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Breaking resonances with Post-Main Sequence Winds

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ABSTRACT

We explore the effects of massive stellar winds on the resonances of planets as a host star transitions into a white dwarf.

1. INTRODUCTION

It has become clear over the past two decades that pollution by metals of the atmospheres of solitaty white dwarf stars (WDs) can be explained via accretion of material from it's planetary system (ref Zuckerman+03, Jura08, Klein+10, etc). Additionally, the ν_6 resonance has been shown to be an effective mechanism for delivering rocky asteroids to within the WD's tidal radius (Smallwood et al. 2021).

However, there are now three examples of WD's that are accreting ice-rich material: G200-39, GD 378, and G238-44 (Xu et al. 2017; Klein et al. 2021; Johnson et al. 2022, , respectively). Of these three, G200-39 and G238-44 are believed to be the result accretion of exo-Kuiper belt objects (exo-KBOs) (GD 378 is best explained by an icy moon of a planet with a strong magnetic field, see Doyle et al. (2021)).

The delivery mechanism of exo-KBOs to a WD remains an open question. Bonsor et al. (2011) investigated the ability of a single planet to scatter KBOs in towards the host star.

Chen et al. (2019) suggested the ν_8 resonance as a mechanism to deliver water to early Earth. It seems that this could also plausibly explain pollution by KBOs, except that objects in this resonance are not expected to survive for a star's main-sequence lifetime. Instead, a WD system would require that objects are put into this resonance during the star's transition from main-sequence to white dwarf.

In the solar system, the largest population of KBOs to investigate as candidates for the ν_8 resonance are the plutinos, or objects in a 3:2 mean-motion resonance with Neptune. The 3:2 resonance is very stable and these objects are expected survive for the full lifetime of the host star (Nesvorný & Roig 2000).

Given this stability, the 3:2 resonance is not expected to break provided that mass loss is sufficiently slow (i.e. the mass loss timescale is much longer than the orbital periods of the objects). However, in this study we investigate the additional effects of massive stellar winds that are the result of mass loss. Effectively, the central mass "seen" by an object orbiting a star shedding it's outer layers is the mass of that star plus the mass of the wind interior to the object. Effectively, the wind imposes a position-dependent time delay on the central mass. We investigate the effects of this differential potential on the resonances of objects in a 3:2 mean-motion resonance with Neptune.

2. METHODS

2.1. *The central force of an isotropic massive wind*

Let $M(t)$ be the mass of the central object. We will assume that change in M be due to an isotropic wind with some speed $v(r,t)$

Let $t_{\rm wind}(r,t)$ be the travel time for wind that arrives to a radius r at time t. For a planet at r the mass interior to its orbit is $M(t - t_{\rm wind})$.

In this study we will assume that mass loss occurs exponentially with an e-folding time of τ. If mass loss begins at $t = 0$ then

$$M = M_0 e^{-t/\tau} \tag{1}$$

and

$$\dot{M} = \frac{-1}{\tau} M_0 e^{-t/\tau} \tag{2}$$

We will also assume that the wind travels at its escape velocity at all times. For a particle that leaves the star at a time t_0

$$v^2 = \frac{2GM(t_0)}{r} \tag{3}$$

For $M = M_\odot$ this corresponds to 617 km/s at 1 $R_\odot$ and 7.7 km/s at 30 AU.

Recognizing that $v = \frac{\mathrm{d}r}{\mathrm{d}t}$ and that $r = R$ at $t = t_0$, we have

$$r^{\frac{3}{2}} - R^{\frac{3}{2}} = \frac{3}{2}\sqrt{2GM(t_0)}(t - t_0) \tag{4}$$

or

$$t_{\rm wind} = \frac{2(r^{\frac{3}{2}} - R^{\frac{3}{2}})}{3\sqrt{2GM(t - t_{\rm wind})}} \tag{5}$$

We now define $M_{\rm eff}(r,t)$ as the mass "seen" by a planet at radius r at time t.

$$M_{\rm eff}(r,t) = M(t - t_{\rm wind}) \tag{6}$$

If M changes exponentially as in Equation 1 then

$$M_{\rm eff}(r,t) = M_0 e^{-(t-t_{\rm wind})/\tau} \tag{7}$$

Our expression for $t_{\rm wind}$ then becomes

$$t_{\rm wind} = \frac{2(r^{\frac{3}{2}} - R^{\frac{3}{2}})}{3\sqrt{2GM_0}} \tag{8}$$

Appendix A shows the results of solving for $t_{\rm wind}$ in Equation 8 is

$$t_{\rm wind} = 2\tau W\left(\frac{r^{\frac{3}{2}} - R^{\frac{3}{2}}}{3\tau\sqrt{2GM_0}} + e^{t/2\tau}\right) \tag{9}$$

Where W is the Lambert W function.

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APPENDIX

A. SOLVING FOR T_{WIND}

We now solve for t_{wind} in Equation 8. Moving the exponential from the denominator we see

$$\frac{t_{\mathrm{wind}}}{2\tau} e^{-(t-t_{\mathrm{wind}})/2\tau} = \frac{2}{3} \frac{r^{\frac{3}{2}} - R^{\frac{3}{2}}}{2\tau\sqrt{2GM_0}} \tag{A1}$$

Where the factor of $1/2\tau$ has been introduced to make each side dimensionless. Taking the natural logarithm of both sides of Equation A1 and rearranging, we have

$$\ln\left(t_{\mathrm{wind}}/2\tau\right) + \frac{t_{\mathrm{wind}}}{2\tau} = \ln\left(A(r)\right) + \frac{t}{2\tau} \tag{A2}$$

Where $A(r)$ is the right hand side of Equation A1. This can easily be put into the form $xe^x = b$ where $x = t_{\mathrm{wind}}/2\tau$ and

$$b = \frac{r^{\frac{3}{2}} - R^{\frac{3}{2}}}{3\tau\sqrt{2GM_0}} + e^{t/2\tau}$$

The solution to this equation is $x = W(b)$ where W is the Lambert W function. We see then that we can solve for t_{wind} as a function of r and t:

$$t_{\mathrm{wind}} = 2\tau W\left(\frac{r^{\frac{3}{2}} - R^{\frac{3}{2}}}{3\tau\sqrt{2GM_0}} + e^{t/2\tau}\right) \tag{A3}$$