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Estimating Real Production *and* Expenditures Across Nations: A Proposal for Improving Existing Practice

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June 2004

Abstract

In this paper we propose a new approach to international comparisons of real GDP measured from the output-side. The traditional Gary-Khamis system to measure real GDP from the expenditure-side is modified to include differences in the terms of trade between countries. It is shown that this system has a strictly positive solution under mild assumptions. On the basis of a set of domestic final output, import and export prices and values for 14 European countries and the U.S. it is shown that differences between real GDP measured from the expenditure and output-side can be substantial, especially for small open economies.

JEL-code: F41, O47

* We thank Bettina Aten, Brian Easton, Erwin Diewert, Prasada Rao, Marshall Reinsdorf, Bart van Ark and seminar participants at Dartmouth University for helpful comments, along with Edwin Stuivenwold and Gerard Ypma for exceptional research assistance. Financial support was provided by the National Science Foundation.

1. Introduction

From its inception, the Penn World Tables (PWT), building on the International Comparisons Program (ICP) of the United Nations, has sought to compare the *standard of living* of individuals in different countries. That is, the term “real GDP per capita” as reported in the PWT is intended to represent the ability to purchase goods and services by a representative agent in the economy. The same is true of benchmark comparisons as published by the United Nations, Eurostat or OECD. But this expenditure-side interpretation of real GDP is quite different from the uses to which benchmark ICP and PWT data are frequently applied, such as in the “millions” of growth regressions (Sala-i-Martin, 1997), where “real GDP” is intended to reflect the production-side of the economy.¹

The goal of this paper is to carefully distinguish the production or output-side measure of real GDP, denoted $real\ GDP^o$, from the expenditure-side measure, denoted $real\ GDP^e$. Real GDP^o measures the production possibilities of an economy, whereas real GDP^e measures real expenditures adjusted for the trade balance.² The reason these concepts were not distinguished in the ICP and PWT is that they treat the net foreign balance in an unsatisfactory way. While there may have been some data justifications for that treatment in benchmark studies of the 1970s, this is no longer the case. The treatment of exports and imports proposed in this paper will not only remove the ambiguity presently surrounding real GDP^e and real GDP^o in the ICP and PWT, but provides a rich new international measure, namely the difference between them. Essentially, these two concepts differ by the *terms of trade* in the economy, i.e. the prices at which goods are exported and imported.

¹ There are many other applications where expenditure-side PPP's are used when output-side PPP's are really needed. Recent examples are Bernard and Jones (1996), Jones and Hall (1999) and Acemoglu and Ventura (2002).

² Note that we do not deal with differences in *nominal* GDP which arise when measured from either the expenditure,

In the first section of the paper we provide a brief background on how the ambiguous treatment of foreign trade arose during the development of the various benchmark studies and the extensions in PWT. In section 2, the distinction between real GDP on the output-side and expenditure-side is set out conceptually, followed by a discussion of how they can be separately measured in time series data. In order to incorporate these concepts into the PWT, however, we need to have a *cross-country* measure of their difference.

To achieve this, we propose a measure of the purchasing power parity for outputs (PPP^o) rather than expenditures (PPP^e). Currently the cornerstone of the ICP and PWT is the PPP for final expenditures (PPP^e), used to deflate nominal national income to obtain real GDP^e. This is described in Section 3. If instead we want to deflate nominal GDP to obtain *real GDP*^o, then we need to use PPP^o , which incorporates prices for exports and imports including both final goods and intermediate inputs. In section 4 we show how PPP^o can be computed using the Geary (1958)-Khamis (1970,1972) (GK) system, and further broken down into separate PPP's for final expenditures, exports and imports. It should be emphasized that by working at the level of entire economies, rather than sectors, some of the difficulties with measuring output-based PPP's are avoided. In particular, we find that while the international prices of intermediate inputs (from trade statistics) are used, the corresponding domestic prices of intermediates are not needed at all. In this sense, our use of trade data provides a *short cut* to obtain output-based deflators for the entire economy.³

income or output side. In the remainder of the paper we assume that nominal GDP is given.

³ The International Comparisons Project (ICOP) at the University of Groningen constructs real GDP by sector from the output side using the industry-of-origin approach. This requires comparison of sectoral output prices across countries (see van Ark and Timmer, 2004, for example). Sectoral real output can be aggregated to obtain real output GDP, but this has only been done for a limited number of countries so far. The short-cut proposed here, which uses international rather than domestic prices of intermediates, is much easier to implement for a larger set of countries.

In section 5, we provide an empirical application of our techniques. The data used for this illustration are from the 1996 and 1999 OECD benchmark comparison and trade data for 14 European countries plus the United States. It would be desirable to apply the methods to other benchmark comparisons and to time series as in PWT, but that is future work. It seems to us very useful to gain some quantitative insight into the importance of this change in method at an early stage so that if it is not second order effect, then international organizations can begin considering its adaptation.

It turns out that real GDP⁰ differs significantly from real GDP^e for a number of European countries. For example, Portugal and Greece both have low terms of trade when measured by our cross-country measure, which means that real GDP^e is less than real GDP⁰, by 4% or more in 1996 and 1999. Ireland also has a low terms of trade in 1996, with real GDP^e less than real GDP⁰, but its export PPP grows rapidly over 1996-99 which substantially narrows the gap between these two GDP measures. Since real GDP^e is close to that reported by the OECD, we are finding that countries with low terms of trade have *higher* real GDP when estimated from the output-side than currently reported: these countries are more productive than the existing cross-country real GDP figures suggest. This applies especially to Portugal, Greece, and Ireland, and to a lesser extent to Austria, Belgium, Italy, the Netherlands and Spain. Our results suggest that the difference between real GDP measured from the expenditure and output side diminishes when countries become richer, as terms of trade improve. Germany and the U.K. are prominent examples of the reverse pattern, whereby they have a high terms of trade and therefore real GDP^e exceeds real GDP⁰. Additional conclusions and directions for further research are given in section 6.

2. Present Practice

When the International Comparison Program began in 1968 there was some experience with purchasing power comparisons, including the expenditure side efforts of Gilbert and Kravis (1954) for the OEEC, and several bilateral comparisons with the USSR for the Comecon countries, and the study of Paige and Bombach (1959).⁴ Paige and Bombach carried out a comparison from the output side between the U.S. and the U.K. in which they encountered a number of difficulties, one of which was the need for double deflation for purposes of comparing value-added across sectors. That was a formidable task at the time and has become easier only to the extent that data are more available. Further Paige and Bombach realized that to make full comparisons it was necessary to collect prices on both final and intermediate products. Comparisons from the expenditure side appeared to require much less in the way of price collection. Thus when the United Nations International Comparisons Project began in 1968 it endorsed comparisons from both the output and expenditure sides, but recognized that it would be easier to carry out expenditure side comparisons.

While the expenditure side comparison does appear much easier than output side of national accounts, this is in part because the net foreign balance was treated in an *ad hoc* fashion. In benchmark comparisons the practice in Eurostat and the OECD has been to value the net foreign balance at exchange rates. Unfortunately, use of the exchange rate of one country, a practice in the EU, is not base-country invariant. The present practice in PWT is to value the net foreign balance at the *PPP for final goods expenditures*, which is simple and is base country invariant. In many discussions over the years, it was generally recognized that the appropriate

⁴ Still earlier work had been carried out by Colin Clark (1940) who prepared PPP and real GDP comparisons for a wide variety of countries which he was to expand and improve in a 1957 edition.

way to value trade would be at export and import prices. But certainly in the early years of the ICP this did not appear feasible from the standpoint of available data.

What was not really appreciated in these discussions is that valuing exports and imports at comparable prices was the only way to accurately compare the real output across countries, a central point of this paper. And once a comparison is made of real GDP on the output-side, it is possible to compare it to real GDP on the expenditure-side across countries. This difference turns out to not only be of analytic interest, but also to be a very revealing indicator of the extent to which the terms of trade differentially impact countries within any group like the OECD.

The distinction between real GDP on the output and expenditure-side can be illustrated by a simple diagram in a two-good economy, shown in Figure 1. We suppose that the PPF is shifting out due to technological change. At unchanged prices, production would increase from point A to point B. Suppose, however, that the relative price of good 1 falls due to its increased supply, so that the new prices are shown by the slope of the line P_3P_3 . Production now occurs at B' rather than B. We have drawn the case where the budget lines P_1P_1 and P_3P_3 are both tangent to an indifference curve U, at points C and D, indicating that the utility of the representative consumer is unchanged.

In the case we have illustrated, the production points A and B' lie on the same ray from the origin so that the *relative outputs* of the two goods are unchanged. This means that any index of real output would be identical, and would simply equal $OB'/OA > 1$, which is the proportional increase in both outputs. This is the increase in real GDP^o as measured on the output-side.

Now suppose we pose a different question, and ask what has happened to the welfare of the representative consumer, with indifference curve shown by U in Figure 1. An exact measure of consumer welfare, or real GDP^e measured on the expenditure-side, would be *unchanged* since

the consumer has the same utility at the two prices. This occurs because there has been fall in the price of the exportable good 1. The change in real GDP^e could be measured by the change in nominal expenditure deflated by an *exact consumer price index*, constructed with the same prices as real GDP^o but using *consumption quantities* rather than *production quantities* in the index.

This illustrates the most important difference between real GDP^e and real GDP^o, i.e. the use of consumption quantities rather than production quantities in the index. The difference between these is exports and imports, of course, but for production quantities we also need to include the imports and exports of all *intermediate inputs*, as well as their prices. These data are not currently used by the ICP, which restricts its attention to final goods.

This distinction between real GDP on the expenditure and output-side is recognized by the United Nations 1993 System of National Accounts (SNA), which proposes three measures of the former, which it calls real Gross Domestic Income (GDI).⁵ One definition is:

$$\text{Real GDI} = (\text{Nominal GDP})/(\text{Final expenditures price index}), \quad (1)$$

as compared to,

$$\text{Real GDP} = (\text{Nominal GDP})/(\text{GDP price index}). \quad (2)$$

The “final expenditures price index” in (1) is constructed over components of (C+I+G). By *excluding* imports and export prices within this price index, changes in the terms of trade (which affect nominal GDP) are then reflected in real GDI. This is demonstrated by Diewert and Morrison (1986), and Kohli (2004) has recently shown that these indexes for Switzerland lead to a much higher growth of real GDI than real GDP, due to its terms of trade improvement. We

⁵ See <http://unstats.un.org/unsd/sna1993/introduction.asp>, paragraph 16.154. The other definitions of real GDI depend on the deflator used for (X-M). Other than the final expenditures price index, we might use the import price index, export prices index, or an average of these, as recommended by Geary; see Neary (1997).

avoid the term “real GDI,” because it is suggestive of the income-approach to measuring GDP (through adding up the earnings of factors) which we do not use. Instead, we use real GDP^e to reflect the expenditure-side concept like (1). This is essentially the same as “real expenditure”, except that by using nominal GDP rather than nominal expenditures in the numerator of (1) we are adjusting for the net foreign balance.

Now we come to the key question motivating this paper: which concept does the PWT use as “real GDP” – the output-side measure real GDP^o, or the expenditure-side measure real GDP^e? It turns out that the answer is unclear: the ICP constructs real GDP^e in *benchmark years*, but then to interpolate between these years, the PWT must reconcile these changes in real GDP^e with the national accounts data reports on countries real GDP growth. Since national accounts real GDP growth is closer to real GDP^o, which is being compared to benchmark estimates of real GDP^e, the distinction between these becomes lost in the reconciliation.⁶ In other words, the differences between real GDP^e (using ICP data) and real GDP^o (using national accounts data) is eliminated, and the year-to-year growth of real GDP from the national accounts is used to impute real GDP in the PWT for all years in-between the benchmark years. The fact that the distinction between real GDP^e and GDP^o is not clearly made is a limitation of previous versions of the PWT that future revisions intend to improve upon.⁷

⁶ Real GDP in the national accounts always includes a measure of terms of trade losses or gains, although the deflator used might differ (see previous footnote). The process of reconciling the initial PWT estimates (in benchmark years) and the national accounts data for each country is described in Summers and Heston (1991, pp. 339-341).

⁷ For example the title of the Phase III report of the ICP was *World Product and Income* (Kravis, Heston and Summers, 1982). This clearly implied a treatment of GDP converted at PPPs of countries as interchangeable with what is termed *output* and *expenditure* in this paper. Further the provision of output per worker figures in PWT, though provided because users were taking per capita real GDP as a labor productivity measure, clearly implies that the real GDP number is an output measure.

3. Measurement of real GDP on the Expenditure Side

Suppose there are $i = 1, \dots, M$ final goods, such as the categories of goods currently collected by the ICP, of which the first M_0 are non-traded. These final goods may also be used as intermediate inputs, and there are another $i = M+1, \dots, M+N$ goods that are exclusively intermediate inputs; for convenience we treat these all as traded internationally. To treat domestic demand, trade and production in a consistent framework, an input-output analysis must be used. In this framework the fundamental equality is between total demand and total supply of each good. For each country $j = 1, \dots, C$, denote final demand by q_{ij} , intermediate demand by z_{ij} , output by y_{ij} , exports by x_{ij} and imports by m_{ij} , for $i = 1, \dots, M+N$. We assume that all of these quantities are *nonnegative*, but many can be zero: in particular, the intermediate inputs $i = M+1, \dots, M+N$ have $q_{ij} = 0$, and the non-traded goods $i = 1, \dots, M_0$ have $x_{ij} = m_{ij} = 0$. Total demand in country j is given by $q_{ij} + x_{ij} + z_{ij}$, and total supply by $y_{ij} + m_{ij}$. Hence the equality between demand and supply is:

$$q_{ij} + x_{ij} + z_{ij} = y_{ij} + m_{ij}, \quad i = 1, \dots, M+N. \quad (3)$$

Re-arranging terms, we obtain:

$$q_{ij} + x_{ij} - m_{ij} = y_{ij} - z_{ij}, \quad i = 1, \dots, M+N. \quad (4)$$

where we refer to $y_{ij} - z_{ij}$ as “net output” of each good, i.e. gross output minus intermediate demand.

Multiplying by prices and summing over goods $i = 1, \dots, M+N$, nominal GDP can be measured either from the expenditure side (left-hand side of (4)) or from the production side (right-hand side), where the units are the national currency. We will distinguish the prices $p_{ij} > 0$

for domestic consumption, used to multiply q_{ij} , $i = 1, \dots, M+N$, from those for imports and exports, $p_{ij}^m > 0$ and $p_{ij}^x > 0$ respectively.⁸ In the System of National Accounts (SNA), it is recommended that export prices be measured *net of tariffs and freight, including any subsidy to the buyer but not to the seller*, i.e. as the f.o.b. (freight on board) price in the exporting country.⁹ It is also recommended that import prices be measured by the f.o.b. prices, though the SNA recognizes that this is not always possible and c.i.f. (cost, insurance, and freight) import prices might be used instead. In either case, the import prices are *net of tariffs*.¹⁰

With these conventions for p_{ij}^m and p_{ij}^x , let $X_j = \sum_{i=M_0+1}^{M+N} p_{ij}^x x_{ij}$ and $M_j = \sum_{i=M_0+1}^{M+N} p_{ij}^m m_{ij}$ denote the value of exports and imports at tariff-free prices, so that nominal GDP measured on the expenditure side is:

$$\text{Nominal GDP}_j^e \equiv \sum_{i=1}^M p_{ij} q_{ij} + (X_j - M_j). \quad (5)$$

Using (4), we can re-write (5) as:

$$\begin{aligned} \sum_{i=1}^M p_{ij} q_{ij} + (X_j - M_j) &= \sum_{i=1}^{M+N} p_{ij} [(y_{ij} - z_{ij}) - (x_{ij} - m_{ij})] + \sum_{i=M_0+1}^{M+N} (p_{ij}^x x_{ij} - p_{ij}^m m_{ij}) \\ &= \sum_{i=1}^{M+N} p_{ij} (y_{ij} - z_{ij}) + \sum_{i=M_0+1}^{M+N} [(p_{ij} - p_{ij}^m) m_{ij} - (p_{ij} - p_{ij}^x) x_{ij}], \end{aligned} \quad (6)$$

where the first line is obtained by using $q_{ij} = (y_{ij} - z_{ij}) - (x_{ij} - m_{ij})$ for the final goods $i=1, \dots, M$, whereas the intermediates have $q_{ij} = 0$, so that $[(y_{ij} - z_{ij}) - (x_{ij} - m_{ij})] = 0$ for $i=M+1, \dots, M+N$.

⁸ In principle we should also distinguish producer from consumer prices, which can differ due to taxes and retail margins, but do not incorporate that distinction here.

⁹ See <http://unstats.un.org/unsd/sn1993/introduction.asp>, paragraphs 6.235, 6.237 and 15.35.

¹⁰ We will use the convention that if the importing country hires the freight services of another country, then we use the c.i.f. import prices (importing both the good and the freight services), but if it uses its own carriers then the f.o.b. prices are used instead (since the freight charges are a part of domestic GDP).

Then the second line follows because $x_{ij} = m_{ij} = 0$ for the non-traded goods $i=1, \dots, M_0$. We can interpret $(p_{ij} - p_{ij}^m)$ as import tariffs (subsidies if negative), and $(p_{ij} - p_{ij}^x)$ as exports subsidies (taxes if negative). So the final summation on the second line is interpreted as import revenue less export subsidies.¹¹ Adding this to the value of net output $\sum_{i=1}^{M+N} p_{ij}(y_{ij} - z_{ij})$ as in (6) gives us nominal GDP measured on the production or output side:¹²

$$\text{Nominal GDP}_j^o = \sum_{i=1}^{M+N} p_{ij}(y_{ij} - z_{ij}) + \sum_{i=M_0+1}^{M+N} [(p_{ij} - p_{ij}^m)m_{ij} - (p_{ij} - p_{ij}^x)x_{ij}], \quad (7)$$

which clearly equals nominal GDP measured on the expenditure side, from (6).

The real counterpart to GDP measured on the expenditure-side in the PWT is obtained according to the Geary-Khamis (GK) system, as follows. We compute the “reference prices” π_i^e for final goods and the purchasing power parities PPP_j^e for each country from the simultaneous equations:

$$\pi_i^e = \sum_{j=1}^C (p_{ij} / PPP_j^e) q_{ij} / \sum_{j=1}^C q_{ij}, \quad i=1, \dots, M, \quad (8)$$

and,

$$PPP_j^e = \sum_{i=1}^M p_{ij} q_{ij} / \sum_{i=1}^M \pi_i^e q_{ij}, \quad j=1, \dots, C. \quad (9)$$

In (8), the nominal prices p_{ij} of final goods are deflated by the PPP exchange rate, and then averaged across countries. The exchange rates are obtained from (9), as the ratio of nominal to real final expenditure, where real expenditure is evaluated using the reference prices.

The system (8)-(9) are $M+C$ equations, but are linearly dependent: by re-arranging (8)

¹¹ If the country is using its own transportation services for imports, so f.o.b. export prices are used for p_{ij}^m , then what we are calling tariff revenue includes the transportation services, which are a part of GDP.

¹² The SNA makes it clear that when measuring the trade balance in (5) at exporter prices, then tariffs less export subsidies need to be added to the value of net output as in (6) and (7) to obtain equality between GDP^e and GDP^o .

and summing over final goods, or by re-arranging (9) and summing over countries, we obtain:

$$\sum_{i=1}^M \sum_{j=1}^C \pi_i^e q_{ij} = \sum_{i=1}^M \sum_{j=1}^C p_{ij} q_{ij} / PPP_j^e, \quad (10)$$

so that world real expenditure at the reference prices π_i equals the sum of countries' expenditures deflated by the PPP exchange rates. This is the key *aggregation property* that the GK system is designed to satisfy. With this, the system (8)-(9) has $M+C-1$ independent equations to determine $M+C-1$ unknowns, so a normalization must be chosen. We will use the normalization that world real expenditure equals the sum of countries' expenditures converted to dollars using the *nominal* exchange rates.¹³ Letting E_j denote the local currency price of US\$ in each country j , this normalization is:

$$\sum_{i=1}^M \sum_{j=1}^C p_{ij} q_{ij} / PPP_j^e = \sum_{i=1}^M \sum_{j=1}^C p_{ij} q_{ij} / E_j. \quad (11)$$

With a normalization like (11), the fact that $q_{ij} \geq 0$ in (8)-(9), with $\sum_{j=1}^N q_{ij} > 0$, ensures that there is a positive solution for π_i^e and PPP_j^e (Prasada Rao, 1971, Diewert, 1999).

Subtracting from real expenditure the trade balance deflated by the PPP exchange rate, we obtain what is called real GDP in the PWT, and what we shall call real GDP^e:

$$\text{Real GDP}_j^e \equiv \sum_{i=1}^M \pi_i^e q_{ij} + (X_j - M_j) / PPP_j^e. \quad (12)$$

The idea of adjusting for the trade balance (evaluated at expenditure prices) is that we are interested in the purchasing power of the economy without borrowing from or lending to the future, i.e. using current resources. We can regard $(X_j - M_j)$ as an income transfer which is

¹³ Notice that this normalization differs from the one made in PWT 6.1 where the PPP for U.S. GDP was set as the numeraire. Actually, the procedure proposed here is equivalent to setting the PPP for 'world' GDP to one.

deflated by the PPP for domestic final goods, before evaluating real GDP on the expenditure side. This suggests an alternative normalization to PPP_j^e to that shown in (11), namely:

$$\sum_{i=1}^M \sum_{j=1}^C [p_{ij}q_{ij} + (X_j - M_j)] / PPP_j^e = \sum_{i=1}^M \sum_{j=1}^C [p_{ij}q_{ij} + (X_j - M_j)] / E_j \quad (13)$$

In (13) we are specifying that world real GDP^e equals world nominal GDP, where the latter is evaluated at the nominal exchange rates E_j .

4. Measurement of Real GDP on the Output Side

In order to construct the output-side real GDP, it is necessary to use the *net domestic output* of goods, i.e. $(y_{ij} - z_{ij})$ rather than *final consumption* of goods q_{ij} , i.e. using (7) rather than (5). In general, it is quite difficult to construct “reference prices” for *all goods* (final plus intermediate) across a range of countries, as needed for this type of calculation. This is the task that the International Comparisons Project (ICOP) at the University of Groningen has undertaken for specific sectors and countries, and it would be a Herculean task to do the same for the entire economies of multiple countries. As an alternative, we shall examine whether the data for final goods combined with the detailed data for exports and imports are enough to give us a measure of output-side real GDP, called real GDP^o.

On the output-side, consider the following GK system to construct reference prices for the net outputs, export and imports:

$$\pi_i^o = \sum_{j=1}^C (p_{ij} / PPP_j^o) q_{ij} / \sum_{j=1}^C q_{ij}, \quad i = 1, \dots, M, \quad (14)$$

$$\pi_i^x = \sum_{j=1}^C (p_{ij}^x / PPP_j^o) x_{ij} / \sum_{j=1}^C x_{ij}, \quad i = M_0 + 1, \dots, M + N, \quad (15)$$

$$\pi_i^m = \sum_{j=1}^C (p_{ij}^m / PPP_j^o) m_{ij} / \sum_{j=1}^C m_{ij} , \quad i=M_0+1, \dots, M+N, \quad (16)$$

and,

$$PPP_j^o = \frac{\text{Nominal GDP}_i^o}{\sum_{i=1}^{M+N} \pi_i^o (y_{ij} - z_{ij}) + \sum_{i=M_0+1}^{M+N} [(\pi_i^o - \pi_i^m) m_{ij} - (\pi_i^o - \pi_i^x) x_{ij}]}, \quad j=1, \dots, C. \quad (17)$$

In (14) we construct reference domestic prices for the final goods, and in (15) and (16) we construct reference prices for exports and imports. These are used in the denominator of (17), which defines the purchasing-power-parity PPP_j^o as the ratio of nominal to real GDP_j^o . In order for these definitions to make sense, we make the following assumptions, already alluded to above:

Assumption 1

Quantities are non-negative, $q_{ij}, x_{ij}, m_{ij} \geq 0$, with $\sum_{i=1}^M q_{ij} > 0$, $\sum_{i=M_0+1}^{M+N} x_{ij} > 0$, $\sum_{i=M_0+1}^{M+N} m_{ij} > 0$.

As it is written, however, the system (14)-(17) is incomplete because we have not yet specified the domestic reference prices of the intermediate inputs, i.e. π_i^o for $i=M+1, \dots, M+N$. These prices appear in real GDP_j^o in the denominator of (17). But fortunately, real GDP_j^o can be re-written as:

$$\begin{aligned} \text{Real GDP}_j^o &= \sum_{i=1}^{M+N} \pi_i^o (y_{ij} - z_{ij}) + \sum_{i=M_0+1}^{M+N} [(\pi_i^o - \pi_i^m) m_{ij} - (\pi_i^o - \pi_i^x) x_{ij}] \\ &= \sum_{i=1}^M \pi_i^o q_{ij} + \sum_{i=M_0+1}^{M+N} (\pi_i^x x_{ij} - \pi_i^m m_{ij}), \end{aligned} \quad (18)$$

which follows from (4), using $(y_{ij} - z_{ij}) = q_{ij}$ for the non-traded final goods $i=1, \dots, M_0$, while the intermediate inputs have $q_{ij} = 0$ so that $(y_{ij} - z_{ij}) = (x_{ij} - m_{ij})$ for $i=M+1, \dots, M+N$. Therefore, to compute real GDP_j^o we do not need the domestic reference prices π_i^o at all for the intermediate

inputs! Also, the second line makes it clear that real GDP_i^o is evaluated using reference prices

π_i^x and π_i^m for traded goods that *do not include* any import tariffs or subsidies to exporters. Using

(18), we can re-write PPP_j^o in (17) as,

$$PPP_j^o = \frac{\text{Nominal } GDP_j^o}{\sum_{i=1}^M \pi_i^o q_{ij} + \sum_{i=M+1}^{M+N} (\pi_i^x x_{ij} - \pi_i^m m_{ij})}, \quad j=1, \dots, C. \quad (19)$$

The system (14)-(16) and (19) has the same number of equations as unknowns. It can be confirmed that the equations are linearly dependent, in the sense that summing (14)-(16) or alternatively (19), we obtain:

$$\sum_{j=1}^C \left(\sum_{i=1}^M \pi_i^o q_{ij} + \sum_{i=M_0+1}^{M+N} (\pi_i^x x_{ij} - \pi_i^m m_{ij}) \right) = \sum_{j=1}^C \left(\sum_{i=1}^M p_{ij} q_{ij} + (X_j - M_j) \right) / PPP_j^o. \quad (20)$$

Thus, one normalization is needed, and we will specify that world real GDP deflated by the output-based PPP_j^o equals world GDP evaluated with the nominal exchange rates.

$$\sum_{i=1}^M \sum_{j=1}^C [p_{ij} q_{ij} + (X_j - M_j)] / PPP_j^o = \sum_{i=1}^M \sum_{j=1}^C [p_{ij} q_{ij} + (X_j - M_j)] / E_j. \quad (21)$$

From the normalizations in (13) and (21), world real GDP evaluated on the expenditure-side will equal world real GDP evaluated on the output-side.

Summing up, we have shown that the GK system (14)-(16), (19) and (21) can be used to obtain a cross-country measure of the GDP price deflator from the output-side, which is PPP_j^o .

We have therefore achieved our goal of demonstrating that final goods data, in conjunction with export or import data, can be used to construct real GDP^o on the output side. However, it remains to be shown that this system has a solution. This task is complicated by the fact that real

GDP_j^0 , appearing in (18) and the denominator of (19), is not guaranteed to be positive for all possible reference prices. This can be ruled out by some additional assumptions, as follows.

First, define the *budget shares* for each final, export and import goods as:

$$\theta_{ij}^v \equiv p_{ij}^v v_{ij} / \text{Nominal GDP}_j, \quad v = q, x, m, \quad (22)$$

where $i=1, \dots, M$ for $v = q$; $i = M_0+1, \dots, M+N$ for $v = x, m$; and $j=1, \dots, C$. Notice that these budget shares are measured relative to nominal GDP. In addition, define the *market shares* for each good as:

$$\mu_{ij}^v \equiv v_{ij} / \sum_{k=1}^C v_{ik}, \quad v = q, x, m, \quad (23)$$

where $i=1, \dots, M$ for $v = q$; and $i=M_0+1, \dots, M+N$ for $v = x, m$. The market shares are measured relative to the world quantity of final demand, exports or imports for each good. Denote the column vectors of budget and market shares by θ_j^v and μ_j^v for $v = q, x, m$ and country j . Then our second assumption is:

Assumption 2

For all countries $j, k = 1, \dots, C$, we have $w_{jk} \equiv \theta_j^q \mu_k^q + \theta_j^x \mu_k^x - \theta_j^m \mu_k^m > 0$.

Clearly, this assumption limits the size of the import shares θ_{ij}^m and μ_{ij}^m . It is appropriate to think of w_{jk} as “weights” because it is readily shown that $\sum_{k=1}^C w_{jk} = 1$. While it is easy to construct examples where Assumption 2 is violated for some countries j and k , it is also true that for “reasonable” values of the import budget and market shares, Assumption 2 can hold. (In fact, it will hold for the U.S. and European countries in our sample). Given this, we have:

Theorem

Under Assumptions 1 and 2, the system (14)-(18) has a strictly positive solution for π_i^o , π_i^x , π_i^m , real GDP_j^o and PPP_j^o .

It is worth emphasizing that Assumption 2 is a sufficient condition to obtain this result, but is not necessary. In particular, we show in the Appendix that the vector of real GDP_j^o across countries is a column eigenvector of the matrix $W=[w_{ij}]$, corresponding to an eigenvalue of unity. Under Assumption 2, this is the Frobenius root of W and its corresponding eigenvector is strictly positive, so that real GDP_j^o is strictly positive along with PPP_j^o and all the reference prices, as stated in the Theorem. However, it is quite possible to obtain a strictly positive eigenvector and therefore a positive solution for real GDP_j^o even without having all the elements of W positive, and in this sense, Assumption 2 is sufficient but not necessary.

Notice that real GDP_j^o in (18) can be decomposed as:

$$\begin{aligned} \text{Real } GDP_j^o &= \sum_{i=1}^M \pi_i^o q_{ij} + \sum_{i=M_0+1}^{M+N} (\pi_i^x x_{ij} - \pi_i^m m_{ij}) \\ &= \left(\frac{\sum_{i=1}^M \pi_i^o q_{ij}}{\sum_{i=1}^M p_{ij} q_{ij}} \right) \sum_{i=1}^M p_{ij} q_{ij} + \left(\frac{\sum_{i=M_0+1}^{M+N} \pi_i^x x_{ij}}{\sum_{i=M_0+1}^{M+N} p_{ij}^x x_{ij}} \right) X_i + \left(\frac{\sum_{i=M_0+1}^{M+N} \pi_i^m m_{ij}}{\sum_{i=M_0+1}^{M+N} p_{ij}^m m_{ij}} \right) M_i. \end{aligned} \quad (24)$$

We can define the three ratios appearing in (24) as the inverse of the PPP's for final expenditure, exports and imports:

$$PPP_j^q \equiv \left(\frac{\sum_{i=1}^M p_{ij} q_{ij}}{\sum_{i=1}^M \pi_i^o q_{ij}} \right), \quad PPP_j^x \equiv \left(\frac{\sum_{i=M_0+1}^{M+N} p_{ij}^x x_{ij}}{\sum_{i=M_0+1}^{M+N} \pi_i^x x_{ij}} \right), \quad PPP_j^m \equiv \left(\frac{\sum_{i=M_0+1}^{M+N} p_{ij}^m m_{ij}}{\sum_{i=M_0+1}^{M+N} \pi_i^m m_{ij}} \right). \quad (25)$$

Comparing (12) and (24), it is immediate that the difference between real GDP_j^e and real GDP_j^o is due to the deflation of final expenditure, exports and imports:

$$\begin{aligned} \text{Real } GDP_j^e - \text{Real } GDP_j^o \\ = \left(\frac{PPP_j^q}{PPP_j^e} - 1 \right) \left(\frac{\sum_{i=1}^N p_{ij} q_{ij}}{PPP_j^q} \right) + \left(\frac{PPP_j^x}{PPP_j^e} - 1 \right) \left(\frac{X_j}{PPP_j^x} \right) - \left(\frac{PPP_j^m}{PPP_j^e} - 1 \right) \left(\frac{M_j}{PPP_j^m} \right). \end{aligned} \quad (26)$$

We could hope that PPP_j^e and PPP_j^q are similar, since they are both computed from final expenditures, but with different reference prices. If these two deflators for final expenditure were equal, then either $PPP_j^x > PPP_j^e$ or $PPP_j^m < PPP_j^e$ is needed to have real GDP_j^e exceed real GDP_j^o , and both inequalities holding is sufficient for this. To interpret these conditions, it is useful to first convert export and import prices to U.S. dollars using the nominal exchange rate. Then having export prices above their reference level and import prices below their reference level will contribute towards real GDP_j^e exceeding real GDP_j^o . For example, proximity to markets that allow for higher export prices would work in this direction, but being distant from markets leading to high import prices would work in the opposite direction, raising PPP_j^m and tending to make real GDP_j^e less than real GDP_j^o .

We conclude this section with a warning: the difference between our cross-country measure of real GDP_j^e and real GDP_j^o *does not* measure the gains from trade as compared to autarky. Instead, it should be interpreted as a measure of the gains from trade from having export or import prices that *differ* from the international reference prices π_i^x and π_i^m . Since π_i^x

and π_i^m are themselves *averages* of countries' actual export and import prices, it is impossible for all countries to be gaining (or losing) relative to these averages. What we can expect to find is that some countries – with higher-than-average exports prices or low import prices – will gain in the sense that real GDP_j^e exceeds real GDP_j^o , whereas other countries will have real GDP_j^e less than real GDP_j^o due to low export prices or high import prices.

Furthermore, the difference between real GDP_j^e and real GDP_j^o does not measure the costs of protection in a country. This can be seen from a simple example. Suppose that the reference price vectors for final goods are equal, $\pi_i^e = \pi_i^o$, and furthermore, the traded goods reference prices are proportional to observed prices in country j , $\pi_i^x = \pi_i^m \propto p_{ij}^x = p_{ij}^m$, at which trade is balanced. The example we have in mind is where the domestic reference prices $\pi_i^e = \pi_i^o$ reflect average import tariffs applied by most countries, but the international reference prices $\pi_i^x = \pi_i^m$ reflect tariff-free prices. Since trade is balanced at the international reference prices, it is immediate that real GDP_j^e in (12) equals real GDP_j^o in (18), both of which measure *the value of production or consumption at domestic reference prices*.

This situation is illustrated in Figure 2 for the case where country j has a tariff on good 2, with production at A and consumption at B. The world relative price of good 1 is shown by the slope by the line P_1AC , while the domestic relative price is shown by the slopes of P_2P_2 and P_3P_3 . Suppose initially that country j 's tariff on good 2 is also applied by all other countries, so the domestic reference prices are proportional to domestic prices, $\pi_i^e = \pi_i^o \propto p_{ij}$. Then real GDP_j^e equal to real GDP_j^o would both be measured by the budget line P_3P_3 , which takes domestic revenue of P_2P_2 and adds the tariff revenue. Alternatively, suppose that countries other than j are

engaged in free trade, so domestic reference prices $\pi_i^e = \pi_i^o$ equal international reference prices $\pi_i^x = \pi_i^m$, while only country j has the tariff on good 2. In that case real GDP_j^e equals real GDP_j^o again, and would both be measured by the budget line P_1AC , which is the value of production or consumption at world prices.

We see from this example we see that real GDP^e can equal real GDP^o even when country j 's tariffs range from positive (low gains from trade) to zero (highest gains) to negative (possible losses from trade if subsidies are high enough). This means that observed differences between real GDP^e and real GDP^o cannot be interpreted as either gains from trade or the costs of protection. The reason for that is that both real GDP^e and real GDP^o are “fixed weight” indexes, where the quantities do not respond at all to the reference prices. In contrast, Neary (2004) has developed a theoretical PPP index on the expenditure-side whose weights reflect optimal consumer purchases in response to reference prices. For example, with reference prices given by the slope of P_1AC in Figure 2, then optimal consumption would be at D , and real expenditure to achieve utility U_j would be given by the budget line P_4D . It seems possible that a theoretical PPP index on the output-side could also be developed, so that optimal output would occur at B , and real output would be measured by the budget line P_5B . In this case, real output exceeds real expenditure, and the difference between them reflects the cost of protection. Extending our definitions of PPP and real GDP on the output-side to allow quantities to respond to reference prices, as Neary (2004) does for the consumer, would potentially provide a measure of the costs of protection, but is beyond the scope of this paper. Instead, we turn next to an empirical application of our results.

5. Application to Europe and the U.S.

To illustrate our formulas, we use a dataset for trade in 1996 and 1999 for 14 European countries and the United States. The source for the European data is the Prodcom dataset of Eurostat. This database contains specific product data classified according the 8-digit PRODCOM product list.¹⁴ Using the value and quantity, we constructed unit-values for exports and imports. There are 4,560 products in total, but for many of these the unit-values are not available or they are not traded by the European nations. As a check on the unit-values, we corrected some obvious errors in the units of quantity and then removed outliers. Outliers were defined as those unit-values which are greater than twice or less than half as much as the European average unit-value. The resulting cleaned dataset has 3,240 products in 1999, with an average of 1,885 products exported by each country and 2,347 products imported.

For the United States, export and imports for 1996 and 1999 are available on the Harmonized System (HS) basis, as described in Feenstra, Romalis and Schott (2002). Using a concordance between the 8-digit Prodcom codes and the 6-digit Harmonized System (HS) classification, we converted the U.S. export and import data to the Prodcom codes. On the export side in 1999, 43% of merchandise exports corresponded to a Prodcom code in the cleaned European dataset (the remaining trade was not exported by Europe or did not have adequate unit-values there). Some of these products had differing unit of measurement in the U.S. and Europe, so correcting for these we ended up with 38% of U.S. exports, or 2,350 products, where the unit-values in the U.S. and Europe could be compared. For imports in 1999, 54% of merchandise imports corresponded to a Prodcom code in the cleaned European dataset, and after correcting for different units of measurement, we obtained 41% of U.S. imports accounting for 2,378

¹⁴ The headings of the PRODCOM list are derived from the Harmonized System or the Combined Nomenclature, which thus enables comparisons to be made between production statistics and foreign trade statistics.

products. Thus, the United States is exporting somewhat more products than the average European country in this dataset, and importing about the same number of products.¹⁵

For PPP_j^e we make use of the parities for about 200 expenditure categories provided by the OECD, in 1996 and 1999. Specifically, we compute the “expenditure price levels” for each country, defined as:

$$PL_j^e \equiv PPP_j^e / E_j = \sum_{i=1}^M (p_{ij} / E_j) q_{ij} / \sum_{i=1}^M \pi_i^e q_{ij}, \quad (27)$$

where E_j denotes the local currency price of US\$ in each country j . Unlike the PPP’s, the price levels are unit free, and indicate how the nominal prices in each country (converted to dollars with the nominal exchange rate) compare to the reference prices (also in dollars). These price levels are reported in columns (1) and (2) of Table 1 for 1996 and 1999. Note that between these two years there was an appreciation of the US\$, and a depreciation of all European currencies, except the British pound, as reported in column (3). This change led to a substantial fall in the price levels for European countries when converted to US\$, or (p_{ij}/E_j) as in (27). This reflects the well-known observation that domestic prices do not change as rapidly as nominal exchange rates.¹⁶

In columns (4) and (5) of Table 1 we report the relative output price level PL_j^o for 1996 and 1999, computed as the output-based PPPs divided by the nominal exchange rate, or PPP_j^o / E_j . Note that the normalization used in these columns is identical to that in columns (1)

¹⁵ This does not yet check that the unit-value for each product in U.S. exports or imports lies within one half and twice the magnitude of the average European unit-value. Performing this check, we find that 944 exports unit-values for the United States should be excluded, leaving 1,406 export products, and 834 import unit-values for the U.S. should be excluded, leaving 1,544 import products. However, it turns out excluding these products for the U.S. has a negligible effect on the PPP’s for any of the countries.

¹⁶ The results are close to those found when using PWT 6.1 data, see Appendix Table 1. Differences are caused by the fact that PPPs in PWT are based on a much larger set of countries, including non-OECD, than used here.

and (2), i.e. the value of real GDP among the 15 countries equals the nominal value of GDP in US\$. As can be shown from (13) and (21), this means that a weighted average of the price levels sums to unity. The variance in output price levels is similar to that in expenditure price levels. As to be expected, output and expenditure price levels are close for big countries such as the U.S., U.K. and France. But in small open economies like the Netherlands and Ireland, differences are bigger.

In Tables 2 and 3 output price levels are decomposed for the years 1996 and 1999 respectively. They are decomposed into price levels for final goods, exports and imports, that is:

$$PL_j^q \equiv PPP_j^q / E_j = \sum_{i=1}^M (p_{ij} / E_j) q_{ij} / \sum_{i=1}^M \pi_i^q q_{ij} , \quad (28)$$

$$PL_j^x \equiv PPP_j^x / E_j = \sum_{i=M_0+1}^M (p_{ij} / E_j) x_{ij} / \sum_{i=1}^M \pi_i^x x_{ij} , \quad (29)$$

$$PL_j^m \equiv PPP_j^m / E_j = \sum_{i=M_0+1}^M (p_{ij} / E_j) m_{ij} / \sum_{i=1}^M \pi_i^m m_{ij} . \quad (30)$$

These price levels are reported in columns (1) to (3). The ratio of PL_j^x / PL_j^m is reported as the terms of trade for each country in column (4):

$$TOT_j^x \equiv PL_j^x / PL_j^m , \quad j = 1, \dots, N. \quad (31)$$

Finally, in columns (5)-(7) we report real GDP^e, real GDP^o, and nominal GDP measured in US\$ for each year.

A number of observations can be made. First, the output price levels of final goods in column (1) appear to be very close to the expenditure price levels given in Table 1, which is encouraging. It indicates that the use of a different set of reference prices for final goods does not influence the estimation of a PPP for final goods: PPP_j^q is almost equal to PPP_j^e . The

choice for output prices for final goods π_i^q or expenditure prices π_i^e , where the former is influenced indirectly by exports and imports via the simultaneous system (14)-(18), but the latter is not, is inconsequential.

Second, export price levels are highly correlated with the price levels of final goods, although the variance is much less. In 1996, the highest relative export price levels could be found in Sweden and Germany, while the lowest in Greece, Portugal and Spain. A similar pattern exists in 1999, except that now relative export prices in the U.S. are the highest of all. The latter finding is indicative of a lack of price response by U.S. exporters on the strengthening of the US\$ vis-à-vis the European currencies over the period from 1996 to 1999. Export prices were not fully adjusted downwards to counteract the appreciation. Apart from exchange rates, other factors play an important role in explaining differences in export prices faced by various countries. It is well known that under imperfect competition, exporters can and do charge different prices in various destination markets (Knetter 1989, 1993). Such market segmentation can arise in response to changes in nominal exchange rates, or trade policies of the importer. In addition, it is becoming recognized that countries differ systematically in their qualities and bundles of export goods (Schott, 2004). This would appear to create differences in the relative value of their exports. A country whose exports consist mainly of relatively cheap goods (relative to international price levels of other goods) will have a lower export PPP. Import price levels of countries are much closer to each other than either export or final goods prices. This might be related to the fact that import baskets are much more similar across countries than export baskets.

The resulting terms of trade appear to be especially disadvantageous for countries like Greece, Portugal and Spain, in contrast to Germany, U.K. and the U.S. whose import PPPs are

much lower than their export PPPs. As a result we can see that real GDP^o in column (6) differs significantly from real GDP^e in column (5) for a number of European countries. Portugal and Greece both have real GDP^e levels which are lower than real GDP^o, by at least 4% in 1996 and 1999. Ireland also had a low terms of trade in 1996, with real GDP^e nearly 9% below real GDP^o, but its rapid growth in the export PPP narrowed the gap between these two GDP measures substantially. Austria (in 1999), Belgium, the Netherlands (in 1999) and Spain are other examples of countries in which real GDP^e is at least 2% lower than real GDP^o. Germany and the U.K. are prominent examples of the reverse pattern with real output GDP bigger than real expenditure GDP, due to relative advantageous terms of trade.

In general one can say that countries with lower levels of GDP per capita (evaluated at either expenditure or output prices), also have disadvantageous terms of trade and hence higher differences between real GDP measured from the expenditure and output side. This is shown in Figure 3 in which countries are ranked at their real GDP^e per capita measure in 1996. There is a clear positive relationship between the level of GDP per capita and difference between real GDP^e and GDP^o. A similar pattern can be found for 1999. These findings have clear implications for much of the cross-country regression literature based on GDP measures at international prices from the PWT. For example, in the convergence literature changes in GDP per capita are regressed on initial levels of GDP per capita. A standard result is that the conditional rate of convergence is about 2% per year. However, our results suggest that this rate might be overestimated. Cross-country differences in GDP per capita levels are less when measuring real GDP from the output side than from the expenditure side. It is the former which is the right

concept for convergence analysis, while the latter concept is actually being used in exercises based on PWT data.

In addition, our results suggest that the difference between real GDP measured from the expenditure and output side diminishes when countries become richer, as terms of trade are improving. This finding lends little support to the model by Acemoglu and Ventura (2002) in which convergence is being driven by declining export prices for countries that grow faster. They claim that specialization and trade introduce *de facto* diminishing returns: countries that accumulate capital faster than average experience a worsening of the terms of trade, depressing rates of return to capital and discouraging further accumulation. In contrast, our results suggest that countries which grow faster will enjoy improved terms of trade, hence international trade might be a driver of divergence in the world income distribution. However, before any conclusive statement can be made on this issue our approach need to be applied to a larger set of countries and a longer time period. This will be the topic of further research.

6. Conclusions

In this paper, it has been shown that there is a fundamental difference between real GDP measured from the output side or from the expenditure side in international comparisons. The difference between the two concepts is in the treatment of the terms of trade. Real GDP from the expenditure side represents the ability to purchase goods and services while real GDP from the output-side measures the production possibilities of the economy. It is the latter concept of real GDP which is of interest to many studies of growth and convergence in the world economy. However, available data from the OECD or the Penn World Tables is based on a mix of cross-country expenditure-side measures of real GDP for benchmark years with national growth rates of real GDP based on output-side measures.

In this paper a clear-cut distinction between the two measures is made. It is shown that real GDP from the output-side can be measured using an expanded Geary-Khamis system of international comparisons. The expanded GK-system includes reference price and PPPs not only for domestic final goods, but also for export and imports. It is shown that this expanded system has a strictly positive solution under mild assumptions. An application of the extended GK-system to 14 European countries and the U.S. shows that the quantitative differences between real GDP measured from the output- or expenditure-side are not of a second order. Real GDP^e can overstate real GDP^o by as much as 9%, and the difference is especially large in small and open economies. It also appears that this difference diminishes when countries become richer, as terms of trade PPPs are higher in more advanced countries.

These findings can have serious implications for studies on growth and divergence in the world economy which currently are based on measures of GDP from the expenditure-side, but conceptually refer to measures of GDP from the output-side. However, before any conclusive statement can be made on this issue our approach need to be applied to a larger set of countries and a longer time period, which will be the topic of further research. Using international trade data from the United Nations which covers the period from 1960 to 2000 and a much wider set of countries more will be known about the differences in the effects of terms of trade between countries and over time. The final aim is to implement the approach in this paper to provide measures of real GDP both from the output-side and expenditure-side in the new benchmark version of the Penn World Tables.

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Appendix

Proof of Theorem:

Denoting nominal GDP_j by G_j and real GDP_j^o by R_j^o, j = 1, ..., C, we can substitute PPP_j^o =

R_j^o / G_j in (14)-(16) to obtain the reference prices,

$$\pi_i^o = \sum_{j=1}^C R_j^o \theta_{ij}^q / \sum_{j=1}^C q_{ij}, \quad i=1, \dots, M, \quad (A1)$$

$$\pi_i^x = \sum_{j=1}^C R_j^o \theta_{ij}^x / \sum_{j=1}^C x_{ij}, \quad i=M_0+1, \dots, M+N, \quad (A2)$$

$$\pi_i^m = \sum_{j=1}^C R_j^o \theta_{ij}^m / \sum_{j=1}^C m_{ij}, \quad i=M_0+1, \dots, M+N, \quad (A3)$$

where we are making use of the definitions of budget shares in (22). Also using the market shares in (23), real GDP_j^o can be computed from (18) as,

$$\begin{aligned} R_k^o &= \sum_{i=1}^M \pi_i^o q_{ik} + \sum_{i=M_0+1}^{M+N} (\pi_i^x x_{ik} - \pi_i^m m_{ik}) \\ &= \sum_{j=1}^C \sum_{i=1}^M R_j^o \theta_{ij}^q \mu_{ik}^q + \sum_{k=1}^C \sum_{i=M_0+1}^{M+N} R_j^o (\theta_{ij}^x \mu_{ik}^x - \theta_{ij}^m \mu_{ik}^m) \\ &= \sum_{j=1}^C R_j^o (\theta_j^q \mu_k^q + \theta_j^x \mu_k^x - \theta_j^m \mu_k^m). \end{aligned} \quad (A4)$$

Define w_{jk} as the term in parentheses in the last line of (A4), W = [w_{jk}] as the corresponding CxC matrix, and R^o as the row vector (R₁^o, ..., R_C^o). Then (A4) can be re-written as:

$$R^o W = R^o, \quad (A5)$$

so that the R^o is a row eigenvector of the matrix W.

It is readily checked that each row of W sums to unity: summing w_{jk} over k = 1, ..., C, the market shares μ_k^q , μ_k^x and μ_k^m each sum to a column vector (1, ..., 1)', while post-multiplying

the budget shares by this vector we obtain $\sum_{i=1}^M \theta_{ij}^q + \sum_{i=M_0+1}^{M+N} (\theta_{ij}^x - \theta_{ij}^m) = 1$ for each $j = 1, \dots, C$, by definition of the budget shares in (22). Since W is strictly positive by Assumption 2, from the Frobenius theorem it has a positive eigenvalue that lies in-between the minimum and maximum of its row sums, and the associated row or column eigenvector is strictly positive. Therefore, R^0 is the positive row eigenvector corresponding to the Frobenius eigenvalue of unity. Using this in (14)-(17) we obtain strictly positive solutions for PPP_j^0 and the reference prices. QED

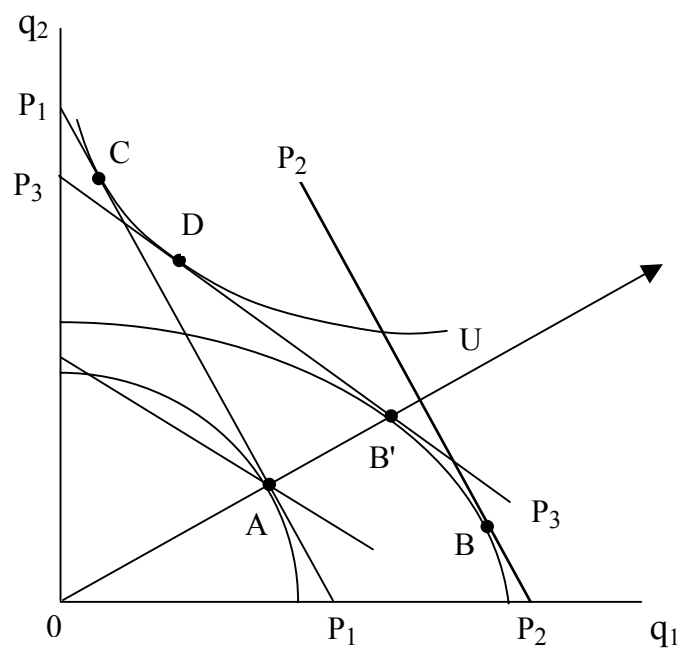


Figure 1

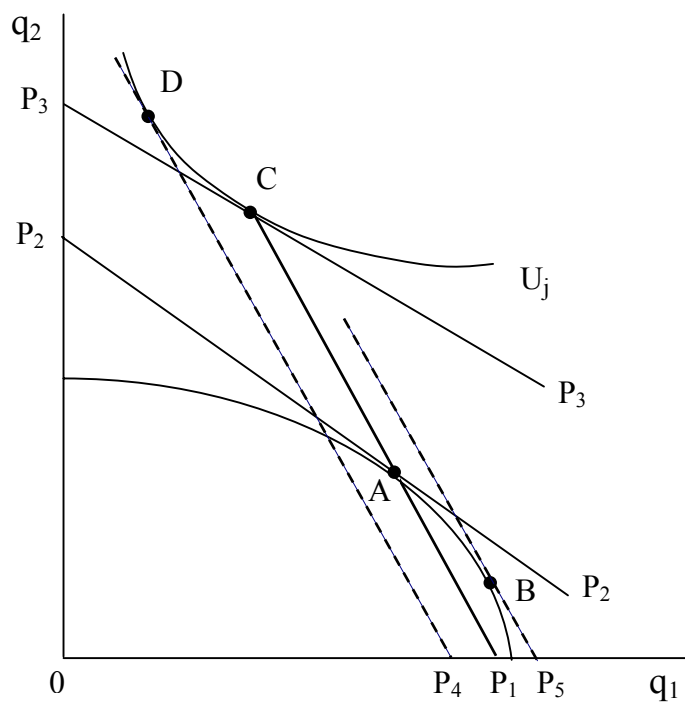


Figure 2

Figure 3: Difference between Expenditure and output-side GDP

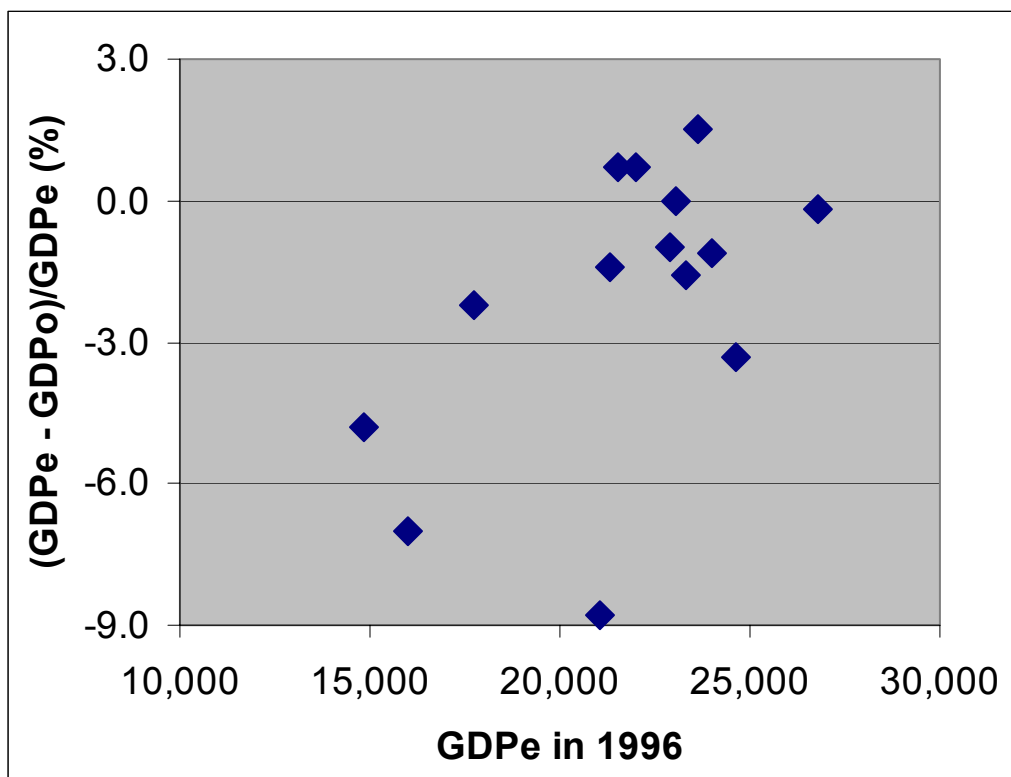


Table 1: Price Levels (PPP/nominal exchange rate)

	Expenditure Price Levels		Deprecia- tion of Currency	Output Price Levels	
	1996 (1)	1999 (2)	1996-99 (3)	1996 (4)	1999 (5)
Austria	118.5	103.3	-19.8	117.5	99.8
Belgium	106.9	98.7	-20.1	103.4	94.2
Denmark	129.2	117.9	-18.5	129.4	114.9
Finland	116.5	107.0	-19.4	114.4	104.9
France	117.2	105.1	-18.5	117.2	103.9
Germany	121.0	105.1	-19.8	122.8	106.7
Greece	79.2	73.0	-23.9	75.4	69.7
Ireland	95.9	101.1	-16.7	87.6	98.1
Italy	93.0	86.1	-16.3	91.6	84.2
Netherlands	106.7	95.2	-20.4	104.3	91.2
Portugal	67.8	64.7	-19.8	63.1	60.0
Spain	86.6	80.5	-20.9	84.5	78.3
Sweden	132.3	116.0	-20.9	131.6	116.8
U.K	94.0	108.7	3.6	94.3	110.7
U.S.	94.1	101.1	0.0	94.7	102.0

Notes:

Columns (1) and (2) are normalized so that the real value of GDP across countries equals the nominal value of GDP in US\$.

Column (3) from IMF, *International Finance Statistics*

Columns (4) and (5) have the same normalization as columns (1) and (2).

Table 2: Price Levels, Real GDP^e and Real GDP^o per-capita, 1996

	Output-side Price Levels, 1996				Real GDP ^e and GDP ^o per-capita			
	Final Goods	Exports	Imports	Terms of Trade	Real GDP ^e	Real GDP ^o	Diff (%)	Nominal \$GDP
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Austria	118.2	113.4	115.3	98.4	23,984	24,246	-1.1	28,621
Belgium	106.7	100.9	105.3	95.8	24,637	25,479	-3.3	26,485
Denmark	129.0	113.5	110.2	103.0	26,788	26,829	-0.2	34,755
Finland	116.2	105.9	108.6	97.5	21,314	21,618	-1.4	24,884
France	117.0	106.1	104.7	101.4	21,996	21,835	0.7	25,962
Germany	120.8	115.3	108.4	106.4	23,651	23,291	1.5	28,785
Greece	79.0	81.0	99.5	81.4	14,857	15,598	-4.8	11,871
Ireland	95.7	90.2	102.8	87.7	21,071	23,117	-8.8	20,330
Italy	92.7	96.6	104.1	92.8	22,912	23,135	-1.0	21,480
Netherlands	106.5	101.9	105.8	96.3	23,307	23,678	-1.6	25,034
Portugal	67.6	85.5	104.0	82.2	16,014	17,227	-7.0	10,978
Spain	86.3	86.6	95.3	90.9	17,712	18,114	-2.2	15,526
Sweden	132.0	118.1	116.5	101.3	23,048	23,043	0.0	30,613
U.K	93.8	100.6	98.5	102.1	21,515	21,358	0.7	20,249
U.S.	94.1	100.6	94.1	106.9	31,476	31,379	0.3	29,442
Average					25,851	25,851	0.0	25,851

Notes:

Columns (1)-(3) decompose the price level taken from Table 1, column (6).

Column (4) equals (2)/(3).

Columns (5) and (6) are computed according to equations (12) and (18), respectively.

Column (7) equals [(5)-(6)]/(5).

Column (8) reports nominal GDP per capita converted to US\$ with the nominal exchange rate.

Table 3: Price Levels, Real GDP^c and Real GDP^o per-capita, 1999

	Output-side Price Levels, 1999				Real GDP ^c and GDP ^o per-capita			
	Final Goods (1)	Exports (2)	Imports (3)	Terms of Trade (4)	Real GDP ^c (5)	Real GDP ^o (6)	Diff (%) (7)	Nominal \$GDP (8)
Austria	103.2	100.7	108.5	92.8	25,147	26,039	-3.4	25,976
Belgium	98.4	94.5	100.2	94.3	24,930	26,110	-4.5	24,606
Denmark	117.5	98.4	101.7	96.7	27,604	28,305	-2.5	32,534
Finland	106.7	99.2	102.9	96.4	23,472	23,942	-2.0	25,113
France	104.8	94.7	97.3	97.4	22,846	23,107	-1.1	24,004
Germany	104.8	107.0	100.7	106.3	24,435	24,068	1.5	25,689
Greece	72.9	73.0	87.3	83.6	16,369	17,138	-4.5	11,942
Ireland	100.9	107.0	112.8	94.9	25,217	25,993	-3.0	25,498
Italy	85.9	87.4	96.0	91.1	23,780	24,324	-2.2	20,485
Netherlands	94.9	89.9	96.0	93.6	26,503	27,648	-4.1	25,218
Portugal	64.6	79.4	96.2	82.5	18,000	19,399	-7.2	11,645
Spain	80.4	79.6	87.7	90.7	18,894	19,425	-2.7	15,207
Sweden	115.7	107.7	104.0	103.5	24,447	24,285	0.7	28,367
U.K	108.4	103.1	96.6	106.7	22,607	22,194	1.9	24,577
U.S.	101.1	107.7	99.2	108.6	33,574	33,280	0.9	33,951
Average					27,425	27,425	0.0	27,425

Notes:

Columns (1)-(3) decompose the price level taken from Table 1, column (6).

Column (4) equals (2)/(3).

Columns (5) and (6) are computed according to equations (12) and (18), respectively.

Column (7) equals [(5)-(6)]/(5).

Column (8) reports nominal GDP per capita converted to US\$ with the nominal exchange rate.

Appendix Table 1: Alternative Expenditure Price Levels (PPP/nominal exchange rate)

	Expenditure Price Levels based on OECD data		Expenditure Price Levels, based on PWT data	
	1996	1999	1996	1999
Austria	118.5	103.3	122.9	107.6
Belgium	106.9	98.7	115.4	102.4
Denmark	129.2	117.9	132.4	121.6
Finland	116.5	107.0	117.0	107.4
France	117.2	105.1	118.2	104.3
Germany	121.0	105.1	126.2	109.7
Greece	79.2	73.0	85.3	80.1
Ireland	95.9	101.1	99.9	101.8
Italy	93.0	86.1	96.1	90.1
Netherlands	106.7	95.2	113.4	101.9
Portugal	67.8	64.7	76.7	70.2
Spain	86.6	80.5	91.5	82.7
Sweden	132.3	116.0	130.0	115.6
U.K.	94.0	108.7	92.3	105.0
U.S.	94.1	101.1	91.6	99.1

Notes:

Columns (1) and (2) are based on OECD data as described in the main text and normalized so that the real value of GDP across countries equals the nominal value of GDP in US\$.

Columns (3) and (4) are alternative expenditure price levels based on PWT 6.1 data, and have the same normalization as columns (1) and (2).