

Math 240: Discrete Structures I (W18) – Assignment 8

Solutions must typed or very neatly written and uploaded to MyCourses no later than **6 pm** on **Saturday, March 31, 2018**. Up to 4 bonus marks will be awarded for solutions typeset in \LaTeX ; both the .tex file and .pdf file must be uploaded.

You may use theorems proven or stated in class, but you must state the theorem you are using. All work must be shown for full marks.

- [14] 1. **Principle of Inclusion-Exculsion .**
- (a) Determine the number of integers from 1 to 1,000,000 which are relatively prime to 990,099. [Note: there is a formula that one can use to calculate this. You may, by all means, read up about it but you may not use it to answer the problem.]
 - (b) Out of a group of 30 people, 20 people play flute, 8 play piano, and 25 play violin. If 20 people play at least two of the three instruments and 6 play all three, how many of the 30 people play none of the three instruments?
- [14] 2. **Pigeon Hole Principle.**
- (a) Let X be a subset of the integers from 1 to 1997 such that $|X| \geq 34$. Show that there exist $x_1, x_2, x_3, x_4, x_5, x_6 \in X$ (not necessarily all distinct) such that $x_1 + x_2 + x_3 = x_4 + x_5 + x_6$.
 - (b) Let n be any positive integer, and let d be a positive integer such that $1 \leq d \leq 9$. Show that some multiple of n can be expressed in base 10 using only the digits 0 and d .
- [12] 3. **Graphs.** For each of the following degree sequences, either show that no graph exists having that degree sequence or construct an example showing that a graph does exist with that degree sequence. If you give an example, give a set representation of its vertices and edges as well as a drawing of the graph.
- (a) 0, 2, 3, 3, 4, 5, 6, 6, 7
 - (b) 0, 2, 3, 3, 4, 5, 6, 7, 8
 - (c) 1, 2, 3, 3, 4, 5, 6, 7, 8
 - (d) 1, 2, 3, 3, 4, 5, 6, 8, 8