MATHSCI Problems

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1. Suppose x_1 and x_2 are the roots of

$$x^2 + x - 7 = 0.$$

Without solving for the roots, find

(a) $x_1^2 + x_2^2$.

(b) $x_1^3 + x_2^3$.

(c) $x_1^4 + x_2^4$.

Hint. It's possible to find sums and products of the roots without finding the roots themselves.

2. What is the difference between boiling and evaporation? Think about the similarities and differences. Focus on the essential features of both. Make a distinction between what is essential and what is not essential.

3. There are three fundamental circuit elements: resistor, capacitor and inductor. Why are they considered to be fundamental? Can we design other circuit elements that are not resistors, capacitors or inductors and build circuits out of them? Can there be other kinds of fundamental circuit elements or are these three unique? Why?

Comment. I don't expect an immediate answer to this. It may take weeks, months or years of thinking. Just keep this question in mind as you learn about electricity and electric circuits. Take your time thinking about this.

4. An orchestra is about to begin playing Beethoven's Fifth Symphony. There is a live audience, and the performance is also being broadcast over the radio. Your friend is $8,000$ kilometers away, listening on the radio. You are in the audience. How far must you sit from the orchestra so that you and your friend hear the opening notes at exactly the same time?

5. If b and c are odd, prove that

$$x^2 + bx + c = 0$$

cannot have two integer solutions.

6. Prove that $n^4 + 4$ is never prime, for all natural numbers $n > 1$.

Comment. A number m is prime if it has no factors except 1 and m. A number m is composite (not prime) if it has a factor other than 1 or m. So this problem asks you to show that $m = n^4 + 4$ is always composite. You can do this by showing that m has a factor that is not 1 and not m.