

# Trigonometry and Constructive Geometry

Training problems for M2 2018 term 1

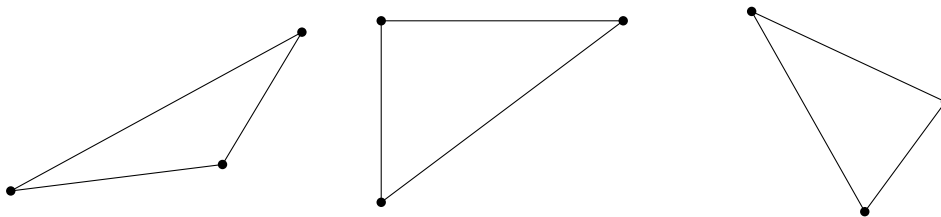
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## 1 Labeling geometrical figures

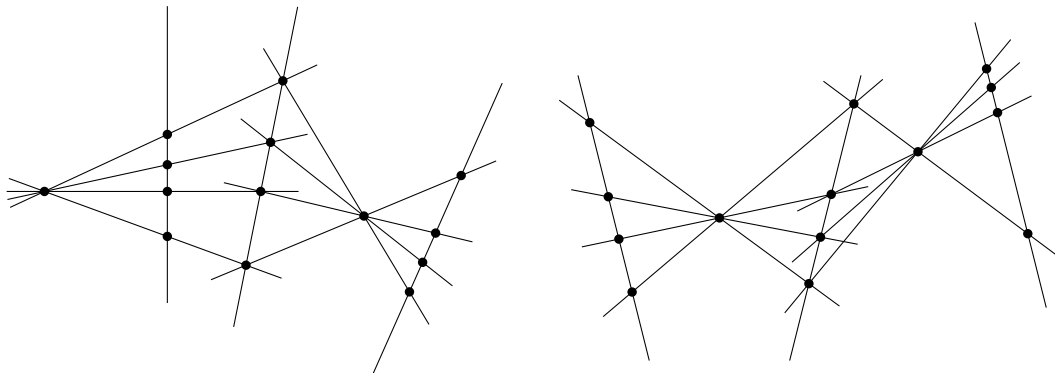
1. Practice writing Greek letters.

$\alpha\beta\gamma\delta\epsilon\theta\lambda\mu\pi\phi\psi$  \_\_\_\_\_

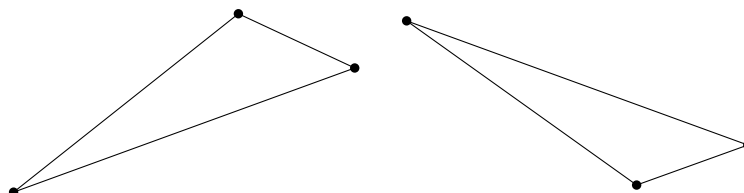
2. Label the sides, angles and vertices of these triangles using the classical method, in counterclockwise order.



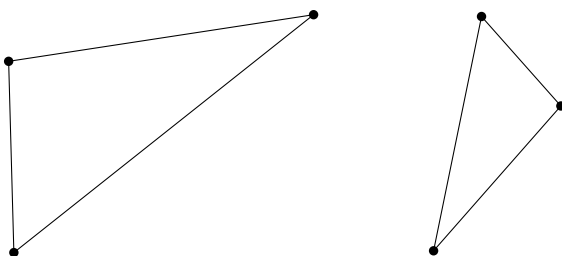
3. Use primes to label the figure in a logical way.



4. These triangles are congruent. Label them using the classical method. Use primes. Write down relationships between angles and sides.

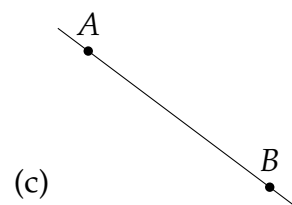
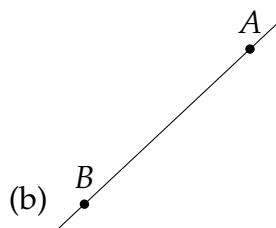
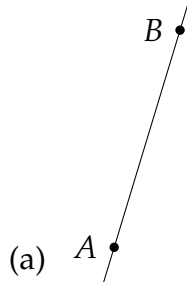


5. These triangles are similar. Label them using the classical method. Write down relationships between angles and between sides. What is the zoom factor? Is it bigger or smaller than 1?

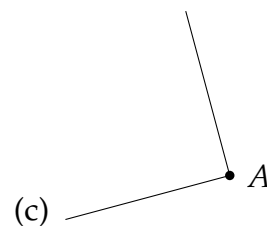
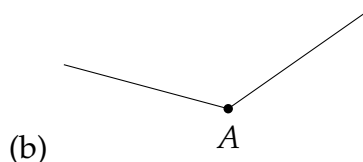
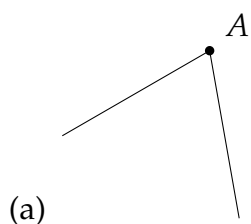


## 2 Congruence and similarity

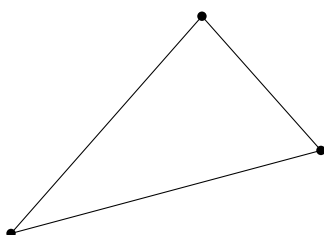
6. Copy these segments using ruler and compass. Don't erase your construction lines and arcs. Label your work



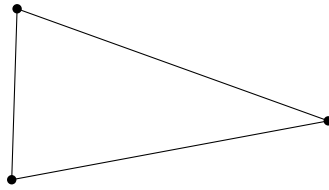
7. Use a ruler and compass to copy the angle at A. Don't erase your arcs or construction lines. Label your work.



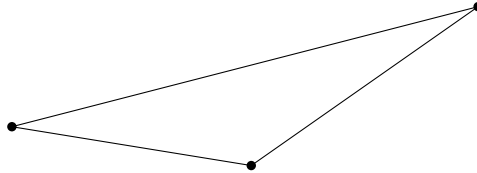
8. Make a congruent copy of this triangle by SSS. Use ruler and compass. Don't erase your construction lines. Label your work



9. Use ruler and compass to make a congruent copy of this triangle by SAS. Label your work. Explain which sides and angle you have copied.



10. Use ruler and compass to make a congruent copy of this triangle by ASA. Label your work. Explain which side and angles you have copied.



11. Use a ruler and compass to construct a counterexample for AAA. Construct two triangles where AAA is true, but not congruent. Use ruler and compass. Label your triangles and write down all the relationships. Is the zoom factor bigger or smaller than 1?

12. Give a counterexample for ASS, SSA. Show that having ASS true leads to two solutions, one congruent, the other not congruent. Use a ruler and compass. Don't erase your construction lines.

13. Prove the parallelogram area formula

$$\text{area} = \text{base} \times \text{height}$$

by doing these steps:

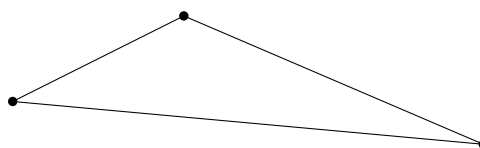
- (a) Construct a parallelogram by ruler and compass.
- (b) Cut the parallelogram into two triangles.
- (c) Copy the two triangles into separate figures using ruler and compass. Label them using classical labelling and primes.
- (d) Use SSS to prove that the two triangles are congruent. Explain why each step is true.
- (e) Write a conclusion.

14. Do the same parallelogram proof as in problem 13 but using SAS.

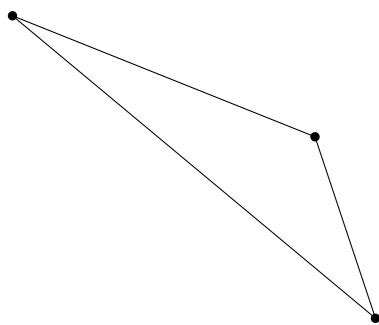
15. Prove the parallelogram area formula using AAS. Follow the steps of problem 13.

16. Prove the parallelogram area formula using ASA. Follow problem 13.

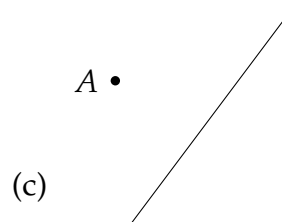
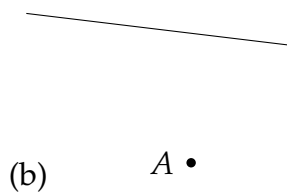
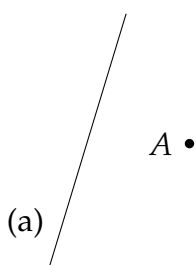
17. Use AAA to make a *smaller* similar copy of this triangle. Do it with ruler and compass. Label your work. Write down the relationships between sides and angles.



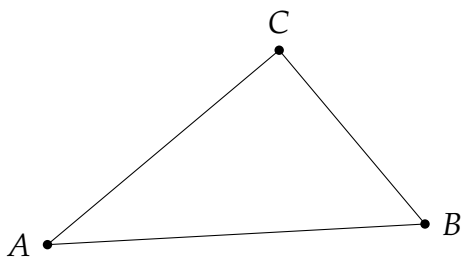
18. Use AAA to make a *larger* similar copy of this triangle. Use ruler and compass. Label your work. Write down relationships between sides and angles.



19. Construct  $90^\circ$  perpendicular lines going through point  $A$ . Use ruler and compass. Don't erase construction lines and arcs.



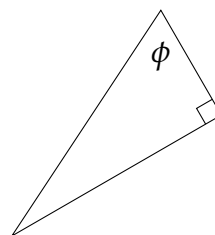
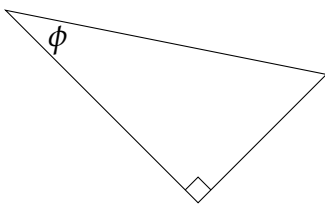
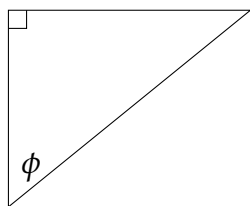
20. Here is a right ( $90^\circ$ ) triangle. We usually label the vertex with the right angle as  $C$  and the longest side as  $c$ . Label the triangle and construct the altitude line  $h$  at  $C$  using ruler and compass.



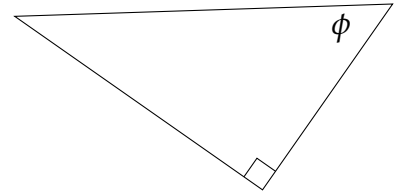
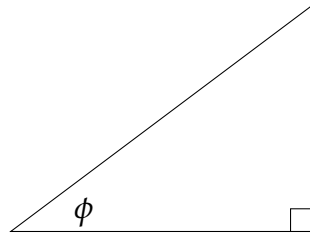
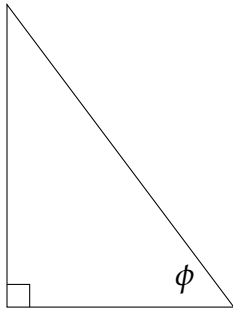
21. Construct a  $90^\circ$  triangle using ruler and compass. Let  $C$  be the  $90^\circ$  vertex. Also construct the altitude line at  $C$ . Label the vertices, angles and sides of your figure. Don't erase your arcs or construction lines.

### 3 $\phi-90^\circ$ triangles

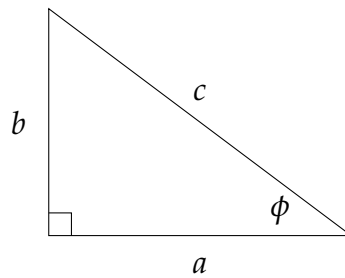
22. Fill in the missing angle.



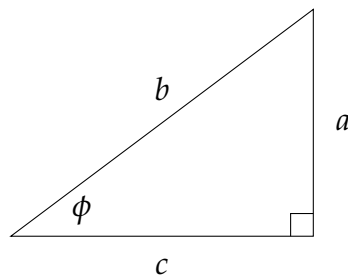
23. Fill in the missing angle and label the sides with proper trigonometric names: hypotenuse, adjacent side, opposite side.



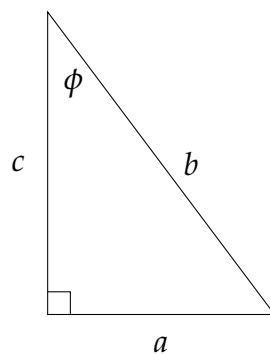
24. Here is a  $\phi-90^\circ$  triangle. What is special about side  $a$ ? Make a list of things.



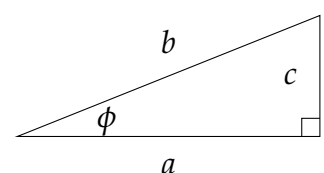
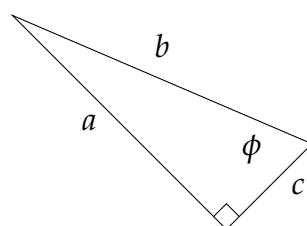
25. Here is a  $\phi-90^\circ$  triangle. What is special about side  $c$ ? Make a list of things.

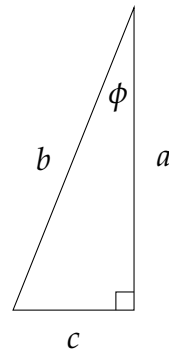
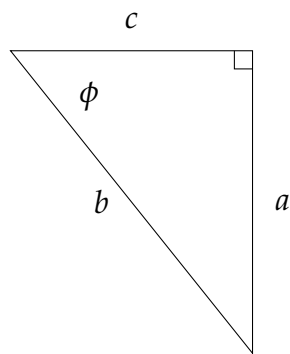


26. Here is a  $\phi-90^\circ$  triangle. What is special about side  $b$ ? Make a list of things.

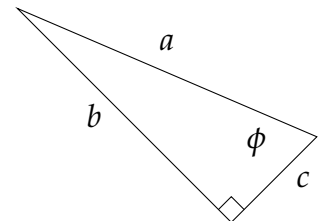
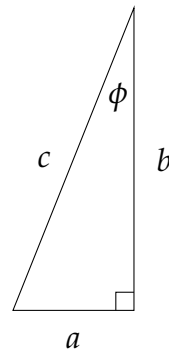
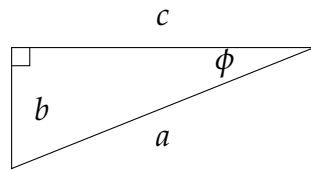
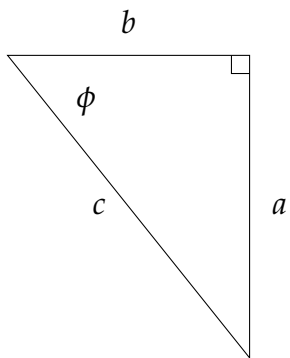


27. Find  $\sin$ ,  $\cos$ ,  $\tan$  in terms of sides  $a$ ,  $b$ ,  $c$ .

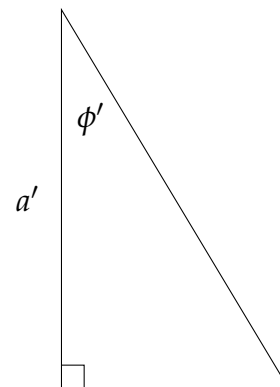
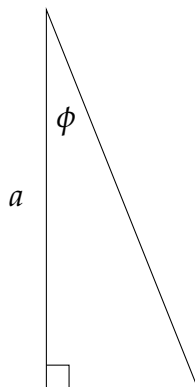




28. Find  $\csc$ ,  $\sec$ ,  $\cot$  in terms of sides  $a$ ,  $b$ ,  $c$ .



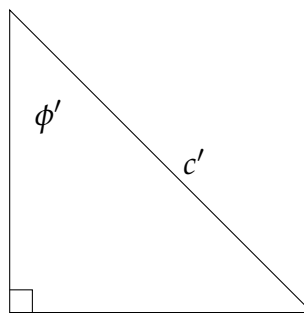
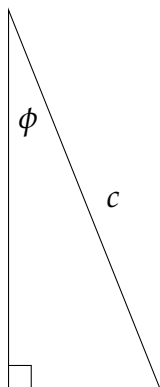
29. Consider the following triangles where  $a = a'$  and  $\phi < \phi'$ .



- (a) Which is bigger,  $\tan \phi$  or  $\tan \phi'$ ?  
 (b) Which is bigger,  $\cot \phi$  or  $\cot \phi'$ ?

Explain why!

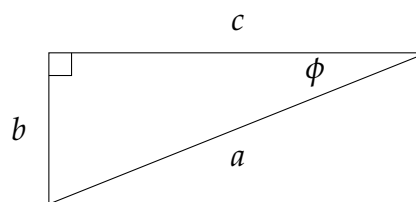
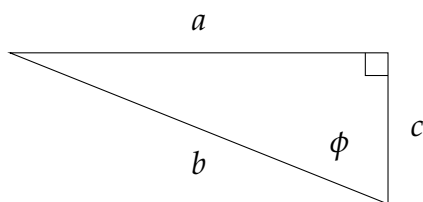
30. Consider the following triangles where  $c = c'$  and  $\phi < \phi'$ .



- (a) Which is bigger,  $\sin \phi$  or  $\sin \phi'$ ?  
 (b) Which is bigger,  $\cos \phi$  or  $\cos \phi'$ ?

Explain why!

31. Label the missing angle, label all sides with proper names and find all trigonometric ratios  $\sin$ ,  $\cos$ ,  $\tan$ ,  $\csc$ ,  $\sec$ ,  $\cot$  in terms of sides  $a$ ,  $b$ ,  $c$ .



32. Why do we have six of these trigonometric ratios,  $\sin$ ,  $\cos$ ,  $\tan$ ,  $\csc$ ,  $\sec$  and  $\cot$ ? Why are they important? What is so special about them?

33. Explain how we were able to calculate the distance to the star 61 Cygni by using trigonometry.

34. Draw a  $\phi$ - $90^\circ$  triangle, label it, and find a relationship between  $\sin$ ,  $\cos$  and  $\tan$ .

35. Draw a  $\phi$ - $90^\circ$  triangle and label it. Use Pythagoras's law to find a relationship between  $\sin$ ,  $\cos$  and 1.

36. Draw a  $\phi$ - $90^\circ$  triangle and label it. Use Pythagoras's law to find a relationship between  $\tan$ ,  $\sec$  and 1.

37. Draw a  $\phi$ - $90^\circ$  triangle and label it. Use Pythagoras's law to find a relationship between  $\cot$ ,  $\csc$  and 1.

## 4 Special angles

38. Change these angles from radians into degrees. Figure it out by drawing little circles.

- |               |                |                 |                |
|---------------|----------------|-----------------|----------------|
| (a) $\pi/2$ . | (c) $7\pi$ .   | (e) $3\pi/4$ .  | (g) $2\pi/3$ . |
| (b) $\pi/6$ . | (d) $5\pi/8$ . | (f) $5\pi/12$ . | (h) $3\pi/2$ . |

39. Change these angles from degrees into radians. Figure it out by drawing little circles.

- |                   |                   |                   |                    |
|-------------------|-------------------|-------------------|--------------------|
| (a) $75^\circ$ .  | (c) $285^\circ$ . | (e) $225^\circ$ . | (g) $195^\circ$ .  |
| (b) $300^\circ$ . | (d) $120^\circ$ . | (f) $720^\circ$ . | (h) $22.5^\circ$ . |

40. Sketch a  $\pi/6$ – $\pi/3$ – $\pi/2$  triangle. Make the hypotenuse 1. Label all the angles and the lengths of the sides.

41. Sketch a  $45^\circ$ – $45^\circ$ – $90^\circ$  triangle. Make the hypotenuse 1. Label all the angles and the lengths of the sides.

42. Check that  $(\sin \phi)^2 + (\cos \phi)^2 = 1$  for  $\phi = 30^\circ$ .

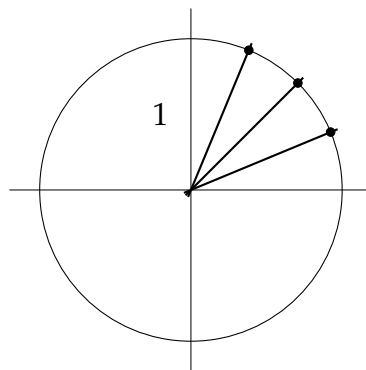
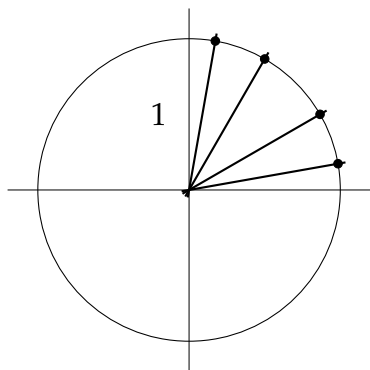
43. Check that  $(\sec \phi)^2 - (\tan \phi)^2 = 1$  for  $\phi = \pi/4$ .

44. Check that  $(\csc \phi)^2 - (\sec \phi)^2 = 1$  for  $\phi = 60^\circ$ .

## 5 The unit circle diagram

45. Construct (with ruler and compass): axes, unit circle, angle  $\phi$ .

46. Sketch the sine and cosine lines and label them.



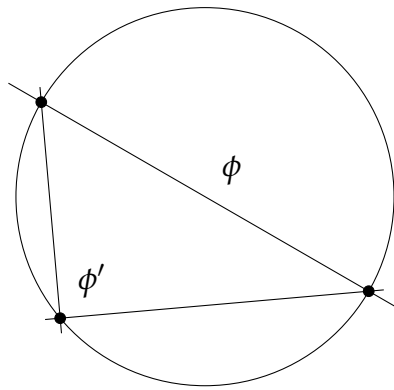
47. Construct with ruler and compass: axes, the unit circle, angle  $\phi$ ,  $\cos \phi$  line and  $\sin \phi$  line.

## 6 Trigonometric identities

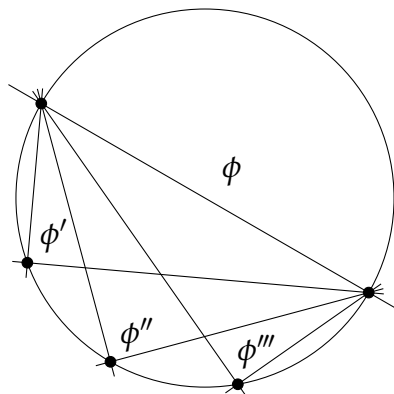
48. What do we mean by a *trig identity*? Give some examples.

49. Given that the line through the circle is a center line. What are the angles  $\phi$  and  $\phi'$ ?

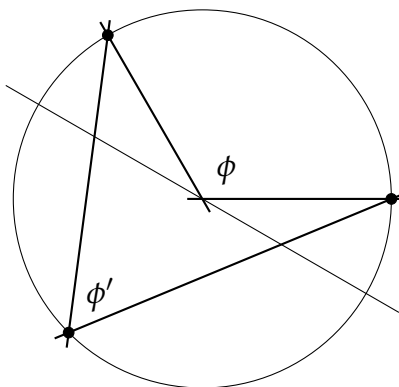




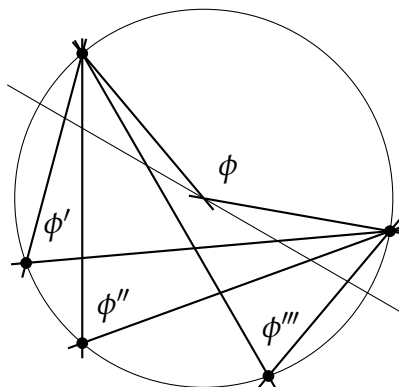
50. What are the angles  $\phi$ ,  $\phi'$ ,  $\phi''$ ,  $\phi'''$ ? The line through the circle is a center line.



51. What is the relationship between angles  $\phi$  and  $\phi'$ ?



52. What are the relationships between angles  $\phi$ ,  $\phi'$ ,  $\phi''$ ,  $\phi'''$ ?



53. Prove the very beautiful circle-angle theorem:

$$\phi' = \frac{\phi}{2}.$$

