## **Arithmetic and Combinatorics**

Training problems for M1 2018 term 1

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## 1 Calculating prodigies

- **1.** "He who refuses to do arithmetic is doomed to talk nonsense." Which famous computer scientist said that? You can use the internet to find out.
- 2. Which famous American calculating prodigy became an astronomer when he grew up?
- **3.** This prodigy was from Germany. He could multiply 100-digit numbers in his head. Who was he?
- **4.** Write down a few interesting things about the life of Jedediah Buxton. Where was he from? When was he born? How did he die? What did he do?
- **5.** What do the amazing powers of calculating prodigies prove about the human mind? Tell me some opinions.

## 2 Euclidean division

- **6.** Who was Euclid and where was he from? When did he live?
- 7. Use the terms dividend, divisor, remainder and quotient to label the parts of the expression

$$29 = 4 \times 6 + 5$$
.

Choose the divisor carefully. Is it 4 or is it 5? Check that  $0 \le r < d$ . Is it true? Is this a correct expression of Euclidean division?

**8.** Check that  $0 \le r < d$ . Is it true?

$$111 = 9 \times 11 + 12$$

Is this a correct expression of Euclidean division?

- **9.** Label the parts of these expressions with the terms *dividend*, *divisor*, *remainder* and *quotient*.
  - (a) 101/39.
  - (b)  $m = q \times d + r, \ 0 \le r < d.$
  - (c)  $59 = 5 \times 11 + 4$ .

- (d)  $a = b \times c + d$ ,  $0 \le d < c$ , d > b.
- (e)  $r \div s$ .
- (f) 39/101.

<b>10.</b>	Which	number	is th	e divisor	and	which	one is	the c	quotient?

(a) 
$$42 = 11 \times 3 + 9$$
.

(c) 
$$69 = 10 \times 6 + 9$$
.

(b) 
$$99 = 7 \times 13 + 8$$
.

(d) 
$$23 = 4 \times 5 + 3$$
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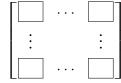
**11.** Do by Euclidean division. Label all the parts of your expressions. Give a clear answer in terms of two numbers.

- (a) 99/91.
- (c) 1001/651.
- (e) 19/1.
- (g) 0/15.

- (b) 919/7.
- (d) 1/19.
- (f) 17/17.
- (h) 15/0.

**12.** These matrices are filled with consecutive integers 0, 1, 2, . . . Figure out what goes into the boxes.

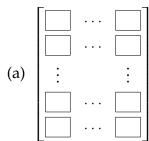
- (a)  $7 \times 7$  matrix.
- (b)  $10 \times 10$  matrix.
- (c)  $n \times n$  matrix.

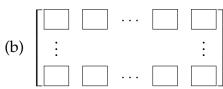


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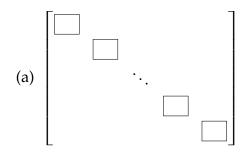
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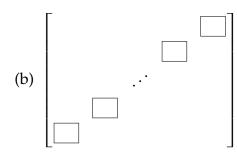
**13.** Each matrix is  $n \times n$  and is filled with consecutive integers  $0, 1, 2, \ldots$  Figure out what goes into the boxes.



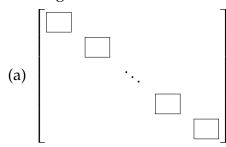


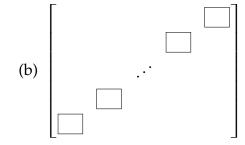
**14.** For a  $6 \times 6$  matrix filled with consecutive integers  $0, 1, 2, \ldots$ , find out what goes into the boxes.





**15.** Now, these are  $n \times n$  square matrices filled with consecutive integers 0, 1, 2, . . . Figure out what goes into the boxes.





Numbers in base-d

<b>16.</b> Fill in the boxes for a 100	× 100 matrix of cons	ecutive integers 0, 1, 2,
<b>17.</b> Consider a 30 × 30 matrix Give the row and column. Us		ive integers starting at 0. Where is 611?
		gers starting at zero. Find the row and
<b>19.</b> Given an $80 \times 80$ matrix of the given position:	of consective number	rs beginning at 0, find the number is at
(a) row 25, column 68.	(	b) row 68, column 25.
<ul><li>20. In a 5000 × 5000 matrix of column 599?</li><li>3 Numbers in base</li></ul>		0,1,2,, what number is at row 991,
<b>21.</b> What is an algorithm? Ex	plain it in your own	words. Give some examples.
<b>22.</b> Make base-10 Euclidean of	livision tables for the	se numbers.
(a) 3351.	(b) 4096.	(c) 12801.
23. Change these numbers in	to base-2 by making	d = 2 Euclidean division tables.
(a) 331.	(b) 409.	(c) 1280.
<b>24.</b> Make Euclidean division	tables with dividend	m = 3721, using these divisors:
(a) $d = 10$ .	(b) $d = 4$ .	(c) $d = 2$ .
<b>25.</b> Use Euclidean division ta	bles to change 757 in	to these bases:
(a) base-5.	(b) base-3.	(c) base-7.
symbols like A, B, etc. Anoth	ner way is to use Tay	er than 9. One way is to use alphabetical lor digits like $(10)$ , $(11)$ . Sometimes it's s better to use Taylor digits. Write down
27. How many different digit	s are there in base-d?	Write them down.

28. Write down all the digits of base-20. Use Taylor's idea for digits bigger than 9.

4 Numbers in base-d

- 29. How many different digits are there in base-16? Write them down...
  - (a) using capital letters.

- (b) using Taylor digits.
- **30.** Change these base-16 numbers from alphabetical symbols to Taylor digits.
  - (a) FE199A6.
- (b) 123ABCD12.
- (c) D00E00F0C.
- (d) 99FF11BB0A.
- 31. Change these base-16 Taylor digit numbers into numbers using alphabetical symbols.
  - (a) (15)(14)(13)0.
- (b) 53(10)10(10)5.
- (c) (11)111(11)(11).
- (d) (14)(12)4(15).

- **32.** Count the digits.
  - (a) 3DCF918B.
- (c) (123)(456)789.
- (e) (89)(71)2(14)(11). (g) 785(21)871.

- (b) 348(11)1(13).
- (d) FFA000A1.
- (f) (123321).
- (h) 11(11)1(111).

- 33. Change 39101 into base-20. Use Taylor digits.
- 34. Change 569112 into base-100. Use Taylor digits.
- 35. Change 569112 into base-1000. Use Taylor digits.
- 36. Now that you see the idea for base-100, 1000, etc, it is very easy to fill in this table without doing any calculations at all...

base-10	891283481785
base-100	
base-1000	
base-10000	
base-1000000	

**37.** This is Pascal's triangle. You can make it as big as you like.

					1					
				1		1				
			1		2		1			
		1		3		3		1		
	1		4				4		1	
1		5		10		10		5		1

Change these numbers into base-9 by using Euclidean division tables. Think about the patterns you see in the digits.

(a) 1.

(b) 10.

- (c) 100.
- (d) 1000.

38. Change 10000 into base-9. Use Pascal's triangle to guess the answer. Check your answer by Euclidean division table. Is your guess right?

<b>39.</b> Change these patterns you see.	numbers into base-99 by	Euclidean division ta	bles. Think about the
(a) 1.	(b) 100.	(c)	10000.
<b>40.</b> Change these	numbers into base-99 by u	sing Pascal's triangle a	nd Taylor digits.
(a) 1000000.	(b) 100000	000. (c)	10000000000.
<b>41.</b> Change these	numbers into base-999 by	Euclidean division. Th	ink about the patterns.
(a) 1.	(b) 1000.	(c)	1000000.
<b>42.</b> Change these	numbers into base-999 by	Pascal's triangle and Ta	aylor digits.
(a) $10^9$ .	(b) $10^{12}$ .	(c) $10^{15}$ .	(d) $10^{24}$ .

- 43. Change  $10^{24}$  into base-9999 by using Pascal's triangle and Taylor digits.
- 44. Change  $10^{48}$  into base-999999 using Pascal's triangle and Taylor digits.

## 4 Computer science arithmetic

- **45.** Why do we use base-2, 4, 8, 16, etc. in the design of computers? Give two reasons.
- **46.** Make a counting table from 0 to 15 in base-2.

base-10	base-2	base-10	base-2
0		8	
1		9	
2		10	
3		11	
4		12	
5		13	
6		14	
7		15	

**47.** Make a counting table from 0 to 15 in base-4.

base-10	base-4	base-10	base-4
0		8	
1		9	
2		10	
3		11	
4		12	
5		13	
6		14	
7		15	

**48.** Fill in this table. One digit in base-4, 8, 16, etc., is equivalent to how many base-2 digits?

kind of digit	equivalent digits in base-2
one base-4 digit	
one base-8 digit	
one base-16 digit	
one base-32 digit	
one base-64 digit	
one base-256 digit	

49. Fill in this table. One digit in base-16, 64, etc., is equivalent to how many base-4 digits?

kind of digit	equivalent digits in base-4
one base-16 digit	
one base-64 digit	
one base-256 digit	

**50.** Fill in this table. One digit in base-64 or base-512, etc., is equivalent to how many base-8 digits?

kind of digit	equivalent digits in base-8
one base-64 digit	
one base-512 digit	