Assignment 6

I have read and I understand the plagiarism provisions in the General Regulations of the University Calendar for the current year, found at here. I have also completed the Online Tutorial on avoiding plagiarism 'Ready Steady Write', located here.

Exercise 1

Part (a)

Let L be the language over the alphabet $A = \{ a, l, p \}$ consisting of all words containing both a and l. Write down the algorithm of a Turing machine that decides L. Process the following strings according to your algorithm: p, al, pap, pla, and $aappla$.

Solution

The following algorithm decides language L over alphabet A:

1. If the current cell is p, move right and go to step 1. If the current cell is a, move right and go to step 2. If the current cell is l, move right and go to step 3. Otherwise, REJECT.
2. If the current cell is p or a, move right and go to step 2. If the current cell is l, ACCEPT. Otherwise, REJECT.
3. If the current cell is p or l, move right and go to step 3. If the current cell is a, ACCEPT. Otherwise, REJECT.

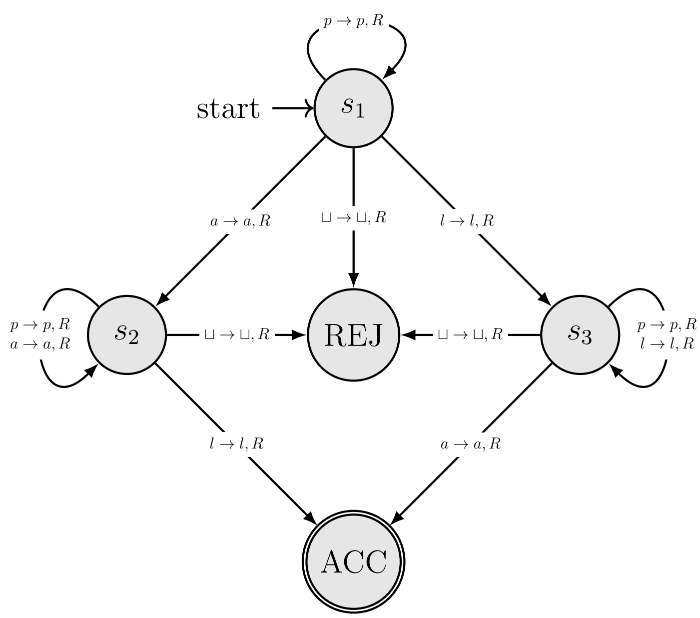
Here are how the provided strings are processed by the above algorithm:

- $p: \ \epsilon s_1 p \to p s_1 \epsilon \to REJECT$
- $al: \ \epsilon s_1 al \to a s_2 l \to ACCEPT$
- $pap: \ \epsilon s_1 pap \to p s_1 ap \to pa s_2 p \to pap s_2 \epsilon \to REJECT$
- $pla: \ \epsilon s_1 pla \to p s_1 la \to pl s_3 a \to ACCEPT$
- $aappla: \ \epsilon s_1 aappla \to a s_2 appla \to aa s_2 pppla \to aap s_2 ppla \to aapp s_2 pla \to aappp s_2 la \to ACCEPT$

Part (b)

Write down the transition diagram of the Turing machine from part (a) carefully labelling the initial state, the accept state, the reject state, and all the transitions specified in your algorithm.

Solution



Exercise 2

Let the alphabet $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Write down the algorithm of an enumerator that prints out EXACTLY ONCE every string in the language $L = \{\ 3m + 1 \mid m \in \mathbb{N}\ \}$ that is EVEN.

Solution

Note that $L = \{\ 3m+1 \mid m \in \mathbb{N}\ \}$ that is even $= \{m \equiv 4 \mod 6 \mid m \in \mathbb{N}\ \}$. Let M be a Turing machine that decides language L. The algorithm of M would be as follows:

1. If the current cell is 4, move right and go to step 5. If the current cell is $\sqcup$ or this is the initial state and the current cell is 0, REJECT. Otherwise, move right and go to step 4.
2. If the current cell is 0 or 6, move right and go to step 5. If the current cell is $\sqcup$, REJECT. Otherwise, move right and go to step 4.
3. If the current cell is 2 or 8, move right and go to step 5. If the current cell is $\sqcup$, REJECT. Otherwise, move right and go to step 4.
4. If the current cell is 0, 3, 6 or 9, move right and go to step 1. If the current cell is 1, 4 or 7, move right and go to step 2. If the current cell is 2, 5 or 8, move right and go to step 3. Otherwise, REJECT.
5. If the current cell is 0, 3, 6 or 9, move right and go to step 1. If the current cell is 1, 4 or 7, move right and go to step 2. If the current cell is 2, 5 or 8, move right and go to step 3. Otherwise, ACCEPT.

Note that M always decides if a string of length n characters belongs to L within n steps. Therefore, it is very easy to print every string in L exactly once by running M on every string in A^*. As proven in lectures, A^* is countably infinite. As such, A^* is enumerable as a sequence like $A^* = \{w_1, w_2, w_3, \ldots\}$. Then the enumerator which prints every string in L exactly once =

1. Repeat the following for $i = 1, 2, 3, \ldots$
2. Run M for $\#w_i$ steps on input w_i.
3. Print out w_i if it is accepted.

Exercise 3

Part (a)

Prove that the emptiness testing problem for phrase structure grammars (PSG's) given by the language

$$E_{PSG} = \{ \ \langle G \rangle \mid \text{G is a phrase structure grammar and } L(G) = \emptyset \ \}$$

is Turning-decidable.

Solution

Let there be a Turing machine such as
$M =$ on input $\langle G \rangle$

1. Mark all terminal symbols in G.
2. Repeat until no new symbol gets marked:
3. Mark non-terminal $\langle T \rangle$ if G contains a rule $r_1, \ldots, r_i, \langle T \rangle, r_{i+2}, \ldots, r_j \rightarrow w_1, \ldots, w_k$ where every symbol except $< T >$ has already been marked.
4. If the start symbol $\langle S \rangle$ is marked, REJECT. Otherwise, ACCEPT.

The phase structure grammar will generate at least one string if the start symbol $\langle S \rangle$ is markable by the Turing machine M. Therefore, M rejects G if $L(G) \neq \emptyset$ and as such E_{PSG} is Turing-decidable.