201533661
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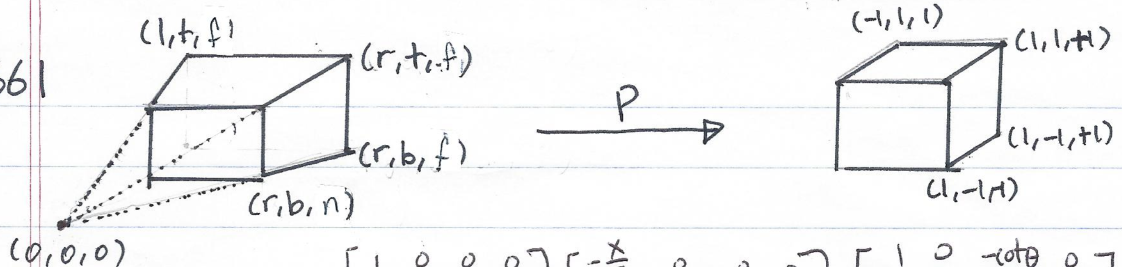

$$P = N\cdot S\cdot H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha & \beta \\ 0 & 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} -\frac{x}{z} & 0 & 0 & 0 \\ 0 & -\frac{y}{z} & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -\cot\theta & 0 \\ 0 & 1 & -\cot\phi & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

definition
- N (normalization): $(x, y, z, 1) \xrightarrow{N} (x, y, \alpha\cdot z+\beta, 0)$
- S (Scaling): $(x, y, z, 1) \xrightarrow{S} (\frac{x}{-z}, \frac{y}{-z}, -\frac{z}{z}, 1)$
- H (Shearing): $(x, y, z, 1) \xrightarrow{H(\theta,\phi)} (x - z\cot\theta, y - z\cot\phi, z, 1)$

* θ is angle between $(+x, -z)$ axis at top view
ϕ is angle between $(+y, -z)$ axis at side view.

$$P = (N\cdot S)\cdot H = \begin{bmatrix} -\frac{x}{z} & 0 & 0 & 0 \\ 0 & -\frac{y}{z} & 0 & 0 \\ 0 & 0 & +\alpha & \beta \\ 0 & 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & -\cot\theta & 0 \\ 0 & 1 & -\cot\phi & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

if we put right-left $(r-l)$ for x,
top - bottom $(t-b)$ for y,
2·near $(2\cdot n)$ for z, we can get result below. (far is to f)

$$P = \begin{bmatrix} \frac{2n}{r-l} & 0 & 0 & 0 \\ 0 & \frac{2n}{t-b} & 0 & 0 \\ 0 & 0 & \alpha & \beta \\ 0 & 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & -\cot\theta & 0 \\ 0 & 1 & -\cot\phi & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

to make near plane mapped to $z=-1$ and far plane to $z=1$,
we can set $\alpha = -\frac{near+far}{far-near}$, $\beta = \frac{2near\cdot far}{far-near}$

then $$P = \begin{bmatrix} \frac{2n}{r-l} & 0 & 0 & 0 \\ 0 & \frac{2n}{t-b} & 0 & 0 \\ 0 & 0 & -\frac{n+f}{f-n} & \frac{2nf}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & -\cot\theta & 0 \\ 0 & 1 & -\cot\phi & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

as angle we set as θ, ϕ, we can express $\cot\theta$ and $\cot\phi$ with (r, l, n, f, t, b).

$\cot\theta = \frac{r+l}{2\cdot near}$ $\qquad \cot\phi = \frac{t+b}{2\cdot near}$

then $$P = \begin{bmatrix} \frac{2n}{r-l} & 0 & 0 & 0 \\ 0 & \frac{2n}{t-b} & 0 & 0 \\ 0 & 0 & -\frac{n+f}{f-n} & \frac{2nf}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & -\frac{r+l}{2n} & 0 \\ 0 & 1 & -\frac{t+b}{2n} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2n}{r-l} & 0 & -\frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & -\frac{t+b}{t-b} & 0 \\ 0 & 0 & -\frac{n+f}{f-n} & \frac{2nf}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$