Theory of Interest - Homework #12

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The problems will be left out. I will only provide the solutions. I will be rounding to the hundred thousandths place in the form of percentage.

(8.2) Arbitrage

8.2.1

Mr. Ralbracht has the opportunity to borrow up to $\$20,000$ for two years at an effective interest rate of 2.5%. This same guy has the opportunity to invest in a Treasury note to provide 3.72% annual effective yield.

Let us suppose that Mr. R get a loan for the amount $\$X$ where $0 < X < 20,000$, then we can see that he will have to pay back $X(1.025)^2$ back in two years.

Next it is given that that Mr. R can buy a Treasury note that gives the investor 3.72% yield. Furthermore if Mr. R were to invest $\$X$ into these treasury bonds, then he would have a result of $\$X(1.0372)^2$.

We conclude there exist an arbitrage opportunity, one that will result in 0% risk for profit. This is because Mr. R has to pay back the loan which is less than his profit

$$X(1.025)^2 < X(1.0372)^2$$

(8.3) The term structure of interest rates

8.3.1

Let the price of a one-year, two-year, and three-year $\$10,000$ zero-coupon bond by $\$9,765$, $\$9,428$, and $\$8,986.82$, respectively. We want to find all the forward rates for these prices.

Recall the formula generated in class for forward rates:

$$(1 + f_{[s,t]})^{t-s} = (1 + r_t)^t (1 + r_s)^{-s} \tag{1}$$

First we start off by calculating the spot rates. Recall that the spot rate t is just the forward rate from $[0, t]$. If you don't believe me then verify it for yourself with the equation above setting $s = 0$. The spot rates are much more simple, since they are the annual effective yield of a zero coupon bond lasting t years. Furthermore our spot rates are

$$f[0,1] = \left(\frac{10000}{9765}\right) - 1 \approx 0.02406554$$
$$f[0,2] = \left(\frac{10000}{9428}\right)^{1/2} - 1 \approx 0.02988851$$
$$f[0,3] = \left(\frac{10000}{8986.82}\right)^{1/3} - 1 \approx 0.036250259$$

Using these spot rates and equation (1) above we can calculate for the remaining forward rates

$$\begin{array}{lll} f[0.1] = 2.40655\% & f[0.2] = 2.98885\% & f[0.3] = 3.62503\% \\ f[1.2] = 3.57446\% & f[1.3] = 4.23969\% & \\ f[2.3] = 4.90919\% & & \end{array}$$

8.3.2

We are given all the information for a three-year bond, a two-year zero coupon bond, and a one-year zero coupon bond except the yield rates. We want to calculate the spot rates, in this case of no arbitrage, or equivalently our yield rates. We have enough information to calculate r_1 directly, however without r_1 we cannot calculate r_2, since the two-year bond is not a zero bond, but instead a bond with annual payments. The same is the case for calculating for r_3.

Furthermore we calculate for the first spot rate (we are dividing the redemption amount by the price) to find

$$r_1 = \frac{1000}{974} - 1 \approx 2.66941\%$$

Now with this information we can calculate r_2 using the price equality for a two-year Bond. We calculate for the present value and set it equal to the price

$$988 = \frac{30}{1+r_1} + \frac{1030}{(1+r_2)^2}$$

Solving for r_2 we find that

$$r_2 \approx 3.64757\%$$

Lastly we solve for r_3 in the same way we solved for r_2. We calculate the present value and set it equal to the price to find

$$990 = \frac{40}{1+r_1} + \frac{40}{(1+r_2)^2} + \frac{1040}{(1+r_3)^3}$$

Solving for r_3 we find that

$$r_3 \approx 4.40624\%$$

8.3.3

For this next problem we are given all the spot rates, and asked to calculate the price of a \$1000 6% par-value bond with semiannual coupons. Furthermore as our previous problem we calculate the present value of the coupon payments and redemption payment to solve for the price to find the price. *Note: the payments are semiannual and spot rates are annual interest rates

$$\begin{aligned} P &= \frac{30}{(1+r_1)^{1/2}} + \frac{30}{(1+r_2)^1} + \frac{30}{(1+r_3)^{3/2}} + \frac{1030}{(1+r_4)^2} \\ &= \frac{30}{(1.019)^{1/2}} + \frac{30}{(1.023)^1} + \frac{30}{(1.0265)^{3/2}} + \frac{1030}{(1.0305)^2} \\ &= \$1,057.82 \end{aligned}$$

8.3.4

We have two things going on in this problem. We have the opportunity for a one-year zero coupon bond and a two-year zero coupon bond with spot rates 1.9% and 3.2% respectively. We also have the option for a two year bond with annual coupons with an yield rate of 3.05% annually. From this information we can find that there will be two separate prices yielding in arbitrage. To be more specific suppose we have a \$1000 10% par-bond with annual coupons.

Then we can calculate the price using the zero coupons giving us a price of

$$P = \frac{30}{1.019} + \frac{1030}{(1.032)^2} = \$1,131.07$$

On the other hand the original price of the bond with 3% yield gives us the price

$$P = \frac{30}{1.03} + \frac{1030}{(1.03)^2} = \$1,133.94$$

So what are we going to do? Sell the second bond for $1,133.95 and buy the first one for $1,131.07. Assuming there is no risk for default we just earned a profit of $2.88 by doing nothing, and our remaining payments should cancel out. Hell we might even give the person who owes us the coupon payment the address for the payments we need to pay for the bond, regardless they will be the same amount. Now if we really wanted to get rich we would do this as many times as possible to earn even more money.