Permutations and Groups

Training problems for M2 2018 term 2

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1. Write down all the different permutations of uv.

2. Write down all the different permutations of abc.

3. Write down all the different permutations of $wxyz$.

4. I have five boxes colored red, green, blue, yellow, and orange. I have five balls colored red, green, blue, yellow and orange. How many different ways can I arrange the balls into the boxes, with one ball in each box?

5. I want to arrange 10 different people in a row. How many ways can I do this?

6. Prove that the number permutations of m objects is $m!$.

7. Prove that the number of permutation machines having m boxes per row is $m!$.

8. How many elements are in...

(a) S_2? (b) S_3? (c) S_4? (d) S_5? (e) S_7?

9. What is the difference between a permutation symbol and a permutation machine?

10. Write the permutation symbol that does the given permutaion.

(a) $abc \to bac$. (b) $bac \to abc$. (c) $abcd \to badc$. (d) $badc \to abcd$.

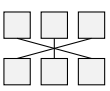
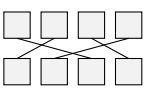
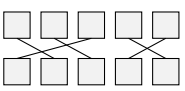
11. Draw the permutation machine that does the given permutaion.

(a) $abc \to cab$. (b) $cab \to abc$. (c) $abcd \to dcba$. (d) $dcba \to abcd$.

12. Change from permutation symbol to permutation machine.

(a) $\begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 1 & 3 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 5 & 4 & 3 & 1 \end{pmatrix}$

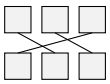
13. Change from permutation machine to permutation symbol.

(a)  (b)  (c)  (d)  .

14. Apply the permutation symbol to the objects. What is the result?

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} abc.$$

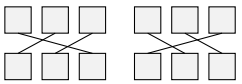
15. Put the objects into the permutation machine. What is the result?

abc  .

16. Apply the permutations to the objects. What happens?

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} abc.$$

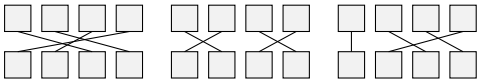
17. Put the objects into the permutation machines. What happens?

abc  .

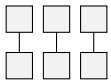
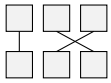
18. Apply the permutation symbols to the objects.

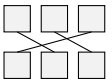
$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 4 & 1 \end{pmatrix} abcd.$$

19. Put the objects into the permutation machines. What do you get?

$abcd$ 

20. Fill in this table for the elements of S_3.

symbol	machine
	
$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$	
	

symbol	machine
$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$	
	
$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$	

21. Use these standard definitions for S_3 permutation symbols...

$$e = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \quad t_1 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \quad t_2 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$$
$$t_3 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \quad s_1 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \quad s_2 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$$

...to fill in this mini S_3 multiplication table:

	e	s_1	s_2
e			
s_1			$s_1 s_2$
s_2			

The entry $s_1 s_2$ tells you how to combine the symbols. Take s_1 from the leftmost column, and then put s_2 from the top row.

22. Use the standard S_3 definitions from problem **21** to construct the full S_3 multiplication table:

	e	t_1	t_2	t_3	s_1	s_2
e						
t_1						$t_1 s_2$
t_2						
t_3						
s_1						
s_2						

The entry $t_1 s_2$ tells you how to combine the symbol from the leftmost column (t_1), with the symbol from the top row (s_2).

23. Define the symbols e and t and use them to construct multiplication tables for S_2 and S_1. How many elements do S_2 and S_1 have?

24. Here are standard definitions for S_4 permutation symbols:

$$e = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} \quad t_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 3 & 4 \end{pmatrix} \quad t_2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 2 & 4 \end{pmatrix} \quad t_3 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix}$$

$$t_4 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 3 & 1 \end{pmatrix} \quad t_5 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 3 & 2 \end{pmatrix} \quad t_6 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 4 & 3 \end{pmatrix} \quad s_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix}$$

$$s_2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix} \quad s_3 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 3 & 2 \end{pmatrix} \quad s_4 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix} \quad s_5 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 1 & 3 \end{pmatrix}$$

$$s_6 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 4 & 1 \end{pmatrix} \quad s_7 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 2 & 3 \end{pmatrix} \quad s_8 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \end{pmatrix} \quad r_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$$

$$r_2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix} \quad r_3 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix} \quad r_4 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix} \quad r_5 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}$$

$$r_6 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix} \quad r_7 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix} \quad r_8 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix} \quad r_9 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$$