-swap, as common narratives elaborated (Moramudali, 2020). Sri Lanka eventually declared a suspension on payment on most foreign debt from April 12, 2022. Whether Sri Lanka was indeed already under the extreme "brink of default" during 2015 is a major gap in the literature of sovereign default that has not yet been investigated.

² From AidData Project ID #33409, "China Eximbank provides \$306.7 million buyer's credit loan for Phase I of Hambantota"

³ This narrative originates from Chellaney (2017). Brautigam (2020), however, provides a different narrative. She mentioned that "*The proceeds were used to increase Sri Lanka's US dollar reserves in 2017-18 with a view to the repayment of maturing international sovereign bonds ...Therefore, the sale of Hambantota was originally a fire sale designed to raise money to deal with larger debt problems.*"

Pakistan

Similar to Sri Lanka, we observe the abrupt increase on the debt to China, as it already has a relatively high debt to other official creditors. Figure 3b shows the change in composition of creditors to Pakistan. Pakistan is the centerpiece of the China Pakistan Economic Corridor (CPEC), a 3000 km corridor that connects China with the Arabian sea. CPEC serves as an important network as it reduces the passage for China's energy import from the Middle Eastern countries (Wikipedia contributors, 2023).

China has launched enormous amounts of infrastructure in Pakistan after 2015. These items include a deep water port, road and rail lines, and most importantly, energy sector projects. Up to 2018, the estimation of the total value of projects under the CPEC is \$62 billion, out of which around \$33 billion is allocated for energy projects (Hurley, Morris and Portelance, 2019). China is expected to finance about 80 percent of this amount. The private investments for energy projects in Pakistan will be financed by the Exim Bank of China at an interest rate of 5-6%. Private Independent Power Producers (IPP) will be responsible for constructing the energy projects under CPEC, instead of the governments of China or Pakistan. In turn, the government of Pakistan will be legally bound to buy electricity from these companies at rates that were agreed upon before. However, despite this significant investment, some projects have already been cancelled, such as three major road projects that were cancelled at the end of 2017 (Hurley, Morris and Portelance, 2019).

In the case of Pakistan, the sudden increase of debt to China draws the attention of researchers and journalists. For example, a report from the Financial Time titled "Pakistan is on the brink" states that Pakistan is following Sri Lanka into default. Given the recent frequent analogy drawn between Pakistan and Sri Lanka, it is essential to analyze Pakistan from the perspective of the sovereign default model.

Chapter 4: Analytic Model

So far, most discussions on China's debt trap have been limited to narrative statistics. To objectively assess this issue, we adopt the up-to-date sovereign debt default model in the literature as a tool for our empirical analysis. Na et al. (2018) proposes a model to study the Argentine economy; Hinrichsen (2021) uses the model to study the enforcement of sovereign debt under war reparations. The model in my thesis strictly follows Na et al. (2018) and Hinrichsen (2021).

International debt often lacks enforcement, and governments hold the decision of whether to repay the debt or default, based on the comparison of future values (Eaton and Gersovitz, 1981). Therefore, default can be considered an optimal policy for a country that faces unsustainable debt levels. By defaulting, the country avoids the burden of paying interest on the debt, but it also faces the consequence of being excluded from the international credit market for a period of time. As a result, the country would have to rely solely on its own financial resources until it regains access to international credit markets. Moreover, studies have pointed out that sovereign debt defaults are often accompanied by a devaluation of the currency; Reinhart (2002) refers to this phenomenon as "Twin Ds." Empirical analysis by Na et al. (2018) further observes that the devaluation rate often decreases after the time of default, suggesting that the Twin Ds phenomenon is the joint result of an optimal policy. They proposed a model that incorporates two key frictions: limited commitment to repay external debts and downward nominal wage rigidity. It is a decentralized version of the Eaton-Gersovitz sovereign debt model. The model predicts that default will occur only after a series of increasingly negative output shocks. Prior to

default, domestic absorption experiences a severe contraction, which leads to a decline in demand for labor. However, due to downward nominal wage rigidity, real wages fail to adjust downward, resulting in involuntary unemployment. To prevent this situation, the optimal policy is to devalue the domestic currency, thereby reducing the real value of wages. As a result, both the model and the data show that default episodes are usually accompanied by significant currency devaluations (Na, Schmitt-Grohé, Uribe and Yue, 2018).

Therefore, for the sovereign debt model, I closely follow Na et al. (2018) to examine the set of conditions under which default is the optimal decision. The calibrated model will then serve as a benchmark metric that allows us to investigate whether China has potentially trapped heavily indebted poor countries into default, using the approach proposed by Hinrichsen (2021).

4.1 Households

The model assumes that the economy is populated by a large number of representative households who maximize their expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t), \quad (1)$$

where $\beta \in (0, 1)$ denotes the discount factor, and c_t represents the consumption good, which is composed of tradable consumption c_t^T and nontradable consumption c_t^N . Assume that c_t follows an aggregate technology

$$c_t = A(c_t^T, c_t^N), \quad (2)$$

where A is an increasing, concave, and linearly homogeneous function that captures characteristics such as the ratio or elasticity of substitution between tradable and nontradable consumption. The period utility function $U(c_t)$ follows the standard assumption, which is

a strictly increasing and strictly concave function.

Assume that the household only has access to the one-period and non-state-contingent bond. The household spends on consumption of tradable and nontradable goods, along with their debt which comes due in the current period. Its resources consist of labor incomes, dividend incomes, lump-sum transfers from the government, and incomes from borrowing from foreign lenders. The household is also endowed with tradable goods, which follow a stochastic process. The budget constraint of the representative household is then

$$P_t^T c_t^T + P_t^N c_t^N + P_t^T d_t = P_t^T \tilde{y}_t^T + W_t h_t + (1 - \tau_t^d) P_t^T q_t^d d_{t+1} + F_t + \Phi_t, \quad (3)$$

where $P_t^T (P_t^N)$ denotes the nominal price of tradable (nontradable) goods, d_t the bond denominated in tradable goods which is due in period t , q_t the price of debt to be repaid at $t + 1$, \tilde{y}_t^T the endowment of tradable goods to the household, W_t the nominal wage, h_t the hours worked, τ_t^d the tax on debt, F_t a lump-sum transfer from the government, and finally Φ_t the nominal profits from owning firms. The household's working hour is bounded by an upper limit

$$h_t \leq \bar{h}, \quad (4)$$

and it takes the working hour h_t as given.

Further, denote the relative price of nontradable in terms of tradable goods as $p_t \equiv \frac{P_t^N}{P_t^T}$, we have the following budget constraint,

$$c_t^T + p_t c_t^N + d_t = \tilde{y}_t^T + w_t h_t + (1 - \tau_t^d) q_t^d d_{t+1} + f_t + \phi_t, \quad (5)$$

where $w_t = \frac{W_t}{P_t^T}$, $f_t = \frac{F_t}{P_t^T}$, $\phi_t = \frac{\Phi_t}{P_t^T}$ are the variables expressed in price of tradable goods. The household's problem is to choose $\{c_t, c_t^T, c_t^N, d_{t+1}\}$ such that its utility (1) is maximized subject to the budget constraints (2) – (5) and the no-Ponzi-game debt limit.

The Lagrangian for the household is

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \{ U(A(c_t^T, c_t^N)) + \lambda_t [\tilde{y}_t^T + w_t h_t + (1 - \tau_t^d) q_t^d d_{t+1} + f_t + \phi_t - c_t^T - p_t c_t^N - d_t] \}.$$

The first order equations are the following:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_t^T} &= A_1(c_t^T, c_t^N) U'(c_t) - \lambda_t = 0 \\ \frac{\partial \mathcal{L}}{\partial c_t^N} &= A_2(c_t^T, c_t^N) U'(c_t) - \lambda_t p_t = 0 \\ \frac{\partial \mathcal{L}}{\partial d_{t+1}} &= (1 - \tau_t^d) q_t^d \lambda_t - E_t \lambda_{t+1} = 0, \end{aligned}$$

where λ_t is the Lagrange multiplier. $A_1(\cdot, \cdot) = \frac{\partial A}{\partial c_t^T}$ and $A_2(\cdot, \cdot) = \frac{\partial A}{\partial c_t^N}$ is respectively the first derivative of the aggregation function with respect to tradable and nontradable consumption. The first order conditions can be concluded as

$$p_t = \frac{A_2(c_t^T, c_t^N)}{A_1(c_t^T, c_t^N)} \quad (6a)$$

$$\lambda_t = U'(c_t) A_1(c_t^T, c_t^N) \quad (6b)$$

$$(1 - \tau_t^d) q_t^d \lambda_t = \beta E_t \lambda_{t+1}. \quad (6c)$$

4.2 Firms

Perfectly competitive firms produce nontradable goods y_t^N according to the production technology

$$y_t^N = F(h_t), \quad (7)$$

where F is strictly increasing and strictly concave. Each firm maximizes its profit by choosing the amount of labor. Profit is given by

$$\Phi_t(h_t) = P_t^N F(h_t) - W_t h_t, \quad (8)$$

and the optimal labor demand is then

$$P_t^N F'(h_t) = W_t.$$

Dividing both side by the price of tradable goods, and define $w_t \equiv \frac{W_t}{P_t^T}$ as the real wage in terms of tradable goods, the first order condition can be written as

$$p_t F'(h_t) = w_t. \quad (9)$$

4.3 Downward Nominal Wage Rigidity

The key assumption in Schmitt-Grohé and Uribe (2016) and Na et al. (2018) is the downward nominal wage rigidity. As the wage is unable to be adjusted to a lower level, involuntary unemployment is inevitable, hence the government has the incentive to allow devaluation. The model imposes a lower bound to the growth rate of nominal wage

$$W_t \geq \gamma W_{t-1}, \quad \gamma > 0. \quad (10)$$

This implies that the growth rate $\frac{W_t - W_{t-1}}{W_{t-1}} \geq \gamma - 1$. When this inequality is unbinding ($W_t > \gamma W_{t-1}$), the economy is fully employed ($h_t = \bar{h}$). However, if the condition binds, the economy might have unemployment ($h_t < \bar{h}$). This relationship can be written as the following equation

$$(\bar{h} - h_t)(W_t - \gamma W_{t-1}) = 0. \quad (11)$$

4.4 Government

We assume here that, under the lack of enforcement in the international credit market, the government has the option to benevolently free up domestic balance sheet by choosing to default or not. Denote I_t as the indicator of whether the government chooses to honor its

debt in period t . If the government repays in this period ($I_t = 1$), the country will be able to borrow in the following period, hence $d_{t+1} > 0$. However, if the government chooses to default ($I_t = 0$), then the country will enter the status of financial autarky and is unable to have any sovereign debt in the next period, hence $d_{t+1} = 0$. The above scenario can be written as a slackness condition

$$(1 - I_t)d_{t+1} = 0. \quad (12)$$

To model the duration of financial exclusion, assume that once the country is in bad standing in the international credit market, it can regain reputation and access to financial markets with probability $\theta \in [0, 1)$, and remain in bad standing with probability $1 - \theta$. This implies that the country has an average exclusion duration of $\frac{1}{\theta}$ periods¹.

Assume that the government distributes the proceeds from the debt tax to households as a lump-sum payment. If the government honors the debt, it repays d_t , but if the government decides to default, it will not make any payments to foreign lenders, and instead will return any payments made by households directly to them. The budget constraint for the government can then be expressed as

$$f_t = \tau_t^d q_t^d d_{t+1} + (1 - I_t)d_t, \quad (13)$$

where $f_t \equiv \frac{F_t}{P_t^T}$ is the lump-sum transfer in terms of tradable goods. Right-hand side of the equation states that the transfer to households will include d_t when $I_t = 0$, which is when the country decides to default. Nevertheless, the transfer of debt tax will be zero after default since $d_{t+1} = 0$ when $I_t = 1$, according to Equation (12).

¹ The expected exclusion period $= \sum_{t=1}^{\infty} t\theta(1 - \theta)^{t-1} = \theta \sum_{t=1}^{\infty} t(1 - \theta)^{t-1} = \frac{1}{\theta}$.

4.5 Foreign Lenders

The behavior of foreign lenders is not explicitly modeled in this framework, but as all rational agents, the expected marginal benefit of lending to the domestic country must be equivalent to the opportunity cost of funds. Let r^* represent the opportunity cost for the foreign lenders; this could be the world interest rate. Since q_t is the price of debt that repays one unit of d_{t+1} tomorrow, the return on the debt is $\frac{1}{q_t}$. The lenders take the risk of default into consideration, therefore, the expected return will actually be lower. Assume that foreign lenders are risk neutral and don't require risk premium, this gives

$$\frac{\Pr(I_{t+1} = 1 \mid I_t = 1)}{q_t} = 1 + r^*. \quad (14)$$

Equivalently, the equation can be written as

$$I_t \left[q_t - \frac{E_t I_{t+1}}{1 + r^*} \right] = 0.$$

4.6 Competitive Equilibrium

Under equilibrium, the households' consumption equals the production of firms

$$c_t^N = y_t^N. \quad (15)$$

The tradable goods are purely endowed exogenously under an AR(1) process

$$\ln(y_t^T) = \rho \ln(y_{t-1}^T) + \mu_t, \quad (16)$$

where $\mu_t \stackrel{\text{iid}}{\sim} \mathcal{N}(0, \sigma_\mu^2)$ is an i.i.d. shock, and $|\rho| \in [0, 1)$ is the autocorrelation parameter. When the country decides to default, it is in bad standing, hence it faces an output loss defined by $L(y_t^T)$. The loss function is non-negative and increasing in the tradable goods.

The endowment of tradable goods to the household is then

$$\tilde{y}_t^T = \begin{cases} y_t^T - L(y_t^T) & \text{if } I_t = 0 \\ y_t^T & \text{otherwise} \end{cases} \quad (17)$$

When the country defaults ($I_t = 0$), the endowment decreases.

Price of debt offered by foreign lenders q_t should be equal to the price of the domestic debt q_t^d , but only during the good standing

$$I_t(q_t^d - q_t) = 0. \quad (18)$$

The market clearing condition can be established by combining various equations, including the household budget constraint (3) and (4), the firm's production function (7) and profit equation (8), the government's constraint on debt (12) and lump-sum return (13), and conditions from (15), (17), and (18). Eventually, the clearing condition for tradable goods is

$$c_t^T = y_t^T - (1 - I_t)L(y_t^T) + I_t(q_t d_{t+1} - d_t) \quad (19)$$

Assume that the law of one price applies to tradable goods. The foreign currency price of tradable goods is denoted as P_t^{T*} , while the nominal exchange rate is represented by \mathcal{E}_t . The law of one price states that the price of tradable goods in the domestic currency is equal to the foreign currency price multiplied by the nominal exchange rate.

$$P_t^T = P_t^{T*} \mathcal{E}_t$$

This implies that the price of a tradable good should be the same in both domestic and foreign currency terms in an efficient market. Without loss of generality, the foreign-currency price of the tradable goods is normalized to 1 ($P_t^{T*} = 1$), hence the nominal

price for tradable goods can be expressed as the nominal exchange rate

$$P_t^T = \mathcal{E}_t. \quad (20)$$

For convenience, also define the devaluation rate of domestic currency as

$$\epsilon_t \equiv \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}} = \frac{P_t^T}{P_{t-1}^T}. \quad (21)$$

The conditions are now sufficient to define a competitive equilibrium.

Definition 1 (Competitive Equilibrium in Na et al. (2018)). A competitive equilibrium is a set of stochastic process $\{c_t^T, h_t, w_t, d_{t+1}, \lambda_t, q_t, q_t^d\}$ satisfying

$$c_t^T = y_t^T - (1 - I_t)L(y_t^T) + I_t(q_t d_{t+1} - d_t), \quad (22)$$

$$(1 - I_t)d_{t+1} = 0, \quad (23)$$

$$\lambda_t = U'(A(c_t^T, F(h_t)))A_1(c_t^T, c_t^N), \quad (24)$$

$$(1 - \tau_t^d)q_t^d \lambda_t = \beta E_t \lambda_{t+1}, \quad (25)$$

$$I_t(q_t^d - q_t) = 0, \quad (26)$$

$$\frac{A_2(c_t^T, F(h_t))}{A_1(c_t^T, F(h_t))} = \frac{w_t}{F'(h_t)}, \quad (27)$$

$$w_t \geq \gamma \frac{w_{t-1}}{\epsilon_t}, \quad (28)$$

$$h_t \leq \bar{h}, \quad (29)$$

$$(h_t - \bar{h}) \left(w_t - \gamma \frac{w_{t-1}}{\epsilon_t} \right) = 0, \quad (30)$$

$$I_t \left[q_t - \frac{E_t I_{t+1}}{1 + r^*} \right] = 0, \quad (31)$$

given processes $\{y_t^T, \epsilon_t, \tau_t^d, I_t\}$ and initial conditions w_{-1} and d_0 .

As proven by Na et al. (2018), if the government is able to set the devaluation rate and the tax on debt freely, then the stochastic process of the variables $\{c_t^T, h_t, d_{t+1}, q_t\}$ can be

determined by the process of $\{y_t^T, I_t\}$ and the initial debt level d_0 .

As discussed previously, the decision of I_t is an optimal policy for the government due to lack of commitment to repay in the international credit market. Furthermore, the default decision of the government in the next period $t + 1$ is also affected by the current decision. To see this argument, first notice that the default decision in $t + 1$ is determined by the state variables $\{y_{t+1}^T, d_{t+1}\}$. However, d_{t+1} is determined in period t , which means that the government in period t understands that it is able to affect the default decision in $t + 1$ via the choice of d_{t+1} . As y_{t+1}^T follows a first-order Markov process, the expected value of y_{t+1}^T is a function of y_t^T , hence the expected value for the default decision on period t is actually a function of y^T and d_{t+1} . Recall that the price for the debt q_t is related to the probability of default in the next period, according to Equation (14), it can be expressed in the contemporary variables

$$q_t = q(y_t^T, d_{t+1}). \quad (32)$$

On the one hand, this provides us the economic intuition that the government internalizes the fact that its choice of debt in the next period can affect the price of the debt. On the other hand, this allows us to clarify the dependencies of variables in the value function.

4.7 Default Decision

Following the standard Eaton-Gersovitz framework, this model considers the following three value functions: value of continuing to repay the debt v^c , value of being in good standing v^g , and value of being in bad standing v^b .

Under the period of being in good financial standing, the value for the government to continue repaying the debt is the maximum value of the utility gained by the households this period, plus the discounted value of being in a good financial standing, subject to the

households' budget constraints. Formally,

$$\begin{aligned}
v^c(y_t^T, d_t) = & \max_{\{c_t^T, h_t, d_{t+1}\}} \{U(A(c_t^T, F(h_t))) + \beta E_t v^g(y_{t+1}^T, d_{t+1})\} \\
\text{s.t. } & c_t^T + d_t = y_t^T + q(y_t^T, d_{t+1})d_{t+1} \\
& h_t \leq \bar{h}.
\end{aligned} \tag{33}$$

Where the first constraint is obtained by setting $I_t = 1$ in Equation (19), and the second is the constraint on working hour.

If the country is in bad standing, the consumption on tradable goods experiences a loss. The government has probability θ of regaining access to international financial markets, and probability $1 - \theta$ of continuing in bad standing. During the period in bad standing, the country obtains no international borrowing, hence, the state variable for debt is excluded. Formally,

$$\begin{aligned}
v^b(y_t^T) = & \max_{\{h_t\}} \{U(A(y_t^T - L(y_t^T), F(h_t))) + \beta E_t [\theta v^g(y_{t+1}^T, 0) + (1 - \theta)v^b(y_{t+1}^T)]\} \\
\text{s.t. } & h_t \leq \bar{h}.
\end{aligned} \tag{34}$$

The tradable consumption $c_t^T = y_t^T - L(y_t^T)$ again follows Equation (19) by setting $I_t = 0$, and is substituted explicitly into the value function.

If the country is in good standing, the government has the freedom to choose which is best for the country: to continue or to default. The decision is made by comparing the value functions of the two scenarios, given the current output shock for tradable goods and the current level of debt

$$v^g(y_t^T, d_t) = \max \{v^c(y_t^T, d_t), v^b(y_t^T)\}. \tag{35}$$

Define the default set $D(d_t)$ as the set of tradable-output levels y_t^T examined by the

government in period t , in which the government's optimal respond is to default. Formally,

$$D(d_t) = \{y_t^T : v^b(y_t^T) > v^c(y_t^T, d_t)\}. \quad (36)$$

In other words, given a current debt level d_t , if the government observes that y_t^T is inside $D(d_t)$, it chooses to default.

Under rational expectations, the foreign lenders recognize the default set, hence the price for debt is determined by Equation (14), given by

$$q(y_t^T, d_{t+1}) = \frac{1 - \Pr\{y_{t+1}^T \in D(d_{t+1}) \mid y_t^T\}}{1 + r^*}. \quad (37)$$

Note that the price of debt enters the value function of continuing, $v^c(y_t^T, d_t)$.

It is obvious that the optimal labor supply is $h_t = \bar{h}$ since all functions, F, A, U , are monotonic, which implies that under the freedom to choose the devaluation rate and the tax on debt, the government can ensure full employment. Denote $w^f(c_t^T)$ the equilibrium wage function under full employment given the consumption of tradable goods. Combining Equation (9) and the Euler equation in (6a) and impose the optimal policy $h_t = \bar{h}$, we have

$$w_t = w^f(c_t^T) \equiv \frac{A_2(c_t^T, F(\bar{h}))}{A_1(c_t^T, F(\bar{h}))} F'(\bar{h}). \quad (38)$$

Knowing that the wage has downward nominal rigidity, the government sets the devaluation rate accordingly. The downward rigidity (11) states that

$$\gamma \leq \frac{W_t}{W_{t-1}} = \frac{w_t}{w_{t-1}} \frac{P_t^T}{P_{t-1}^T} = \epsilon \frac{w_t}{w_{t-1}},$$

where the second equal sign comes from Equation (21). Substitute the wage under full employment, we get

$$\epsilon_t \geq \gamma \frac{w_{t-1}}{w^f(c_t^T)}. \quad (39)$$

This is the family of optimal devaluation policies. Following Na et al. (2018) and Hinrichsen (2021), we assume that the government chooses the minimal devaluation target that stabilizes nominal wages, that is, $\epsilon_t = \gamma \frac{w_{t-1}}{w^f(c_t^T)}$.

Chapter 5: Empirical Results

Typically, a model under the Eaton-Gersovitz framework does not have an analytical solution. Therefore, the optimal default set defined by Equation (36), as well as the value functions and the policy functions, must be obtained numerically via the technique of value function iteration. This requires the assignment of functional forms as well as structural parameters that matches the economy. I follow the functional forms and the calibration approach introduced in Na et al. (2018) and Hinrichsen (2021).

5.1 Calibration

Functional Forms

Following Na et al. (2018), the time unit is assumed to be one quarter, and the periodic utility function is assumed to be the constant relative risk aversion (CRRA) type

$$U(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma}, \quad (40)$$

where σ is the inverse of elasticity of intertemporal substitution of the consumption. The aggregator function for tradable and non-tradable consumption takes the constant elasticity of substitution (CES) form

$$c_t = A(c_t^T, c_t^N) = \left[a (c_t^T)^{1-\frac{1}{\xi}} + (1-a) (c_t^N)^{1-\frac{1}{\xi}} \right]^{\frac{1}{1-\frac{1}{\xi}}}. \quad (41)$$

The CES aggregator states that the share of tradable consumption is $a \in [0, 1]$, and the elasticity of substitution between the tradable and non-tradable consumption is ξ . Moreover, following the literature, to make the consumption of tradable goods c_t^T and the external debt d_t independent of the outputs in the nontradable sector in the equilibrium, assume that the inter- and intratemporal elasticity of substitution is equivalent (See Uribe and Schmitt-Grohé, 2017, Chapter 9.5). That is,

$$\xi = \frac{1}{\sigma}. \quad (42)$$

The production technology for the nontradable goods follows a simple form

$$y_t^N = F(h_t) = h_t^\alpha. \quad (43)$$

The loss-function in Equation (17) is positive and increasing with y_t^T , and following Chatterjee and Eyigungor (2012), I adopt the quadratic form with two parameters

$$L(y_t^T) = \max \left\{ 0, \delta_1 y_t^T + \delta_2 (y_t^T)^2 \right\}. \quad (44)$$

This is also adopted in Na et al. (2018). In this setting, if we set $\delta_1 < 0$ and $\delta_2 > 0$, the output-loss increases as y_t^T increases, indicating that the more a country is endowed, the more it loses during default.

Calibration of Sri Lanka

The model is calibrated to Sri Lanka before 2008, when the Chinese government started to provide the increasing amount of loans. China started to provide loans to Sri Lanka in 2005, and during 2006 to 2008, the debt amount to China remains to be around \$1 billion, which is roughly 2.9% of GDP. Starting from 2009, debt to China has increased to \$3 billion, and it reached \$7.5 billion in 2014, accounting for 9.5% of GDP (see Figure 3a).

I proxy the output process of Equation (16) by the detrended log-real-GDP of Sri Lanka from 1980 to 2021. Considering that the seasonality in the quarterly data for Sri Lanka might impose a higher volatility estimated in the AR(1) process, I follow Hinrichsen (2021) and estimate the annual data over 1980 to 2021. I obtain the cyclical component of the output by filtering the time series with an HP-filter with smoothing parameter λ set to 100. Estimation of the AR(1) on the cyclical component thus yields $\rho = 0.9114$ and $\sigma_u = 0.0123$ ¹. Figure 4 presents the decomposition of the log-real-GDP.

The global risk-free world interest rate r^* is set to match the U.S. 3-month treasury bill rate during 1990 to 2007², which is roughly 4% annually, or 1% for one quarter. This is in line with Chatterjee and Eyigungor (2012) and Na et al. (2018). The probability of reentry is difficult to assess due to the lack of data. As a result, following Chatterjee and Eyigungor (2012) and Hinrichsen (2021), I set the probability of reentry to 0.0385, which implies that the country will be in default on average for about 6.5 years.

The labor share is set as $\alpha = 0.65$ based on the calibration in Jegajeevan (2016), which matches the estimation of labor share in Duma (2007). The share of tradable consumption is approximated by the share of tradable output in total output, as suggested in Uribe and Schmitt-Grohé (2017). Calculating the mean of tradable-to-GDP ratio over 1980 to 2021, I set the value as $a = 0.36$. The elasticity of substitution between tradable and nontradable goods ξ is set as 0.5 following Uribe and Schmitt-Grohé (2017), which is close to the cross-country estimation of 0.44 by Stockman and Tesar (1995). According to the assumption in Equation (42), $\sigma = 1/\xi = 2$. The calibration on the two parameters ξ and σ is inline with most real-business-cycles (Uribe and Schmitt-Grohé, 2017; Na et al., 2018). Nominal wage rigidity is set as $\gamma = 1.109$ based on the empirical estimation of downward wage

¹ Since the AR(1) estimation is conducted on annual data, the estimated coefficients must be quarterized. Specifically, $\rho = 1 - \frac{1-\hat{\rho}}{4}$, and $\sigma_u = \frac{\hat{\sigma}}{\sqrt{4}}$, where $\hat{\rho}$ and $\hat{\sigma}$ are the estimated parameters for the AR(1) via OLS.

² Quarterly averaged data retrieved from FRED. The span of 1990 to 2007 is set to match the periods of the Great Moderation, when the business cycle fluctuation is reduced (Hakkio, 2013). The average of 3-month treasury bill rate over 1990 to 2007 is 4.10%.

rigidity in 2014 by Matschke and Nie (2022)³.

Following Na et al. (2018), the rest of the parameters $(\beta, \delta_1, \delta_2)$, which is respectively the subjective discount factor and the two parameters for the loss function, is chosen to match three equilibrium outcomes⁴: (i) the average debt-to-GDP ratio in periods of good standing is 65% per quarter; (ii) the frequency of default is 2.6 times per century; and (iii) the average output loss is 7% per year conditional on being in financial autarky. The following justifies this choice of targets.

- (i) The average debt-to-GDP ratio to be targeted is motivated by the fact that the average annual debt-to-GDP ratio of Sri Lanka in the data is about 44%. The value is calculated by averaging the nominal external-debt-to-GDP ratio over 2001 to 2008⁵.

Multiplying this by an average of 37% haircut⁶ implies that about 16.28% of the debt is unsecured annually⁷. Since we are dealing with a model with quarterly period, this results in the 65% debt-to-GDP ratio targeted during calibration.

- (ii) Sri Lanka suspended all debt payment in April 2022, but else there is no other default episode within the periods of calibration. Note that Sri Lanka interacted with Paris Club in 2005 for a moratorium due to the tsunami of December 2004, but this event is not counted as a default according to global rating systems such as S&P Global (Kraemer et al., 2022). Due to this uncertainty of determining a default episode, I set the default frequency to be 2.6 as the benchmark target, which is the frequency of Argentina following Na et al. (2018). Attempts on counting default episodes based

³ Matschke and Nie (2022) estimates downward wage rigidity by measuring the annualized average gross real wage growth during an unemployment cycle for each country.

⁴ In particular, I use the surrogate optimization solver in Matlab to search for the optimal values for the three parameters. Essentially, VFI must be proceeded for each triplet of the parameters to obtain the targeting equilibrium outcomes. Details are mentioned in section 5.2.

⁵ Data source: International Debt Statistics. The period of year is chosen to be 8 years before China's increasing support of loans. The time span of 8 years is inline with that in Uribe and Schmitt-Grohé (2017).

⁶ This is the average sovereign haircut between 1970 and 2010 (Cruces and Trebesch, 2013). The haircut data for the current Sri Lanka sovereign default is not available since it is still under restructuring.

⁷ In the model, we assume that the country defaults on 100% of the debt, hence this approach is necessary to handle the case of a haircut.

on a different definition will be conducted as a robustness check in later section.

- (iii) Na et al. (2018) adopts a growth accounting approach proposed by Zarazaga (2012) to calculate the output loss associated with the default. Applying this method to Sri Lanka, however, suggests that the output increases along with default, indicating that the method is not suitable in this case. See Appendix A for more detail. I therefore follow Na et al. (2018) and set the output loss to be 7%, matching the case of Argentina.

Table 1 summarizes the calibrated parameters and their sources.

Calibration of Pakistan

The calibration strategy for Pakistan is similar to that of Sri Lanka. The parameters for the output process is obtained from the cyclical component of the HP-filter on the annual log-real-GDP for Pakistan from 1980 to 2021, which yields $\rho = 0.9008$ and $\sigma_u = 0.0111$ (see Figure 4). The risk-free interest rate remains to be 4% annually, hence $r = 1\%$. Pakistan defaulted on January 1999, completed its debt restructuring on December 1999 (Kraemer et al., 2022), but gained partial reaccess (flows > 0) in 2004, and full reaccess (flow $> 1\%$ of GDP) in 2006 (Trebesch, 2011, Table 5.6). The model adopted in my thesis does not distinguish between partial or full reaccess, hence the reentry period is set as the first year Pakistan gain positive flow of debt. Accordingly, the reentry period is set to 6 years (24 quarters), and $\theta = 0.0417$.

The labor share is set as 0.4 to match the capital share in real GDP, following Rehman et al. (2020). The share of tradable consumption is calibrated according to the tradable-to-GDP ratio over 1980 to 2021, which gives $a = 0.33$. The intratemporal elasticity of substitution of consumption $\xi = 0.5$ and $\sigma = 2$, following the same justification for Sri Lanka (Rehman et al., 2020; Uribe and Schmitt-Grohé, 2017). Nominal wage rigidity is set as $\gamma = 1.048$ based on the empirical estimation of downward wage rigidity in 2014 by Matschke and Nie (2022).

Finally, the triplet $(\beta, \delta_1, \delta_2)$ is also chosen to match the three equilibrium results: (i) the average debt-to-traded-GDP ratio in periods of good standing is 44% per quarter; (ii) the frequency of default is 2.6 times per century; and (iii) the average output loss is 7% per year conditional on being in financial autarky.

- (i) The average debt-to-GDP-ratio between 2006 and 2013⁸ is 30% according to the International Debt Statistics. Multiplying it with an average 37% haircut ratio⁹ estimated in Cruces and Trebesch (2013) yields an 11.1% of annual unsecured debt, which gives an 44.4% unsecured debt-to-GDP ratio quarterly.
- (ii) Default record according to Kraemer et al. (2022) and Uribe and Schmitt-Grohé (2017) shows that in Pakistan there is only one default episode in 1999. However, Pakistan sought debt relief from Paris Club several times during the 1970s and 1980s (Kundi, 2016), indicating that if we only consider the event of 1999, we might underestimate the default frequency¹⁰. Following the same logic in Sri Lanka, since determining a default episode is not straightforward in this case, I set the default frequency to be 2.6 times per century as the benchmark target, following Na et al. (2018). A robustness check with a different approach of determining default episodes will be provided in later section.
- (iii) The capital-output ratio rose from 1.36 to 1.4 after the default episode, which suggests that the approach of Zarazaga (2012) is also not suitable in this case. See

⁸ Following the same logic as with Sri Lanka, I choose an 8-years window before the increasing amount of loans from China in 2013.

⁹ Cruces and Trebesch (2013) estimates that the haircut ratio for the 1999 default of Pakistan is about 15%. This yields an 18% unsecured debt-to-GDP ratio to be targeted, which is too low for the model. I therefore adopt the average haircut ratio instead.

¹⁰ The Government of Pakistan (2008) reports a comprehensive list of debt restructuring events in Pakistan. In May 1972, Pakistan interacted with the Paris Club for a short-term debt relief of \$234 million following the separation of East Pakistan. In June 1974, an arrangement with member countries of Aid-to-Pakistan Consortium agreed to provide another debt relief of \$650 million. By 1981, political crisis worsen the debt service, hence a debt relief was granted by the Consortium. The nuclear test in 1998 by Pakistan led to international sanction and embargo, causing Pakistan to fail its debt service and had to seek debt restructuring with the Paris Club. The followed debt treatments (in 1999 and 2001) from the Paris Club were under the “Houston term”, which differs from the previous debt relief in a sense that it granted a smaller concessional interest rate and a longer grace periods (Siddiqui et al., 2001).

Appendix A for more details¹¹. As a result, I again follow Na et al. (2018) and set the target average output loss to be 7%.

Table 2 summarizes the calibrated parameters for Pakistan.

5.2 Numerical Computation

The approximated equilibrium is obtained by conducting value function iteration over an $n_y \times n_d$ discretized and equally spaced state space, where $n_y = 200$ is the number of grids for the output process and $n_d = 200$ is the number of grids for the debt (Na et al., 2018). Denote $[\underline{y}^T, \bar{y}^T]$ as the lower and upper bound of output grid. Following Uribe and Schmitt-Grohé (2017), this is set as $[-4.2\sigma_u, 4.2\sigma_u]$. Also following the authors, since the average debt levels for both countries do not exceed 150%, the upper bound for debt is set as 1.5, therefore the debt range for the value function iteration is $[\underline{d}, \bar{d}] = [0, 1.5]$.

I follow Schmitt-Grohé and Uribe (2016) and discretize the AR(1) process for output by constructing a transition probability matrix over the grids of the output. A time series of 10 million observations was generated based on Equation (16). Each observation is then assigned to the nearest grid point among the 200 discrete values of $\ln y^T$. The discretized series is analyzed to calculate the probabilities of transitioning from one discrete state to another in consecutive periods. To obtain the transition probability matrix, a 200×200 matrix is initialized with zeros. For each pair of consecutive observations, the corresponding element in the matrix is incremented by 1. After considering all the observations, the matrix is normalized by dividing each row by the sum of its elements. This results in the estimated transition probability matrix, which effectively captures the covariance matrices of order 0 and 1 (Uribe and Schmitt-Grohé, 2017).

Value function iteration is then conducted for a given set of calibrated parameters,

¹¹ In the calculation, I first consider the period of partial reaccess to the credit market in 2004 as the end of autarky, following Trebesch (2011). The attempt to calculate output loss by defining the emergence of default as full reaccess to financial market in 2006 instead of partial reaccess in 2004 is examined, but it yields an output loss of 0.25%, which is too subtle compared to cross-country studies of output loss during default (Borensztein and Panizza, 2009).

which is composed of the set of fixed parameters $(\rho, \sigma_u, r^*, \theta, \alpha, a, \xi, \gamma)$ and the tuple $(\beta, \delta_1, \delta_2)$. The default dynamic and moments can then be conducted once the value function converges. Following Uribe and Schmitt-Grohé (2017) and Na et al. (2018), a simulation of the model based on the policy function is conducted sequentially¹² for 1.1 million iterations, of which the first 0.1 million periods are discarded.

A critical procedure of the calibration is to select the tuple $(\beta, \delta_1, \delta_2)$ such that it matches the three targets described in section 5.1, which are the default frequency, debt-to-GDP ratio, and output loss. In particular, the tuple is chosen to minimize the Euclidean distance between the target value and the corresponding moments generated by the dynamic simulation¹³. Since the value function iteration and dynamic simulation are time-consuming processes, I use the surrogate optimization algorithm in MATLAB as the searching algorithm, as it is suitable for minimizing time-consuming objective functions.

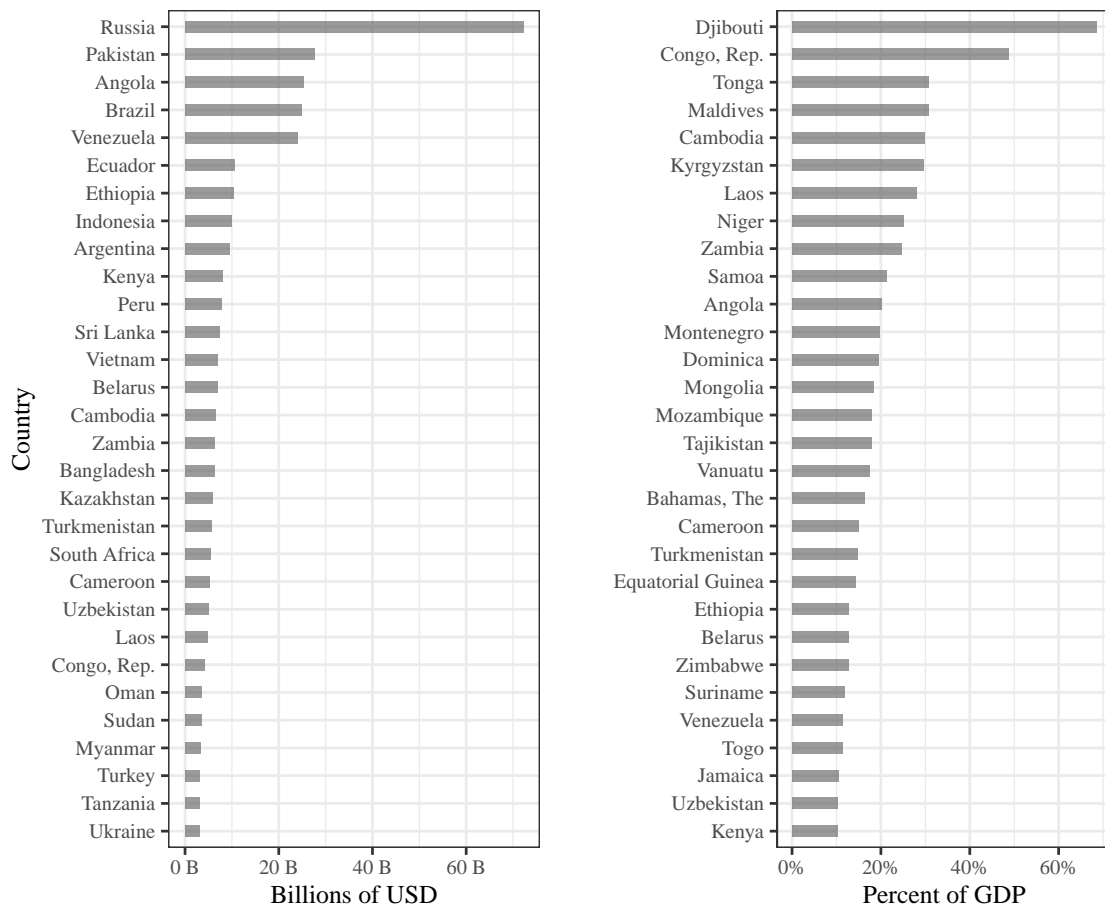
5.3 Results

¹² The main randomness comes from the AR(1) output. Once y_{t+1}^T is determined, along with determination of the debt level for next period d_{t+1} , we have the state (y_{t+1}^T, d_{t+1}) for $t + 1$, which is a grid-point. Values for the rest of the endogenous variables such as c_{t+1}, I_{t+1} can then be obtained by extracting the corresponding value from the state grid-point on the policy function.

¹³ Default frequency is evaluated by taking the mean of $\{I_t\}_{t \in T}$ and multiply it by 100×4 quarters per century. Debt-to-GDP ratio is calculated by taking the mean of $\{d_t/y_t^T\}_{t \in \text{Good T}}$, which is the average debt-to-tradable-output ratio condition on being in good standing. Output loss is evaluated by taking the mean of $\{(y_t^T - \tilde{y}_t^T)/y_t^T\}_{t \in \text{Bad T}}$, which is the average output difference between the endowment and the actual received output, divided by the endowment, over periods of bad standings.

Chapter 6: Conclusion and Discussion

Figure 1: Debt to China Statistic by Country in 2017



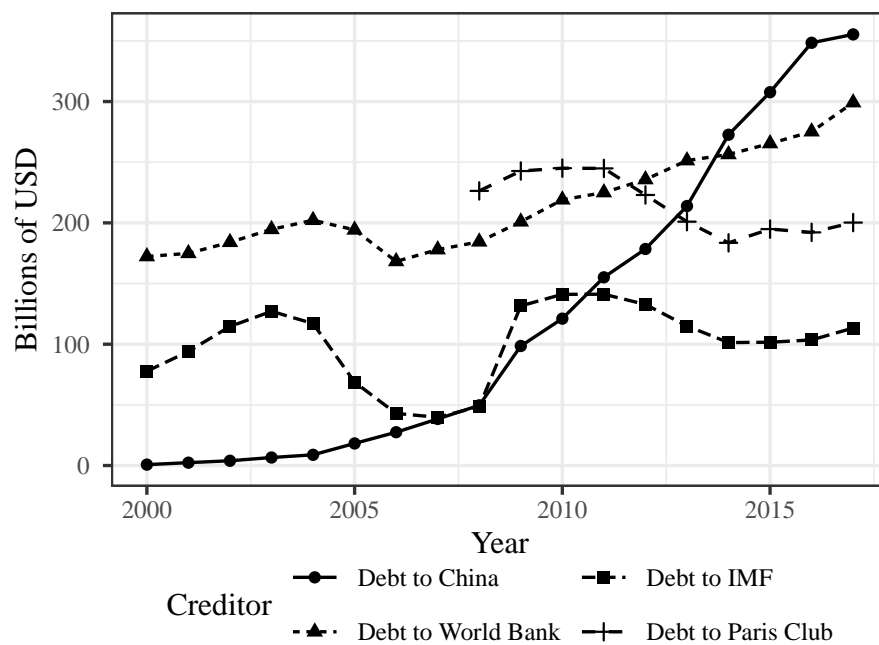
(a) Top 30 Debtor by Total Debt in USD

(b) Top 30 Debtor by Dept-to-GDP Ratio

Source: Horn, Reinhart and Trebesch (2021) database

Note: The figure on the left presents the top 30 countries in amount of total external debt to China in 2017. The figure on the right compares by the China-debt-to-GDP ratio.

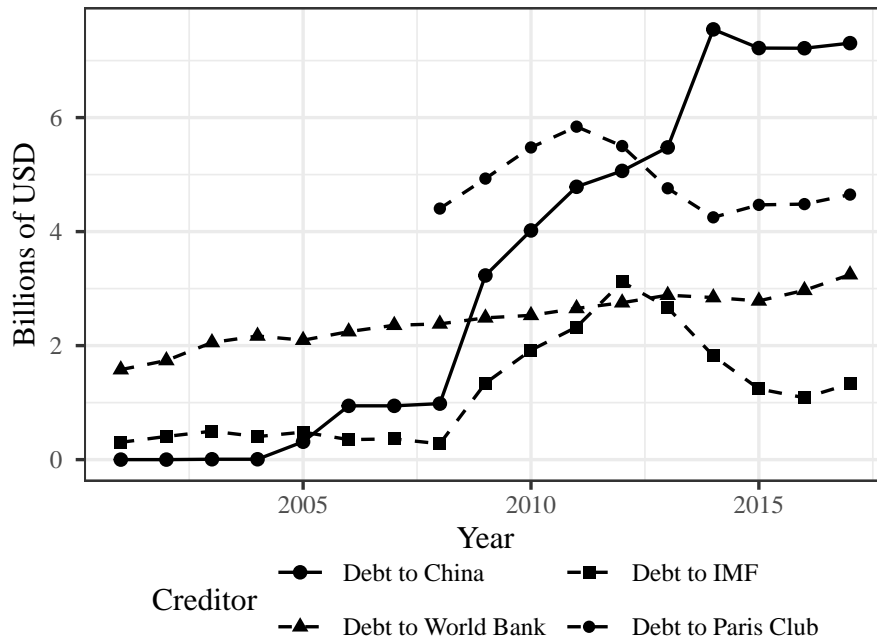
Figure 2: Change of Aggregate Public Debt for Different Official Creditors



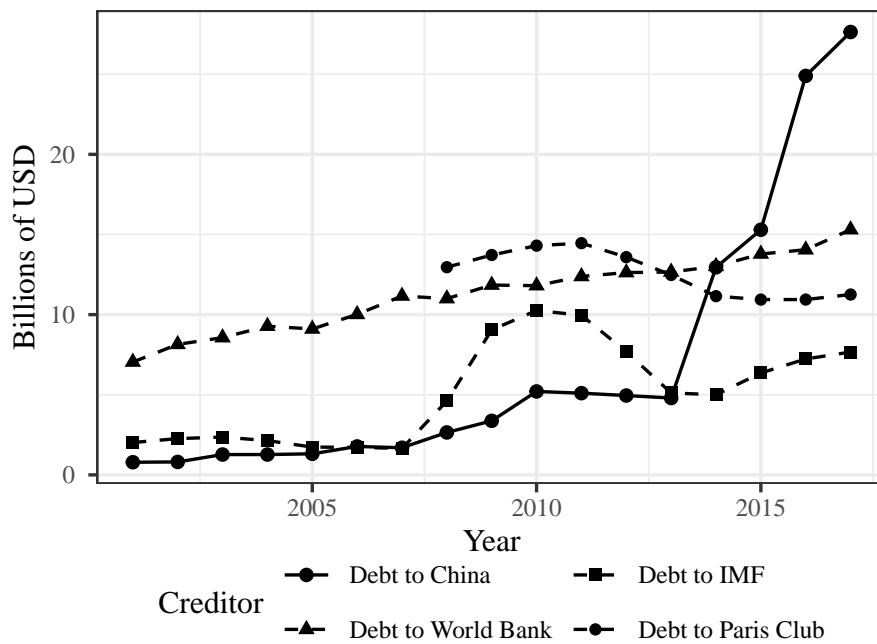
Source: Horn, Reinhart and Trebesch (2021) database

Note: The figure shows the change in the aggregate external public debt that the developing countries owed to different official creditors. These include China, World Bank (excluding China), IMF, and all 22 Paris Club governments. It is obvious that China had become the largest official creditors in the world according to the estimation of Horn, Reinhart and Trebesch (2021).

Figure 3: Debt to Main Creditors



(a) Sri Lanka

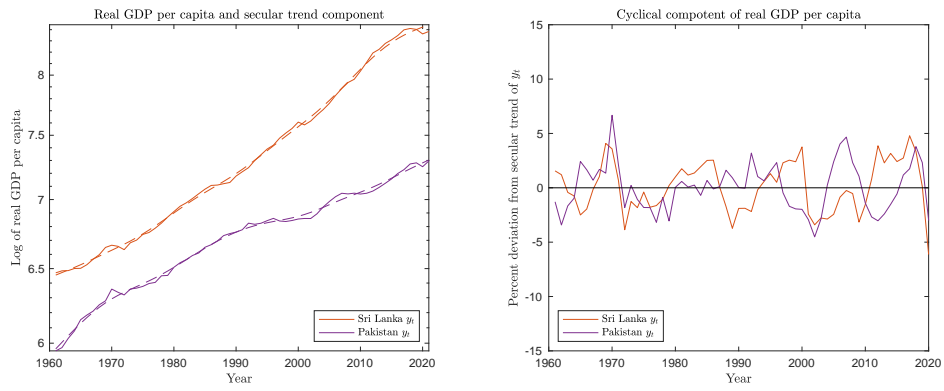


(b) Pakistan

Source: Horn, Reinhart and Trebesch (2021) database

Note: The figure shows the change in the external public debt that Sri Lanka and Pakistan owed to different official creditors. These include China, World Bank (excluding China), IMF, and all 22 Paris Club governments.

Figure 4: Decomposition of log-real-tradable-GDP for Sri Lanka and Pakistan



Source: World Bank national accounts data.

Note: The cyclical component in the right is obtained by the HP-filter with smoothing parameter $\lambda = 100$ for the log-real-GDP in the left. The quarterized AR(1) estimation for Sri Lanka yields $(\rho, \sigma_u) = (0.9114, 0.0123)$, and for Pakistan yields $(\rho, \sigma_u) = (0.9008, 0.0111)$

Table 1: Calibration for Sri Lanka

Parameter	Description	Value	Source
ρ	Autocorrelation of output	0.9114	Estimation of AR(1) on GDP
σ_u	Standard deviation of output	0.0123	Estimation of AR(1) on GDP
r^*	Risk-free rate	0.01	U.S. 3-month treasury bill rate
θ	Probability of reentry	0.0385	Chatterjee and Eyingongor (2012)
α	Labor share in non-tradable goods sector	0.65	Jegajeevan (2016)
a	Share of tradable consumption	0.35	Share of tradable goods in GDP
ξ	Intratemporal elasticity of substitution of consumption	0.5	Na et al. (2018)
σ	Inverse of intertemporal elasticity of substitution of consumption	2	$1/\xi$
γ	Downward wage rigidity	1.109	Matschke and Nie (2022)
β	Discount factor	(...)	Self-estimated
δ_1	Coefficient of the linear term in loss function	(...)	Self-estimated
δ_2	Coefficient of the quadratic term in loss function	(...)	Self-estimated
\bar{h}	Labor endowment	1	Normalized to 1

Note: The time unit is one quarter. AR(1) is performed on annual tradable GDP data but quarterized following the approach of Hinrichsen (2021).

Table 2: Calibration for Pakistan

Parameter	Description	Value	Source
ρ	Autocorrelation of output	0.9008	Estimation of AR(1) on GDP
σ_u	Standard deviation of output	0.0111	Estimation of AR(1) on GDP
r^*	Risk-free rate	0.01	3 month treasury bill rate
θ	Probability of reentry	0.0417	Trebesch (2011)
α	Labor share in non-tradable goods sector	0.4	Rehman et al. (2020)
a	Share of tradable consumption	0.33	Share of tradable goods in GDP
ξ	Intratemperal elasticity of substitution of consumption	0.5	Na et al. (2018)
σ	Inverse of intertemperal elasticity of substitution of consumption	2	$1/\xi$
γ	Downward wage rigidity	1.048	Matschke and Nie (2022)
β	Discount factor	(...)	Self-estimated
δ_1	Coefficient of the linear term in loss function	(...)	Self-estimated
δ_2	Coefficient of the quadratic term in loss function	(...)	Self-estimated
\bar{h}	Labor endowment	1	Normalized to 1

Note: The time unit is one quarter. AR(1) is performed on annual tradable GDP data but quarterized following the approach of Hinrichsen (2021).

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Appendix A: Output Loss

The following demonstrates the calculation of output loss associated with the default using the growth accounting approach proposed by Zarazaga (2012).

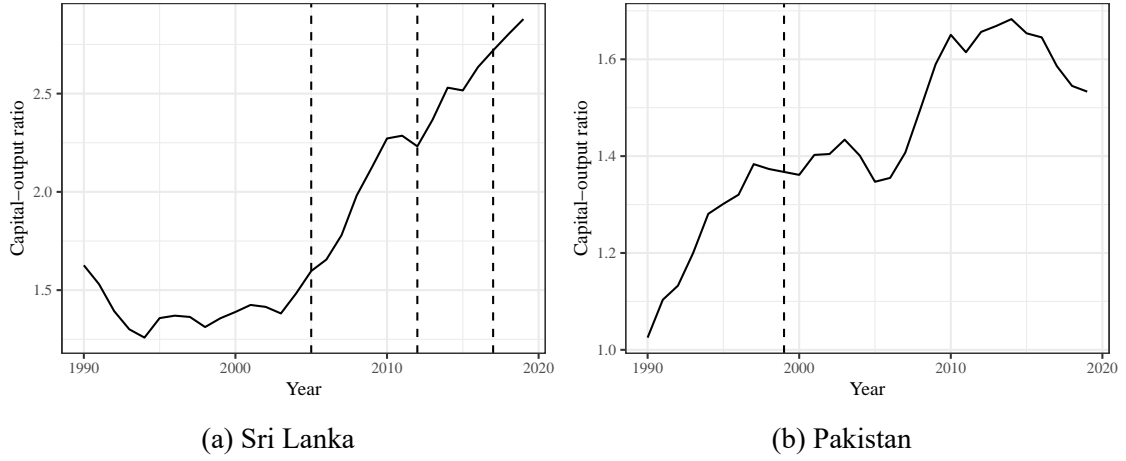
Following Zarazaga (2012), assume that the production function follows the form $y_t = h_t^\alpha k_t^{1-\alpha}$ where y_t denotes output, k_t denotes physical capital, and h_t denotes employment. This implies that by the relationship $\frac{y_t}{h_t} = \left(\frac{k_t}{h_t}\right)^{\frac{1-\alpha}{\alpha}}$. If the capital-output ratio before the default episode $\kappa_b = \frac{k_b}{y_b}$ falls to $\kappa_a = \frac{k_a}{y_a}$ after the default episode, the output per worker would be $\Delta = \left[\left(\frac{\kappa_a}{\kappa_b}\right)^{\frac{1-\alpha}{\alpha}} - 1 \right] \times 100$ percent higher. If we ascribe all the observed decrease in capital to the sovereign default, we conclude that the output loss is on average $\frac{\Delta}{2}\%$ per worker during the period. Note that $\left(\frac{\kappa_a}{\kappa_b}\right)^{\frac{1-\alpha}{\alpha}} - 1 > 0$ if and only if $\kappa_a > \kappa_b$. This implies that there is an output loss only if the capital-output-ratio decreases. As argued in Zarazaga (2012), the output loss is associated with a trough in the capital-output-ratio.

Using data from the Penn World Table, the annually capital-output ratio is calculated by dividing the capital stock at current PPPs (variable *cn*) by the output-side real GDP at current PPPs (variable *cgdp*), both in million 2017 U.S. dollar.

In the case of Sri Lanka, capital-output ratio associated with the three default episodes (2005, 2012, 2017) recorded in the BoC-BoE Sovereign Default Database all increase, even for the consecutive years, as shown in Figure 5a. It is therefore unreasonable to attribute all the variations of the output to the sovereign default episodes.

For the case of Pakistan's biggest default in 1999, according to previous calibration, $\alpha = 0.4$, which implies that by the relationship $\frac{y_t}{h_t} = \left(\frac{k_t}{h_t}\right)^{\frac{3}{2}}$. Pakistan gained partial reaccess (debt flow > 0) and emerged from financial autarky in 2004 (Trebesch, 2011),

Figure 5: Capital-Output Ratio, 1990 to 2020



Source: Penn World Table

Note: The solid line represents the capital-output ratio for Sri Lanka and Pakistan. Default episode examined is plotted in vertical dashed lines.

therefore the growth accounting will be conducted within 1999 to 2004. The capital-output-ratio was about 1.36 in 1999. It rose to about 1.43 in 2003, and slowly fell to 1.4 in 2004, as shown in Figure 5b. Following the exact same logic, since $\frac{k_t}{y_t}$ rose from 1.36 to 1.4 between 1999 and 2004, the output per worker actually increased, which is contradicting our assumption of an output loss. Alternatively, if we consider the year of full reaccess as the end of default (debt flow > 1% GDP), which is in 2006 (Trebesch, 2011), the calculation yields that the output per worker would have been $\left[\left(\frac{1.36}{1.355} \right)^{\frac{3}{2}} - 1 \right] \times 100 = 0.5\%$ higher, which gives an average output loss of 0.25%. The value is too low compared to the average output loss of 5.5% according to cross-country studies (Uribe and Schmitt-Grohé, 2017; Borensztein and Panizza, 2009), which indicates that it is also unreasonable to ascribe all the effect on output to capital. Hence, we conclude that the estimation of output cost of the default for Pakistan following Zarazaga (2012) is also not applicable.

Appendix B: Properties of the Default Set

In Chapter 4, I introduce the default set given by the decentralized Eaton-Gersovitz model by Na et al. (2018). It is worth examining some properties of the default set, as it justifies visually the empirical results of the thesis. Despite the fact that the default set can only be established through numerical computation instead of an analytical expression, some properties of the default set can still be obtained without knowing the exact form of the set.

Recall that the default set is defined as

$$D(d_t) = \{y_t^T : v^b(y_t^T) > v^c(y_t^T, d_t)\},$$

which is the set of output levels within which the country is best to default given a debt level d_t . I derive the following three properties regarding the default set under the model specifications of Na et al. (2018):

1. If the default set is not empty, then the trade balance deficit is less than the output loss. That is, $q(y_t^T, d_{t+1})d_{t+1} - d_t < -L(y_t^T)$.
2. If $y_1 \in D(d_t)$ and $\underline{y} \leq y_2 \leq y_1$, then $y_2 \in D(d_t)$
3. The default set $D(d_t)$ is an interval $[\underline{y}, y^*(d_t)]$, where $y^*(d_t)$ is increasing in d_t .

Here, \underline{y} denotes the lower bound on the endowment level during numerical computation.

Similar properties are proved in Arellano (2008) and Uribe and Schmitt-Grohé (2017) for the centralized version of Eaton-Gersovitz model.

Proposition 1. If $D(d_t) \neq \emptyset$, then $q(y_t^T, d_{t+1})d_{t+1} - d_t < -L(y_t^T)$ for all d_{t+1} .

Proof. The proof is by contradiction. Suppose that $q(y_t^T, \tilde{d}_{t+1})\tilde{d}_{t+1} - d_t > -L(y_t^T)$ for some \tilde{d}_t , according to the definition of $v^c(d_t, y_t^T)$,

$$\begin{aligned} v^c(d_t, y_t^T) &= \max_{d_{t+1}, h_t} \{U(A(y_t^T + q_t(y_t^T, d_{t+1})d_{t+1} - d_t, F(h_t))) + \beta E_t v^g(y_{t+1}^T, d_{t+1})\} \\ &\geq U\left(A(y_t^T + q_t(y_t^T, \tilde{d}_{t+1})\tilde{d}_{t+1} - d_t, \bar{h})\right) + \beta E_t v^g(y_{t+1}^T, \tilde{d}_{t+1}) \\ &\geq U\left(A(y_t^T - L(y_t^T), \bar{h})\right) + \beta E_t v^b(y_{t+1}^T) \\ &\equiv v^b(y_t^T). \end{aligned}$$

For third line, the first term holds due to the fact that both the utility function and the aggregation function is strictly increasing and concave, and the second term holds due to the definition of $v^g = \max\{v^b, v^c\}$.

This result, however, yields a contradiction since if $v^c(d_t, y_t^T) \geq v^b(y_t^T)$ for all possible endowments of tradable output, then we have $D(d_t) = \emptyset$ by definition. Therefore, we conclude that if under the certain debt level, the default set is not empty, then we must have $q(y_t^T, d_{t+1})d_{t+1} - d_t < -L(y_t^T)$ for all d_{t+1} . \square

Proposition 2. If $y_1 \in D(d_t)$ and $\underline{y} \leq y_2 \leq y_1$, then $y_2 \in D(d_t)$.

Proof. Consider the difference between the value function under bad standings $v^b(y_t^T)$ and the value function of continuing to repay its debt $v^c(y_t^T, d_t)$ as $\Delta(y_t^T, d_t) \equiv v^b(y_t^T) - v^c(y_t^T, d_t)$. By definition, any tradable output in the default set $y_t^T \in D(d_t)$ satisfies $\Delta(y_t^T, d_t) > 0$.

Consider the first derivative of the difference function $\Delta_{y,d} = \frac{\partial \Delta}{\partial y_t^T} = v_y^b(y) - v_y^c(y, d)$.

Recall that

$$v^c(y, d) = \max_{\{d'\}} \{U(A(y + q(y, d')d' - d, F(1))) + \beta E_t v^g(y', d')\}$$

$$v^b(y) = U(A(y - L(y), F(1))) + \beta E_t [\theta v^g(y', 0) + (1 - \theta)v^b(y')].$$

The notation for tradable output y_t^T is simplified as y and y_{t+1}^T as y' , and similarly d_t is simplified as d and d_{t+1} as d' . Also, from previous discussion we know that the optimal working hours is $h_t^* = \bar{h}$ when the government chooses to devalue during default as its optimal policy, which is normalized to unity for simplicity. Applying the envelope theorem on $v^c(y, d)$,

$$v_y^c \equiv \frac{\partial v^c}{\partial y} = \frac{\partial}{\partial y} U[A(y + q(y, d')d' - d, F(1))]$$

$$= (1 + q_y(y, d')d') A_1(c_c, F(1)) U'(A(c_c, F(1))),$$

where $q_y(y, d') \equiv \frac{\partial q}{\partial y}$, $c_c \equiv y + q(y, d')d' - d$. This is easily derived by the chain rule. As for $v^b(y)$,

$$v_y^b \equiv \frac{\partial v^b}{\partial y} = [1 - L'(y)] A_1(c_b, F(1)) U'(A(c_b, F(1))),$$

where $L' \equiv \frac{\partial L}{\partial y}$ and $c_b \equiv y - L(y)$. For the sake of simplicity, the second parameter for the aggregation function $A(\cdot, \cdot)$ and its derivative $A_1(\cdot, \cdot)$ will be simplified by showing only the first argument since the second argument is always $F(1)$.

Accordingly, the difference function

$$\begin{aligned}
\Delta_y &= v_y^b(y) - v_y^c(y, d) \\
&= \left[1 - L'(y)\right] A_1(c_b) U'(A(c_b)) - \left(1 + q_y(y, d') d'\right) A_1(c_c) U'(A(c_c)) \\
&= A_1(c_b) U'(A(c_b)) - A_1(c_c) U'(A(c_c)) \\
&\quad - L'(y) A_1(c_b) U'(A(c_b)) - q_y(y, d') d' A_1(c_c) U'(c_c). \tag{45}
\end{aligned}$$

Note that $A(\cdot, \cdot)$ and $U(\cdot)$ are both concave and increasing by assumption. This implies that if $c_1 < c_2$, then (i) $A(c_1) < A(c_2)$, (ii) $A_1(c_1) > A_1(c_2) > 0$, and (iii) $U'(c_1) > U'(c_2) > 0$. Together, it implies that $U'(A(c_1)) > U'(A(c_2))$. Furthermore, since $\frac{A_1(c_1)}{A_1(c_2)} > 1$ and $\frac{U'(A(c_1))}{U'(A(c_2))} > 1$, we have

$$\frac{A_1(c_1) U'(A(c_1))}{A_1(c_2) U'(A(c_2))} > 1 \implies A_1(c_1) U'(A(c_1)) > A_1(c_2) U'(A(c_2)). \tag{46}$$

The first two terms in Equation (45) resembles this relationship. Since

$$c_b \equiv y - L(y) > y + q(y, d') d' - d \equiv c_c$$

according to Proposition 1, by Equation (46)

$$A_1(c_b) U'(A(c_b)) - A_1(c_c) U'(A(c_c)) < 0.$$

The third term in Equation (45) is negative since the loss function is assumed to be non-negative and nondecreasing (Na et al., 2018), hence $L'(y) > 0$. The marginal price of debt offered by foreign lenders $q_y(y, d')$ is positive since a better condition of output today y yields a higher output tomorrow y' due to the AR(1) nature of output, which in turn decreases the probability of default tomorrow. As a result, the price of bond increases.

Overall, we have

$$\Delta_y(y, d) < 0$$

if the default set is not empty. That is, $v^b(y) - v^c(y, d)$ is a decreasing function of y .

When default is an optimal policy under the tuple (y_1, d) , which means that $y_1 \in D(d)$, then by definition $v^b(y_1) > v^c(y_1, d)$. For any given $y_2 \leq y_1$, since $v^b(y) - v^c(y, d)$ is decreasing in y , we also have $v^b(y_2) > v^c(y_2, d)$, hence $y_2 \in D(d)$. \square

Proposition 3. The default set $D(d_t)$ is an interval $[\underline{y}, y^{T*}(d_t))$, where $y^{T*}(d_t)$ is increasing in d_t .

Proof. An output is in the default set if $v^b(y_t^T) - v^c(y_t^T, d_t) > 0$. It is trivial that as the output goes to infinity, the country has no incentive to default hence $v^c(\infty, d_t) > v^b(\infty)$. By the intermediate value theorem, it is obvious that there exist some y^{T*} such that $\Delta(y^{T*}, d_t) = v^b(y^{T*}) - v^c(y^{T*}, d_t) = 0$, where $y^{T*} = y^{T*}(d_t)$ is the upper bound of default set that depends on the current debt level. Since $\Delta_y(y_t^T, d_t) = v_y^b(y_t^T) - v_y^c(y_t^T, d_t) < 0$ when $D(d_t) \neq \emptyset$, all values such that $y_t^T < y_t^{T*}$ has $\Delta(y_t^T, d_t) < 0$. This proves that the default set is an interval¹.

Taking the total derivative of the upper limit with respect to the debt level using the equation $\Delta(y^{T*}(d_t), d_t) = 0$, we get

$$\frac{dy^{T*}(d_t)}{dd_t} = -\frac{\frac{\partial \Delta}{\partial d_t}}{\frac{\partial \Delta}{\partial y^T}} = -\frac{-v_d^c(y^{T*}(d_t), d_t)}{v_y^b(y^{T*}) - v_y^c(y^{T*}, d_t)}.$$

We know that $v_y^b(y^{T*}) - v_y^c(y^{T*}, d_t) < 0$. Applying the envelope theorem to $v^c(y_t^{T*}, d_t)$, we get

$$\frac{\partial v^c}{\partial d_t} = -A_1 \left[y_t^{T*} + q_t(y_t^{T*}, d_{t+1})d_{t+1} - d_t \right] U' \left[A(y_t^{T*} + q_t(y_t^{T*}, d_{t+1})d_{t+1} - d_t) \right] < 0.$$

¹ The lower bound of the interval is the lowest level of endowment \underline{y}

Eventually,

$$\frac{dy^{T^*}(d_t)}{dd_t} > 0. \quad (47)$$

This result implies that as the debt level increases (decreases), the upper bound of the default set should be strictly increasing (decreasing). \square

These results match those of Arellano (2008) and Uribe and Schmitt-Grohé (2017). As discussed by Na et al. (2018), when optimal devaluation and taxation policies are implemented, the equilibrium allocation in the economy aligns with that of the centralized Eaton-Gersovitz model. Therefore, it is not surprising that the default set in the decentralized economy exhibits similar behavior to that observed in a traditional centralized economy.