Dimensional Analysis Training Problems

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1 Mass, length, time

1. What are the three fundamental dimensions of mechanics and what are the MKS units for them?

2. Make drawings of fluids flowing in pipes and explain the meaning of $\dot{m}$.

3. Use drawings and explain the meaning of $\dot{A}$.

4. Use drawings and interpret the meaning of $\dot{V}$.

5. Explain the meaning of $\dot{\rho}$ and give an example.

6. Pipe 1 can fill a tank in time t_1. Pipe 2 can do it in time t_2. How fast can they do it together?

(a) What is the key idea here ($\dot{x}$, $\dot{A}$, etc.)?
(b) Find a formula for t, the time it takes for them to do it together.
(c) If $t_1 = 8\,\text{hr}$ and $t_2 = 6\,\text{hr}$ what is t?

7. Three pipes can each fill a tank in times t_1, t_2 and t_3. How long does it take for them to fill the tank if they do it together?

8. Jim and Bob are shovelling a huge mound of dirt. Jim can do it in time t_1. Bob can do it in time t_2. What is the key idea here? Find the time it takes for both of them to do it together.

9. Jim and Bob work painting white lines on roads. It takes Jim 32 hours to finish painting the downtown road. It takes Bob only 24 hours to do the same job. If they start at opposite ends and do the work together, how long does it take?

(a) What is the key quantity here?
(b) Find a formula for t, the time it takes for them to do it together.
(c) Put the numbers into your formula and give the answer in hours and minutes.

10. Jim and Bob are painting walls. Jim can paint a wall in 6 hours. Bob can paint the same wall in 4 hours. How fast can they do it together?

(a) What is the key quantity?
(b) Find a formula for the time it takes for them to do it together.
(c) Get a numerical answer.

11. Five guys are working on inflating a large balloon. The times that it takes for each one to fill the balloon on his own are $t_1, t_2, \ldots t_5$. What is the key idea here? How long does it take to fill the balloon if all five work on it together?

12. Fill in this table.

Quantity	Dimensions	MKS units
$\dot{x}$		
$\dot{A}$		
$\dot{V}$		
$\dot{m}$		
$\dot{\rho}$		
Newton's dot, $\dot{}$		
Two Newton dots, $\ddot{}$		

13. Guess a relationship between ρ, $\dot{m}$, v and A by examining the dimensions of these quantities.

14. Use the *abc* method to find a relationship between the quanitites in **13**.

15. Interpret the meaning of the relationship in **13**. Use drawings.

16. Consistent or inconsistent? Use the square bracket notation $[q]$ for dimensions of q.

(a) $\dfrac{v}{\dot{m}} = \dfrac{1}{\rho A}$.

(b) $\dfrac{\dot{m}}{\rho} = \dfrac{v}{A}$.

17. Air flows through a $0.01\,\mathrm{m}^2$ pipe with a velocity of $2\,\mathrm{m/s}$. What is the mass flow rate through the pipe? The density of air is $1.23\,\mathrm{kg/m}^3$.

18. Liquid mercury flows through a pipe with area $0.005\,\mathrm{m}^2$ at $20\,\mathrm{cm/s}$. What is the mass flow rate through the pipe? The density of mercury is $13,500\,\mathrm{kg/m}^3$.

19. A jet plane coveres a distance of 900 km in 30 min. What is the velocity of the plane in MKS units?

20. Consistent or inconsistent?

(a) $m = \dfrac{m_0}{\sqrt{1 - v^2}}$.

(b) $m = \dfrac{m_0}{\sqrt{1 - \dfrac{v^2}{c^2}}}$.

where c is the speed of light.

2 Acceleration, force, energy

21. Fill in this table.

Quantity	Dimensions	MKS units
distance, x		
velocity, $\dot{x}$, v		
acceleration, a, $\dot{v}$, $\ddot{x}$		
force, F		
$\dot{p}$ (momentum dot)		
pressure, p		
Work, W		
Kinetic energy, T		
$\dot{W}$ (work dot)		
Power, P		

22. Use drawings of cars to explain the difference between:
(a) $a = 0$.
(b) $a > 0$.
(c) $a < 0$.

23. A car accelerates from 10 m/s to 20 m/s in 5 seconds. What is the acceleration of the car?

24. If the car in **23** has a mass of 500 kg, what was the force on the car that caused the acceleration?

25. What is the gravitational force on an 80 kg man standing on the surface of the Earth?

26. Find a relationship between distance x, acceleration a and time t. Take the dimensionless factor (F) to be $1/2$.

27. Consistent or inconsistent? Use square bracket notation $[q]$.

(a) $x = \frac{1}{2}a^2t$.

(b) $x = \frac{1}{2}at^2$.

28. Consistent or inconsistent?

(a) $Fx = \frac{1}{2}mv^2$.

(b) $Fv = \frac{1}{2}ma^2$.

29. The atmospheric pressure of air at the Earth's surface is about 101,000 Pa.
(a) What is the gravitational force exerted on a 1 m^2 area of floor by all the air sitting on top of it?
(b) What is the mass of the air sitting on that 1 m^2 of floor?

30. Suppose you have a tank of liquid. What is the pressure at a depth h down into the liquid? This should depend on density, the acceleration of gravity, and of course h.
(a) Assume that there is no atmosphere pushing down on the liquid. Use the *abc* method to find a relationship between pressure, density, g and h. Take the dimensionless factor (F) to be 1.
(b) Change your relationship so that it takes into consideration atmospheric pressure p_0 pushing down onto the liquid.

31. Consistent or inconsistent? Do them carefully using the square bracket $[q]$ notation.
(a) $p = p_0 + \rho gh$.
(b) $\rho = \rho_0 + pgh$.
where p is pressure.

32. Consistent or inconsistent?

$$p = \frac{1}{2}\rho v^2 + \rho gh$$

where g is the acceleration of gravity.

33. A tank of liquid mercury is sitting with the top open to the atmosphere. Atmospheric pressure is 101,000 Pa. Use Pascal's law to determine the pressure 150 cm below the surface. The density of liquid mercury ρ_{Hg} is 13,500 kg/m^3.

34. What are the dimensions of the spring constant k? Figure it out from the formula for spring force. What are the SI-MKS units of k?

35. A man pushes 25 m against a force of 500 N. How much work did he do?

36. Consistent or inconsistent?

(a) $E = \frac{1}{2}k^2x$.

(b) $E = \frac{1}{2}kx^2$.

Where E is the energy in the spring, k is the spring constant, and x is how much you compress the spring.

37. A spring has a constant of $k = 1{,}000{,}000$ N/m. How much energy is in the spring if I compress it by 5 cm? Remember to change cm into MKS units.

38. A 10 kg frozen turkey is shot from a catapult at 15 m/s. What is the momentum of the turkey?

39. Use the definition of momentum, $p = mv$ and the formula for kinetic energy to find kinetic energy in terms of m and p only, no velocity v.

40. Use the formula you found in **39** to find the kinetic energy of the turkey in **38**.

41. Show how you can get the usual version of Newton's law, $F = ma$, from the more powerful version, $F = \dot{p}$.

42. Show that if there are no forces on a mechanical system, then the momentum does not change. Give a simple argument that explains why this is true.

43. Use your imagination and determine a relationship between time, work and power. Think about machines doing work and draw some cartoons. That will help you figure it out.

44. The kilowatt-hour is a unit used by electric companies. It's a kilowatt times an hour. What kind of unit is this? What are the dimensions? What is $1\,\text{kW}\cdot\text{hr}$ in MKS units?

45. A horsepower is a unit of power often used to describe engines and big machines. One horsepower is about 745.7 Watts. An Alfa-Romeo Quadrifoglio has a 500 hp engine. What is that in MKS units?

46. A machine can do $120,000$ Joules of work in 1 minute. What is the power of this machine?

47. How much energy does a 60 W lightbulb use if you leave it on for 12 hours?

3 Electrical dimensions

48. What is current? Draw a picture comparing electrical current to pipe flow.

49. Explain the idea of voltage by making drawings comparing water going through a pipe uphill and current going through a resistor.

50. 20 C of charge flows though a wire in 4 seconds. What is the current?

51. Show how you can discover Ohm's law for yourself by drawing some pictures and thinking about how current flows through resistances.

52. Use Ohm's law to find the dimensions and SI-MKS units of resistance.

53. Fill in this table.

Quantity	Dimensions	MKS units
Charge, q		
Current, I, $\dot{q}$		
Voltage, V		
Resistance, R		

54. A current of $4,$ A flows through a $250\,\Omega$ resistor. What is the voltage across the resistor?

55. Apply a voltage of 100 V across a resistance of $25\,\Omega$. What current do you get? Try to do this calculation only with fundamental units kg, m, s, C.

56. Use the *abc* method to find a relationship between power P and resistance R and current I. Take the dimensionless factor (F) to be simply 1.

57. Interpret the meaning of the formula in **56** by thinking about your toaster.

58. Consistent or not?

(a) $P = R^2 I$.
(b) $P = RI^2$.

59. 5 Amperes of current flows through a 250 Ω resistor. How much power is lost as heat?

60. How much energy is lost in **59** if the current is left on for 24 hours?

61. Coulomb discovered a way to calculate the electric force between two charges. Let's say two charges, q_1 and q_2 are distance r apart. The electric force is

$$F = k\frac{q_1 q_2}{r^2}$$

where k is Coulomb's constant. Find the dimensions of k. Find the SI-MKS units of k.

62. One electron-volt is the charge on one electron multiplied by one volt. It is used a lot in high-energy physics. Can you explain what *one electron-volt* means? What is it supposed to measure? What are the dimensions of electron-volt? Can you change electron-volt into MKS units?