M1 Training Problems

Ted Szylowiec

tedszy@gmail.com

1 Linear equations, sketches and exact plots

1. Solve $(4x-2)-(2x-1)=3x$. How many solutions does it have?

2. Solve $(3x-2)-(2x-1)=x$. How many solutions does it have?

3. Solve $(4x-1)-(2x-1)=2x$. How many solutions does it have?

4. Solve for x. $5x+6=3x+2$. How many solutions does it have?

5. Solve for x. $5x+2=5x+2$. How many solutions does it have?

6. Solve for x. $3x+1=3x-1$. How many solutions does it have?

7. Sketch freehand, no ruler: $y=x$, $y=-x$. Put them on the same axes. Remember to label everything.

8. Sketch freehand, no ruler: $y=2x$, $y=-2x$. Put them on the same axes.

9. Sketch freehand, no ruler.

$$y=\frac{x}{2}, \quad y=-\frac{x}{2}.$$

Put them on the same axes.

10. Sketch freehand, no ruler. Label everything.

$$y=2x+1.$$

11. Sketch freehand, no ruler. Label everything.

$$y=-2x-1.$$

12. Sketch freehand, no ruler. Label everything.

$$y=-\frac{x}{2}+3.$$

13. Sketch freehand, no ruler. Label everything.

$$y=\frac{x}{2}-3.$$

14. Make an exact plot. Find the x and y intercepts. Show your work. Use a ruler.

$$y = 3x - 2.$$

15. Make an exact plot.

$$y = -\frac{x}{3} + 1.$$

16. Consider $y = 3x - 2$.

(a) Make a freehand sketch, no ruler.
(b) Make an exact plot, with a ruler. Find intercepts.

17. Consider the equation

$$2x + 3 = -\frac{x}{2} + 1.$$

(a) Solve for x by algebra. How many solutions does it have?
(b) Make exact plots of the left-hand side and right-hand side of the equation. Show where the solutions are.

18. Consider the equation

$$2x + 3 = 2x - 1.$$

(a) Solve for x by algebra. How many solutions does it have?
(b) Make exact plots of the left-hand side and right-hand side of the equation. Show where the solutions are. Make sure your plots match your algebra.

19. Consider the equation

$$-2x + 1 = -2x + 1.$$

(a) Solve for x by algebra. How many solutions does it have?
(b) Make exact plots of the left-hand side and right-hand side of the equation. Show where the solutions are. Make sure your plots tell the same story as your algebra.

20. Consider the equation

$$3x - 6 = x = \frac{x}{3} + 2.$$

Does this equation have a solution? Make exact plots of the left-hand side, the right-hand side, and the middle, all on the same axes. Show where the solution is, if there is one.

21. Consider the equation

$$\frac{x}{2} + 1 = 2x - 3 = -x + 4.$$

Use exact plots of left-hand side, right-hand side and middle to figure out if this has solutions. Show where the solutions are, if there are any.

22. Consider the three-way equation

$$x + 5 = 2x + 6 = 3x + 7.$$

Make exact plots of the LHS, RHS and middle. Show where the solutions are, if any.

2 One equation, many unknowns

23. The not-so-interesting cases for solutions of equations are when there are no solutions or anything is a solution. But what about the interesting ones? Draw the interesting cases for these equations:

(a) One unknown, $ax = b$.
(b) Two unknowns, $ax + by = c$.
(c) Three unknowns, $ax + by + cz = d$.

24. Fill in this table.

Equations and unknowns	What can happen?
One equation, one unknown	No solutions. Anything is a solution. All solutions are on one point.
One equation, two unknowns. $ax + by = c$	
One equation, three unknowns. $ax + by + cz = d$	

25. Consider the equation $ax = b$. By choosing numbers for a and b you can give examples for the different cases.

(a) Give an example where it has no solution.
(b) Give an example where anything is a solution.
(c) Give an example where there is one solution on one point.

26. Consider $ax + by = c$. Choose numbers for a, b, c and give examples for the following different cases:

(a) Give an example where it has no solution.
(b) Give an example where anything is a solution.
(c) Give an example where all solutions are on a line.

27. Consider $ax + by + cz = d$. Choose numbers for a, b, c, d and give examples for these different cases:

(a) Give an example where it has no solution.
(b) Give an example where anything is a solution.
(c) Give an example where all solutions are on a plane.

28. Consider the line $ax + by = c$. Find the x-intercept. Show how you found it.

29. Consider the line $ax + by = c$. Find the y-intercept. Show how.

30. Consider the line $ax + by = c$. Find the slope. Don't just write the answer. Show how.

31. Consider the equation $2x + y = 1$. Make an exact plot of this by the *abc* method. Find the x-intercept and y-intercept. Use a ruler. Label everything: axes, line, intercepts.

32. Make an exact plot of

$$-\frac{x}{2} + y = 3.$$

by the *abc* method.

33. Make an exact plot of $2y - x = -2$ by the *abc* method. Be careful.

34. Consider $3x - 2y = 1$.

(a) Make an exact plot by the *abc* method. Use a ruler and label everything.
(b) Find the slope.

35. Consider $\frac{x}{2} + \frac{y}{3} = 1$.

(a) Make an exact plot by the *abc* method. Use a ruler. Label everything.
(b) Find the slope.

3 Two equations, two unknowns

36. Give an example of how two lines can join to make a point. Draw.

37. Draw an example of two lines having no point in common.

38. Draw an example of two lines that join on an infinite number of points.

39. Draw an example of two planes having no points or lines in common.

40. Draw an example of two planes joining one one line.

41. Draw an example of two planes joining on an infinite number of different lines.

42. Draw an example of three planes joining to make a line.

43. Draw an example of three planes joining to make exactly one point.

44. Consider the system of equations:

$$\begin{aligned} x + 2y &= 1 \qquad (1) \\ x - y &= 2 \qquad (2) \end{aligned}$$

Plot (1) and (2) on the same axes by *abc* method and show where they are both true.

45. Consider the system

$$\begin{aligned} 2x - y &= 1 \qquad (1) \\ x + y &= 2 \qquad (2) \end{aligned}$$

(a) Make exact plots of (1) and (2) by *abc* method and show where the solution is.

(b) Find the solution by algebra.
(c) Does your work in (a) match your work in (b)?

46. Consider the system

$$x - \frac{y}{3} = 1 \qquad (1)$$
$$\frac{x}{2} + y = 2 \qquad (2)$$

(a) Make exact plots of (1) and (2) by *abc* method and show where the solution is.
(b) Find the solution by algebra.
(c) Does your work in (a) match your work in (b)?

4 Cramer's method

5 Properties of determinants

47. Begin with $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$. Swap the rows. What happens to the determinant?

48. Begin with $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$. Swap the columns. What happens to the determinant?

49. Begin with $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$. Flip it over the long diagonal. What happens to the determinant?

50. Begin with $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$. Flip it over the other long diagonal. What happens?

51. Start with $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$. Add row 1 to row 2. What happens to the determinant?

52. Start with $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$. Add row 2 to row 1. What happens to the determinant?

53. Start with $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$. Add column 1 to column 2. What happens?

54. Start with $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$. Add column 2 to column 1. Does the determinant change?

55. Start with $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$. First add column 2 to column 1. Then add row 2 to row 1. Does the determinant change?

56. Start with $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$. First swap rows. Then after that, swap the columns. Does the determinant change?

57. Start with $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$. Multiply column 1 by -1. What happens to the determinant?

58. Start with $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$. Multiply column 2 by -1. What happens to the determinant?

59. Start with $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$. Multiply row 1 by -1. What happens to the determinant?

60. Start with $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$. Multiply row 2 by -1. What happens to the determinant?

61. Start with $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$. Multiply all the elements a, b, c and d by -1. What happens to the determinant?

62. Start with $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$. Multiply row 1 by m and row 2 by n. What happens to the determinant? How does it change?

63. Start with $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$. Multiply column 1 by m and column 2 by n. What happens to the determinant? How does it change?

64. Start with $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$. Multiply row 1 by k and row 2 by $1/k$. What happens to the determinant? Does it change?

65. Start with $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$. Multiply column 1 by k and column 2 by $1/k$. What happens to the determinant? Does it change?

6 Applications of determinants

66. Given that $\frac{a}{b} = \frac{c}{d}$ prove that $\frac{d}{b} = \frac{c}{a}$. Use determinant properties.

67. Given that $\frac{a}{b} = \frac{c}{d}$ prove that $\frac{a}{c} = \frac{b}{d}$. Use determinant properties.

68. Given that $\frac{a}{b} = \frac{c}{d}$ prove that $\frac{b}{a} = \frac{d}{c}$. Use determinant properties.

69. Given that $\frac{a}{b} = \frac{c}{d}$ prove that $\frac{a}{c} = \frac{a+b}{c+d}$. Use determinants.

70. Given that $\frac{a}{b} = \frac{c}{d}$ prove that $\frac{a+b}{b} = \frac{c+d}{d}$. Use determinants.

71. Given that $\frac{a}{b} = \frac{c}{d}$ prove that $\frac{a+b}{c+d} = \frac{b}{d}$. Use determinants.

72. Given that $\frac{a}{b} = \frac{c}{d}$ prove that $\frac{a}{b} = \frac{c-a}{d-b}$. Use determinants.

73. Given that $\frac{a}{b} = \frac{c}{d}$ prove that $\frac{a}{b} = \frac{c+2a}{d+2b}$. Use determinants.

74. Given that $\frac{a}{b} = \frac{c}{d}$ prove that $\frac{a+c}{b+d} = \frac{a-c}{b-d}$. Use determinants.

75. Given that $\frac{a}{b} = \frac{c}{d}$ prove that $\frac{a+b}{a-b} = \frac{c+d}{c-d}$. Use determinants.