# Blood vessel modelling

Teddy Groves

# The problem in my words

# Big picture questions

→ How do age and disease affect brain vessel health?

→ What are precapillary sphincters for?

# Experimental protocol

Baseline $\rightarrow$ Hypertension $\rightarrow$ Recovery $\rightarrow$ Ablation $\rightarrow$ Hypertension 2

# Measurements

- → Diameter before & after whisker stimulation (slow, measures responsiveness)
- → Diameter and center point (fast, FTed to measure pulsatility)
- → Blood pressure at femoral artery
- → Speed (measures velocity and total flux)

# Specific questions 1: whisker stimulation

How do ageing and sphincter ablation affect whisker stimulation response?

a) Does the response depend on age and/or vessel type?

b) Does the effect of sphincter ablation on the response depend on age?

# Specific questions 2: pulsatility

How do age, treatment, pressure and vessel type affect pulsatility of diameter (Pd) and center position (Pc)?

a) Is there an overall age difference? If so, is it explained by pressure?

b) Does the effect of sphincter ablation depend on age?

c) What is the effect of pressure?

d) Does the effect of treatment (also on diameters) vary by vessel type?

# Modelling approach

# Bayesian multilevel regression modelling

1. A generative model expressing topologically how measurables $\hat{y}$ depend on parameters $\theta$. i.e. $f$ s.t. $\hat{y} = f(\theta)$
2. A measurement model probabilistically connecting measurables with measurements $y$. i.e. $p(y \mid \hat{y})$
3. A prior model probabilistically expressing non-experimental information about parameters. i.e. $p(\theta)$

# Multilevel models

In a multilevel model, some parameters express second or higher order information.

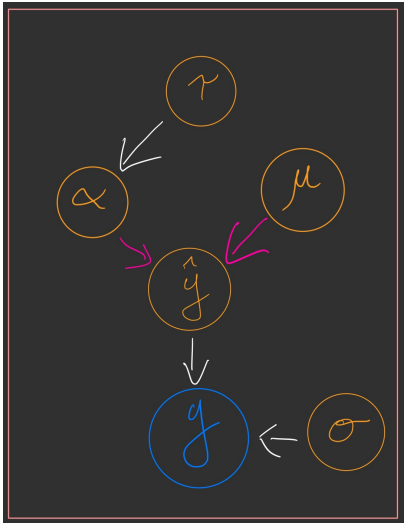
For example, in this model $\tau$ represents information about the parameters $\alpha$.

$$y_i \sim N(\hat{y}_i, \sigma)$$
$$\hat{y}_i = \mu + \alpha_{mouse(i)}$$
$$\alpha \sim N(0, \tau)$$

Multilevel models generalise random/mixed effect models.

# Representing Bayesian multilevel models with graphs

- → Node = parameter (orange) or measurement (blue)
- → Edge = full dependence (pink) or probabilistic dependence (white)
- → N.B. **Not** the same as "Bayesian networks"
- → See [1] for more about graphical models.



# Reading about this area

[1] Bayesian graphical models case study with general introduction/discussion.

[2] Paper about “Bayesian workflow”.

https://betanalpha.github.io/assets/case_studies/hierarchical_modeling.html Abstract mathemtical discussion of multilevel modelling in general.

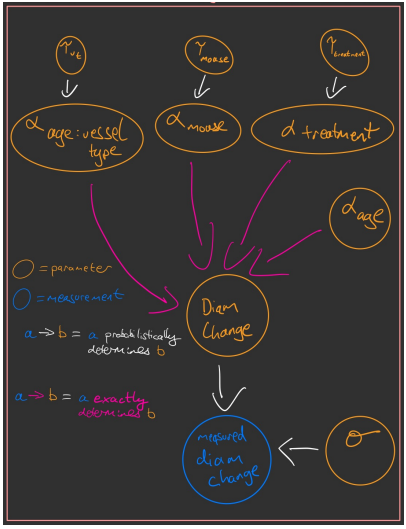
# Why is this approach appropriate here?

- → We are interested in group-level effects (treatment, mouse, vessel type) and don't know the population parameters (e.g. how much to expect mouse effects to vary).
- → We have ample but not massive data, so regularisation from priors is useful.
- → We need a fairly complex model; Bayesian multilevel models scale better with complexity.

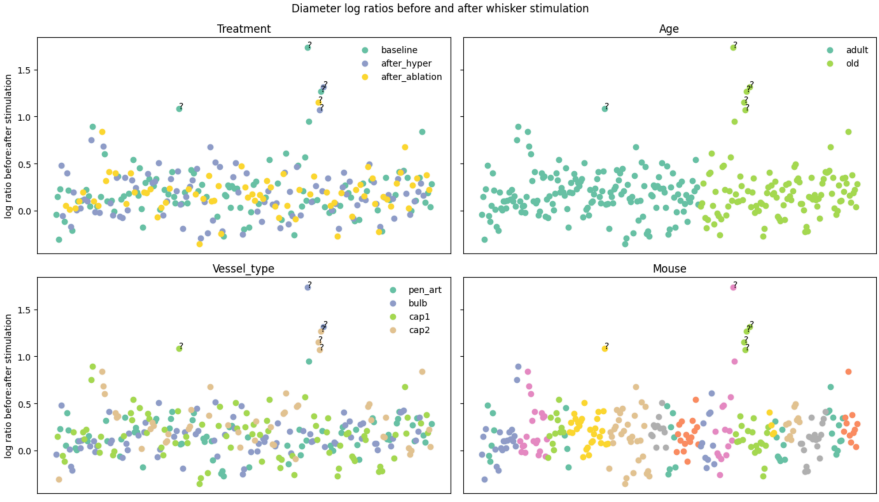
# Plan

# Whisker stimulation model

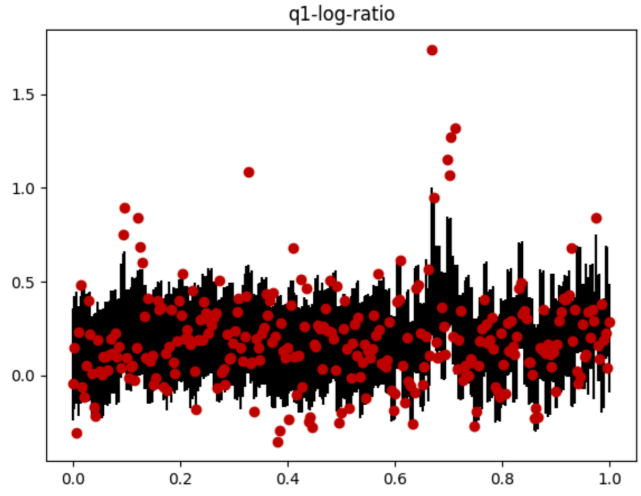
- → Measurable: $\ln \frac{diamafter}{diambefore}$
- → Measurement model: linear regression
- → Parameters: Note that $\alpha_{age:vessel\ type}$ depends on age



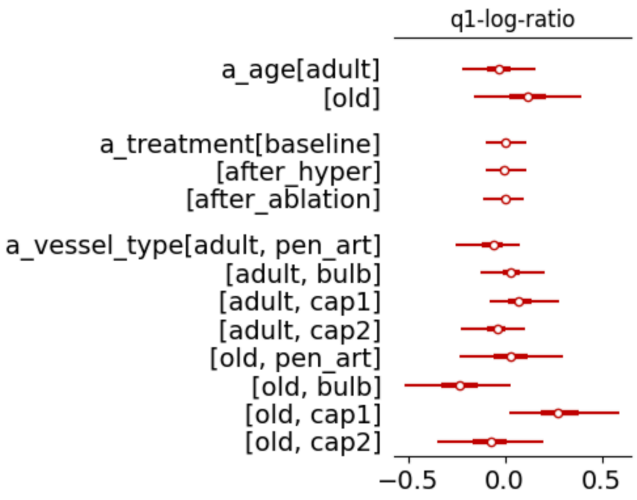
# Whisker stimulation log ratios



# Model fit

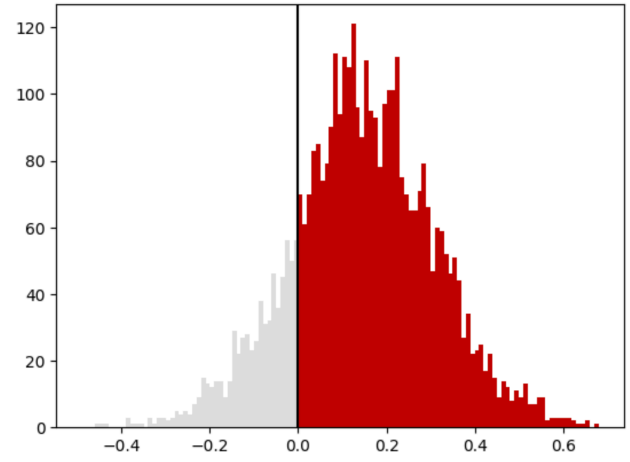


# Non-mouse effects

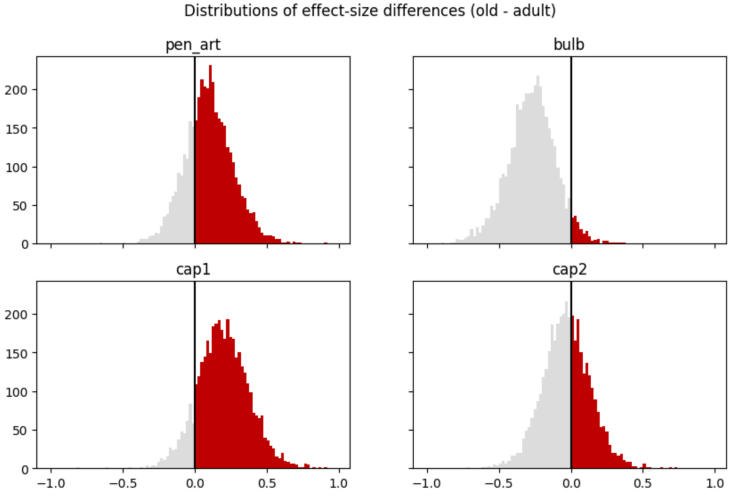


# Age effect

Probability of positive effect: 0.82525



# Vessel type effect by age



# References


[1] D. J. Spiegelhalter, "Bayesian Graphical Modelling: A Case-Study in Monitoring Health Outcomes," *Journal of the Royal Statistical Society Series C: Applied Statistics*, vol. 47, no. 1, pp. 115–133, Mar. 1998, doi: 10.1111/1467-9876.00101.

[2] A. Gelman *et al.*, "Bayesian workflow," *arXiv:2011.01808 [stat]*, Nov. 2020, Available: https://arxiv.org/abs/2011.01808