Homework 4

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12.4 Let $H = \pm 1, \pm 1$ be the subgroup of $G = \mathbb{C}^X$ of fourth roots of unity. Describe the cosets of H in G explicitly. Is G/H isomorphic to G?

M.6 Let $a = (a_1, ..., a_k)$ and $b = (b_1, ..., b_k)$ be points in k-dimensional space $\mathbb{R}^k$. A *path* from a to b is a continuous function on the unit interval $[0, 1]$ with values in $\mathbb{R}^k$, a function $X : [0, 1] \to \mathbb{R}^k$, sending $t \rightsquigarrow X(t) = (x_1(t), ..., x_k(t))$, such that $X(0) = a$ and $X(1) = b$. If S is a subset of $\mathbb{R}^k$ and if a and b are in S, define $a \sim b$ if a and b can be joined by a pth lying entirely in S.

(a) Show that $\sim$ is an equivalence relations on S. Be careful to check that any paths you construct stay within the set S.

(b)