Math 240: Discrete Structures I (W18) – Assignment 2

Solutions must typed or very neatly written and uploaded to MyCourses no later than **6 pm** on **Saturday, February 10, 2018**. Up to 4 bonus marks will be awarded for solutions typeset in LaTeX; both the .tex file and .pdf file must be uploaded.

You may use theorems proven or stated in class, but you must state the theorem you are using.

[15] 1. **Proofs with sets.** Let A, B, C be arbitrary sets. For each of the following statements, either prove it is true (without a Venn diagram) or give a counterexample to show that it is false.

(a) $(A \setminus B) \setminus C = A \setminus (B \cup C)$

(b) $\Big(A \oplus B = A \oplus C \text{ and } A \cap B = A \cap C\Big) \Rightarrow B = C$

(c) $(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D)$

2. **Relations.**

[11] (a) Determine whether or not each relation is reflexive, symmetric, antisymmetric, and/or transitive. For each property, if the relation has that property, prove it. If it doesn't have that property, give a counterexample. State if the relation is a total order, partial order but not a total order, or neither; justify your answer.

i. $\mathcal{R} = \{(X, Y) \in (\mathcal{P}(A))^2 \mid X \cap Y \neq \emptyset\}$ where A is some arbitrary set

ii. $\mathcal{R} = \{(a, b) \in \mathbb{N}^2 \mid a \text{ divides } b\}$ (a divides b means that there is some integer k such that $b = ka$)

[6] (b) For $a, b \in \mathbb{R} \setminus \{0\}$, define $a \sim b$ iff $\frac{a}{b} \in \mathbb{Q}$. Prove that $\sim$ defines an equivalence relation on $\mathbb{R} \setminus \{0\}$. Show that $\left[\frac{9-\sqrt{5}}{1-\sqrt{5}}\right] = \left[\frac{2}{3-6\sqrt{5}}\right]$.

[8] 3. **Proof techniques.** Prove the following statements using the method of your choice (direct proof, proof of the contrapositive, proof by contradiction).

(a) Let $a, b \in \mathbb{R}$. If $a \in \mathbb{Q}$ and $b \notin \mathbb{Q}$, then $a \pm b \notin \mathbb{Q}$.

(b) If the average of 4 distinct integers is 10, then at least one of the integers is greater than 11.