Theory of Interest - Homework #4

Thomas Lockwood

February 4, 2016

Section 2.4

Problem 2.4.1

Payments of \$3000 now and \$8000 eight years from now are equivalent to a payment of \$10000 four years from now at either rate i or rate j. Find $|i-j|$. Explain why the yield rate is not unique in this case. If we let $x=(1+i)^4$ we get the polynomial:

$$3000x^2 - 10000x + 8000 = 0$$

Solving the above polynomial gives us the roots $x = 8/6$ or $x = 2$ giving us the interest values $i \approx 0.074568832$ or $i \approx 0.189207115$. Therefore

$$|i-j| \approx .114637183$$

Going to the important fact 2.4.9 in our book the hypothesis for a unique solution requires that all $B_{t_i}(i)$ is positive or negative. However the payment of \$10000 of sets our parity making the terms positive and negative. Thus we cannot find a unique solution.

Problem 2.4.2

Success, Inc. enters into a financial arrangement in which it agrees to pay \$100, 000 today and \$101, 000 two years from now in exchange for \$200, 000 on year from now. Show that there is no yield rate that can be assigned to this two-year transaction. If we let $x=(1+i)$ we get the polynomial (for simplicity sake we will reduce the terms of the polynomial by a factor of 1000):

$$100x^2 + 200x + 101 = 0$$

However this polynomial has complex roots! We can verify this by looking at the determinant:

$$b^2 - 4ac = 200^2 - 4(100)(101) = 200^2 - (200)(202) < 0$$

as we can see the determinant is less than 0, thus there are no real solutions!

Problem 2.4.3

Sigmund, Inc. agrees to pay \$150,000 today and \$40,000 four years from today in return for \$210,000 two years from today. We want to find what the yield rate for this four-year financial arrangement is. If we let $x = (1+i)^2$ we get the polynomial:

$$150x^2 - 210x + 40 = 0$$

We find that the roots to this polynomial are $x \approx .227418437$ or $x \approx 1.172981563$ giving us the interest rates

$$i \approx 0.082858053 \text{ or } i \approx -0.523115908$$

Problem 2.4.4

Firms A, B, C, and D enter into a financial arrangement. Money flush firm A will pay expanding firms B and C each \$1,000,000 today. B will pay D \$2,200,000 three years from today. C will pay B \$800,000 two years from today and D \$350,000 two years from today. Finally, D will pay A \$3,200,000 six years from today. We want to calculate the yield rate or interest rate, to the nearest hundredth of a percent, that each firm experiences over the period of their involvement. We can do this by using our handy dandy BA II Plus Calculator. For the firm A we press the [CF] button and the corresponding values:

$$\begin{aligned} &CF_0 = -2,000,000 && \\ &C_{01} = 0 && F_{01} = 5 \\ &C_{02} = 3,200,000 && F_{02} = 1 \end{aligned}$$

In order to find the yield rate we hit the [IRR] then [CPT] buttons on our calculator. We find that the interest rate is:

$$i_A \approx 0.081483747$$

Next solving for B we use the same method except we will use the corresponding values:

$$\begin{aligned} &CF_0 = 1,000,000 && \\ &C_{01} = 0 && F_{01} = 1 \\ &C_{02} = 800,000 && F_{02} = 1 \\ &C_{03} = -2,200,000 && F_{03} = 1 \end{aligned}$$

Once we compute our yield rate we find that:

$$i_B \approx 0.09751232353$$

Next we shall solve for the yield rate for firm C by using the corresponding values:

$$\begin{aligned} &CF_0 = 1,000,000 && \\ &C_{01} = 0 && F_{01} = 1 \\ &C_{02} = -800,000 - 350,000 = -1,150,000 && F_{02} = 1 \end{aligned}$$

This gives us the yield rate:

$$i_C \approx 0.07238052948$$

Last we will solve for the yield rate for firm D by using the corresponding values:

$$\begin{aligned} &CF_0 = 350,000 && \\ &C_{01} = 2,200,000 && F_{01} = 1 \\ &C_{02} = 0 && F_{02} = 1 \\ &C_{03} = -3,200,000 && F_{03} = 1 \end{aligned}$$

Giving us the yield rate:

$$i_D \approx 0.7495010347$$

Problem 2.4.8

[recommended for those with a BA II Plus calculator] On January 1, Ezequiel opens an account at Friendly Bank. His opening deposit for \$50 and he makes deposits at the end of each quarter for four years, then makes no more deposits. He closes his account after seven years for \$3423.28. We want to find his annual yield rate if his quarterly deposits were \$60 for the first year, \$75 for the second year, \$50 for the third year, and \$300, \$450, \$800, and \$240 in the fourth year. Using our [CF] function on our calculator we input the corresponding values:

$$\begin{aligned}
&CF_0 = 50 & & \\
&C_{01} = 60 & &F_{01} = 4 \\
&C_{02} = 75 & &F_{02} = 4 \\
&C_{03} = 50 & &F_{03} = 4 \\
&C_{04} = 300 & &F_{04} = 1 \\
&C_{05} = 450 & &F_{05} = 1 \\
&C_{06} = 800 & &F_{06} = 1 \\
&C_{07} = 240 & &F_{07} = 1 \\
&C_{08} = 0 & &F_{08} = 11 \\
&C_{09} = 3423.28 & &F_{09} = 1
\end{aligned}$$

Giving us the quarterly yield rate $i_4 \approx 0.01751583335$, however we want to know the annual yield rate. No sweat we can just easily convert between the two giving us:

$$i = (1 + i_4)^4 - 1 \approx 0.07192575$$

Section 2.5

Problem 2.5.1

Angela loans Kathy $\$8,000$. Kathy repays the loan by paying \$6000 at the end of one and half years and \$4000 at the end of three years. The money recieved at $t = 1\frac{1}{2}$ is immediately reinvested at an annual effective interest rate of 6%. We want to find Kathy's annual effective rate of interest and Angela's annual yield. We can solve for Kathy's 3/2 annual effective interest by using our [CF] button on our BA II Plus calculator and inputing the corresponding values:

$$\begin{aligned}
&CF_0 = 8000 & & \\
&C_{01} = -6000 & &F_{01} = 1 \\
&C_{02} = -4000 & &F_{02} = 1
\end{aligned}$$

This gives us the value $i_{3/2} \approx 0.17539$. However we want the annual effective interest rate. We just simply convert to annual interest to find that

$$i_K = (1 + i_{3/2})^{2/3} - 1 \approx 0.113751046$$

Next we want to find Angela's annual yield. We can solve for it by inputting the corresponding values in out [CF] function on our calculator:

$$\begin{aligned} &CF_0 = -8000 && \\ &C_{01} = 0 && F_{01} = 2 \\ &C_{02} = 6000(1.06)^{3/2} + 4000 && F_{02} = 1 \end{aligned}$$

Giving us the annual yield rate:

$$i_A \approx 0.09654635707$$

Problem 2.5.2

Kurt loans Randy \$14000. Randy repays the loan by paying Kurt \$4000 at the end of one year and \$6000 at the end of two years and as well as at the end of three years. The money received at $t = 1$ and $t = 2$ is immediately reinvested at an annual effective interest rate of 3%. Correct to the nearest tenth of a percent. We want to find Randy's annual effective interest rate and Kurt's annual yield. We can solve for Randy's annual effective interest rate by using the [CF] function on our BA II Plus calculator and using the following inputs:

$$\begin{aligned} &CF_0 = 0 && \\ &C_{01} = -4000 && F_{01} = 1 \\ &C_{02} = -6000 && F_{02} = 2 \end{aligned}$$

Furthermore we find, by using [IRR] and [CPT] buttons, that the interest rate is:

$$i_R \approx 0.06547093648$$

We do the same to solve for Kurt's annual yield by inputting the following:

$$\begin{aligned} &CF_0 = 14000 && \\ &C_{01} = -4000 && F_{01} = 1 \\ &C_{02} = -6000 && F_{02} = 2 \end{aligned}$$

Furthermore we find that the interest rate is:

$$i_K \approx 0.05466234658$$

Section 2.7

For this chapter I will use $i_t w$ for the time weighted interest and i as the annual interest.

Problem 2.7.1

On January 1, 1988, Antonio invests \$9400 in an investment fund. On January 1, 1989 his balance is \$10600 and he deposits \$2400. On July 1, 1989 his balance is \$14400 and he withdraws \$1000. On January 1, 1992 his balance is \$$P$. We want to express his annual time-weighted yield as a function of P. We have that:

$$(1+i_{tw})^4 = \left(\frac{10600}{9400}\right)\left(\frac{14400}{13000}\right)\left(\frac{P}{13400}\right)$$

Furthermore doing algebra to solve for i_{tw} we find that:

$$i_{tw} = \sqrt[4]{\left(\frac{10600}{9400}\right)\left(\frac{14400}{13000}\right)\left(\frac{P}{13400}\right)} - 1 = \sqrt[4]{\frac{15,264}{163,748,000}P} - 1 = f(P)$$

Problem 2.7.2

Arthur buys \$2000 worth of stock. Six months later, the value of the stock has risen to \$2200 and Arthur buys another \$1000 worth of stock. After another 8 months, Arthur's holdings are worth \$2700 and he sells off \$800 of them. Ten months later, Arthur finds that his stock has a value of \$2100. We want to (a) compute the annual time-weighted yield rate of the stock over the two-year period, (b) compute the annual dollar-weighted yield for Arthur over the two-year period, and (c) explain whether the answer in (a) or (b) should be larger and why.

(a) In order to find the time-weighted yield rate we solve the following equation

$$i_{tw} = \sqrt{\left(\frac{2200}{2000}\right)\left(\frac{2700}{3200}\right)\left(\frac{2100}{1900}\right)} - 1 \approx 0.012828894$$

(b) In order to find the annual dollar-weighted yield for Arthur we will use our BA II Plus calculator's [CF] function and input the following values: Note*: we will be finding the monthly IRR, and then we shall convert it to annual.

$$\begin{aligned}
&CF_0 = -2000 && \\
&C_{01} = 0 && F_{01} = 5 \\
&C_{02} = -1000 && F_{02} = 1 \\
&C_{03} = 0 && F_{03} = 7 \\
&C_{04} = 800 && F_{04} = 1 \\
&C_{05} = 0 && F_{05} = 9 \\
&C_{06} = 2100 && F_{06} = 1
\end{aligned}$$

Then we use our lovely [IRR] + [CPT] buttons to find that the monthly dollar-weighted yield rate for Arthur is:

$$\frac{i^{(12)}}{12} \approx -0.00175939899$$

And converting to annual yield rate we find that:

$$i = (1 + \frac{i^{(12)}}{12})^{12} - 1 \approx -0.020909679$$

(c) As we can see the interest in (a) was greater than the interest in (b). This will not always be the case. His dollar-weighted yield rate was lower than the time-weighted yield rate. We can see this as the randomness of the stock market, and if Arthur would have invested his money in a better way he could have gotten more money. In fact if he didn't touch his money he would have gotten more money. He did not beat the stock market with his choices of buying and selling stocks.

Problem 2.7.3

Bright Future Investment Fund has a balance of $\$1,205,000$ on January 1. On May 1, the balance is $\$1,230,000$. Immediately after this balance is noted, $\$800,000$ is added to the fund. If there are no further contributions to the fund for the year and the time-weighted annual yield for the fund is 16%, then we want to find what the fund balance is at the end of the year. We use the equation:

$$(1 + i_{tw}) = \left(\frac{1,230,000}{1,205,000}\right)\left(\frac{P}{2,030,000}\right)$$

Next rearranging the equation and solving for P and find that:

$$P = (1.16)\left(\frac{1,205,000}{1,230,000}\right)2,030,000 \approx \$2,306,938.211$$

Section - Chapter 2 Review

Problem 2.R.1

Sohail makes an initial investment of $\$20,000$. In return, he receives $\$4000$ at the end of one year and another $\$18000$ at the end of three years.

(a) Assuming that the investment is made at simple interest rate r, we want to write down an equation of value for the investment and find r. We bring the values to time 0 and use the following equation:

$$20000 = \frac{4000}{1+r} + \frac{18000}{1+3r}$$

and solving for r we get the polynomial:

$$60r^2 + 50r - 2 = 0$$

Which we use the quadratic formula to find the roots, and we find that it gives us only one positive value for r:

$$r \approx 0.038244802$$

(d) Next we want to use the [CF] function in our BA II Plus calculator to compute the [IRR] to the nearest millionth. We plug in the following values on our [CF] chart:

$$\begin{array}{ll} CF_0 = -20,000 & \\ C_{01} = 4000 & F_{01} = 1 \\ C_{02} = 0 & F_{02} = 1 \\ C_{03} = 18,000 & F_{01} = 1 \end{array}$$

And after pressing our [IRR] + [CPT] combination we find that:

$$i \approx 0.03697017203 \approx 3.697017\%$$

Problem 2.R.3

Elyse invests \$16,312 at $t = 0$. In return, she gets \$8000 at $t = 1$ and \$10,000 at $t = 2$. Half of the time 1 payment, she reinvests at an annual effective interest rate of 5%. We want to find her annual yield rate for the two-year period. We use the [CF] function on our calculator and input the following values for the semiannual yield rate:

$$\begin{array}{ll} CF_0 = -16,312 & \\ C_{01} = 0 & F_{01} = 2 \\ C_{02} = 8000(1.05)^{1/2} & F_{02} = 1 \\ C_{03} = 10000 & F_{03} = 1 \end{array}$$

We find that the semi-annual yield rate is:

$$\frac{i^{(2)}}{2} \approx 0.03133126191$$

And converting this to our annual yield rate we find that:

$$i = (1 + \frac{i^{(2)}}{2})^2 - 1 \approx 0.063644172$$

Problem 2.R.4

Sports Manufacturing borrows $1,000,000$ for three years at 6% nominal interest convertible quarterly, and \$500,000 for five years at a 5% effective discount rate. At the end of two years, it makes a \$200,000 three-year loan at 7% annual effective interest. We want to find the annual internal rate of return should Sports Manufacturing report for the combined cash-flows for the 5 year period. As always we run to the [CF] function of our calculator and input the following values:

$$
\begin{aligned}
&CF_0 = 1,000,000 + 500,000 && \\
&C_{01} = 0 && F_{01} = 1 \\
&C_{02} = -200,000 && F_{02} = 1 \\
&C_{03} = -1,000,000(1 + \frac{.06}{4})^{12} && F_{03} = 1 \\
&C_{04} = 0 && F_{04} = 1 \\
&C_{05} = 200,000(1.07)^3 - 500,000(1 - .05)^{-5} && F_{05} = 1
\end{aligned}
$$

and we press [IRR] + [CPT] to find that the yield rate is:

$$i \approx 0.05603487502$$

Problem 2.R.5

Abiyote invested \$24,500 on January 1, 1994 in the Utopia Fund. On May 1, 1995, his balance was \$28,212 and he withdrew \$10,000. On December 1, 1995 his balance was \$15,892, and he deposited \$8000. On January 1, 1997 his balance was \$30,309.

(a) We want to find the annual time-weighted yield for the three year period. We use the equation:

$$i_{tw} = \left(\left(\frac{28,212}{24,500}\right)\left(\frac{15,892}{18,212}\right)\left(\frac{30,309}{23,892}\right)\right)^{1/3} - 1 \approx 0.08426624$$

(b) We want to find an approximate annual dollar-weighted yield received by Abiyote for the next three-year period from January 1, 1994 until January 1, 1997 using (2.6.5) We have that $A = 24,500$, $B = 30,309$, $C_{16/36} = -10,000$, $C_{23/36} = 8000$, and $C = -10,000 + 8000 = -2000$. Using our approximation (2.6.5) we find that:

$$I = B - A - C = 30,309 - 24,500 + 2000 = 7809$$

and furthermore we have:

$$J_3 \approx \frac{I}{A + \sum C_{t_k}(1 - t_k)} = \frac{7809}{24500 - 10,000(1 - \frac{16}{36}) + 8000(1 - \frac{23}{36})} \approx 0.357664122$$

And converting to a single year approximation we find that:

$$J = (1 + J_3)^{1/3} - 1 \approx 0.107296976$$

(c) We want to find the dollar weighted yield received by Abiyote for the three-year period from January 1, 1994 until January 1, 1997, correct to the nearest millionth of a percent. We use the equation:

$$24500 = 10000(1 + i)^{-16/24} - 8000(1 + i)^{-23/24} + 30309(1 + i)^{-3}$$

and solving for i with out calculator we find that:

$$i \approx 0.1068499865$$

(d) We can see that the time-weighted yield rate is lower than the dollar-weighted yield rate. This happened because Abiyote invested their money in a efficient way; that is to say, they put more money in whenever there were gains, and took money out when the losses were at their large. Abiyote would make a good stock broker.