Math 240: Discrete Structures I (W18) – Problem Set 9

These problems are for your own practice, in preparation for the final exam. Your solutions will not be graded, but a solution set will be provided at a later date for your reference.

1. Let $\Delta(G)$ denote the maximum degree of a vertex in a graph G. Prove that a tree T has at least $\Delta(T)$ leaves.
2. Let G be an arbitrary graph which is not Eulerian.
   (a) Show that one can add a new vertex to G, say v, and some number of edges from v to G so that the resulting graph is Eulerian.
   (b) Give an example to show that one cannot make a non-Eulerian graph Eulerian simply by adding edges to the existing vertices.
3. Let G be a graph, and let $e \in E(G)$. Prove that e is in every spanning tree of G if and only if e is a cut edge of G (recall that a cut edge is an edge whose deletion disconnects the graph).

   For the next two problems, let $\psi(H)$ denote the number of connected components of a graph H.
4. Show that a graph G contains at least $|E(G)| - |V(G)| + k(G)$ distinct cycles.
5. Let G be a Hamiltonian graph, and for a set $S \subseteq V(G)$, let $G - S$ denote the graph obtained by deleting S and all incident edges. Prove that $k(G - S) \leq |S|$ for every $S \subseteq V(G)$.
6. A *claw* in a graph G is a vertex v together with three other vertices x_1, x_2, x_3 such that $vx_i \in E(G)$ for each $i \in \{1, 2, 3\}$ but $x_i x_j \notin E(G)$ for any $i, j \in \{1, 2, 3\}$. A *triangle* is a set of three vertices which are all adjacent to one another. Suppose a graph G has no claws and no triangles. Determine what its connected components are and prove your answer.