Arithmetic and Combinatorics

Training problems for M1 2018 term 1

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1 Calculating prodigies

1. *"He who refuses to do arithmetic is doomed to talk nonsense."* Which famous computer scientist said that? You can use the internet to find out.

2. Which famous American calculating prodigy became an astronomer when he grew up?

3. This prodigy was from Germany. He could multiply 100-digit numbers in his head. Who was he?

4. Write down a few interesting things about the life of Jedediah Buxton. Where was he from? When was he born? How did he die? What did he do?

5. What do the amazing powers of calculating prodigies prove about the human mind? Tell me some opinions.

2 Euclidean division

6. Who was Euclid and where was he from? When did he live?

7. Use the terms *dividend*, *divisor*, *remainder* and *quotient* to label the parts of the expression

$$29 = 4 \times 6 + 5.$$

Choose the divisor carefully. Is it 4 or is it 5? Check that $0 \leqslant r < d$. Is it true? Is this a correct expression of Euclidean division?

8. Check that $0 \leqslant r < d$. Is it true?

$$111 = 9 \times 11 + 12$$

Is this a correct expression of Euclidean division?

9. Label the parts of these expressions with the terms *dividend*, *divisor*, *remainder* and *quotient*.

(a) $101/39$.
(b) $m = q \times d + r,\ 0 \leqslant r < d$.
(c) $59 = 5 \times 11 + 4$.
(d) $a = b \times c + d,\ 0 \leqslant d < c,\ d > b$.
(e) $r \div s$.
(f) $39/101$.

10. Which number is the divisor and which one is the quotient?

(a) $42 = 11 \times 3 + 9$.
(b) $99 = 7 \times 13 + 8$.
(c) $69 = 10 \times 6 + 9$.
(d) $23 = 4 \times 5 + 3$.

11. Do by Euclidean division. Label all the parts of your expressions. Give a clear answer in terms of two numbers.

(a) $99/91$.
(b) $919/7$.
(c) $1001/651$.
(d) $1/19$.
(e) $19/1$.
(f) $17/17$.
(g) $0/15$.
(h) $15/0$.

12. These matrices are filled with consecutive integers $0, 1, 2, \ldots$ Figure out what goes into the boxes.

(a) 7×7 matrix.
(b) 10×10 matrix.
(c) $n \times n$ matrix.

$$\begin{bmatrix} \square & \cdots & \square \\ \vdots & & \vdots \\ \square & \cdots & \square \end{bmatrix} \qquad \begin{bmatrix} \square & \cdots & \square \\ \vdots & & \vdots \\ \square & \cdots & \square \end{bmatrix} \qquad \begin{bmatrix} \square & \cdots & \square \\ \vdots & & \vdots \\ \square & \cdots & \square \end{bmatrix}$$

13. Each matrix is $n \times n$ and is filled with consecutive integers $0, 1, 2, \ldots$ Figure out what goes into the boxes.

(a) $\begin{bmatrix} \square & \cdots & \square \\ \square & \cdots & \square \\ \vdots & & \vdots \\ \square & \cdots & \square \\ \square & \cdots & \square \end{bmatrix}$

(b) $\begin{bmatrix} \square & \square & \cdots & \square & \square \\ \vdots & & & & \vdots \\ \square & \square & \cdots & \square & \square \end{bmatrix}$

14. For a 6×6 matrix filled with consecutive integers $0, 1, 2, \ldots$, find out what goes into the boxes.

(a) $\begin{bmatrix} \square & & & & \\ & \square & & & \\ & & \ddots & & \\ & & & \square & \\ & & & & \square \end{bmatrix}$

(b) $\begin{bmatrix} & & & & \square \\ & & & \square & \\ & & \iddots & & \\ & \square & & & \\ \square & & & & \end{bmatrix}$

15. Now, these are $n \times n$ square matrices filled with consecutive integers $0, 1, 2, \ldots$ Figure out what goes into the boxes.

(a) $\begin{bmatrix} \square & & & & \\ & \square & & & \\ & & \ddots & & \\ & & & \square & \\ & & & & \square \end{bmatrix}$

(b) $\begin{bmatrix} & & & & \square \\ & & & \square & \\ & & \iddots & & \\ & \square & & & \\ \square & & & & \end{bmatrix}$

16. Fill in the boxes for a 100×100 matrix of consecutive integers $0, 1, 2, \ldots$

$$\begin{bmatrix} \Box & \Box & \cdots & \cdots & \Box & \Box \\ \Box & \Box & & & \Box & \Box \\ \vdots & & \ddots & \iddots & & \vdots \\ \vdots & & \iddots & \ddots & & \vdots \\ \Box & \Box & & & \Box & \Box \\ \Box & \Box & \cdots & \cdots & \Box & \Box \end{bmatrix}$$

17. Consider a 30×30 matrix filled with consecutive integers starting at 0. Where is 611? Give the row and column. Use Euclidean division.

18. Consider a 99×99 matrix of consecutive integers starting at zero. Find the row and column position of 3333.

19. Given an 80×80 matrix of consective numbers beginning at 0, find the number is at the given position:

(a) row 25, column 68.
(b) row 68, column 25.

20. In a 5000×5000 matrix of consective integers $0, 1, 2, \ldots,$ what number is at row 991, column 599?

21. Make base-10 Euclidean division tables for these numbers.

(a) 3351.
(b) 4096.
(c) 12801.

22. Change these numbers into base-2 by making $d = 2$ Euclidean division tables.

(a) 3351.
(b) 4096.
(c) 12801.

23. What is an algorithm? Explain it in your own words. Give some examples.