

# Syntax of Propositional Logic

## THE AXIOMS of the DEDUCTIVE SYSTEM:

$$(A_1) \quad \varphi \rightarrow (\varphi \rightarrow \varphi) \quad \text{anything!}$$

$$(A_2) \quad (\varphi \rightarrow (\varphi \rightarrow \chi)) \rightarrow ((\varphi \rightarrow \varphi) \rightarrow (\varphi \rightarrow \chi))$$

$$(A_3) \quad (\neg \varphi \rightarrow \neg \varphi) \rightarrow (\varphi \rightarrow \varphi)$$

## THE RULE OF DEDUCTION: (MODUS PONENS)

$$\frac{\varphi, \varphi \rightarrow \psi}{\psi}$$

## TH. OF DEDUCTION (pg. 102: Th. 2.62)

$$\Gamma \cup \{\varphi\} \vdash \psi \iff \Gamma \vdash \varphi \rightarrow \psi$$

P 2.54 (p. 95)

(i) If  $\varphi$  is an axiom  $\Rightarrow \Gamma \vdash \varphi$

(ii) If  $\varphi \in \Gamma \Rightarrow \Gamma \vdash \varphi$

(iii) If  $\Gamma \vdash \varphi$  and  $\Gamma \vdash \varphi \rightarrow \psi \Rightarrow \Gamma \vdash \psi$  (similar to MP)

S 3.4 (iii) - Reduction to Absurd

$$\begin{cases} \Gamma \cup \{\neg \varphi\} \vdash \varphi \\ \Gamma \cup \{\neg \varphi\} \vdash \neg \varphi \end{cases} \Rightarrow \Gamma \vdash \varphi$$

P 2.55 (p. 97)

(i)  $\vdash \varphi \Rightarrow \Gamma \vdash \varphi$

P 2.61 (p. 101)

$$\vdash \varphi \rightarrow \varphi$$

P 2.64 (p. 105)

$$\begin{cases} \Gamma \vdash \varphi \rightarrow \psi \\ \Gamma \vdash \varphi \rightarrow \chi \end{cases} \Rightarrow \Gamma \vdash \varphi \rightarrow \chi$$

P 2.68 (p. 108)

$$\begin{cases} \Gamma \cup \{\varphi\} \vdash \psi \\ \Gamma \cup \{\neg \varphi\} \vdash \psi \end{cases} \Rightarrow \Gamma \vdash \psi$$

P 2.63

$$\vdash (\varphi \rightarrow \psi) \rightarrow ((\varphi \rightarrow \chi) \rightarrow (\varphi \rightarrow \chi))$$

P 2.65

$$\vdash (\varphi \rightarrow (\psi \rightarrow \chi)) \rightarrow (\varphi \rightarrow (\varphi \rightarrow \chi))$$

P 2.66

$$\Gamma \cup \{\neg \varphi\} \vdash \neg (\varphi \rightarrow \varphi) \Rightarrow \Gamma \vdash \varphi$$

P 2.69 (p. 109)

$$\vdash \varphi \wedge \psi \vdash \varphi$$

$$\vdash \varphi \wedge \psi \vdash \psi$$

$$\vdash \varphi, \psi \vdash \varphi \wedge \psi$$

$$\vdash \varphi, \psi \vdash \chi \iff \vdash \varphi \wedge \psi \vdash \chi$$