Assignment 1

January 25, 2019

(a) $Depth - first$ **search always expands at least as many nodes as** A^* **search with an admissible heuristic.**

False. Depth-first search may possibly, sometimes, BY GOOD LUCK, expand fewer nodes than A* search with an admissible heuristic. E.g., it is logically possible that sometimes, by good luck, depth-first search may march directly to the goal with no back-tracking.

(b) $h(n) = 0$ **is an admissible heuristic for the 15-Puzzle (https://en.wikipedia.org/wiki/15 puzzle).**

True. $h(n) = 0$ is the heuristic function, and a heuristic value of 0 is extremely optimistic value, as it would never over-estimate the distance to the goal from a given node.

(c) $Breadth - first$ **search is complete even if zero step costs are allowed.**

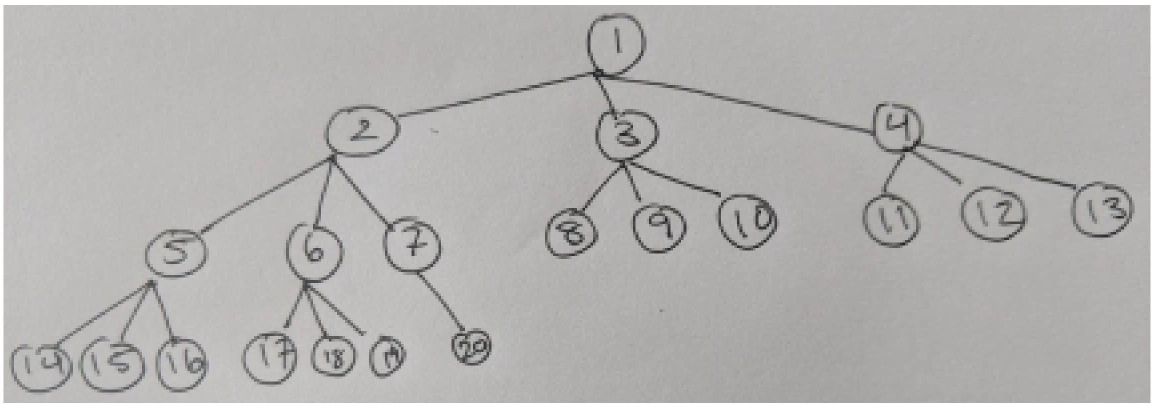
True, because if a goal exists, the goal is at finite depth and will be found in $O(b^{s+1})$ time. Complete meaning that BFS will find a goal IFF one exists. BFS is not necessarily optimal, meaning that BFS will not find a lowest-cost goal when one exists. Therefore, the step costs are not important when it comes to the algorithm being complete.

(d) Assume that a rook can move on a chessboard any number of squares in a straight line, vertically or horizontally, but cannot jump over other pieces. Manhattan distance is an admissible heuristic for the problem of moving the rook from square A to square B in the smallest number of moves.

False. Since a rook can move across the board, or cover multiple cells in a single move, the Manhattan distance would be pessimistic, not optimistic, and may over-estimate the the optimal path.

2 Uninformed search: Consider a state space where the start state is 1 and the successor function for state i (where i = 1, 2, ...) returns three states: $3i-1,\quad 3i,\quad 3i+1$**.**

a) Draw the state space graph for states 1 to 20.



b) Suppose the goal state is 17. List the order in which nodes will be visited for:

(i) breadth-first search
[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17] – found goal

(ii) depth-limited search with depth limit 2
[1, 2, 5, 6, 7, 3, 8, 9, 10, 4, 11, 12, 13] – won't find goal

(iii) iterative deepening search.
level 0) [1]
level 1) [1, 2, 1, 3, 1, 4]
level 2) [1, 2, 5, 6, 7, 1, 3, 8, 9, 10, 1, 4, 11, 12, 13]
level 3) [1, 2, 5, 14, 15, 16, 1, 2, 6, 17] – found goal

3 Approximate A^* search: Let $h^*(x)$ be the shortest distance between x and a goal state t. Let $h(\cdot)$ be a heuristic that over-estimates $h^*(x)$ by at most $\cdot$, that is, for all states x, $h(x) \leq h^*(x) + \cdot$ (but still assigns value 0 to goal states). Show that A^* tree search using h finds a goal state whose cost is at most $\cdot$ more than the optimal goal. Formally, if t is the goal state returned by A^*, then $g(t) \leq h^*(s) + \epsilon$.

Suppose A^* search is not optimal. It may pop a sub-optimal goal, t off the fringe before t^* (optimal goal). If s is the start node, t^* is the lowest-cost goal node, and t is a sub-optimal goal, we have the relationship, $g(t^*) < g(t)$. If $h^*(x)$ is the lowest cost from s to t, we have $h^*(s) = g(t^*)$. The heuristic function $h(x)$ is approximate, so we have the relationship $h(x) \leq h^*(x) + \epsilon$. With these relationships, we have $f(t) = g(t) + h(t)$, but since $h(t)$, the heuristic cost, of a goal is 0, we have $f(t) = g(t)$.

Suppose some node, n, is in the optimal sub-path of t^*, n would be on the fringe, along with t, implying $f(n) \geq f(t)$ means that t is on the optimal path. However, if this is the case, we would pop the sub-optimal t off the fringe before n, which is a direct contradiction of A^* search. If A^* popped t off the fringe first, it would have to be the case that t is optimal for it to be popped before n. Since A^* is optimal, it will always return the optimal goal.

4 Solve the following 8-puzzle using steepest ascent hill-climbing

Initial State

2	3	
1	8	4
7	6	5

Goal State

1	2	3
8		4
7	6	5

1. State the heuristic used.

Manhattan distance is the chosen heuristic. This is better than the misplaced tile heuristic because the Manhattan heuristic takes into account how "out-of-place" each cell is.

2. List the steps taken. For example if the first three moves consist of moving the 3 tile, then the 8 tile, then the 6 tile. Your answer would start like: 3,8,6

$[3 \rightarrow, 2 \rightarrow, 1 \uparrow, 8 \leftarrow]$