As mentioned previously, Pakistan is ~~craving~~ solutions for its power supply, but is not under the financial status of realizing ~~it~~. It is reasonable to argue that without the loans and projects initiated by China, Pakistan would not have undertaken these energy projects, and its debt would not have accumulated dramatically from other official creditors.

Another aspect of uncertainty when assessing the counterfactual debt-to-tradable-GDP ratio is the impact on GDP following China's investment in infrastructure. According to estimates from Bandiera and Tsiropoulos (2020), using a long-term growth model, additional investment from BRI accounted for 1.93% of GDP, resulting in a 0.41% increase in GDP compared to the baseline model in 2020. This effect is five times larger than that observed in Sri Lanka. Although estimations for the period between 2013 and 2017 have not been conducted, a preliminary observation can be made from Figure 7b. Nominal tradable GDP demonstrates increasing growth after 2013, despite the total debt level ~~increasing~~ at a faster pace. Since this observation is not sufficient for causal inference, further examination of the infrastructure's contribution to GDP is a topic worthy of future research.

5.4 Robustness Check

5.4.1 Default Probability

In the model of Na et al. (2018), the price of debt offered by foreign lenders in period t takes the probability of the economy defaulting in the next period into consideration, given that the economy does not default in the current period and is in good financial standing, as depicted in Equation (14). As a result, it is worthy to also examine the conditional probability of defaulting under the current state.[20]

The conditional probability of default $\Pr(I_{t+1} = 0 \mid I_t = 1)$ is equivalent to the prob-

[20] As suggested by several committees, the "unconditional" default probabilities under states are able to provide more insights to the issue of debt trap compared to a clear-cut region of the default set. However, the model is unable to evaluate such concept as the default decision is deterministic under given state variables. In turn, I report the probability of defaulting in the consecutive period "conditional" on the economy ~~is~~ currently in good financial standing. This provides a slightly different interpretation on the risk of defaulting, but is still able to grasp the concept of uncertainty under different states of an economy.

ability that the output in the next period does not fall into the default set:

$$p(y_t^T, d_{t+1}) = \Pr\big(y_{t+1}^T \in D(d_{t+1}) \mid y_t^T\big).$$

This conditional probability of default is a function of debt in next period d_{t+1} and the current tradable output y_t^T. In numerical computation, this is obtained by multiplying the transition probability matrix of tradable output with the default policy function, which is a matrix over the state space of output and debt.

Figure 11 demonstrates the default probability over each state in the model for both Sri Lanka and Pakistan. The darker region represents a lower probability of defaulting in the next period, condition on the debt level unchanged. Notice that for some states that are under the default set in Figure 8, the default probability might not be 100%. This is because if for some reason the economy is unable to default (even if it is optimal to do so), then there is a chance that the output shocks upward, and the economy moves back to the non-default set. As a result, Figure 11 provides a fuzzy set of default risk among the boundary of default set. The output-debt ratios from the observed data as well as the datapoints that remove the debt to China ~~are also shown~~ in Figure 11. The calculations are equivalent to those described in Section 5.3.

Reexamining the datapoints of Sri Lanka under Figure 11a, it is obvious that during 2012 to 2017, Sri Lanka is wandering around the fuzzy region. Among all years that are in the default set, only 2015 and 2016 ~~is~~ under the level of high default risk. This justifies the conclusion that the new president of Sri Lanka ~~is~~ under great pressure on having to renegotiate with China or other creditors. When removing the debts to China, all years appear in the lowest level on the scale, indicating that the default probabilities in the absence of China are zero.

One might be curious how the default probability behave when debt from other creditor is removed in the case of Sri Lanka. From Figure 3a, debts to China exceed all other main creditors after 2013. A quick observation on Figure 11a suggest~~ed~~ that excluding other creditors instead will in general lead to a higher debt-to-tradable-GDP ratio, and therefore the default probability will remain in the fuzzy zone of default.[21]

As for the case of Pakistan, the results do not provide further insight ~~compared~~ that from the default set. In 2010 and 2017, the default probability is solidly 100%, in-line with the conclusion elaborated in the previous section. During 2013 to 2017 after ~~the~~ CPEC initiated, the default probability increased rapidly. The default probability in 2010 does not drop significantly in the absence of debt to China, while for years after 2015 the probabilities of the corresponding datapoints yield almost 0% for all periods.[22] This concludes that the interpretations of the debt-trap diplomacy using a clear-cut default set ~~is~~ robust under ~~a~~ representation with more information about the uncertainty.

5.4.2 Filtering Method

The value function iteration of the model depends on the persistency and volatility of the AR(1) process for the tradable output, namely (ρ, σ_u). The parameters are obtained with the cyclical component after conducting the HP-filter. In addition, the x-coordinate of the datapoints plotted on the default set is also obtained with the cyclical component, representing deviation from the trend. One may be concerned that the method of detrending might yield a different conclusion. In Na et al. (2018) and Hinrichsen (2021), the output process is detrended using the log-quadratic filter.[23] Generally, the volatility for the cyclical component obtained with log-quadratic filter is higher than that of HP-filter (Uribe and Schmitt-Grohé, 2017).

Figure 6 illustrates the decomposition of the (per capita) tradable output using the log-quadratic filter. It yields $(\rho, \sigma_u) =$ (0.9325, 0.0266) for Sri Lanka and $(\rho, \sigma_u) =$ (0.9239, 0.0174) for Pakistan. Compared to the unconditional standard deviation of 4.37% for Sri Lanka's per capita tradable output using the HP-filter, the log-quadratic filter yields 7.38%.

[21] Take 2016 for instance. Removing the debt from the second-highest creditor(s), in this case the aggregated debt stock from members of the Paris Club, yields a debt-to-tradable-GDP ratio of 199% (haircut and quarter adjusted), which corresponds to about 45% chance of defaulting in the next period. ~~Meanwhile,~~ removing China yields a debt-to-tradable-GDP ratio of 187%, which corresponds to merely 5% chance of defaulting in the next period. Since the direction of ratio by removing other creditors is trivial and obvious, I do not list all the results on the graph.

[22] Removing debt from the second-highest creditor, which is the World Bank, yields a debt-to-tradable-GDP ratio of 101%. According to the figure, the probability is still close to zero.

[23] Specifically, assume that real GDP can be expressed by the cyclical component and trend (secular) component $y_t = y_t^s + y_t^c$. The components are estimated by running OLS: $y_t = a + bt + ct^2 + \epsilon_t$, and then setting $y_t^c = \epsilon_t$ and $y_t^s = a + bt + ct^2$ (Uribe and Schmitt-Grohé, 2017).