Arithmetic and Combinatorics

Training problems for M1 2018 term 1

Ted Szylowiec
tedszy@gmail.com

1 Calculating prodigies

1. *"He who refuses to do arithmetic is doomed to talk nonsense."* Which famous computer scientist said that? You can use the internet to find out.

2. Which famous American calculating prodigy became an astronomer when he grew up?

3. This prodigy was from Germany. He could multiply 100-digit numbers in his head. Who was he?

4. Write down a few interesting things about the life of Jedediah Buxton. Where was he from? When was he born? How did he die? What did he do?

5. What do the amazing powers of calculating prodigies prove about the human mind? Tell me some opinions.

2 Euclidean division

6. Who was Euclid and where was he from? When did he live?

7. Use the terms *dividend*, *divisor*, *remainder* and *quotient* to label the parts of the expression

$$29 = 4 \times 6 + 5.$$

Choose the divisor carefully. Is it 4 or is it 5? Check that $0 \leqslant r < d$. Is it true? Is this a correct expression of Euclidean division?

8. Check that $0 \leqslant r < d$. Is it true?

$$111 = 9 \times 11 + 12$$

Is this a correct expression of Euclidean division?

9. Label the parts of these expressions with the terms *dividend*, *divisor*, *remainder* and *quotient*.

(a) $101/39$.
(b) $m = q \times d + r,\ 0 \leqslant r < d$.
(c) $59 = 5 \times 11 + 4$.
(d) $a = b \times c + d,\ 0 \leqslant d < c,\ d > b$.
(e) $r \div s$.
(f) $39/101$.

10. Which number is the divisor and which one is the quotient?

(a) $42 = 11 \times 3 + 9$.
(b) $99 = 7 \times 13 + 8$.
(c) $69 = 10 \times 6 + 9$.
(d) $23 = 4 \times 5 + 3$.

11. Do by Euclidean division. Label all the parts of your expressions. Give a clear answer in terms of two numbers.

(a) $99/91$.
(b) $919/7$.
(c) $1001/651$.
(d) $1/19$.
(e) $19/1$.
(f) $17/17$.
(g) $0/15$.
(h) $15/0$.

12. These matrices are filled with consecutive integers $0, 1, 2, \ldots$ Figure out what goes into the boxes.

(a) 7×7 matrix.

$$\begin{bmatrix} \Box & \cdots & \Box \\ \vdots & & \vdots \\ \Box & \cdots & \Box \end{bmatrix}$$

(b) 10×10 matrix.

$$\begin{bmatrix} \Box & \cdots & \Box \\ \vdots & & \vdots \\ \Box & \cdots & \Box \end{bmatrix}$$

(c) $n \times n$ matrix.

$$\begin{bmatrix} \Box & \cdots & \Box \\ \vdots & & \vdots \\ \Box & \cdots & \Box \end{bmatrix}$$

13. Each matrix is $n \times n$ and is filled with consecutive integers $0, 1, 2, \ldots$ Figure out what goes into the boxes.

(a) $$\begin{bmatrix} \Box & \cdots & \Box \\ \Box & \cdots & \Box \\ \vdots & & \vdots \\ \Box & \cdots & \Box \\ \Box & \cdots & \Box \end{bmatrix}$$

(b) $$\begin{bmatrix} \Box & \Box & \cdots & \Box & \Box \\ \vdots & & & & \vdots \\ \Box & \Box & \cdots & \Box & \Box \end{bmatrix}$$

14. For a 6×6 matrix filled with consecutive integers $0, 1, 2, \ldots$, find out what goes into the boxes.

(a) $$\begin{bmatrix} \Box & & & & \\ & \Box & & & \\ & & \ddots & & \\ & & & \Box & \\ & & & & \Box \end{bmatrix}$$

(b) $$\begin{bmatrix} & & & & \Box \\ & & & \Box & \\ & & \iddots & & \\ & \Box & & & \\ \Box & & & & \end{bmatrix}$$

15. Now, these are $n \times n$ square matrices filled with consecutive integers $0, 1, 2, \ldots$ Figure out what goes into the boxes.

(a) $$\begin{bmatrix} \Box & & & & \\ & \Box & & & \\ & & \ddots & & \\ & & & \Box & \\ & & & & \Box \end{bmatrix}$$

(b) $$\begin{bmatrix} & & & & \Box \\ & & & \Box & \\ & & \iddots & & \\ & \Box & & & \\ \Box & & & & \end{bmatrix}$$

16. Fill in the boxes for a 100×100 matrix of consecutive integers $0, 1, 2, \ldots$

$$\begin{bmatrix} \square & \square & \cdots & \cdots & \square & \square \\ \square & \square & & & \square & \square \\ \vdots & & \ddots & \iddots & & \vdots \\ \vdots & & \iddots & \ddots & & \vdots \\ \square & \square & & & \square & \square \\ \square & \square & \cdots & \cdots & \square & \square \end{bmatrix}$$

17. Consider a 30×30 matrix filled with consecutive integers starting at 0. Where is 611? Give the row and column. Use Euclidean division.

18. Consider a 99×99 matrix of consecutive integers starting at zero. Find the row and column position of 3333.

19. Given an 80×80 matrix of consective numbers beginning at 0, find the number is at the given position:

(a) row 25, column 68. (b) row 68, column 25.

20. In a 5000×5000 matrix of consective integers $0, 1, 2, \ldots,$ what number is at row 991, column 599?

3 Numbers in base-d

21. What is an algorithm? Explain it in your own words. Give some examples.

22. Make base-10 Euclidean division tables for these numbers.

(a) 3351. (b) 4096. (c) 12801.

23. Change these numbers into base-2 by making $d = 2$ Euclidean division tables.

(a) 331. (b) 409. (c) 1280.

24. Make Euclidean division tables with dividend $m = 3721$, using these divisors:

(a) $d = 10$. (b) $d = 4$. (c) $d = 2$.

25. Use Euclidean division tables to change 757 into these bases:

(a) base-5. (b) base-3. (c) base-7.

26. There are two ways we can handle digits bigger than 9. One way is to use alphabetical symbols like A, B, etc. Another way is to use Taylor digits like (10), (11). Sometimes it's better to use alphabetical symbols. Other times it is better to use Taylor digits. Write down some reasons why.

27. How many different digits are there in base-d? Write them down.

28. Write down all the digits of base-20. Use Taylor's idea for digits bigger than 9.

29. How many different digits are there in base-16? Write them down...

(a) using capital letters. (b) using Taylor digits.

30. Change these base-16 numbers from alphabetical symbols to Taylor digits.

(a) FE199A6. (b) 123ABCD12. (c) D00E00F0C. (d) 99FF11BB0A.

31. Change these base-16 Taylor digit numbers into numbers using alphabetical symbols.

(a) (15)(14)(13)0. (b) 53(10)10(10)5. (c) (11)111(11)(11). (d) (14)(12)4(15).

32. Count the digits.

(a) 3DCF918B. (b) 348(11)1(13). (c) (123)(456)789. (d) FFA000A1. (e) (89)(71)2(14)(11). (f) (123321). (g) 785(21)871. (h) 11(11)1(111).

33. Change 39101 into base-20. Use Taylor digits.

34. Change 569112 into base-100. Use Taylor digits.

35. Change 569112 into base-1000. Use Taylor digits.

36. Now that you see the idea for base-100, 1000, etc, it is very easy to fill in this table without doing any calculations at all...

base-10	891283481785
base-100	
base-1000	
base-10000	
base-1000000	

37. This is Pascal's triangle. You can make it as big as you like.

$$\begin{array}{ccccccccccc} & & & & & 1 & & & & & \\ & & & & 1 & & 1 & & & & \\ & & & 1 & & 2 & & 1 & & & \\ & & 1 & & 3 & & 3 & & 1 & & \\ & 1 & & 4 & & 6 & & 4 & & 1 & \\ 1 & & 5 & & 10 & & 10 & & 5 & & 1 \end{array}$$

Change these numbers into base-9 by using Euclidean division tables. Think about the patterns you see in the digits.

(a) 1. (b) 10. (c) 100. (d) 1000.

38. Change 10000 into base-9. Use Pascal's triangle to guess the answer. Check your answer by Euclidean division table. Is your guess right?

39. Change these numbers into base-99 by Euclidean division tables. Think about the patterns you see.

(a) 1. (b) 100. (c) 10000.

40. Change these numbers into base-99 by using Pascal's triangle and Taylor digits.

(a) 1000000. (b) 100000000. (c) 10000000000.

41. Change these numbers into base-999 by Euclidean division. Think about the patterns.

(a) 1. (b) 1000. (c) 1000000.

42. Change these numbers into base-999 by Pascal's triangle and Taylor digits.

(a) 10^9. (b) 10^{12}. (c) 10^{15}. (d) 10^{24}.

43. Change 10^{24} into base-9999 by using Pascal's triangle and Taylor digits.

44. Change 10^{48} into base-999999 using Pascal's triangle and Taylor digits.

4 Computer science arithmetic

45. Why do we use base-2, 4, 8, 16, etc. in the design of computers? Give two reasons.

46. Make a counting table from 0 to 15 in base-2.

base-10	base-2
0	
1	
2	
3	
4	
5	
6	
7	

base-10	base-2
8	
9	
10	
11	
12	
13	
14	
15	

47. Make a counting table from 0 to 15 in base-4.

base-10	base-4
0	
1	
2	
3	
4	
5	
6	
7	

base-10	base-4
8	
9	
10	
11	
12	
13	
14	
15	

48. Fill in this table. One digit in base-4, 8, 16, etc., is equivalent to how many base-2 digits?

kind of digit	equivalent digits in base-2
one base-4 digit	
one base-8 digit	
one base-16 digit	
one base-32 digit	
one base-64 digit	
one base-256 digit	

49. Fill in this table. One digit in base-16, 64, etc., is equivalent to how many base-4 digits?

kind of digit	equivalent digits in base-4
one base-16 digit	
one base-64 digit	
one base-256 digit	

50. Fill in this table. One digit in base-64 or base-512, etc., is equivalent to how many base-8 digits?

kind of digit	equivalent digits in base-8
one base-64 digit	
one base-512 digit	