Edward (Ted) Watters **Final Project**
Email: ewatter4@jhu.edu
Course: EN.625.722.3VL Probability,Stochastic Process II
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Professor: Dr. Woolf

1 Abstract

Bond credit ratings can be modeled as discrete finite state absorbing Markov chains. Using this, the predictive nature of assigned credit ratings can be compared with the eventual absorbing states of bond default or bond maturity.

General findings:

- Credit ratings are useful estimators of default likelihood
- Default rates associated with credit rating data collected by Mergent tracks with published 3rd party estimates.
- There are challenges with data set size, high number of states, and multiple low probability states which cause issues with numerical stability when implementing standard analytical methods. In these cases, simulations are helpful

Additionally, bond credit ratings, and their eventual default, maturity, or current state as an active security, occur in predictable sequences. A Hidden Markov Model was used to model these sequences.

General findings:

- Because of state clumping (e.g. investment grade securities stay investment grade), states repeating themselves, and absorbing states, approximately only 1 hidden state is required for every 2 observed states.
- It is likely that a model may fail cross validation because of initial observed states that occur in the test set, but not in the training set. This is because of several observed states having low initial probabilities.

All non-proprietary project files are located at: `https://github.com/tedwatters/BondCredit`

2 Data Sources

Rating History and Default status were exclusively pulled from Wharton's Research Data Services (WRDS) Mergent Fixed Income Security Database (FISD)[2]. In particular, data from the Bond Issue and Bond Ratings tables were merged in MySQL. This was a challenge, as the total table has approximately 1.2 million records and over 40 columns. I needed to change my temporary file partition to allow for the up to 80 GB of temporary tables created. Optimizing this MySQL query is future work.

WRDS data requires specific access, and the raw files are not located on the project github.

This data needed to be filtered and interpreted to be useful for the project. The steps used are available in the github repository. Applicable files include the .sql files, as well as the first portion of the markov model.py file. Specific adjustments included:

- Adding a state index value, using the ratings date.
- Adding default and maturity states to the time series

- Removing duplicate data
- Removing errant states
- Creating a rule set for how to handle ”No Rating” states

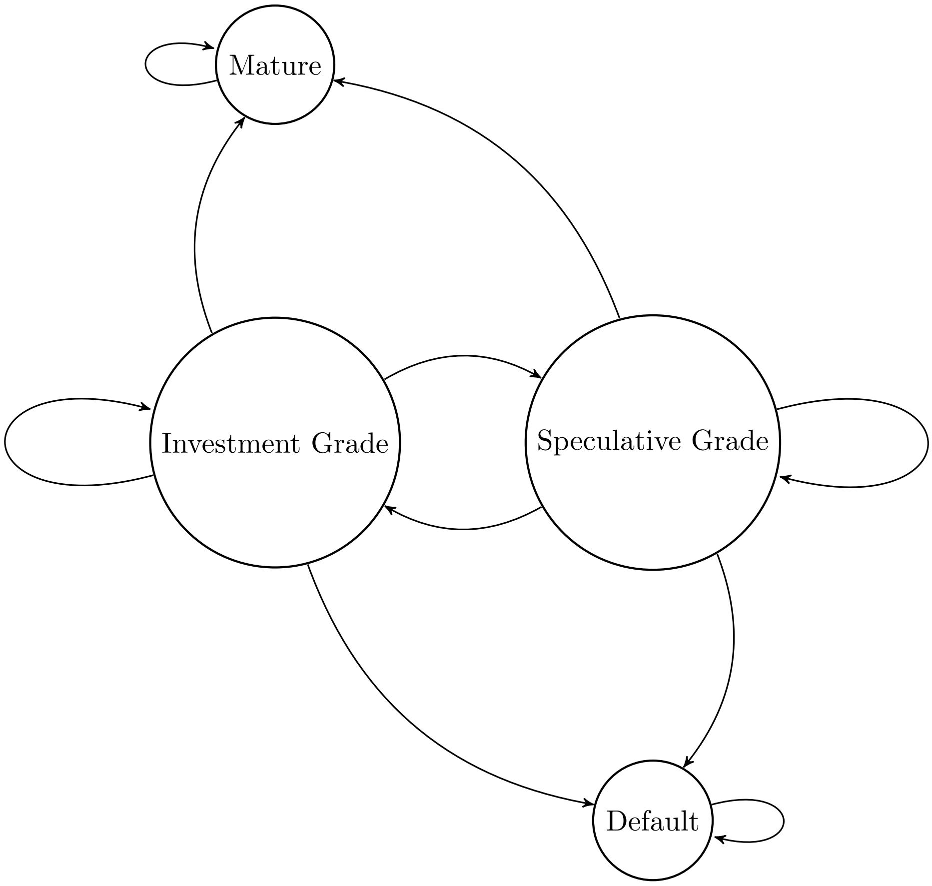
The data set does have some limitations in coverage of securities[3]

3 Background

The bond credit system is well described on Wikipedia [1].

- There are 3 main credit rating agencies - Fitch, Moody's, and Standard and Poor's
- Each credit rating agency has its own nomenclature of ratings, but the general system is similar between agencies.

Consider a simple model of the system with only 2 rating states. A bond is rated by the credit agencies until it either defaults or matures.



This is a discrete finite state absorbing Markov chain. The step index is simply whenever another rating occurs, which may or many not be at specified time intervals. Rating adjustments can also take place based on events.

In practice, we need a separate chain for each rating agency, and each rating agency has a multitude of states within the investment and speculative grades.

For example, the transition probability matrix for Fitch is:

Table 1: Fitch - Transition Probability Matrix (**P**)

	A	A+	A-	AA	AA+	AA-	AAA	B	B+	B-	BB	BB+	BB-	BBB	BBB+	BBB-	C	CC	CCC	CCC+	CCC-	DEFAULT	MATURE
A	0.967	0.007	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.018
A+	0.065	0.886	0.001	0.000	0.000	0.008	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.039
A-	0.025	0.001	0.893	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.006	0.030	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.045
AA	0.000	0.000	0.000	0.941	0.000	0.012	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.046
AA+	0.000	0.000	0.000	0.035	0.760	0.003	0.060	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.142
AA-	0.026	0.037	0.000	0.003	0.000	0.874	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.060
AAA	0.000	0.000	0.000	0.000	0.000	0.000	0.908	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.092
B	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.653	0.091	0.088	0.016	0.002	0.024	0.000	0.000	0.000	0.001	0.003	0.023	0.028	0.005	0.001	0.065
B+	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.087	0.702	0.028	0.022	0.000	0.097	0.000	0.000	0.000	0.001	0.000	0.002	0.002	0.007	0.000	0.053
B-	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.109	0.029	0.673	0.016	0.000	0.004	0.000	0.000	0.000	0.009	0.008	0.049	0.042	0.009	0.000	0.051
BB	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.004	0.015	0.000	0.754	0.073	0.081	0.001	0.000	0.008	0.000	0.000	0.000	0.001	0.000	0.000	0.062
BB+	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.004	0.000	0.060	0.800	0.013	0.004	0.000	0.072	0.002	0.000	0.000	0.000	0.000	0.000	0.044
BB-	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.016	0.048	0.004	0.078	0.017	0.768	0.000	0.000	0.003	0.000	0.000	0.003	0.000	0.000	0.001	0.062
BBB	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.000	0.881	0.034	0.036	0.000	0.002	0.000	0.000	0.000	0.000	0.043
BBB+	0.006	0.000	0.022	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.028	0.901	0.006	0.000	0.000	0.000	0.000	0.000	0.000	0.035
BBB-	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.003	0.032	0.001	0.058	0.002	0.865	0.000	0.000	0.000	0.000	0.000	0.000	0.036
C	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.000	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.713	0.077	0.000	0.000	0.012	0.090	0.100
CC	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.007	0.000	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.175	0.572	0.070	0.000	0.105	0.011	0.056
CCC	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.010	0.002	0.085	0.000	0.000	0.000	0.000	0.000	0.000	0.039	0.048	0.658	0.075	0.031	0.002	0.050
CCC+	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.057	0.000	0.048	0.000	0.000	0.000	0.000	0.000	0.000	0.030	0.017	0.029	0.768	0.017	0.000	0.034
CCC-	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.007	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.138	0.065	0.014	0.022	0.435	0.246	0.072
DEFAULT	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
MATURE	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000

Key observations are:

- The absorbing states DEFAULT and MATURE
- The likelihood that rating state remains the same between steps is high
- There are many near zero values in the matrix. This makes the matrix ill conditioned, so developing numerically stable solutions for certain matrix operations may not be possible. Solutions by simulation may be preferred.

4 Absorption State Analysis

Discrete finite state absorbing Markov chains have certain properties[4]. For instance, the transition probabilty matrix can be broken down into:

$$\mathbf{P} = \begin{Bmatrix} \mathbf{Q} & \mathbf{R} \\ \mathbf{0} & \mathbf{I}_s \end{Bmatrix}$$

Where $\mathbf{Q}$ is $t \times t$, $\mathbf{R}$ is $t \times s$, and $\mathbf{I}_s$ is $s \times s$.

Using this, the absorption probability matrix [5] can be calculated as:

$$\mathbf{M} = (\mathbf{I} - \mathbf{Q})^{-1}\mathbf{R}$$

Unfortunately, calculating the inverse requires a well conditioned $\mathbf{Q}$. For $(\mathbf{I} - \mathbf{Q})$ associated with the Fitch transition probability matrix, the condition number is 78. Since the log of the condition number is approximately an upper bound for the precision in the matrix [6], then the matrix would only be accurate to 4.4. This is clearly unacceptable for probabilities. Also, when creating an analytical solution, the following matrix was created in Python. This absorption probability matrix uses Gaussian elimination, but similar results were acquired when using numpy's inverse function.

Table 2: Fitch - Absorption Probability Analytical Solution ($\mathbf{M}$)

	DEFAULT	MATURE
A	794628.025	1412570.006
A+	-325558.619	-864594.402
A-	1005973.858	2097764.771
AA	-21023.344	-49520.118
AA+	10125.897	8686.682
AA-	-504900.533	-986777.336
AAA	6.405	4.335
B	16.345	36.021
B+	1615.927	3374.777
B-	-79.532	-159.987
BB	-4160.873	-8595.739
BB+	-9985.929	-21827.831
BB-	-2037.083	-4236.812
BBB	95256.711	149351.360
BBB+	-4698984.552	-10716974.023
BBB-	-10991.659	-19277.581
C	-1.965	-4.232
CC	-3.661	-7.339
CCC	-13.484	-26.026
CCC+	-14.550	-28.300
CCC-	-1.161	-3.043

When using a simulation to arrive at the absorption probability matrix, results were much more reasonable:

Table 3: Fitch - Absorption Probability - Simulation

	DEFAULT	MATURE	RECURRENT
A	0.003	0.997	0.000
A+	0.000	1.000	0.000
A-	0.006	0.994	0.000
AA	0.000	1.000	0.000
AA+	0.000	1.000	0.000
AA-	0.000	1.000	0.000
AAA	0.000	1.000	0.000
B	0.013	0.987	0.000

	DEFAULT	MATURE	RECURRENT
B+	0.000	1.000	0.000
B-	0.000	1.000	0.000
BB	0.000	1.000	0.000
BB+	0.000	1.000	0.000
BB-	0.011	0.989	0.000
BBB	0.000	1.000	0.000
BBB+	0.003	0.997	0.000
BBB-	0.001	0.999	0.000
C	0.482	0.518	0.000
CC	0.158	0.842	0.000
CCC	0.041	0.959	0.000
CCC+	0.000	1.000	0.000
CCC-	0.769	0.231	0.000

Again, these results are specific to Fitch. However, values for Moody's and Standard and Poor's were similar.

These values generally track with historical default rates. However, please note that the data sets that are being used may not be the same (e.g. global vs. US, corporate only vs. corporate + municipal, etc.). Additionally, the absorptive probability that was calculated in the Markov Model measures if the bond **will ever default** before it matures, given that it is in a certain state now. Figure 1 is specific to a **1-year period**.

Figure 1: Standard and Poor's Historical Default Rates[1]

Standard & Poor's One-Year Global Structured Finance Default Rates By Refined Rating Category, 1978-2008

Year	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC to C
1993	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6.25		0
1994	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.85	0	0
1995	0	0	0	0	0	0	0	0	0.43	0	0	0.98	0	0	0.95	0	52.63
1996	0	0	0	0	0	0.15	0	0	0	0	0	0.61	12.50		0	0	31.03
1997	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	20.69
1998	0	0	0	0	0	1.04	0.91	0	0.19	0	0	1.03	0	0	2.34	0	22.58
1999	0	0	0	0	0	0	0.77	0	0	0.39	0	0	0	0	1.54	0	19.35
2000	0	0	0	0	0	0	0	0	0.11	0	0	0.61	0	0	2.19	0	5.26
2001	0.05	0	0	0	0	0.12	0	2.22	0	0.86	0.83	0.55	0.91	2.00	2.69	3.27	26.87
2002	0	0	0.06	0	0.27	0.14	0	1.77	0.19	0.70	1.26	2.03	1.12	2.50	3.60	23.24	27.03
2003	0	0	0	0	0.19	0.03	0.16	0.20	0.60	0.50	0.75	0.84	1.43	3.28	1.64	5.15	32.58
2004	0	0	0	0	0	0	0	0	0.16	0.17	0.50	0.81	0.29	0.79	2.23	3.56	13.79
2005	0	0	0	0	0	0	0	0	0.08	0.06	0.15	0.14	0.45	0.33	1.34	2.53	16.08
2006	0	0	0	0	0	0	0	0	0.06	0.20	0	0.33	0.36	0.26	0.36	1.42	19.18
2007	0.04	0.03	0.07	0.08	0	0.10	0.21	0.48	0.47	1.27	5.07	1.61	1.53	0.68	1.55	1.47	24.11
2008	0.53	0.35	0.57	1.15	1.15	0.87	1.42	2.27	1.26	3.45	5.60	4.21	5.07	8.53	12.84	10.28	56.92

Summary statistic	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC to C
Mean	0.02	0.01	0.02	0.05	0.06	0.08	0.14	0.37	0.16	0.38	3.56	0.81	1.24	1.22	2.18	2.83	16.73
Median	0	0	0	0	0	0	0	0	0	0	0	0.61	0	0.26	1.55	0	17.63
Minimum	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Maximum	0.53	0.35	0.57	1.15	1.15	1.04	1.42	2.27	1.26	3.45	57.14	4.21	12.50	8.53	12.84	23.24	56.92
Standard Deviation	0.09	0.07	0.10	0.23	0.23	0.24	0.35	0.76	0.29	0.78	12.39	1.02	2.90	2.20	2.93	5.59	16.60

5 Hidden Markov Model (Fitch)

Because of computing time constraints, I only created a hidden markov model for the Fitch rating data. This included ratings for approximately 150,000 securities.

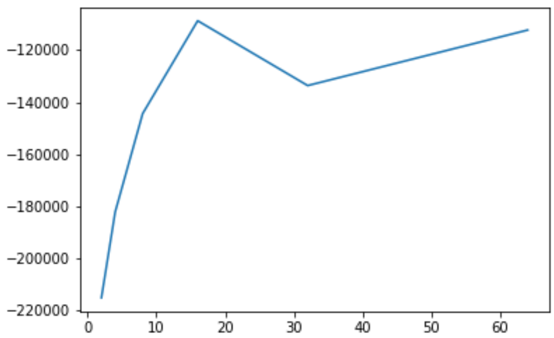
The sequence of observed states used were the ratings states. The sequences are of variable length, depending on if the issue matured, defaulted, or is still active. Additionally, since ratings do not have a fixed periodicity, some securities have more ratings than others.

The hidden states are modeled as unknown hyper parameters. To determine the number of hidden states needed, I performed a 5-fold cross validation using between 10 and 15 iterations when fitting for each state.

To make the model, I leaned heavily on the Deep Learning Courses Hidden Markov Model in Python online class.[7][8]

My hypothesis was that there would be less hidden states than observed states because of data clumping, and the fact that the model is absorptive as $n \longrightarrow \infty$. To test this hypothesis, I randomly split the sequences into 5 groups. I also assigned integer values to each rating state. The code for this is in the hidden markov model python file on the github repository (`https://github.com/tedwatters/BondCredit`). There are approximately 32 observed states for each rating agency, so I tested the number of hidden states from 2 to 64. Figure below shows log likelihood of the test set compared to the number of states chosen.

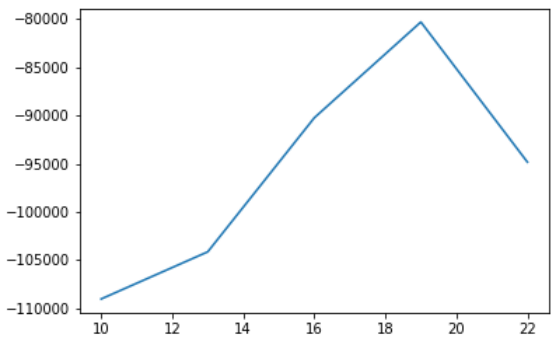
Figure 2: Log Likelihood and Number of Hidden States (Fitch data, 5-fold, Test 1, 10 iterations)



There was a global maximum at 16 states for this run . It is possible there may be another maximum at ¿64 states; however, compute time would make a model with this number of states impractical for this project. Based on the state sampling, the optimal number of states is between 9 and 31 (inclusive). Two additional tests were run to look closer at this region. The best estimate of number of hidden states from test 2 was 19 states, which informed values of m tested for the 3rd run, as well as the cross validation.

And tightening the range more tightly around the maximum at 19 hidden states: At this point, I began the process

Figure 3: Log Likelihood and Number of Hidden States (Fitch data, 5-fold, Test 2, 10 iterations)



of looking at cross validation. Since partition 0 was used for the initial range investigating, the graph remains the same for this number of hidden states.

Starting with test partition 1, I started to see errors where log likelihood returned at $-\infty$. Specifically, the

Figure 4: Log Likelihood and Number of Hidden States (Fitch data, 5-fold, Test 3, 15 iterations)

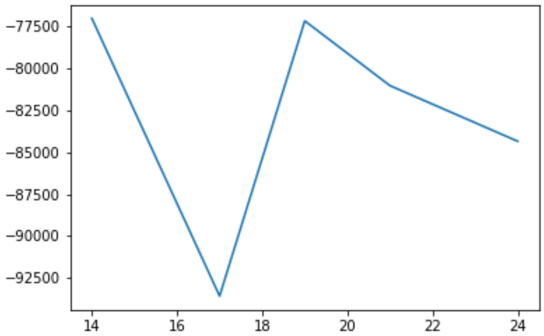
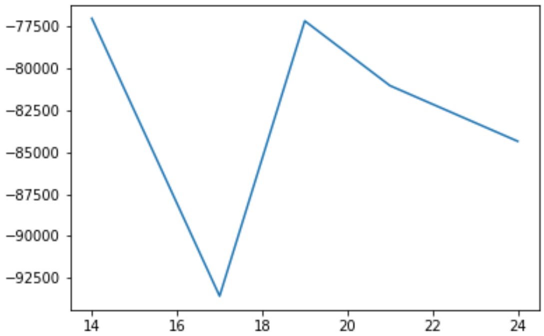


Figure 5: Log Likelihood and Number of Hidden States (Fitch data, 5-fold, Cross validation (Test Set 0), 15 iterations)



error was when scaling α' using the equation[9]:

$$\hat{\alpha}(t,i) = \frac{\alpha'(t,i)}{c(t)}$$

where:

$$c(t) = \sum_{i=1}^{M} \alpha'(t,i)$$

and:

$$\alpha'(t,i) = \pi_i B(j, x(1))$$

The problem was that $c(t)$ was being calculated as 0 for certain partitions, and returning a divide by zero error when calculating. The driver behind this issue is that when initializing the forward-backward algorithm, the following condition must be met:

$$\text{unique initiation states, test set} \subseteq \text{unique initiation states, training set}$$

In this case, only test partitions 1 and 3 met this criteria, thereby causing the scaled forward-backward algorithm to fail. The root cause for this is that are some very rare initial observation states that may get sampled in the test set, but are excluded from the training set.

Looking at the final π, $\mathbf{A}$, $\mathbf{B}$ generated from test set 4, we can make some generalizations:

- π - There are some high (0.581) and low (0.000) initial hidden states. This checks with our understanding that some initial observed states are very common, such as "NR" (0.701), and some almost never occur, such as "DEFAULT". Since the probability of observed state "NR" is higher than the highest probability of the initial hidden states, this suggests that multiple hidden states contribute to initial "NR" ratings. This makes sense because certain sequences such as "NR" $\longrightarrow$ "MATURE" are very common, but so are other "NR" to other rating sequences.

Figure 6: Log Likelihood and Number of Hidden States (Fitch data, 5-fold, Cross validation (Test Set 1), 15 iterations)

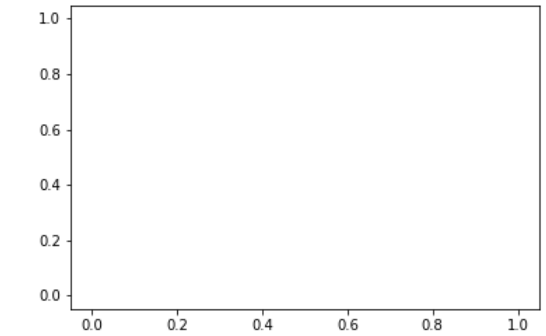


Figure 7: Log Likelihood and Number of Hidden States (Fitch data, 5-fold, Cross validation (Test Set 2), 15 iterations)

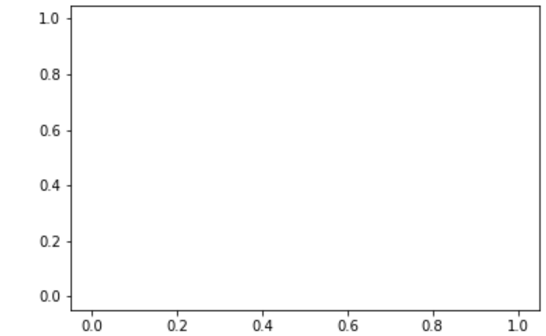
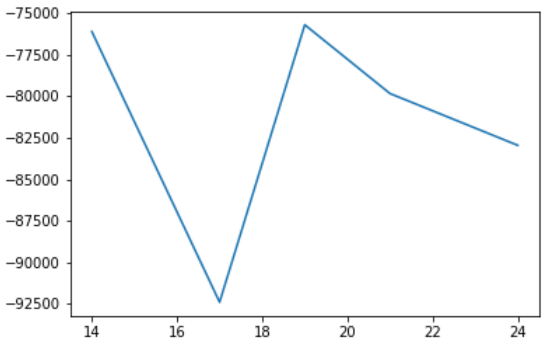


Figure 8: Log Likelihood and Number of Hidden States (Fitch data, 5-fold, Cross validation (Test Set 3), 15 iterations)



- **A** and **B** - In just looking at the Markov chain, it was shown that there is clumping of states. For instance, if a bond was AAA rated before, it will not likely default. It is very likely that it keeps the AAA rating, or goes to another investment grade rating. In the hidden and observed state matrices, The are some very high probabilities in certain rows which indicate state clumping and high predictive power.

Table 4: Fitch HMM - π

0.025
0.046
0.031

0.025
0.000
0.015
0.138
0.020
0.581
0.015
0.094
0.000
0.013
0.024
0.000

Table 5: Fitch HMM - **A**

0.901	0.000	0.000	0.029	0.001	0.000	0.017	0.000	0.006	0.000	0.013	0.023	0.009	0.002
0.000	0.941	0.000	0.013	0.000	0.027	0.000	0.000	0.000	0.000	0.010	0.000	0.000	0.009
0.000	0.000	0.000	0.395	0.000	0.000	0.000	0.000	0.000	0.000	0.579	0.000	0.000	0.025
0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.006	0.000	0.000	0.021	0.940	0.000	0.009	0.000	0.012	0.000	0.005	0.000	0.001	0.006
0.000	0.206	0.000	0.222	0.000	0.493	0.000	0.000	0.001	0.000	0.027	0.000	0.000	0.051
0.000	0.000	0.000	0.000	0.003	0.000	0.975	0.000	0.001	0.000	0.016	0.005	0.000	0.001
0.000	0.000	0.000	0.518	0.000	0.000	0.000	0.000	0.000	0.000	0.423	0.000	0.000	0.060
0.008	0.001	0.000	0.000	0.025	0.001	0.004	0.000	0.936	0.000	0.000	0.000	0.025	0.000
0.000	0.000	0.000	0.571	0.000	0.000	0.000	0.000	0.000	0.000	0.282	0.000	0.000	0.147
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
0.006	0.000	0.000	0.018	0.003	0.000	0.040	0.000	0.018	0.000	0.012	0.891	0.001	0.010
0.012	0.000	0.000	0.000	0.002	0.000	0.000	0.000	0.021	0.000	0.007	0.000	0.958	0.000
0.000	0.004	0.000	0.405	0.000	0.001	0.000	0.000	0.035	0.000	0.124	0.000	0.000	0.431

Figure 9: Log Likelihood and Number of Hidden States (Fitch data, 5-fold, Cross validation (Test Set 4), 15 iterations)

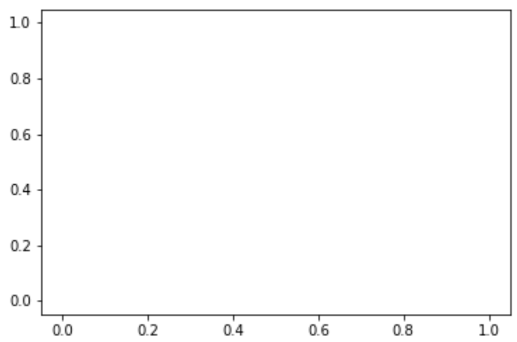


Table 6: Fitch HMM - **B**

0.000	0.005	0.000	0.000	0.000	0.001	0.000	0.645	0.000	0.000	0.000	0.004	0.000	0.000	0.000	0.109	0.000	0.135	0.000	0.092	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.999	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.002	0.000	0.000	0.006	0.000	0.002	0.000	0.002	0.000	0.008	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.002	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.001	0.000	0.000	0.404	0.000	0.545	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.015	0.010	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.999	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.002	0.000	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.002	0.000	0.000	0.004	0.000	0.000	0.040	0.000	0.000	0.000	0.063	0.068	0.000	0.053	0.000	0.000	0.000	0.000	0.000	0.004	0.001	0.000	0.689	0.000	0.000	0.000	0.027
0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.006	0.000	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.038	0.960	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.010	0.000	0.000	0.526	0.000	0.000	0.431	0.000	0.001	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

6 Further Work

- Optimize MySQL table merge. Temporary tables took approximately 80GB of hard drive storage during the merge. There may be a more efficient query.
- Create Hidden Markov Model based on adjusted data set. Specifically:
  - Make steps monthly vs arbitrary rating times (which could be based on initial issue, review based on changes in economic outlook, or just annually)
  - Add monthly yield data. This would be used as the observed state.
  - Monthly rating data can be the first known hidden set of states. The hypothesis those bonds with a higher credit rating would have a lower yield.
  - Create a hyper-parameterized model using just the observed yield data, and an additional model that includes the rating data
- Attempt to add additional data sources to validate the WRDS database. Also, attempt to include a larger sphere of fixed income securities.
- Utilize cloud computing to:
  - Increase the number of iterations run for the hidden markov model during fitting. I ran between 10 and 15 iterations per state in this project. A concern would be that the number of hidden states here is chosen based on quickness of goodness of fit to converge, rather than goodness of fit at convergence.
  - Increase the number of states tested during cross validation
  - Increase the number of folds (partitioning into test/ training sets) used during cross validation
- Further investigate and generalize the conditions which can lead to failure of a trained hidden markov model to be unable to find the likelihood of a test set.

7 Appendix - Raw Data

Table 7: Fitch - Initial State Probability (π)

	RATING	INITIAL˙STATE˙PROBABILITY
0	A	0.021
1	A+	0.012
2	A+(EXP)	0.000
3	A-	0.010
4	A-(EXP)	0.000
5	AA	0.006
6	AA+	0.002
7	AA-	0.017
8	AA-(EXP)	0.000
9	AAA	0.185
10	B	0.001
11	B+	0.001
12	B+(EXP)	0.000
13	B-	0.001
14	B-(EXP)	0.000
15	BB	0.003
16	BB(EXP)	0.000
17	BB+	0.003
18	BB+(EXP)	0.000
19	BB-	0.003
20	BBB	0.013
21	BBB(EXP)	0.000
22	BBB+	0.009
23	BBB-	0.011
24	C	0.000
25	CC	0.000
26	CCC	0.000

	RATING	INITIAL˙STATE˙PROBABILITY
27	CCC+	0.000
28	CCC-	0.000
29	DEFAULT	
30	MATURE	
31	NR	0.701

Table 8: Moody's - Initial State Probability (π)

	RATING	INITIAL˙STATE˙PROBABILITY
32	A	0.000
33	A1	0.014
34	A2	0.023
35	A3	0.025
36	Aa1	0.004
37	Aa2	0.008
38	Aa3	0.016
39	Aaa	0.382
40	B1	0.007
41	B2	0.005
42	B3	0.007
43	Ba1	0.004
44	Ba2	0.004
45	Ba3	0.005
46	Baa	
47	Baa1	0.019
48	Baa2	0.019
49	Baa3	0.015
50	C	
51	Ca	0.000
52	Caa1	0.004
53	Caa2	0.002
54	Caa3	0.000
55	DEFAULT	
56	MATURE	
57	NR	0.436

Table 9: Standard and Poor's - Initial State Probability (π)

	RATING	INITIAL˙STATE˙PROBABILITY
58	A	0.026
59	A+	0.014
60	A-	0.021
61	AA	0.003
62	AA+	0.236
63	AA-	0.015
64	AAA	0.128
65	B	0.006
66	B+	0.006
67	B-	0.005
68	BB	0.005
69	BB+	0.007
70	BB-	0.006
71	BBB	0.020
72	BBB+	0.016
73	BBB-	0.014
74	C	0.000
75	CC	0.000

	RATING	INITIAL˙STATE˙PROBABILITY
76	CCC	0.001
77	CCC+	0.003
78	CCC-	0.000
79	D	0.000
80	DEFAULT	
81	MATURE	
82	NR	0.467

Table 10: Fitch - Transition Probability Matrix ($\mathbf{P}$)

	A	A+	A-	AA	AA+	AA-	AAA	B	B+	B-	BB	BB+	BB-	BBB	BBB+	BBB-	C	CC	CCC	CCC+	CCC-	DEFAULT	MATURE
A	0.967	0.007	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.018
A+	0.065	0.886	0.001	0.000	0.000	0.008	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.039
A-	0.025	0.001	0.893	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.006	0.030	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.045
AA	0.000	0.000	0.000	0.941	0.000	0.012	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.046
AA+	0.000	0.000	0.000	0.035	0.760	0.003	0.060	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.142
AA-	0.026	0.037	0.000	0.003	0.000	0.874	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.060
AAA	0.000	0.000	0.000	0.000	0.000	0.000	0.908	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.092
B	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.653	0.091	0.088	0.016	0.002	0.024	0.000	0.000	0.000	0.001	0.003	0.023	0.028	0.005	0.001	0.065
B+	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.087	0.702	0.028	0.022	0.000	0.097	0.000	0.000	0.000	0.001	0.000	0.002	0.002	0.007	0.000	0.053
B-	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.109	0.029	0.673	0.016	0.000	0.004	0.000	0.000	0.000	0.009	0.008	0.049	0.042	0.009	0.000	0.051
BB	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.004	0.015	0.000	0.754	0.073	0.081	0.001	0.000	0.008	0.000	0.000	0.000	0.001	0.000	0.000	0.062
BB+	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.004	0.000	0.060	0.800	0.013	0.004	0.000	0.072	0.002	0.000	0.000	0.000	0.000	0.000	0.044
BB-	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.016	0.048	0.004	0.078	0.017	0.768	0.000	0.000	0.003	0.000	0.000	0.003	0.000	0.000	0.001	0.062
BBB	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.000	0.881	0.034	0.036	0.000	0.002	0.000	0.000	0.000	0.000	0.043
BBB+	0.006	0.000	0.022	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.028	0.901	0.006	0.000	0.000	0.000	0.000	0.000	0.000	0.035
BBB-	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.003	0.032	0.001	0.058	0.002	0.865	0.000	0.000	0.000	0.000	0.000	0.000	0.036
C	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.000	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.713	0.077	0.000	0.000	0.012	0.090	0.100
CC	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.007	0.000	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.175	0.572	0.070	0.000	0.105	0.011	0.056
CCC	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.010	0.002	0.085	0.000	0.000	0.000	0.000	0.000	0.000	0.039	0.048	0.658	0.075	0.031	0.002	0.050
CCC+	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.057	0.000	0.048	0.000	0.000	0.000	0.000	0.000	0.000	0.030	0.017	0.029	0.768	0.017	0.000	0.034
CCC-	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.007	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.138	0.065	0.014	0.022	0.435	0.246	0.072
DEFAULT	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
MATURE	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000

Table 11: Moody's - Transition Probability Matrix (**P**)

	A1	A2	A3	Aa1	Aa2	Aa3	Aaa	B1	B2	B3	Ba1	Ba2	Ba3	Baa	Baa1	Baa2	Baa3	C	Ca	Caa1	Caa2	Caa3	NR	DEFAULT	MA
A1	0.479	0.211	0.062	0.000	0.050	0.056	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.14
A2	0.099	0.662	0.065	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.055	0.003	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.11
A3	0.000	0.109	0.630	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.130	0.022	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.10
Aa1	0.038	0.005	0.000	0.327	0.276	0.093	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.25
Aa2	0.099	0.003	0.001	0.043	0.472	0.180	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.20
Aa3	0.134	0.265	0.001	0.000	0.032	0.399	0.000	0.000	0.000	0.000	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.16
Aaa	0.000	0.000	0.000	0.001	0.001	0.000	0.498	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.50
B1	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.424	0.117	0.040	0.003	0.023	0.221	0.000	0.000	0.002	0.003	0.000	0.000	0.009	0.002	0.002	0.001	0.000	0.15
B2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.185	0.360	0.190	0.000	0.003	0.015	0.000	0.000	0.000	0.004	0.000	0.002	0.054	0.011	0.004	0.000	0.001	0.17
B3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.049	0.166	0.359	0.000	0.002	0.003	0.000	0.000	0.001	0.003	0.000	0.005	0.165	0.041	0.015	0.000	0.001	0.19
Ba1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.008	0.007	0.002	0.461	0.112	0.035	0.000	0.005	0.007	0.261	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.10
Ba2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.025	0.012	0.007	0.209	0.416	0.144	0.000	0.000	0.005	0.045	0.000	0.000	0.003	0.001	0.000	0.001	0.001	0.13
Ba3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.134	0.030	0.017	0.012	0.197	0.433	0.000	0.000	0.000	0.004	0.001	0.000	0.008	0.002	0.010	0.000	0.000	0.15
Baa																									
Baa1	0.001	0.001	0.270	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.001	0.001	0.000	0.000	0.434	0.157	0.009	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.12
Baa2	0.000	0.000	0.048	0.000	0.000	0.000	0.000	0.001	0.001	0.000	0.008	0.004	0.003	0.000	0.145	0.537	0.138	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.11
Baa3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.008	0.002	0.000	0.062	0.013	0.006	0.000	0.007	0.183	0.597	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.12
C	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.490	0.000	0.008	0.020	0.024	0.000	0.208	0.24
Ca	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.030	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.215	0.359	0.010	0.013	0.063	0.000	0.073	0.23
Caa1	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.003	0.016	0.196	0.001	0.001	0.001	0.000	0.000	0.000	0.000	0.004	0.025	0.362	0.184	0.047	0.001	0.004	0.15
Caa2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.005	0.026	0.001	0.000	0.000	0.000	0.000	0.002	0.000	0.032	0.094	0.134	0.387	0.181	0.000	0.005	0.13
Caa3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.005	0.008	0.000	0.000	0.002	0.000	0.000	0.000	0.002	0.113	0.212	0.018	0.119	0.323	0.000	0.055	0.14
NR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.99
DEFAULT	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.00
MATURE	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.00

Table 12: Moody's - Transition Probability Matrix (**P**)

	A	A+	A-	AA	AA+	AA-	AAA	B	B+	B-	BB	BB+	BB-	BBB	BBB+	BBB-	C	CC	CCC	CCC+	CCC-	D	DEFAULT	MATURE
A	0.041	0.100	0.498	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.005	0.073	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.280
A+	0.593	0.090	0.006	0.000	0.000	0.080	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.004	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.222
A-	0.224	0.002	0.082	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.015	0.448	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.226
AA	0.002	0.021	0.000	0.006	0.015	0.547	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.008	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.400
AA+	0.000	0.000	0.000	0.001	0.241	0.009	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.747
AA-	0.134	0.463	0.000	0.041	0.000	0.025	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.336
AAA	0.000	0.000	0.000	0.003	0.430	0.000	0.053	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.514
B	0.000	0.000	0.000	0.001	0.000	0.001	0.000	0.058	0.325	0.318	0.007	0.004	0.033	0.004	0.003	0.003	0.000	0.007	0.009	0.049	0.007	0.003	0.001	0.168
B+	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.280	0.064	0.043	0.075	0.010	0.342	0.004	0.004	0.001	0.001	0.000	0.003	0.006	0.002	0.002	0.000	0.164
B-	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.310	0.058	0.068	0.005	0.000	0.009	0.001	0.000	0.000	0.001	0.014	0.058	0.271	0.030	0.010	0.001	0.163
BB	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.064	0.004	0.077	0.377	0.275	0.004	0.001	0.045	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.152
BB+	0.001	0.000	0.001	0.000	0.000	0.000	0.001	0.000	0.001	0.000	0.192	0.070	0.061	0.023	0.001	0.367	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.282
BB-	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.071	0.267	0.005	0.297	0.062	0.088	0.001	0.002	0.010	0.000	0.016	0.000	0.005	0.004	0.002	0.000	0.168
BBB	0.001	0.001	0.011	0.000	0.000	0.000	0.000	0.000	0.002	0.000	0.003	0.007	0.000	0.074	0.249	0.350	0.000	0.000	0.000	0.001	0.000	0.000	0.001	0.300
BBB+	0.032	0.001	0.271	0.000	0.000	0.003	0.000	0.000	0.000	0.000	0.000	0.003	0.002	0.213	0.043	0.048	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.384
BBB-	0.000	0.001	0.006	0.000	0.000	0.002	0.000	0.002	0.000	0.001	0.051	0.133	0.020	0.460	0.018	0.066	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.239
C	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.004	0.026	0.000	0.009	0.030	0.880	0.000	0.047
CC	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.004	0.000	0.004	0.000	0.002	0.000	0.000	0.000	0.000	0.138	0.015	0.029	0.052	0.069	0.621	0.006	0.060
CCC	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.010	0.000	0.015	0.000	0.004	0.001	0.003	0.000	0.001	0.056	0.156	0.037	0.197	0.295	0.110	0.000	0.114
CCC+	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.042	0.004	0.273	0.003	0.000	0.000	0.002	0.001	0.000	0.004	0.049	0.277	0.051	0.092	0.036	0.001	0.162
CCC-	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.003	0.000	0.023	0.000	0.000	0.000	0.000	0.000	0.000	0.181	0.246	0.115	0.034	0.018	0.254	0.006	0.120
D	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.000	0.009	0.000	0.000	0.000	0.000	0.000	0.000	0.010	0.039	0.060	0.018	0.114	0.248	0.167	0.333
DEFAULT	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
MATURE	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000

Table 13: Fitch - Absorption Probability Analytical Solution (**M**)

	DEFAULT	MATURE
A	794628.025	1412570.006
A+	-325558.619	-864594.402
A-	1005973.858	2097764.771
AA	-21023.344	-49520.118
AA+	10125.897	8686.682
AA-	-504900.533	-986777.336
AAA	6.405	4.335
B	16.345	36.021
B+	1615.927	3374.777
B-	-79.532	-159.987
BB	-4160.873	-8595.739
BB+	-9985.929	-21827.831
BB-	-2037.083	-4236.812
BBB	95256.711	149351.360
BBB+	-4698984.552	-10716974.023
BBB-	-10991.659	-19277.581
C	-1.965	-4.232
CC	-3.661	-7.339
CCC	-13.484	-26.026
CCC+	-14.550	-28.300
CCC-	-1.161	-3.043

Table 14: Moody's - Absorption Probability Analytical Solution (**M**)

	DEFAULT	MATURE
A1		
A2		
A3	-1.808	-3183.073
Aa1	-2.962	-5216.019
Aa2	-0.548	-965.413
Aa3	-0.722	-1270.749
Aaa	-1.192	-2100.123
B1	-0.002	-2.394
B2	-2.432	-4328.667
B3	-1.239	-2260.327
Ba1	-0.762	-1457.754
Ba2	-14.303	-25186.801
Ba3	-8.242	-14530.063
Baa	-4.015	-7108.219
Baa1	-47340.512	-83316700.328
Baa2	-4.490	-7904.814
Baa3	-9.489	-16705.095
C	-25.050	-44093.437
Ca	0.406	-43.590
Caa1	0.219	-133.118
Caa2	-0.302	-735.407
Caa3	-0.066	-446.072
NR	0.110	-301.433

Table 15: Standard and Poor's - Absorption Probability Analytical Solution (**M**)

	DEFAULT	MATURE
A	0.002	0.998

	DEFAULT	MATURE
A+	0.001	0.999
A-	0.001	0.999
AA	0.001	0.999
AA+	0.000	1.000
AA-	0.001	0.999
AAA	0.000	1.000
B	0.056	0.944
B+	0.035	0.965
B-	0.086	0.914
BB	0.014	0.986
BB+	0.007	0.993
BB-	0.027	0.973
BBB	0.003	0.997
BBB+	0.001	0.999
BBB-	0.005	0.995
C	0.274	0.726
CC	0.258	0.742
CCC	0.189	0.811
CCC+	0.131	0.869
CCC-	0.226	0.774
D	0.293	0.707

Table 16: Fitch - Absorption Probability - Simulation

	DEFAULT	MATURE	RECURRENT
A	0.003	0.997	0.000
A+	0.000	1.000	0.000
A-	0.006	0.994	0.000
AA	0.000	1.000	0.000
AA+	0.000	1.000	0.000
AA-	0.000	1.000	0.000
AAA	0.000	1.000	0.000
B	0.013	0.987	0.000
B+	0.000	1.000	0.000
B-	0.000	1.000	0.000
BB	0.000	1.000	0.000
BB+	0.000	1.000	0.000
BB-	0.011	0.989	0.000
BBB	0.000	1.000	0.000
BBB+	0.003	0.997	0.000
BBB-	0.001	0.999	0.000
C	0.482	0.518	0.000
CC	0.158	0.842	0.000
CCC	0.041	0.959	0.000
CCC+	0.000	1.000	0.000
CCC-	0.769	0.231	0.000

Table 17: Moody's - Absorption Probability - Simulation

	DEFAULT	MATURE	RECURRENT
A	0.003	0.997	0.000
A+	0.000	1.000	0.000
A-	0.001	0.999	0.000
AA	0.000	1.000	0.000
AA+	0.000	1.000	0.000
AA-	0.000	1.000	0.000
AAA	0.000	1.000	0.000

	DEFAULT	MATURE	RECURRENT
B	0.004	0.996	0.000
B+	0.000	1.000	0.000
B-	0.000	1.000	0.000
BB	0.000	1.000	0.000
BB+	0.000	1.000	0.000
BB-	0.011	0.989	0.000
BBB	0.000	1.000	0.000
BBB+	0.005	0.995	0.000
BBB-	0.003	0.997	0.000
C	0.470	0.530	0.000
CC	0.158	0.842	0.000
CCC	0.043	0.957	0.000
CCC+	0.000	1.000	0.000
CCC-	0.784	0.216	0.000

Table 18: Standard and Poor's - Absorption Probability - Simulation

	DEFAULT	MATURE	RECURRENT
A	0.002	0.998	0.000
A+	0.000	1.000	0.000
A-	0.002	0.998	0.000
AA	0.000	1.000	0.000
AA+	0.000	1.000	0.000
AA-	0.000	1.000	0.000
AAA	0.000	1.000	0.000
B	0.017	0.983	0.000
B+	0.000	1.000	0.000
B-	0.000	1.000	0.000
BB	0.000	1.000	0.000
BB+	0.000	1.000	0.000
BB-	0.018	0.982	0.000
BBB	0.000	1.000	0.000
BBB+	0.003	0.997	0.000
BBB-	0.002	0.998	0.000
C	0.424	0.576	0.000
CC	0.152	0.848	0.000
CCC	0.045	0.955	0.000
CCC+	0.000	1.000	0.000
CCC-	0.751	0.249	0.000

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