Theory of Interest - Homework #10

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April 15, 2016

(6.2) Bond alphabet soup and the basic price formula

6.2.1

A $\$1,000$ 10% ten-year bond has semiannual coupons. It is purchased new at \$880 and is redeemable at $\$1,020$. We want to find the coupon amount and the effective yield rate per coupon period.

First we calculate the coupon amount

$$Fr = (1000)\left(\frac{.10}{2}\right) = \%50$$

Next we calculate the effective yield rate using our cashflow worksheet on our BA II Plus Calculator. We input the following values

$$\begin{aligned} &CF_0 = -880 \\ &C_{01} = 50 && F_{01} = 19 \\ &C_{02} = 1070 && F_{02} = 1 \end{aligned}$$

By pressing [CPT] + [IRR] we find that

$$i \approx 6.109246\%$$

6.2.2

A $\$2,500$ 6.5% eight-year bond has annual coupons. If it is purchased for $\$2,590$, the investor will anticipate a 5.4% annual yield for the eight-year investment. We want to find the redemption amount on this bond. We can solve for the redemption amount by solving for C in our "FRANK" price value equation

$$P = Fra_{\overline{n}|j} + K \tag{1}$$

where $K = Cv_j^n$. Solving for C we find that

$$C = [P - Fra_{\overline{n}|j}](1+j)^n = [2590 - 162.5a_{\overline{8}|5.4\%}](1.054)^8 \approx \$2,370.69$$

6.2.3

A 6% $1,000 par-value bond maturing in eight years and having semiannual coupons is to be replaced by a 5.5% $1,000 par bond, also with semiannual coupons. Both bonds are bought to yield 5% nominal interest convertible semiannually. We want to find in how many years should the new bond mature. We will answer to the nearest half year.

We first solve for the price of the first bond with equation (1) above with our effective interest rate $j = 2.5\%$

$$P_1 = Fra_{\overline{16}|j} + Cv_j^{16} = 30a_{\overline{16}|2.5\%} + 1000(1.025)^{-16} \approx \$1,065.28$$

Next we will use our BA II Plus Calculator to solve for the number of payments it takes to mature by inputting the following values

$$N = ? \qquad I/Y = 2.5 \qquad PV = -1,065.28 \qquad PMT = 27.5 \qquad FV = 1000$$

By pressing [CPT] + [N] we find that

$$N \approx 43 \text{ months} = 21.5 \text{ years}$$

6.2.5

An investor owns a $3,000 par-value 12% bond with semiannual coupons. The bond will mature at par at the end of fourteen years. The investor decides that a ten-year bond would be preferable. Current yield rates are 6% convertible semiannually. The investor uses the proceeds from the sale of the 12% bond to purchase an 8% bond with semiannual coupons, maturing at par at the end of ten years. We want to find the face value of the 8% bond.

By using equation (1) above we solve for the price of the first bond for the effective interest rate $j = 3\%$

$$P_1 = Fra_{\overline{n}|j} + Cv_j^n = 180a_{\overline{28}|3\%} + 3000(1.03)^{-28} = \$4,688.77$$

Next we use this value for P_2 and solve for our face value F_2. Using equation (1) we solve for F_2 to find (Note: $F_2 = C$)

$$F_2 = \frac{P_1}{[ra_{\overline{n|}j} + v^n]} = \frac{P_1}{[(.04)a_{\overline{20|}3\%} + (1.03)^{-20}]} \approx \$4,081.54$$

6.2.6

Christie DeLeon purchased a ten-year $\$1,000$ bond with semiannual coupons for $\$982$. The bond had a $\$1,100$ redemption payment at maturity, a nominal coupon rate of 7% for the first five years, and a nominal coupon rate of $q\%$ for the final five years. Christie calculated that her annual effective yield for the ten-year period was 7.35%. We want to find q.

For this we use our price formula and split up the coupon payments of the annuities for an effective interest rate $j = (1.0735)^{-1/2} - 1$ to find

$$P = Fra_{\overline{10|}j} + F\frac{q}{2}a_{\overline{10|}j}v^{10} + Cv^{20}$$

Solving for q we find that

$$q = \frac{[P - Fra_{\overline{10|}j} - Cv^{20}]2(1+j)^{10}}{Fa_{\overline{10|}j}} = \frac{[982 - 35a_{\overline{10|}j} - 1100(1+j)^{-20}]2(1+j)^{10}}{1000a_{\overline{10|}j}}$$
$$\approx 0.052161608$$

(6.3) The premium discount formula

6.3.1

A $\$3,000$ 9% twelve-year bond with annual coupons is purchased with a discount of $\$57$ and yields 9.1% if held to maturity. We want to find the price.

We use the following equation to solve for price

$$D = C - P = C(j - g)a_{\overline{n|}j} \tag{2}$$

Using equation (2) we solve for C to find that

$$C = \frac{Fr + \frac{D}{a_{\overline{n|}j}}}{j} = \frac{3000(.09) + \frac{57}{a_{\overline{12|}9.1\%}}}{.091} \approx \$3,054.95$$

Using our value for C we solve for the price using equation (2)

$$P = C - D \approx \$2,997.95$$

6.3.2

A $2,000 11% ten-year bond has semiannual coupons and is sold to yield 5.2% convertible semiannually. The discount on the bond is $83.28. We want to find the redemption amount.

Using the previous problem's equation we solve for C once again to find

$$C = \frac{Fr + \frac{D}{a_{\overline{n}|j}}}{j} = \frac{2000(.055) + \frac{83.28}{a_{\overline{20}|2.6\%}}}{.026} \approx \$4,438.18$$

6.3.3

Alicia bought a newly issued $1,000 20% ten-year bond, redeemable at $1,100 and having yearly coupons. It was bought at a premium with a price of $1,400. Alicia immediately took a constant amount D from each coupon and deposited it in a savings account earning 8% effective annual interest, so as to accumulate the full amount of the premium by a moment after the final deposit. We want to find how much Alicia deposited each year in the 8% account.

We note that the premium S

$$S = P - C \tag{3}$$

And we evaluate the equality from the information above to be

$$S = P - C = Ds_{\overline{n}|j}$$

Solving for D we find that

$$D = \frac{P - C}{s_{\overline{n}|j}} = \frac{300}{s_{\overline{10}|8\%}} \approx \$20.71$$

(6.4) Other pricing formulas for bonds

6.4.1

A $1,000 bond with coupon rate of 8% has quarterly coupons and is redeemable after an unspecified number of years at $957. The bond is bought to yield 12% convertible semiannually. If the present value of the redemption amount is $355.40, we want to find the purchase price using the Makeham formula.

Recall the Makeham formula

$$P = \frac{g}{j}(C - K) + K \tag{4}$$

for the interest rate $j = (1.06)^{1/2} - 1 \approx 0.029563014$ and $g = \frac{Fr}{C} = \frac{20}{957} \approx 0.0208$ we find that solving for the price

$$P = \frac{g}{j}(C - K) + K = \frac{g}{j}(957 - 355.40) + 355.40 \approx \$780.68$$

6.4.2

A $\$20,000$ bond has annual coupons and is redeemable at the end of fourteen years for $\$22,600$. It has a base amount equal to $\$18,450$ when purchased to yield 6%. We want to find its base amount if it were purchased to yield 7%.

We recall the handy dandy equation for the base amount G

$$Gj = Fr \tag{5}$$

Furthermore we notice that

$$G_6(.06) = Fr = G_7(.07)$$

and so we use ratios to find the that

$$\frac{G_7}{G_6}\frac{(.07)}{(.06)} = \frac{Fr}{Fr}$$

and solving for G_7 we get

$$G_7 = \frac{G_6(.06)}{(.07)} = \frac{18450(.06)}{(.07)} \approx \$15,814.29$$

(6.5) Bond amortization schedules

6.5.1

A $\$2,000$ bond with semiannual coupons is redeemable for $\$2,100$ in fifteen years. It has a coupon rate of 6.5%. The bond is purchased to yield 8% per annum compounded semiannually. We want to find the price of the bond, the amount for accumulation of discount in the tenth coupon, and the amount of interest in the tenth coupon payment.

First we want to calculate the price of the bond. We will use our equation (1) above. We have

$$P = Fra_{\overline{n}|j} + K = 2000(.0325)a_{\overline{30}|4\%} + 2100(1.04)^{-30} \approx \$1771.45$$

Next it will help us to solve the principal at time 10 and interest in the 10th coupon payment if we solve for the book value at time 9. Recall the equation for book values

$$B_t = Cga_{\overline{n-t}|j} + Cv^{n-t} \tag{6}$$

and so we calculate the book value at time 9 to be

$$B_9 = 2000(.0325)a_{\overline{21}|4\%} + 2100v^{21} \approx \$1833.45$$

and we our book value at time 9 to find the interest in the 10th payment Recall the equation for interest for the t-th payment

$$I_t = B_{t-1}j \tag{7}$$

And we calculate the interest in the 10th payment to be

$$I_{10} = B_9(.04) \approx \$73.34$$

Next we solve for the principal for the 10th payment. Recall that the equation for principal in the t-th payment for a discount bond is

$$P_t = B_9 - Cg \tag{8}$$

And so we calculate

$$P_{10} = B_9 - (2000)(.0325) \approx \$8.34$$

6.5.2

A bond with a face balue of \$6000 and an annual coupon rate of 12% convertible semiannually will mature in ten years for its face balue. If the bond is priced using a nominal yield rate of 6% convertible semiannually, we want to find what the amount of premium is in this bond and the amount of amortization of premium in the 7th coupon.

First we calculate the premium S by using equation (3)

$$S = C(g-j)a_{\overline{n}|j} = 6000(.06 - .03)a_{\overline{20}|3\%} \approx \$2677.95$$

Next we calculate the book value at time 6 to calculate the premium in the 7th year. We use equation (6) to find

$$B_6 = C(g-j)a_{\overline{14}|} + C = 6000(.06 - .03)a_{\overline{14}|} + 6000 \approx \$8033.29$$

Next we calculate for the premium at time 7

$$P_7 = Cg - B_6 j = 6000(.06) - B_6(.03) \approx \$119.00$$

6.5.3

A fifteen-year bond, which was purchased at a premium, has semiannual coupons. The amount for amortization of the premium in the second coupon is \$977.19 and the amount for amortization of premium in the fourth coupon is \$1,046.79. We want to find the amount of the premium.

We are given P_2 and P_4 and these values are equivalent to

$$P_2 = C(g-j)v^{29}$$

$$P_4 = C(g-j)v^{27}$$

Furthermore if we divide P_4 by P_2 we find that

$$\frac{P_4}{P_2} = (1+j)^2$$

solving for j we get

$$j = \sqrt{\frac{P_4}{P_2}} - 1 \approx .0349998$$

Next we solve for $C(g-j)$ in order to solve for the premium. We find that

$$C(g-j) = P_2(1+j)^{29} = 2650.005$$

and so it follows that the calculating the premium we get

$$S = C(g-j)a_{\overline{30|}} \approx \$48739.14$$