Trigonometry and Constructive Geometry

Training problems for M2 2018 term 1
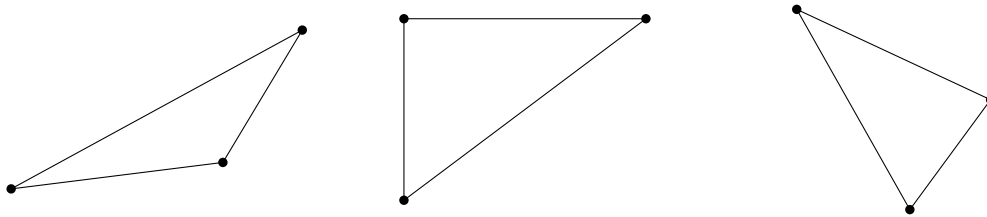
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1 Labeling geometrical figures
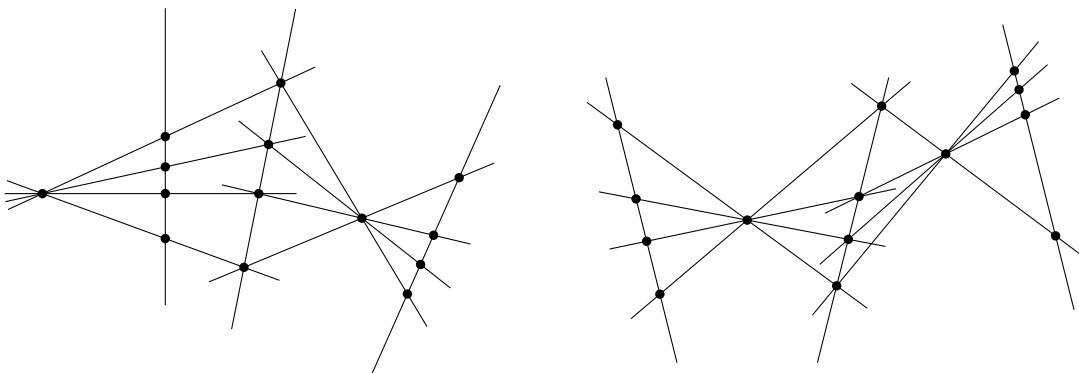
1. Practice writing Greek letters.

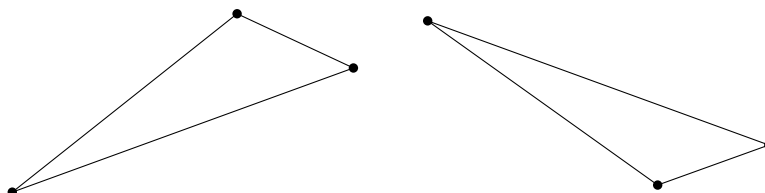$$\alpha\beta\gamma\delta\epsilon\theta\lambda\mu\pi\phi\psi$$

2. Label the sides, angles and vertices of these triangles using the classical method, in counterclockwise order.
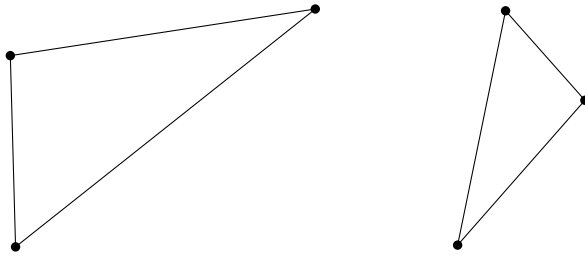
3. Use primes to label the figure in a logical way.

4. These triangles are congruent. Label them using the classical method. Use primes. Write down relationships between angles and sides.
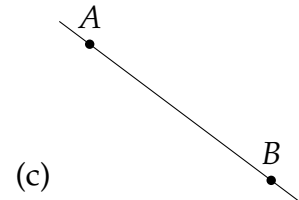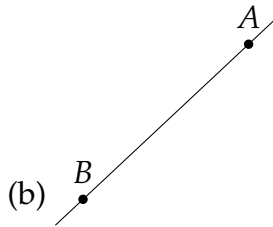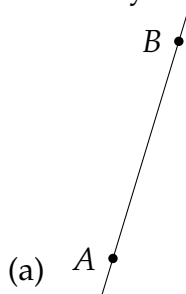
5. These triangles are similar. Label them using the classical method. Write down relationships between angles and between sides. What is the zoom factor? Is it bigger or smaller than 1?
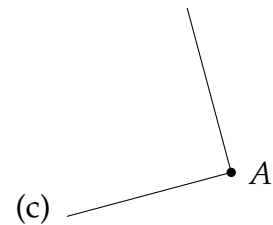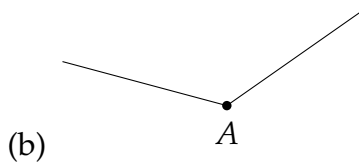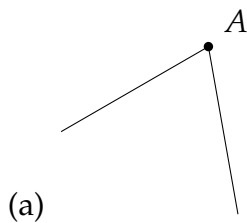
## 2   Congruence and similarity

6. Copy these segments using ruler and compass. Don't erase your construction lines and arcs. Label your work

(a)   (b)   (c)

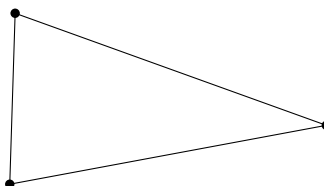**7.** Use a ruler and compass to copy the angle at A. Don't erase your arcs or construction lines. Label your work.

(a)   (b)   (c)

8. Make a congruent copy of this triangle by SSS. Use ruler and compass. Don't erase your construction lines. Label your work
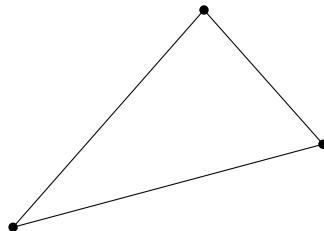
9. Use ruler and compass to make a congruent copy of this triangle by SAS. Label your work. Explain which sides and angle you have copied.

10. Use ruler and compass to make a congruent copy of this triangle by ASA. Label your work. Explain which side and angles you have copied.
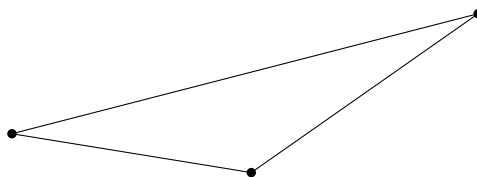
11. Use a ruler and compass to construct a counterexample for AAA. Construct two triangles where AAA is true, but not conguent. Use ruler and compass. Label your triangles and write down all the relationships. Is the zoom factor bigger or smaller than 1?

12. Give a counterexample for ASS, SSA. Show that having ASS true leads to two solutions, one congruent, the other not congruent. Use a ruler and compass. Don't erase your construction lines.

13. Prove the parallelogram area formula

$$\text{area} = \text{base} \times \text{height}$$

by doing these steps:
  (a) Construct a parallelogram by ruler and compass.
  (b) Cut the parallelogram into two triangles.
  (c) Copy the two triangles into separate figures using ruler and compass.
      Label them using classical labelling and primes.
  (d) Use SSS to prove that the two triangles are congruent.
      Explain why each step is true.
  (e) Write a conclusion.

**14.** Do the same parallelogram proof as in problem **13** but using SAS.

15. Prove the paralellogram area formula using AAS. Follow the steps of problem **13**.

**16.** Prove the parallelogram area formula using ASA. Follow problem **13**.

17. Use AAA to make a *smaller* similar copy of this triangle. Do it with ruler and compass. Label your work. Write down the relationships between sides and angles.
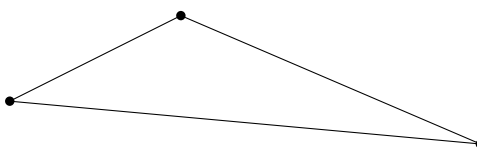
18. Use AAA to make a *larger* similar copy of this triangle. Use ruler and compass. Label your work. Write down relationships between sides and angles.
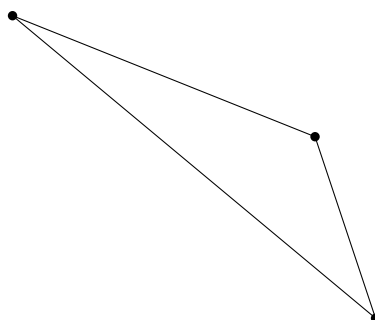
19. Construct 90° perpendicular lines going through point *A*. Use ruler and compass. Don't erase construction lines and arcs.

(a) *A* •

(b) *A* •

(c) *A* •
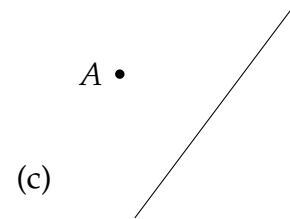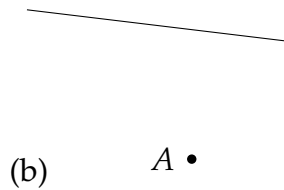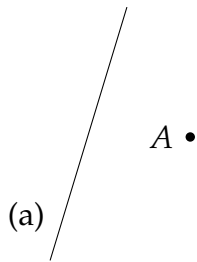
20. Here is a right (90°) triangle. We usually label the vertex with the right angle as *C* and the longest side as *c*. Label the triangle and construct the altitude line *h* at *C* using ruler and compass.

C

A

B
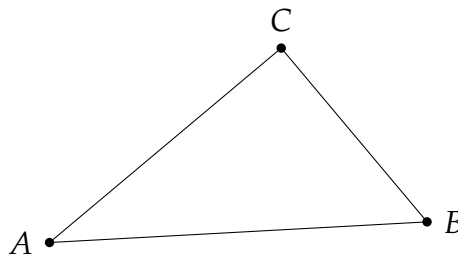
21. Construct a 90° triangle using ruler and compass. Let *C* be the 90° vertex. Also construct the altitude line at *C*. Label the vertices, angles and sides of your figure. Don't erase your arcs or construction lines.
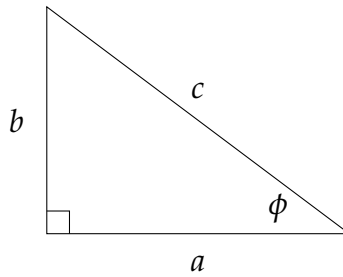
3 ϕ–90° **triangles**
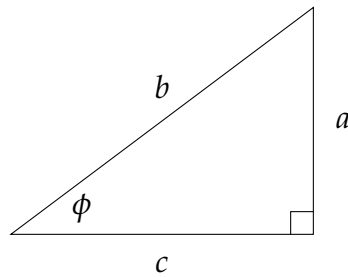
22. Fill in the missing angle.

ϕ

ϕ

ϕ
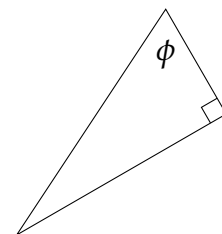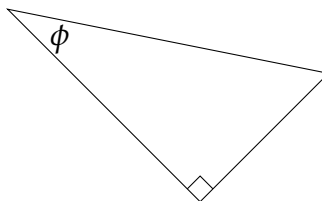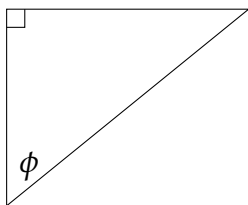
23. Fill in the missing angle and abel the sides with proper trigonometric names: hypotenuse, adjacent side, opposite side.
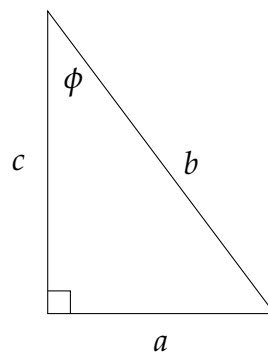
ϕ

ϕ

ϕ
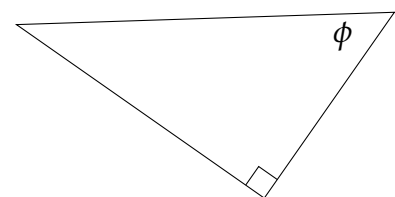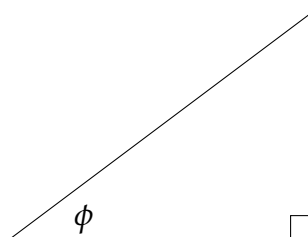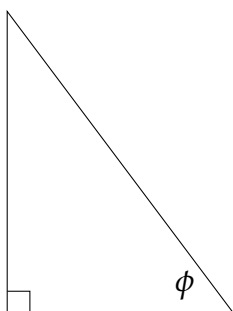
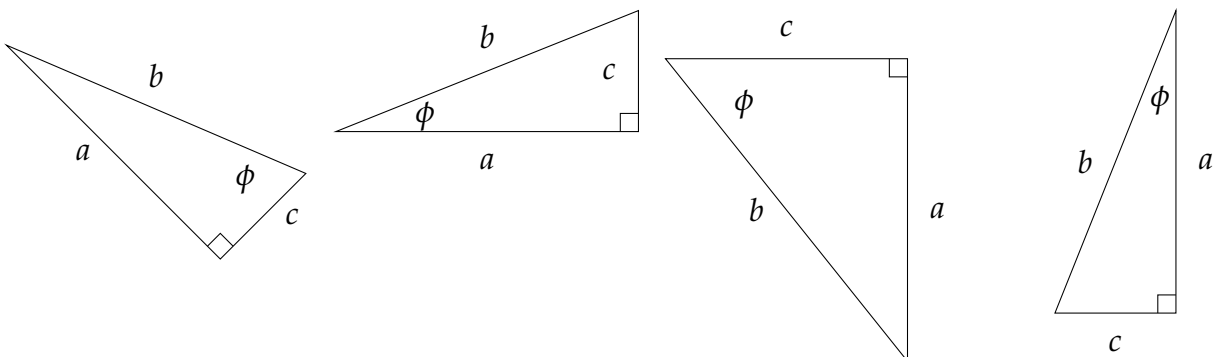**24.** Here is a ϕ–90° triangle. What is special about side a? Make a list of things.

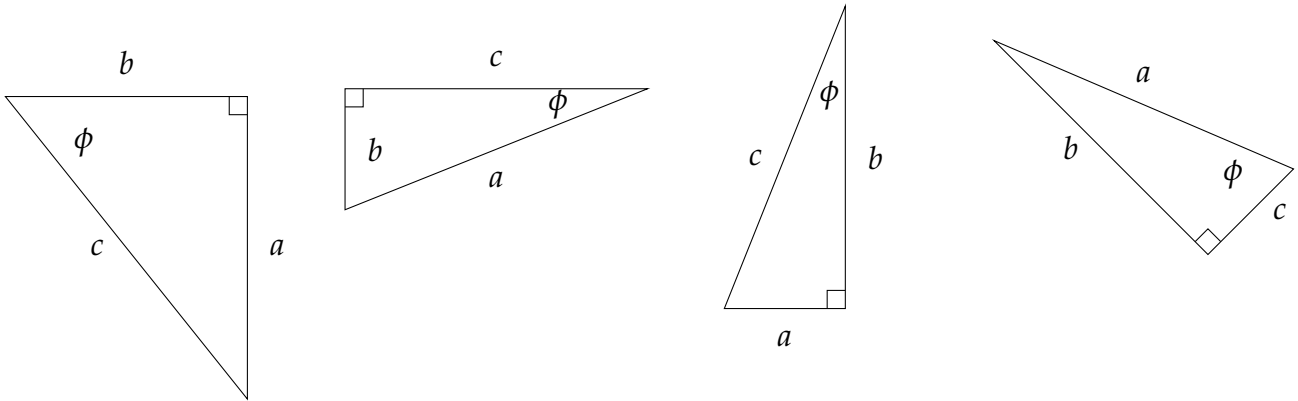**25.** Here is a ϕ–90° triangle. What is special about side c? Make a list of things.

26. Here is a ϕ–90° triangle. What is special about side b? Make a list of things.

27. Find sin, cos, tan in terms of sides a, b, c.

28. Find csc, sec, cot in terms of sides *a*, *b*, *c*.

29. Label the missing angle, label all sides with proper names and find all trigonometric ratios sin, cos, tan, csc, sec, cot. in terms of sides *a*, *b*, *c*.

30. Why do we have six of these trigonometric ratios, sin, cos, tan, csc, sec and cot? Why are they important? What is so special about them?

31. Explain how we were able to calculate the distance to the star 61 Cygni by using trigonometry.

32. Draw a ϕ–90° triangle, label it, and find a relationship between sin, cos and tan.

33. Draw a ϕ–90° triangle and label it. Use Pythagoras's law to find a relationship between sin, cos and 1.

34. Draw a ϕ–90° triangle and label it. Use Pythagoras's law to find a relationship between tan, sec and 1.

35. Draw a ϕ–90° triangle and label it. Use Pythagoras's law to find a relationship between cot, csc and 1.

4   Special angles