

M2 Training Problems

Ted Szylowiec

tedszy@gmail.com

1 Functions, identity, inverses and plots

1. Let $f(x) = 2x + 1$. Find...

- (a) Find $f(f(x))$.
- (b) Find $f(f(f(x)))$.
- (c) Find $f(f(f(f(x))))$.

2. Let $f(x) = 3x^2 + 1$ and $g(x) = 2x - 3$.

- (a) Find $f(g(x))$.
- (b) Find $g(f(x))$.

Are they the same?

3. Let $f(x) = ax + b$.

- (a) Find $f(f(x))$.
- (b) Find $f(f(f(x)))$.

4. Let $f(x) = ax + b$ and $g(x) = cx + d$.

- (a) Find $f(g(x))$.
- (b) Find $g(f(x))$.

Are they the same?

5. Sketch $y = x$ and $y = -x$. Put them on the same axes. Label everything.

6. Sketch $y = 2x$ and $y = -2x$. Put them on the same axes.

7. Sketch these lines on the same axes.

$$y = \frac{x}{2}, \quad y = -\frac{x}{2}.$$

8. Make an exact plot of $y = 3x + 2$ by finding the x -intercept and y -intercept.

9. Make an exact plot of

$$y = -\frac{x}{2} - 1.$$

10. If $f(x)$ and $g(x)$ are linear, show that

- (a) $f(g(x))$ is linear.
- (b) $g(f(x))$ is linear.

11. Let $f(x) = 3x + 2$. Find $f^{-1}(x)$. Do it two ways:

- (a) By $f(f^{-1}(x)) = I(x)$.
- (b) And by $f^{-1}(f(x)) = I(x)$.

12. Let $f(x) = ax + b$. Find $f^{-1}(x)$. Do it two ways:

- (a) By $f(f^{-1}(x)) = I(x)$.
- (b) And by $f^{-1}(f(x)) = I(x)$.

13. Let $f(x) = 2x + 1$. Find $f^{-1}(x)$. Make exact plots of f and f^{-1} . Also draw I .

14. Let $f(x) = -2x + 3$. Find $f^{-1}(x)$. Make exact plots of f and f^{-1} . Also draw I .

15. Consider the function

$$f(x) = -\frac{x}{2} + 3.$$

Find $f^{-1}(x)$. Make exact plots of f and f^{-1} . Also draw I .

16. Sketch the curve $y = x^2$. Use the unit square idea.

17. Let $f(x) = x^2$. Sketch f , I and f^{-1} on the same axes.

18. Let $f(x) = x^2 + 1$. Sketch f , I and f^{-1} on the same axes.

2 Logarithms