

M2 Training Problems

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1 Functions, identity, inverses and plots

1. Let $f(x) = 2x + 1$. Find...

- (a) Find $f(f(x))$.
- (b) Find $f(f(f(x)))$.
- (c) Find $f(f(f(f(x))))$.

2. Let $f(x) = 3x^2 + 1$ and $g(x) = 2x - 3$.

- (a) Find $f(g(x))$.
- (b) Find $g(f(x))$.

Are they the same?

3. Let $f(x) = ax + b$.

- (a) Find $f(f(x))$.
- (b) Find $f(f(f(x)))$.

4. Let $f(x) = ax + b$ and $g(x) = cx + d$.

- (a) Find $f(g(x))$.
- (b) Find $g(f(x))$.

Are they the same?

5. Sketch $y = x$ and $y = -x$. Put them on the same axes. Label everything.

6. Sketch $y = 2x$ and $y = -2x$. Put them on the same axes.

7. Sketch these lines on the same axes.

$$y = \frac{x}{2}, \quad y = -\frac{x}{2}.$$

8. Make an exact plot of $y = 3x + 2$ by finding the x -intercept and y -intercept.

9. Make an exact plot of

$$y = -\frac{x}{2} - 1.$$

10. If $f(x)$ and $g(x)$ are linear, show that

- (a) $f(g(x))$ is linear.
- (b) $g(f(x))$ is linear.

11. Let $f(x) = 3x + 2$. Find $f^{-1}(x)$. Do it two ways:

- (a) By $f(f^{-1}(x)) = I(x)$.
- (b) And by $f^{-1}(f(x)) = I(x)$.

12. Let $f(x) = ax + b$. Find $f^{-1}(x)$. Do it two ways:

- (a) By $f(f^{-1}(x)) = I(x)$.
- (b) And by $f^{-1}(f(x)) = I(x)$.

13. Let $f(x) = 2x + 1$. Find $f^{-1}(x)$. Make exact plots of f and f^{-1} . Also draw I .

14. Let $f(x) = -2x + 3$. Find $f^{-1}(x)$. Make exact plots of f and f^{-1} . Also draw I .

15. Consider the function

$$f(x) = -\frac{x}{2} + 3.$$

Find $f^{-1}(x)$. Make exact plots of f and f^{-1} . Also draw I .

16. Sketch the curve $y = x^2$. Use the unit square idea.

17. Let $f(x) = x^2$. Sketch f , I and f^{-1} on the same axes.

18. Let $f(x) = x^2 + 1$. Sketch f , I and f^{-1} on the same axes.

19. Are there functions that are inverses of themselves? Does there exist any functions with the property $f(x) = f^{-1}(x)$? In other words, f is its own inverse.

- (a) Find one such self-inverse function f .
- (b) Try to find more, as many as you can.

2 Logarithms

20. Draw the fastest-growing function f that you can imagine. Draw $I(x)$ and use it to find the inverse f^{-1} .

21. Draw the slowest-growing function f that you can imagine. Draw the identity $I(x)$ and use it to find the inverse f^{-1} .

22. Given f , tell me about the inverse f^{-1} . Does it grow fast, slowly, very fast etc.?

- (a) f is a fast-growing function.
- (b) f does not grow at all.
- (c) f is a slow-growing function.
- (d) f is a very slow-growing function.
- (e) f is a very fast-growing function.

23. Fill in this table about the behavior of $f(x) = 2^x$ for different values of x .

x	$f(x)$
$x = 0$	$f = 1$
$x > 0$	
$x < 0$	
$x > 1$	
$x \rightarrow \infty$	
$x \rightarrow -\infty$	

24. Plot 2^x , 3^x and 5^x all on the same axes.

25. Consider the function

$$f(n) = \left(1 + \frac{1}{n}\right)^n.$$

Use a calculator. Fill in this table and notice how $f(n)$ becomes closer and closer to the magic number e as $n \rightarrow \infty$.

x	$f(x)$
1	2
2	
5	
10	
100	
1000	

26. Plot 2^x , e^x and 10^x all on the same axes.

27. Plot $f(x) = 2^x$ and the identity line $I(x)$. Use the identity line to draw the inverse $f^{-1}(x) = \log_2 x$.

28. Plot $f(x) = 3^x$, $I(x)$ and $f^{-1}(x) = \log_3 x$ all on the same axes.

29. Fill in this table about the behavior of $g(x) = \log_2 x$ for different values of x .

x	$g(x)$
$x = 1$	$g = 0$
$x > 1$	
$x < 1$	
$x = 2$	
$x \rightarrow \infty$	
$x \rightarrow 0$	

30. Does 2^x ever touch the x -axis? Does $\log_2 x$ ever touch the y -axis?

31. Fill in the table.

x	3^x	x	$\log_3 x$
1		1	
2		9	
3		27	
4		243	
5		59049	

32. Fill in the table.

x	10^x	x	$\log_{10} x$
1		1	
2		10	
3		1000	
4		100,000	
5		10,000,000	

33. Plot $\log_2 x$, $\log_3 x$ and $\log_5 x$ all on the same axes. Label all the important points.

34. Plot $\log_2 x$, $\log_e x$ and $\log_{10} x$ all on the same axes. Label all the important points.

35. The formulas relating f , f^{-1} and I establish the two most important properties of logarithms and exponentials. Use $f(x) = a^x$ and $f^{-1}(x) = \log_a x$ and tell me what these formulas imply:

(a) $f(f^{-1}(x)) = I(x)$.

(b) $f^{-1}(f(x)) = I(x)$.