Assignment 1

Weishi Wang, ID 260540022

January 26, 2018

Problem 1. *Use a truth table to determine if each statement is a tautology, contradiction, or contingency.*

(a) $(P \vee Q) \Rightarrow \neg P$

(b) $(P \Leftrightarrow Q) \wedge (Q \Leftrightarrow R) \Rightarrow (P \Leftrightarrow R)$

(c) $[(P \oplus Q) \oplus \neg Q] \Leftrightarrow P$, *where* $P \oplus Q$ *is defined by the following truth table:*

P	Q	$P \oplus Q$
T	T	F
T	F	T
F	T	T
F	F	F

Solution. Solution for problem 1:

(a)

P	Q	$\neg P$	$P \vee Q$	$(P \vee Q) \Rightarrow \neg P$
T	T	F	T	F
T	F	F	T	F
F	T	T	T	T
F	F	T	F	T

Thus, this is a contingency.

(b)

P	Q	R	$P \Leftrightarrow Q$	$Q \Leftrightarrow R$	$P \Leftrightarrow R$	$(P \Leftrightarrow Q) \wedge (Q \Leftrightarrow R)$	$(P \Leftrightarrow Q) \wedge (Q \Leftrightarrow R) \Rightarrow (P \Leftrightarrow R)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	F	T	T	T
T	F	F	F	T	F	F	T
F	T	T	F	T	F	F	T
F	T	F	F	F	T	T	T
F	F	T	T	F	F	F	T
F	F	F	T	T	T	T	T

Thus, This is a tautology.

(c)

P	Q	$P \oplus Q$	$\neg Q$	$(P \oplus Q) \oplus \neg Q$	$[(P \oplus Q) \oplus \neg Q] \Leftrightarrow P$
T	T	F	F	F	F
T	F	T	T	F	F
F	T	T	F	T	F
F	F	F	T	T	F

Thus, this is a contradiction.

Problem 2. *Verify the following statements using only identities (see the list posted on MyCourese). Show all of your work and name the identity or identites used in each step.*

(a) $[(P \Rightarrow Q) \wedge P] \Rightarrow Q$ *is a tautology*

(b) $\neg(P \wedge Q) \wedge (Q \Rightarrow P) \equiv \neg Q$

(c) $\neg[(P \vee Q) \vee [(Q \vee \neg R) \wedge (P \vee R)]] \equiv \neg P \wedge \neg Q$

Solution. Solution for problem 2:

(a)

$[(\mathrm{P} \Rightarrow \mathrm{Q}) \wedge \mathrm{P}] \Rightarrow \mathrm{Q}$
$\equiv [(\neg\mathrm{P} \vee \mathrm{Q}) \wedge \mathrm{P}] \Rightarrow \mathrm{Q}$ (Conditional)
$\equiv [(\neg\mathrm{P} \wedge \mathrm{P}) \vee (\mathrm{Q} \wedge \mathrm{P})] \Rightarrow \mathrm{Q}$ (Distributive)
$\equiv [(\mathbb{F}) \vee (\mathrm{Q} \wedge \mathrm{P})] \Rightarrow \mathrm{Q}$ (Complement)
$\equiv (\mathrm{Q} \wedge \mathrm{P}) \Rightarrow \mathrm{Q}$ (Identity)
$\equiv \neg(\mathrm{Q} \wedge \mathrm{P}) \vee \mathrm{Q}$ (Conditional)
$\equiv (\neg\mathrm{Q} \vee \neg\mathrm{P}) \vee \mathrm{Q}$ (DeMorgan's laws)
$\equiv \neg\mathrm{Q} \vee \mathrm{Q} \vee \neg\mathrm{P}$ (Associative)
$\equiv \mathbb{T} \vee \neg\mathrm{P}$ (Complement)
$\equiv \mathbb{T}$ (Domination)
Thus, this is indeed a tautology.

(b)

$\neg(\mathrm{P} \wedge \mathrm{Q}) \wedge (\mathrm{Q} \Rightarrow \mathrm{P})$
$\equiv (\neg\mathrm{P} \vee \neg\mathrm{Q}) \wedge (\mathrm{Q} \Rightarrow \mathrm{P})$ (DeMorgan's laws)
$\equiv (\neg\mathrm{P} \vee \neg\mathrm{Q}) \wedge (\neg\mathrm{Q} \vee \mathrm{P})$ (Conditional)
$\equiv \neg\mathrm{Q} \vee (\neg\mathrm{P} \wedge \mathrm{P})$ (Factorization)
$\equiv \neg\mathrm{Q} \vee \mathbb{F}$ (Complement)
$\equiv \neg\mathrm{Q}$ (Identity)

(c)

$\neg[(\mathrm{P} \vee \mathrm{Q}) \vee [(\mathrm{Q} \vee \neg\mathrm{R}) \wedge (\mathrm{P} \vee \mathrm{R})]]$
$\equiv \neg[[(\mathrm{P} \vee \mathrm{Q}) \vee (\mathrm{Q} \vee \neg\mathrm{R})] \wedge [(\mathrm{P} \vee \mathrm{Q}) \vee (\mathrm{P} \vee \mathrm{R})]]$ (Distributive)
$\equiv \neg[(\mathrm{P} \vee \mathrm{Q} \vee \mathrm{Q} \vee \neg\mathrm{R}) \wedge (\mathrm{P} \vee \mathrm{Q} \vee \mathrm{P} \vee \mathrm{R})]$ (Associative)
$\equiv \neg[(\mathrm{P} \vee \mathrm{Q} \vee \neg\mathrm{R}) \wedge (\mathrm{P} \vee \mathrm{Q} \vee \mathrm{R})]$ (Idempotent)
$\equiv \neg[(\mathrm{P} \vee \mathrm{Q}) \vee (\neg\mathrm{R} \wedge \mathrm{R})]$ (Factorization)
$\equiv \neg[(\mathrm{P} \wedge \mathrm{Q}) \vee \mathbb{F}]$ (Complement)

$\equiv \neg(P \vee Q)$ (Identity)
$\equiv \neg P \wedge \neg Q$ (DeMorgan's laws)

Problem 3. *Of the following conditional and biconditional statements, which are true and which are false? Briefly justify your answers.*

(a) π is an integer if and only if $\sqrt{e+3}$ is a vowel.

(b) $0 > 1$ whenever $2 + 2 = 4$.

(c) If (a) implies (b), then pigs cannot fly.

Solution. Solution for problem 3:

(a)

Let P = "$\sqrt{e+3}$ is a vowel" and Q = "π is an integer".
The statement claims that P $\Leftrightarrow$ Q.
P $\Leftrightarrow$ Q $\equiv$ (P $\Rightarrow$ Q) $\wedge$ (Q $\Rightarrow$ P)
$\equiv$ ($\neg$P $\vee$ Q) $\wedge$ ($\neg$Q $\vee$ P)
$\neg$P = "$\sqrt{e+3}$ is not a vowel". This statement is true.
So ($\neg$P $\vee$ Q) is true.
Also, $\neg$Q = " π is not an integer". This statement is also true.
So ($\neg$Q $\vee$ P) is true as well.
Therefore, ($\neg$P $\vee$ Q) $\wedge$ ($\neg$Q $\vee$ P) is true
Thus, P $\Leftrightarrow$ Q is true.
The statement is true.

(b)

Let P = "2 + 2 = 4" and Q = "0 > 1".
The statement claims that P $\Rightarrow$ Q.
P $\Rightarrow$ Q $\equiv$ $\neg$P $\vee$ Q
$\neg$P = "2 + 2 $\neq$ 4", which is false.
Also, since 0 < 1, Q is false.
$\neg$P $\vee$ Q $\equiv \mathbb{F} \vee \mathbb{F} \equiv \mathbb{F}$
Thus, P $\Rightarrow$ Q is false.
The statement is false.

(c)

Let P = "(a)", Q = "(b)" and R = "pigs cannot fly"
The statement claims that (P $\Rightarrow$ Q) $\Rightarrow$ R.
P $\Rightarrow$ Q $\equiv$ $\neg$P $\vee$ Q,
From (a) and (b), we know that (a) is true and (b) is false.
Therefore, P is true and Q is false.

$\neg$P is false.
$\text{P} \Rightarrow \text{Q} \equiv \neg\text{P} \vee \text{Q} \equiv \mathbb{F} \vee \mathbb{F} \equiv \mathbb{F}$
Then,
$(\text{P} \Rightarrow \text{Q}) \Rightarrow \text{R}$
$\equiv \mathbb{F} \Rightarrow \text{R}$
$\equiv \neg\, \mathbb{F} \vee \text{R}$
$\equiv \mathbb{T} \vee \text{R}$
$\equiv \mathbb{T}$
The statement is true.

Problem 4. *Symbolize the following English sentences in logic, using the abbreviation scheme provided.*

(a) "Thunder only happens when it's raining."

T: thunder happens; R: it's raining

(b) "For every positive integer n there is a prime number that is bigger than n but at most 2n."

I(x) : x is a positive integer; P(x) : x is a prime number; B(x,y) : x is bigger than y.

(c) "Goldbach's Conjecture is true if every even integer greater than 2 can be written as the sum of two primes."

G : Goldbach's Conjecture is true; E(x) : x is an even integer; T(x) : x is greater than 2; P(x) : x is the sum of two primes.

Solution. Solution for problem 4:

(a)

$\mathrm{R} \Rightarrow \mathrm{T}$

(b)

$\forall$ n I(n) $\exists$ x [P(x) $\wedge$ B(x,n) $\wedge$ $\neg$ B(x,2n)]

(c)

$\forall$ x [(E(x) $\wedge$ T(x) $\wedge$ P(x)) $\Rightarrow$ G]