Dimensional Analysis Training Problems

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1. What are the three fundamental dimensions of mechanics and what are the MKS units for them?

2. Make drawings of fluids flowing in pipes and explain the meaning of $\dot{m}$.

3. Use drawings and explain the meaning of $\dot{A}$.

4. Use drawings and interpret the meaning of $\dot{V}$.

5. Explain the meaning of $\dot{\rho}$ and give an example.

6. Pipe 1 can fill a tank in time t_1. Pipe 2 can do it in time t_2. How fast can they do it together?

(a) What is the key idea here ($\dot{x}$, $\dot{A}$, etc.)?
(b) Find a formula for t, the time it takes for them to do it together.
(c) If $t_1 = 8\,\text{hr}$ and $t_2 = 6\,\text{hr}$ what is t?

7. Three pipes can each fill a tank in times t_1, t_2 and t_3. How long does it take for them to fill the tank if they do it together?

8. Jim and Bob are shovelling a huge mound of dirt. Jim can do it in time t_1. Bob can do it in time t_2. What is the key idea here? Find the time it takes for both of them to do it together.

9. Jim and Bob work painting white lines on roads. It takes Jim 32 hours to finish painting the downtown road. It takes Bob only 24 hours to do the same job. If they start at opposite ends and do the work together, how long does it take?

(a) What is the key quantity here?
(b) Find a formula for t, the time it takes for them to do it together.
(c) Put the numbers into your formula and give the answer in hours and minutes.

10. Jim and Bob are painting walls. Jim can paint a wall in 6 hours. Bob can paint the same wall in 4 hours. How fast can they do it together?

(a) What is the key quantity?
(b) Find a formula for the time it takes for them to do it together.
(c) Get a numerical answer.

11. Five guys are working on inflating a large balloon. The times that it takes for each one to fill the balloon on his own are $t_1, t_2, \ldots t_5$. What is the key idea here? How long does it take to fill the balloon if all five work on it together?

12. Fill in this table.

Quantity	Dimensions	MKS units
$\dot{x}$		
$\dot{A}$		
$\dot{V}$		
$\dot{m}$		
$\dot{\rho}$		
$\dot{p}$ (momentum dot)		
$\dot{W}$ (work dot)		
Newton's dot, $\cdot$		

13. Guess a relationship between ρ, $\dot{m}$, v and A by examining the dimensions of these quantities.

14. Use the *abc* method to find a relationship between the quanitites in **13**.

15. Interpret the meaning of the relationship in **13**. Use drawings.

16. Consistent or inconsistent? Use the square bracket notation $[q]$ for the dimensions of q.

(a) $\dfrac{v}{\dot{m}} = \dfrac{1}{\rho A}$.

(b) $\dfrac{\dot{m}}{\rho} = \dfrac{v}{A}$.

17. Air flows through a $0.01\,\mathrm{m}^2$ pipe with a velocity of $2\,\mathrm{m/s}$. What is the mass flow rate through the pipe? The density of air is $1.23\,\mathrm{kg/m}^3$.

18. Liquid mercury flows through a pipe with area $0.005\,\mathrm{m}^2$ at $20\,\mathrm{cm/s}$. What is the mass flow rate through the pipe? The density of mercury is $13,500\,\mathrm{kg/m}^3$.

19. Fill in this table.

Quantity	Dimensions	MKS units
distance, x		
velocity, $\dot{x}$, v		
acceleration, a, $\dot{v}$, $\ddot{x}$		
force, F		

20. A jet plane coveres a distance of 900 km in 30 min. What is the velocity of the plane in MKS units?

21. Use drawings of cars to explain the difference between:
(a) $a = 0$.
(b) $a > 0$.
(c) $a < 0$.

22. A car accelerates from 10 m/s to 20 m/s in 5 seconds. What is the acceleration of the car?

23. If the car in **22** has a mass of 500 kg, what was the force on the car that caused the acceleration?

24. What is the gravitational force on an 80 kg man standing on the surface of the Earth?

25. Consistent or inconsistent? Use square bracket notation $[q]$.

(a) $x = \frac{1}{2}a^2t$.

(b) $x = \frac{1}{2}at^2$.

26. Consistent or inconsistent?

(a) $Fx = \frac{1}{2}mv^2$.

(b) $Fv = \frac{1}{2}ma^2$.

27. Consistent or inconsistent? Do them carefully using the square bracket $[q]$ notation.
(a) $p = p_0 + \rho gh$.
(b) $\rho = \rho_0 + pgh$.
where p is pressure.

28. Consistent or inconsistent?

(a) $m = \dfrac{m_0}{\sqrt{1 - v^2}}$.

(b) $m = \dfrac{m_0}{\sqrt{1 - \dfrac{v^2}{c^2}}}$.

where c is the speed of light.

29. A tank of liquid mercury is sitting with the top open to the atmosphere. Atmospheric pressure is 101,000 Pa. Use Pascal's law to determine the pressure 150 cm below the surface. The density of liquid mercury ρ_{Hg} is 13,500 kg/m^3.

30. Use your imagination and determine a relationship between time, work and power. Think about machines doing work and draw some cartoons. That will help you figure it out.

31. The kilowatt-hour is a unit used by electric companies. It's a kilowatt times an hour. What kind of unit is this? What are the dimensions? What is 1 kW $\cdot$ hr in MKS units?

32. A horsepower is a unit of power often used to describe engines and big machines. One horsepower is about 745.7 Watts. An Alfa-Romeo Quadrifoglio has a 500 hp engine. What is that in MKS units?

33. A machine can do 120,000 Joules of work in 1 minute. What is the power of this machine?

34. How much energy does a 60 W lightbulb use if you leave it on for 12 hours?