STU22005 Applied Probability II
Professor Caroline Brophy
Continuous Assessment Sheet 2

Due date: submit before 4pm Friday 5th March 2021.

For this assignment, you must do questions 1 and 2 (you should also do the remaining quesions, but do not submit them). To submit the assignment, you must submit two files:

- File 1: Fill in the answer sheet that accompanies this assignment. You must fill in the answers on the pdf document, save the file, and upload to Blackboard.
- File 2: Take images (e.g., photographs on your phone) of your workings to show how you did the questions. Combine into a **single** pdf document and upload to Blackboard.

NB: you must submit the two files, the answer sheet alone will not be accepted.

1. Consider an independent random sample $Y_1, \ldots, Y_9$ of a normal distribution with unknown mean μ and variance $\sigma^2 = 1$.

   (a) What is $P(|Y_1 - \mu| \leq \frac{1}{2})$?
   (b) What is $P(|\bar{Y} - \mu| \leq \frac{1}{2})$?

   Hint: Begin by defining the distribution of Y_1 and of $Y_1 - \mu$ in (a) and the distribution of $\bar{Y}$ and of $\bar{Y} - \mu$ in (b). Use sketches to understand the 'area under the curve' of interest to assist finding the probability in each case.

2. Experience has shown that the number of accidents to occur along a particular 10-mile stretch of a motorway is a Poisson random variable with a mean of 2 per week. What is the (approximate) probability that there will be less than 100 accidents on this stretch of motorway in a year (assuming 1 year = 52 weeks)?

   Hint: Begin by defnining the distribution of the random variable the number of accidents that occur in one week, call it X_i. Then define the distribution of the random variable X which is the sum of the X_i over 52 weeks.

Below are additional questions that do not need to be handed up.

3. Let $Y_1, \ldots, Y_n$ be an independent random sample from a $\mathrm{N}(\mu, \sigma^2)$. Define:

$$S^2 = \frac{\sum_{i=1}^{n} (Y_i - \bar{Y})^2}{n-1}.$$

   Prove that

$$E(S^2) = \sigma^2$$

4. A drug is to be rated either effective or ineffective by a national drug board. Lab results indicate that the drug is effective 75% of the time and ineffective in the remainder of cases. When effective the drug increases the lifespan of a patient by 5 years. When ineffective the drug causes a complication which decreases the lifespan of the patient by 1 year. As part of a drug trial study you administer the drug to 10,000 patients.

   (a) Calculate the expected value and variance of the lifespan increase for a single patient being treated with this drug.

   (b) What is the probability that the average lifespan increase for the patients in the study will be 3 and a half years or more?

   Hint: Let L be the random variable in part (a), and in part (b), begin by defining the distribution of the random variable $\bar{L}$.