

Arithmetic and Combinatorics Part 2

Training problems for M1 2018 term 2

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1. This is the menu at the Italian restaurant.

<i>Main dishes</i>	<i>Side dishes</i>	<i>Desserts</i>
Fetuccine alfredo	Porcini mushroom bruschetta	Hazelnut tartufo
Pepperoni pizza	Grilled polenta	Cannoli
Three-cheese lasagna	Spinach ricotta gnocci	White chocolate panna cotta
	Radicchio with lemon	Sfogliatelle
	Stuffed artichokes	

I want to get one main dish, one side dish, *and* one dessert. How many ways can I do that? Write the definitions for what you are doing. Be clear. Write the principle. Do the computation.

2. At the Italian restaurant in problem 1, I want to get either one main dish *or* one side dish *or* one dessert. How many ways can I do that? Show definitions, principle, calculation. Do a proper job, don't just write the answer.

3. At the Italian restaurant in 1, I want to get either only a main dish or a side dish and a dessert. How many ways can I do that? Show definitions, principle, calculation.

4. At the Italian restaurant in 1, I want to get either both a main dish and a dessert or both a side dish and a dessert. How many ways can I do that? Definitions. Principle. Calculation.

5. The Boring Book Library has nothing but the most boring books on the dullest topics.

<i>Topic</i>	<i>Number of books</i>
Dishwashing	7
K-Pop	6
Mops and brooms	5
British top hats	4

6. We have 7 girls and 5 boys. How many ways can we make a team by choosing 3 girls and 3 boys? Definitions, principle, calculation.

7. We have 7 girls and 5 boys. We want a small team of only 3 people: either all girls or all boys. How many ways can I do this? Definitions, principle, calculation.

8. How many 3-digit numbers are there? Show definitions, principle, calculation.

9. How many 3-digit numbers can you make using only *even* digits? Show your definitions, combinatorics principle, and your calculations.
10. How many 3-digit numbers can you make using only *odd* digits? Definitions. Principle. Calculation. Be clear. Explain what you are doing. Don't just write an answer.
11. What is a permutation? Explain it.
12. Write down all different permutations of the letters *EFG*. How many are there?
13. Write down all the different permutations of the digits 1234. How many are there.
14. Prove that the number of different ways to arrange k objects in order is $k!$.
15. I have 6 books and I want to arrange them in order on a bookshelf. How many different ways can I do it?
16. I have 6 books and I want to choose 3 to arrange on my bookshelf in order. How many ways can I do this?
17. I have 8 students. I want to choose 3 of them and give them prizes: 1st, 2nd and 3rd place. Does order matter? How many ways can I do this? Do it in two steps and show definitions, principle, calculation.
18. We have n students. We want to choose k of them and arrange them in order. How many ways can we do this? Show your definitions, what principle you use, and the calculation of the final answer.
19. We have 5 girls and 5 boys in our class. I want to choose 2 girls and one boy and give them prizes: 1st place, 2nd place, 3rd place. Does order matter? How many ways can I do this? Definitions (be clear). Principle (what combinatorics principle are you using?) Calculation (get the final answer).
20. We have 5 girls and 5 boys. I want to choose 3 girls and 3 boys to make a team of six. Then I want to choose three in the team to be president, secretary and messenger. Is order important? How many ways can I do this? Show definitions, principle, calculation.
21. We have 6 girls and 5 boys. I want to give three prizes to the girls (1st, 2nd, 3rd) *and* three prizes to the boys (1st, 2nd, 3rd). How many ways can I do this? Definitions. Principle. Calculation.
22. We have 6 consonants *mnpqr*, 5 vowels *aeiou* and 5 digits 12345. I want to make passwords by choosing two of each, a total of 6 symbols. Of course order matters when you make passwords. For example: *2aqur5* and *5ruqa2* are two different passwords. How many such passwords can I make? Definitions. Principle. Calculation.
23. (A) I have n objects. I select k of them in some special order. This one first, then that one, then another one, and so on.
 (B) I have n objects. I select k of them without order, but then I arrange them in some special order later. I put one first, then another one second, and so on.
 Is there a difference between (A) and (B)? Think about it.