STU22005 Applied Probability II
Professor Caroline Brophy
Continuous Assessment Sheet 4

Due date: submit before 4pm Friday 9th April 2021.

For this assignment, you must do question 1 (you should also do the remaining questions, but do not submit them).

For this assignment, there is no answer sheet file.

- Submit a single file: Take images (e.g. photographs on your phone) of your workings to show how you did each part of the question. Combine into a pdf document and upload to Blackboard.

1. It is believed that there are levels of infection of a disease among people in a region that are not displaying any symptoms, but the levels are unknown. A random sample of 12 people (not displaying any symptoms) from the region were tested for the disease on two separate occasions. The number of positive tests out of 12 were: 3 and 2. Assume that the level of asymptomatic infection remains constant over the two occasions, and that any individual selected from the population is equally likely to test positive.

   (a) Give a one-sentence reason why the binomial distribution could be used to model this data. Also state the distribution parameters.

   (b) Write down the likelihood function for these data. Start your answer using the generic form with x_i's and use algebraic manipulations to simplify the function as much as possible. Finish by replacing the generic x_i values with the data values and simplify as much as possible.

   (c) Sketch (by hand) the likelihood function and use your sketch to say roughly what value the function is maximised at indicating the value on your graph. Note, you can use a small number of points (e.g., 10 or 11 points) in your graph and extrapolate in between them to draw the function.

   (d) Derive the maximum likelihood estimate and give its value.

Below are additional questions that do not need to be handed up.

2. Suppose $x_1, \ldots, x_n$ is an observed sample of size n from a Geometric random variable with parameter p. What is the maximum likelihood estimator of p?

3. The following shows the heart rate (in beats/minute) of a person measured throughout the day:

$$73, 75, 84, 76, 93, 79, 85, 80, 76, 78, 80$$

   Assume the data is an iid sample from $\mathrm{N}(\mu, \sigma^2)$ where σ^2 is known. What is the likelihood for μ if

   (a) the whole data is reported?

   (b) only the sample mean $\bar{x}$ is reported?

4. We played the (hopefully fun!) Distribution Game in class, where only stating the distribution was required. Now, work out the probability for each question.

   (a) Suppose you are playing a game of Ludo and so you need to roll a 6 to get your first counter out of home and onto the board. What is the probability that you will get onto the board in between 4 and 6 goes?

   (b) A recent study has shown that 30% of women in a certain large population suffer from anemia (iron deficiency). A random sample of 8 women is taken from the population and tested. What is the probability that three or more of the women in the sample are anemic?

   (c) A basketball player scores a basket from the free throw line with probability 0.45. You start observing a training session on free throw attempts at a random point. What is the probability that her third basket occurs on the sixth shot?

   (d) Suppose that items in a vending machine get stuck on the way out at random with probability 0.03. You arrive at the vending machine, put your money in and select your item. What is the probability you end up not getting your item (and are really annoyed!)?

   (e) Consider a lotto draw with 45 balls where 6 balls are chosen at random. What is the probability of matching 4 balls?