M2 Training Problems

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1 Functions, identity, inverses and plots

1. Let $f(x) = 2x + 1$. Find...
(a) Find $f(f(x))$.
(b) Find $f(f(f(x)))$.
(c) Find $f(f(f(f(x))))$.

2. Let $f(x) = 3x^2 + 1$ and $g(x) = 2x - 3$.
(a) Find $f(g(x))$.
(b) Find $g(f(x))$.
Are they the same?

3. Let $f(x) = ax + b$.
(a) Find $f(f(x))$.
(b) Find $f(f(f(x)))$.

4. Let $f(x) = ax + b$ and $g(x) = cx + d$.
(a) Find $f(g(x))$.
(b) Find $g(f(x))$.
Are they the same?

5. Sketch $y = x$ and $y = -x$. Put them on the same axes. Label everything.

6. Sketch $y = 2x$ and $y = -2x$. Put them on the same axes.

7. Sketch these lines on the same axes.

$$y = \frac{x}{2}, \quad y = -\frac{x}{2}.$$

8. Make an exact plot of $y = 3x + 2$ by finding the x-intercept and y-intercept.

9. Make an exact plot of

$$y = -\frac{x}{2} - 1.$$

10. If $f(x)$ and $g(x)$ are linear, show that
(a) $f(g(x))$ is linear.
(b) $g(f(x))$ is linear.

11. Let $f(x) = 3x + 2$. Find $f^{-1}(x)$. Do it two ways:

(a) By $f(f^{-1}(x)) = I(x)$.
(b) And by $f^{-1}(f(x)) = I(x)$.

12. Let $f(x) = ax + b$. Find $f^{-1}(x)$. Do it two ways:
(a) By $f(f^{-1}(x)) = I(x)$.
(b) And by $f^{-1}(f(x)) = I(x)$.

13. Let $f(x) = 2x + 1$. Find $f^{-1}(x)$. Make exact plots of f and f^{-1}. Also draw I.

14. Let $f(x) = -2x + 3$. Find $f^{-1}(x)$. Make exact plots of f and f^{-1}. Also draw I.

15. Consider the function

$$f(x) = -\frac{x}{2} + 3.$$

Find $f^{-1}(x)$. Make exact plots of f and f^{-1}. Also draw I.

16. Sketch the curve $y = x^2$. Use the unit square idea.

17. Let $f(x) = x^2$. Sketch f, I and f^{-1} on the same axes.

18. Let $f(x) = x^2 + 1$. Sketch f, I and f^{-1} on the same axes.

19. Are there functions that are inverses of themselves? Does there exist any functions with the property $f(x) = f^{-1}(x)$? In other words, f is its own inverse.
(a) Find one such self-inverse function f.
(b) Try to find more, as many as you can.

2 Logarithms