

# Syntax of Propositional Logic

THE AXIOMS of the DEDUCTIVE SYSTEM:

- (A1)  $\varphi \rightarrow (\psi \rightarrow \varphi)$  anything!
- (A2)  $(\varphi \rightarrow (\psi \rightarrow \chi)) \rightarrow ((\varphi \rightarrow \psi) \rightarrow (\varphi \rightarrow \chi))$
- (A3)  $(\neg \varphi \rightarrow \neg \psi) \rightarrow (\varphi \rightarrow \psi)$

THE RULE OF DEDUCTION: (MODUS PONENS)

$$\frac{\varphi, \varphi \rightarrow \psi}{\psi}$$

TH. OF DEDUCTION (pg. 102: Th. 2.62)

$$\Gamma \cup \{\varphi\} \vdash \psi \iff \Gamma \vdash \varphi \rightarrow \psi$$

P 2.54 (p. 95)

(i) If  $\varphi$  is an axiom  $\Rightarrow \Gamma \vdash \varphi$

(ii) If  $\varphi \in \Gamma \Rightarrow \Gamma \vdash \varphi$

(iii) If  $\Gamma \vdash \varphi$  and  $\Gamma \vdash \varphi \rightarrow \psi \Rightarrow \Gamma \vdash \psi$  (similar to MP)

P 2.55 (p. 97)

(ii)  $+ \varphi \Rightarrow \Gamma \vdash \varphi$

P 2.61 (p. 101)

$\vdash \varphi \rightarrow \varphi$

P 2.64 (p. 105)

$\{ \Gamma \vdash \varphi \rightarrow \psi \quad \Gamma \vdash \varphi \rightarrow \chi \} \Rightarrow \Gamma \vdash \varphi \rightarrow \chi$

P 2.68 (p. 108)

$\{ \Gamma \cup \{\varphi\} \vdash \psi \quad \Gamma \cup \{\neg \varphi\} \vdash \psi \} \Rightarrow \Gamma \vdash \psi$

S 3.4 (iii) - Reduction to Absurd

$$\{ \Gamma \cup \{\neg \varphi\} \vdash \psi \quad \{ \Gamma \cup \{\neg \varphi\} \vdash \neg \psi \} \vdash \neg \psi \} \Rightarrow \Gamma \vdash \psi$$

P 2.63

$$\vdash (\varphi \rightarrow \psi) \rightarrow ((\varphi \rightarrow \chi) \rightarrow (\psi \rightarrow \chi))$$

P 2.65

$$\vdash (\varphi \rightarrow (\psi \rightarrow \chi)) \rightarrow (\psi \rightarrow (\varphi \rightarrow \chi))$$

P 2.66

$$\Gamma \vdash \neg \varphi \vdash \neg (\varphi \rightarrow \varphi) \Rightarrow \Gamma \vdash \psi$$

P 2.69 (p. 109)

$$\{ \varphi \wedge \psi \} \vdash \psi$$

$$\{ \varphi \wedge \psi \} \vdash \varphi$$

$$\{ \varphi, \psi \} \vdash \varphi \wedge \psi$$

$$\{ \varphi, \psi \} \vdash \chi \Leftrightarrow \{ \varphi \wedge \psi \} \vdash \chi$$