# SME M1 Training Problems

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Problems that may be harder than usual (or more advanced) are marked with a bullet •. Problems that are possibly even harder (and even more advanced) are marked with two bullets ••. All problems are worth doing! Try to do all of them. Most don't take very long. Some of the problems include answers.

**1.** Solve $(8x - 3) - (7x - 2) = 1$.

*Answer:* $x = 0$.

**2.** Solve $(5x - 4) - (4x - 3) - (3x - 2) = 2x - 1$.

*Answer:* $x = 1/2$.

**3.** Solve for $x$:

$$(1 - 2x) - (2 - 3x) - (3 - 4x) - (4 - 5x) - (5 - 6x) = 10.$$

*Answer:* $x = 23/16$.

**4.** Prove this two-way formula

$$\frac{1}{(n-2)(n-1)} = \frac{1}{n-2} - \frac{1}{n-1}.$$

by taking the right-hand side, doing some algebra, and showing that it is equal to the left-hand side.

**5.** Use a two-way formula similar to the one in problem **4**. Solve for $x$.

$$\frac{x}{10 \cdot 11} + \frac{x}{11 \cdot 12} + \frac{x}{12 \cdot 13} + \frac{x}{13 \cdot 14} + \frac{x}{14 \cdot 15} = 1.$$

*Answer:* $x = 30$.

**6.** Factor the denominators into two parts and use a two-way formula like in problem **4**. Solve for $x$.

$$\frac{x}{90} + \frac{x}{110} + \frac{x}{132} + \frac{x}{156} = \frac{1}{13}.$$

*Answer:* $x = 9/4$.

**7.** Use a two-way fraction formula like in problem **4** and find the sum.

$$\frac{1}{10 \cdot 11} + \frac{1}{11 \cdot 12} + \cdots + \frac{1}{100 \cdot 101}$$

*Answer:* 91/1010.

**8.** Prove that

$$\frac{1}{(n+1)(n+2)(n+3)} = \frac{1}{2}\left(\frac{1}{(n+1)(n+2)} - \frac{1}{(n+2)(n+3)}\right).$$

**9.** Use a three-way formula and solve for $x$:

$$\frac{x}{10\cdot 11\cdot 12}+\frac{x}{11\cdot 12\cdot 13}+\frac{x}{12\cdot 13\cdot 14}+\frac{x}{13\cdot 14\cdot 15}=1.$$

*Answer:* $x = 462$.

**10.** Factor the denominators into three-way products. Use a three-way formula to solve for $x$.

$$\frac{x}{120}+\frac{x}{210}+\frac{x}{504}+\frac{x}{720}=83.$$

*Answer:* $x = 5040$.

**11.** Use a three-way formula to find the sum. You may need to use a calculator.

$$\frac{1}{10\cdot 11\cdot 12}+\frac{1}{11\cdot 12\cdot 13}+\cdots+\frac{1}{99\cdot 100\cdot 101}.$$

*Answer:* 999/222200.

**12.** Prove that

$$\frac{1}{(n-3)(n-2)(n-1)n}=\frac{1}{3}\left(\frac{1}{(n-3)(n-2)(n-1)}-\frac{1}{(n-2)(n-1)n}\right).$$

**13.** Use a four-way formula to solve for $x$.

$$\frac{x}{2\cdot 3\cdot 4\cdot 5}+\frac{x}{3\cdot 4\cdot 5\cdot 6}+\frac{x}{4\cdot 5\cdot 6\cdot 7}+\frac{x}{5\cdot 6\cdot 7\cdot 8}=13.$$

*Answer:* $x = 1008$.

**14.** Factor the denominators into four-way products and use a four-way formula to solve for x.

$$\frac{x}{5040}+\frac{x}{7920}+\frac{x}{11880}=17.$$

*Answer:* $x = 41580$.

**15.** Use a four-way formula to find the sum. You may need to use a calculator.

$$\frac{1}{10\cdot 11\cdot 12\cdot 13}+\frac{1}{11\cdot 12\cdot 13\cdot 14}+\cdots+\frac{1}{98\cdot 99\cdot 100\cdot 101}.$$

*Answer:* 1513/5999400.

**16.** • Study the patterns in these two, three and four-way formulas:

$$\frac{1}{n(n+1)}=\frac{1}{n}-\frac{1}{n+1}$$

$$\frac{1}{n(n+1)(n+2)}=\frac{1}{2}\left(\frac{1}{n(n+1)}-\frac{1}{(n+1)(n+2)}\right)$$

$$\frac{1}{n(n+1)(n+2)(n+3)}=\frac{1}{3}\left(\frac{1}{n(n+1)(n+2)}-\frac{1}{(n+1)(n+2)(n+3)}\right).$$

Now take a guess for a five-way formula beginning like this:

$$\frac{1}{n(n+1)(n+2)(n+3)(n+4)}=\cdots$$

**17.** • Prove your guess for the five-way formula in problem **16**.

**18.** •• Take a guess at a $k$-way formula. It begins like this on the left-hand side:

$$\frac{1}{n(n+1)(n+2)\cdots(n+k-1)} = \cdots.$$

**19.** •• Prove the $k$-way formula that you guessed in problem **18**. You can use the three dots (ellipsis) notation $\cdots$ wherever you need it.

**20.** • Prove these. Use two, three and four-way formulas to change the left-hand side into the right-hand side.

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{(n-1)n}=1-\frac{1}{n}$$

$$\frac{1}{1\cdot 2\cdot 3}+\frac{1}{2\cdot 3\cdot 4}+\cdots+\frac{1}{(n-2)(n-1)n}=\frac{1}{2}\left(\frac{1}{2}-\frac{1}{(n-1)n}\right)$$

$$\frac{1}{1\cdot 2\cdot 3\cdot 4\cdot}+\frac{1}{2\cdot 3\cdot 4\cdot 5}+\cdots+\frac{1}{(n-3)(n-2)(n-1)n}=\frac{1}{3}\left(\frac{1}{6}-\frac{1}{(n-2)(n-1)n}\right).$$

**21.** • Study the patterns in problem **20**. Guess the next one. It will be a sum of five-way terms.

**22.** • Prove answer to problem **21** by using the formula that you guessed in problem **16**.

**23.** •• Examine the patterns in the sums of problems **20** and **21** and take a guess for the $k$-way sum.

**24.** • • • Use your result from problem **18** to prove your answer to problem **23**.

**25.** Figure them out...
(a) $\{\varnothing\} \cap \varnothing$.
(b) $\{\varnothing\} \cap \{\varnothing\}$.
(c) $\{\varnothing\} \cup \varnothing$.
(d) $\{\varnothing\} \cup \{\varnothing\}$.

**26.** Figure these out...
(a) $\{1, \varnothing\} \cap \varnothing$.
(b) $\{1, \varnothing\} \cap \{\varnothing\}$.
(c) $\{1, \varnothing\} \cup \varnothing$.
(d) $\{1, \varnothing\} \cup \{\varnothing\}$.

**27.** Let $A = \{2, 5, 9, 11, 13\}$ and $B = \{1, 9, 11, 14, 21\}$. Figure out $A \cup B$ and $A \cap B$.

**28.** Let $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4, 6, 7, 8\}$ and $C = \{2, 7, 8, 10, 11\}$. Find the following sets:
(a) $(A \cup B) \cap C$.
(b) $A \cup (B \cap C)$.
(c) $(A \cap B) \cup (B \cap C)$.

**29.** • Let $\mathbb{M} = \{-n, n \in \mathbb{N}\}$. Is $\mathbb{M} \cup \mathbb{N}$ the same set as $\mathbb{Z}$? If not, what is the difference? What is $\mathbb{M} \cap \mathbb{N}$?

**30.** • Let $A$ be the set of all integers divisible by 5. Let $B$ be the set of all integers divisible by 3. Figure out the following...
(a) $A \cup B$.
(b) $A \cap B$.

**31.** Draw these intervals on the real line.
(a)

**32.** Solve for $x$. Express your answer using sets and drawings on the real line.
(a) $(5x + 1) = 6x - 5 - (x - 5)$.
(b) $(5x + 1) = 6x - 5 - (x - 6)$.