STU22005 Applied Probability II
Professor Caroline Brophy
Continuous Assessment Sheet 1

Due date: submit before 4pm Friday 19th February 2021.

For this assignment, you must do questions 1 and 2 (but question 3 is not required and should not be handed up). To submit the assignment, you must submit two files:

- File 1: Fill in the answer sheet that accompanies this assignment. You must fill in the answers on the pdf document, save the file, and upload to Blackboard.
- File 2: Take images (e.g., photographs on your phone) of your workings to show how you did the questions. Combine into a **single** pdf document and upload to Blackboard.

NB: you must submit the two files, the answer sheet alone will not be accepted.

1. Carry out the following hypothesis tests assuming independent normally distributed data. Find the test statistic, the critical value (using the Tables) and state your conclusion. Note that the variance is estimated here, not known. The t-distribution should be used in each case to find the critical values.
   (a) $n = 10$, $\bar{y} = 124$, $s = 10$, $H_0 : \mu = 110, H_A : \mu \neq 110$, $\alpha = 0.01$.
   (b) $n = 8$, $\bar{y} = 0.6$, $s = 0.2$, $H_0 : \mu = 0.5, H_A : \mu \neq 0.5$, $\alpha = 0.05$.
   (c) $n = 25$, $\bar{y} = 33.4$, $s = 6.8$, $H_0 : \mu = 30, H_A : \mu > 30$, $\alpha = 0.1$.

2. The scores students get on an examination are normally distributed with mean $\mu = 55$ and variance $\sigma^2 = 100$ and scores are independent of each other. Any mark in excess of 70 is a first. Assume 10 students take the exam and they get scores $Y_1, \ldots, Y_{10}$.
   (a) What is the probability any one of these students gets a first?
   (b) What is the probability exactly one of the 10 students will get a first?
   (c) What is the probability that the average mark for the class will be above 60?

Below is an additional question that does not need to be handed up.

3. We showed in class that if $Y_1, Y_2, \ldots, Y_n$ are independent and identically distributed (IID) as $\mathrm{N}(\mu, \sigma^2)$ that

$$E(\bar{Y}) = \mu \qquad \text{and} \qquad Var(\bar{Y}) = \frac{\sigma^2}{n}$$

   Verify that this is true even if the Y_i are not normally distributed, so long as they are IID with $E(Y_i) = \mu$ and $Var(Y_i) = \sigma^2$, $i = 1, \ldots, n$.