

# Sorting and Hashing

See R&G Chapters:  
9.1, 13.1-13.3, 13.4.2



# Why Sort?

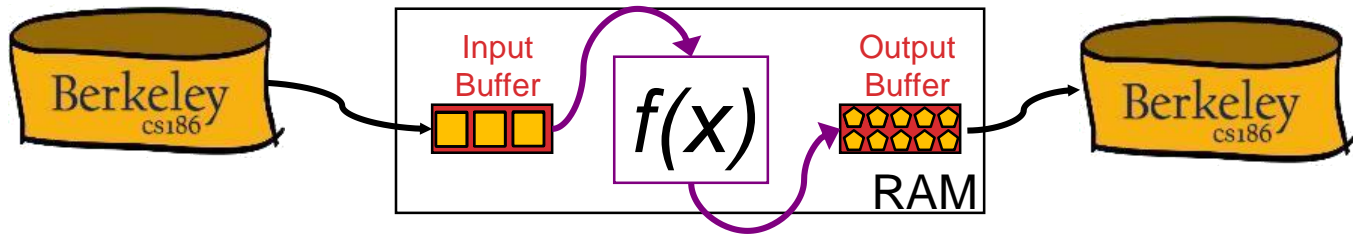
- “Rendezvous”
  - Eliminating duplicates (DISTINCT)
  - Grouping for summarization (GROUP BY)
  - Upcoming sort-merge join algorithm
- Ordering
  - Sometimes, output must be ordered (ORDER BY)
    - e.g., return results ranked in decreasing order of relevance
  - First step in bulk-loading tree indexes
- Problem: sort 100GB of data with 1GB of RAM.
  - why not virtual memory?

# Out-of-Core Algorithms

- Two themes
  1. Single-pass streaming data through RAM
  2. Divide (into RAM-sized chunks) and Conquer

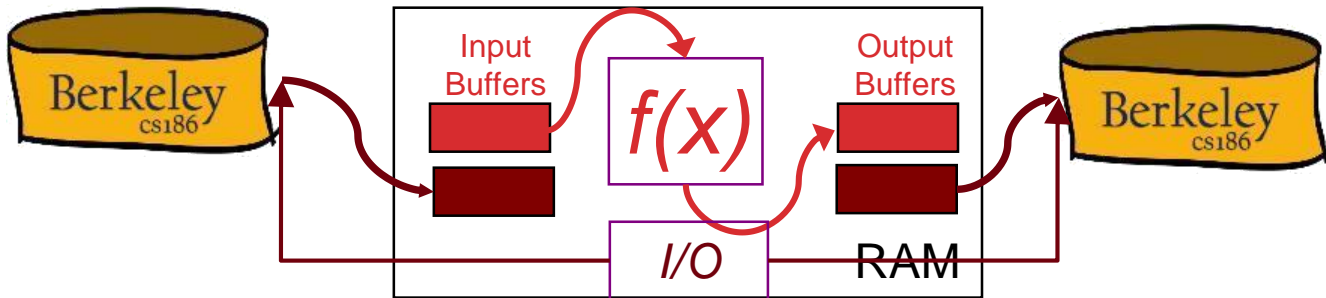
# Single-pass Streaming

- Simple case: “Map”.
  - Goal: Compute  $f(x)$  for each record, write out the result
  - Challenge: minimize RAM, call read/write rarely
- Approach
  - Read a chunk from INPUT to an Input Buffer
  - Write  $f(x)$  for each item to an Output Buffer
  - When Input Buffer is consumed, read another chunk
  - When Output Buffer fills, write it to OUTPUT



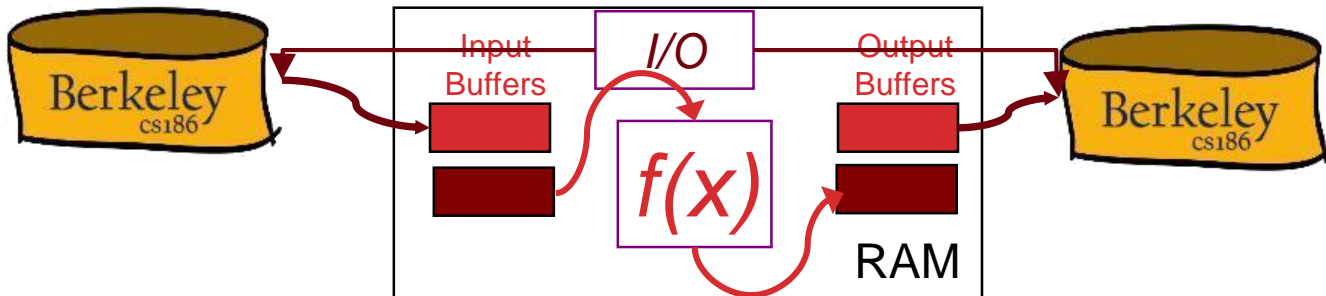
# Better: Double Buffering pt 1

- **Main thread** runs  $f(x)$  on one pair I/O bufs
- 2nd **I/O thread** drains/fills unused I/O bufs in parallel
  - Why is parallelism available?
  - Theme: I/O handling usually deserves its own thread
- Main thread ready for a new buf? Swap!



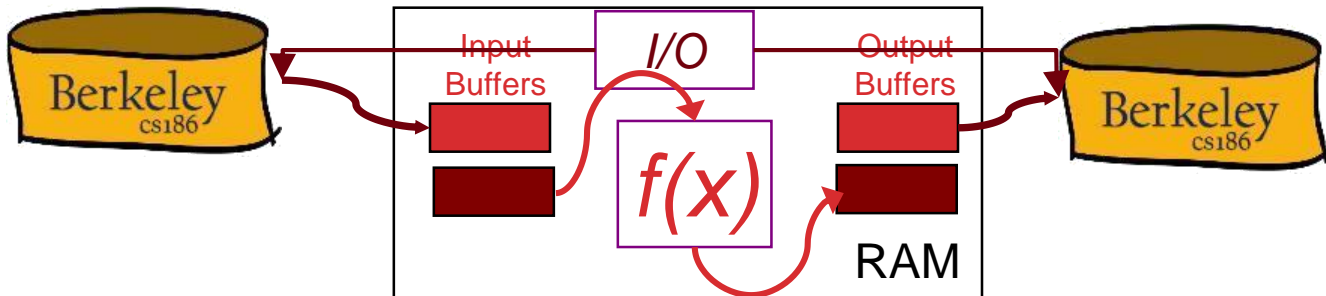
# Better: Double Buffering pt 2

- **Main thread** runs  $f(x)$  on one pair I/O bufs
- 2nd **I/O thread** drains/fills unused I/O bufs in parallel
  - Why is parallelism available?
  - Theme: I/O handling usually deserves its own thread
- Main thread ready for a new buf? Swap!



# Double Buffering applies to all streams

- Usable in any of the subsequent discussion
  - Assuming you have RAM buffers to spare!
  - But for simplicity we won't bring this up again.



# Sorting & Hashing: Formal Specs

## Sorting

- Produce an output file  $F_S$ 
  - with contents  $R$  **stored in order by a given sorting criterion**

## Hashing

- Produce an output file  $F_H$ 
  - with contents  $R$ , **arranged on disk so that no 2 records that have the same hash value are separated by a record with a different hash value.**
  - I.e. matching records are always “stored consecutively” in  $F_H$ .

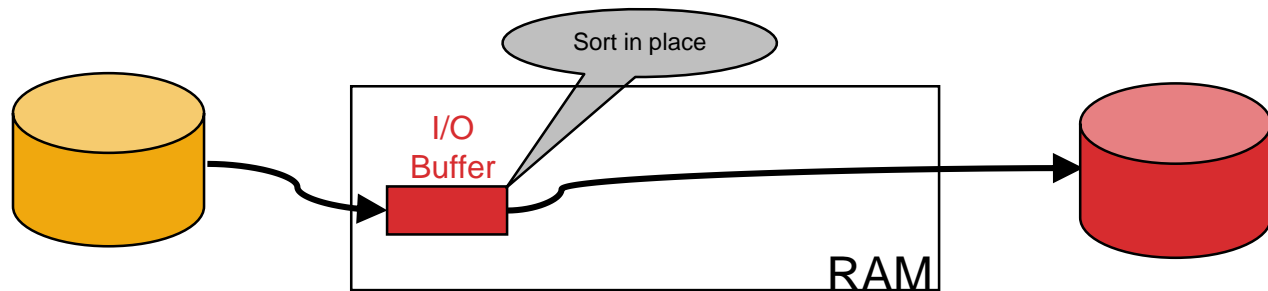
### Given:

- A file  $F$ :
  - containing a multiset of records  $R$
  - consuming  $\mathbf{N}$  blocks of storage
- Two “scratch” disks
  - each with  $\gg N$  blocks of free storage
- A fixed amount of space in RAM
  - memory capacity equivalent to  $\mathbf{B}$  blocks of disk



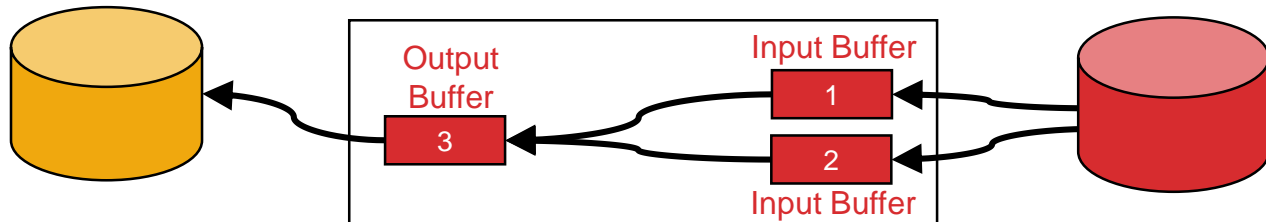
# Sorting: 2-Way (a strawman)

- Pass 0 (conquer a batch):
  - read a page, sort it, write it.
  - only one buffer page is used
  - a repeated “batch job”

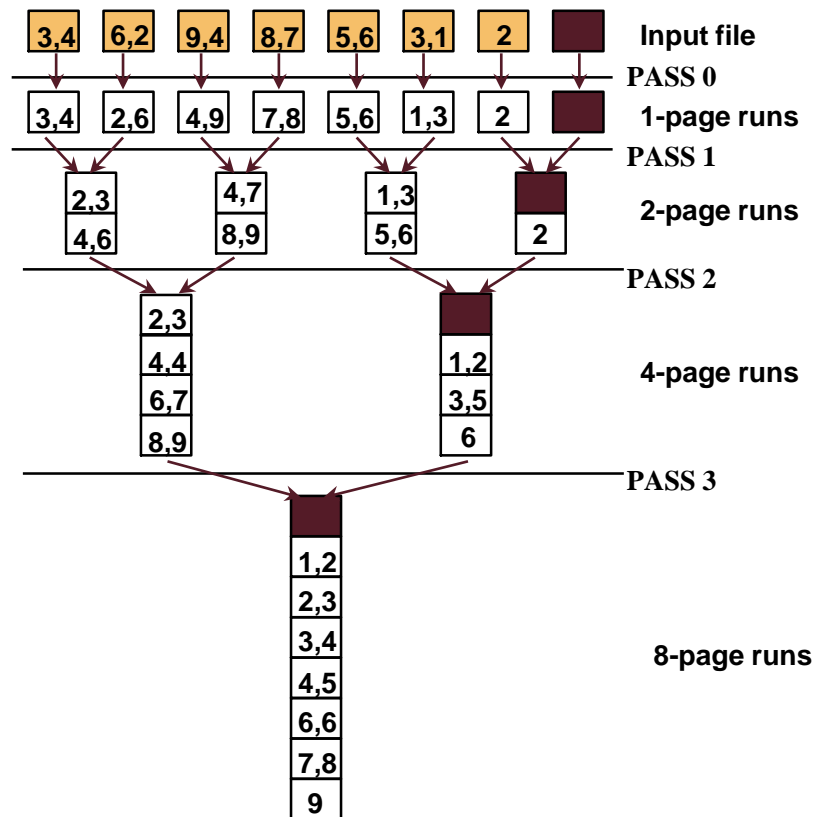


# Sorting: 2-Way (a strawman), cont

- Pass 0 (conquer a batch):
  - read a page, sort it, write it.
  - only one buffer page is used
  - a repeated “batch job”
- Pass 1, 2, 3, ..., etc. (merge via streaming):
  - requires 3 buffer pages
    - note: this has nothing to do with double buffering!
  - merge pairs of runs into runs twice as long
  - a streaming algorithm, as in the previous slide!
    - Drain/fill buffers as the data streams through them



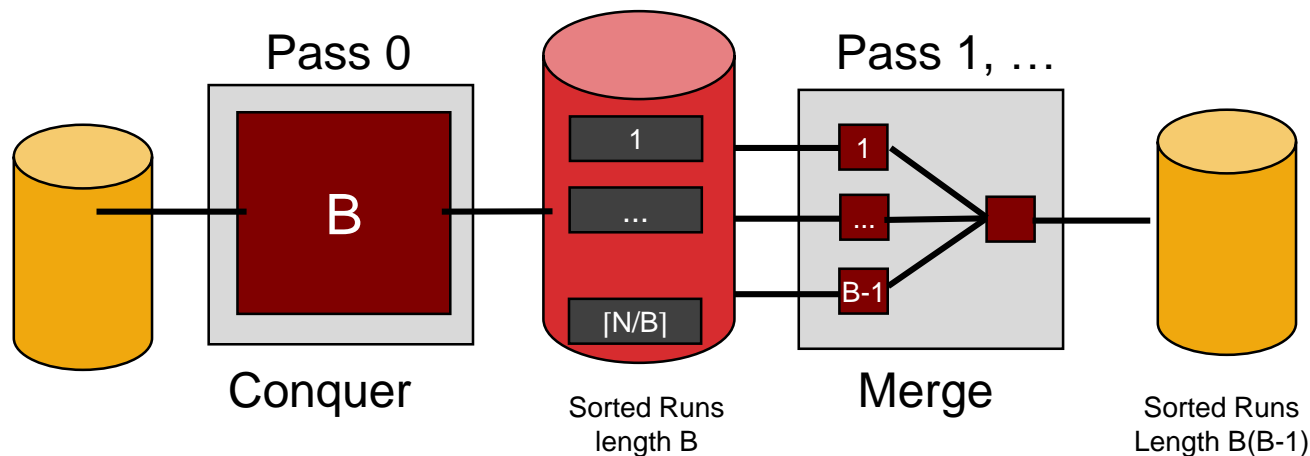
# Two-Way External Merge Sort



- Conquer and Merge:
  - sort subfiles and merge
- Each pass we read + write each page in file ( $2N$ )
- $N$  pages in the file.
  - So, the number of passes is:  $= \lceil \log_2 N \rceil + 1$
- So total cost is:  $2N(\lceil \log_2 N \rceil + 1)$

# General External Merge Sort

- More than 3 buffer pages. How can we utilize them?
  - Big batches in pass 0, many streams in merge passes
- To sort a file with  $N$  pages using  $B$  buffer pages:
  - Pass 0: use  $B$  buffer pages. Produce  $\lceil N / B \rceil$  sorted runs of  $B$  pages each.
  - Pass 1, 2, ..., etc.: merge  $B-1$  runs at a time.



# Cost of External Merge Sort

- Number of passes:  $1 + \lceil \log_{B-1} \lceil N / B \rceil \rceil$
- Cost =  $2N * (\text{\# of passes})$
- E.g., with 5 buffer pages, to sort 108 page file:
  - Pass 0:  $\lceil 108 / 5 \rceil = 22$  sorted runs of 5 pages each
    - last run is only 3 pages
  - Pass 1:  $\lceil 22 / 4 \rceil = 6$  sorted runs of 20 pages each
    - last run is only 8 pages
  - Pass 2: 2 sorted runs, 80 pages and 28 pages
  - Pass 3: Sorted file of 108 pages

Formula check:  $1 + \lceil \log_4 22 \rceil = 1 + 3 \rightarrow \underline{4 \text{ passes}} \quad \checkmark$

# # of Passes of External Sort

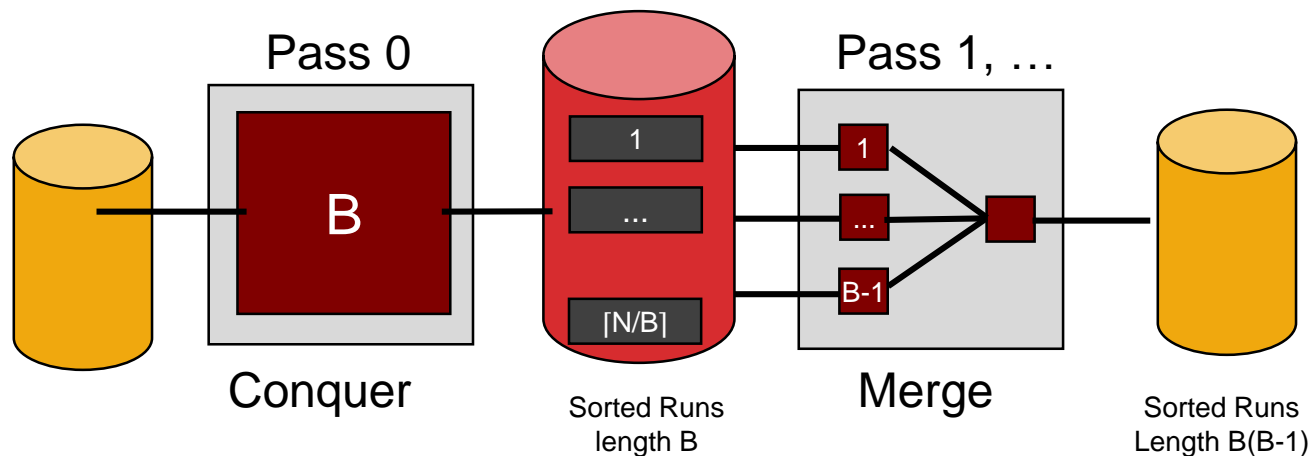


( I/O cost is  $2N$  times number of passes)

N	B=3	B=5	B=9	B=17	B=129	B=257
100	7	4	3	2	1	1
1,000	10	5	4	3	2	2
10,000	13	7	5	4	2	2
100,000	17	9	6	5	3	3
1,000,000	20	10	7	5	3	3
10,000,000	23	12	8	6	4	3
100,000,000	26	14	9	7	4	4
1,000,000,000	30	15	10	8	5	4

# Memory Requirement for External Sorting

- How big of a table can we sort in two passes?
  - Each “sorted run” after Phase 0 is of size  $B$
  - Can merge up to  $B-1$  sorted runs in Phase 1
- Answer:  $B(B-1)$ .
  - Sort  $N$  pages of data in about  $B = \sqrt{N}$  space



# Alternative: Hashing

- Idea:
  - Many times we don't require order
  - E.g.: removing duplicates
  - E.g.: forming groups
- Often just need to rendezvous matches
- Hashing does this
  - But how to do it out-of-core??



# Divide

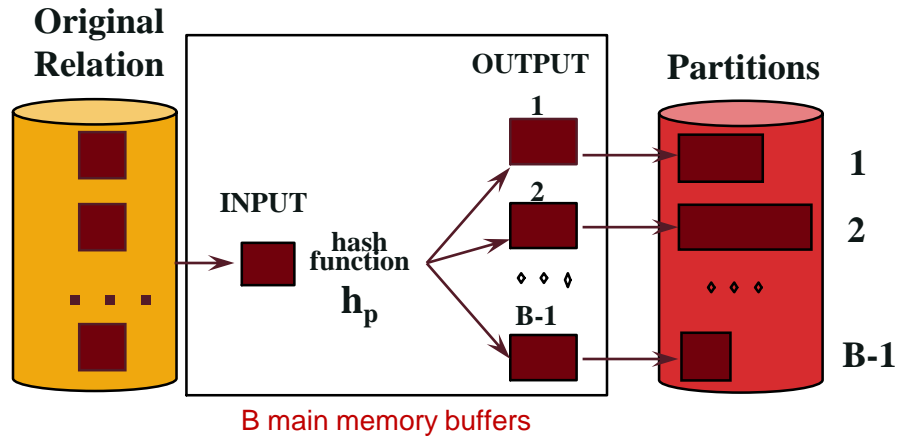
- Streaming Partition (divide):  
Use a hash function  $h_p$  to stream records to disk partitions
  - All matches rendezvous in the same partition.
  - Each partition a mix of values
  - Streaming alg to create partitions on disk:
    - “Spill” partitions to disk via output buffers

# Conquer

- ReHash (conquer):  
Read partitions into RAM hash table one at a time, using hash f'n  $h_r$ 
  - Each bucket contains a small number of distinct values
- Then read out the RAM hash table buckets and write to disk
  - Ensuring that duplicate values are contiguous

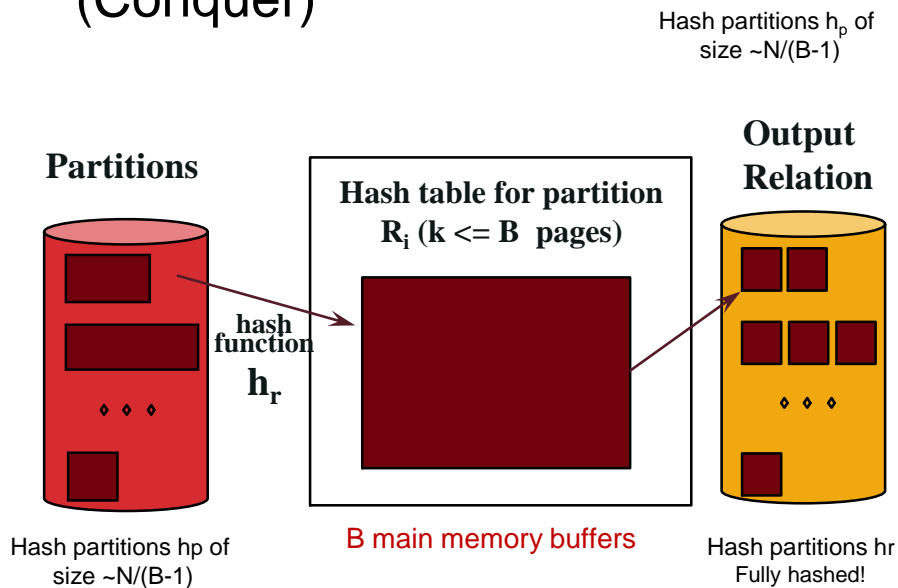
# Two Phases: Divide

- Partition:  
(Divide)

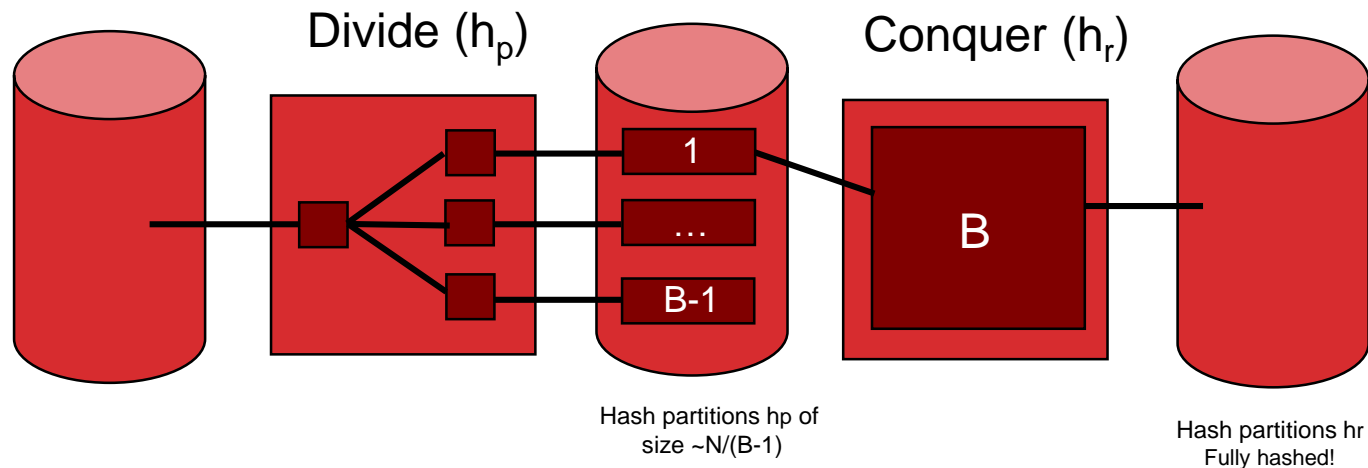


# Two Phases: Conquer

- Rehash:  
(Conquer)



# Cost of External Hashing



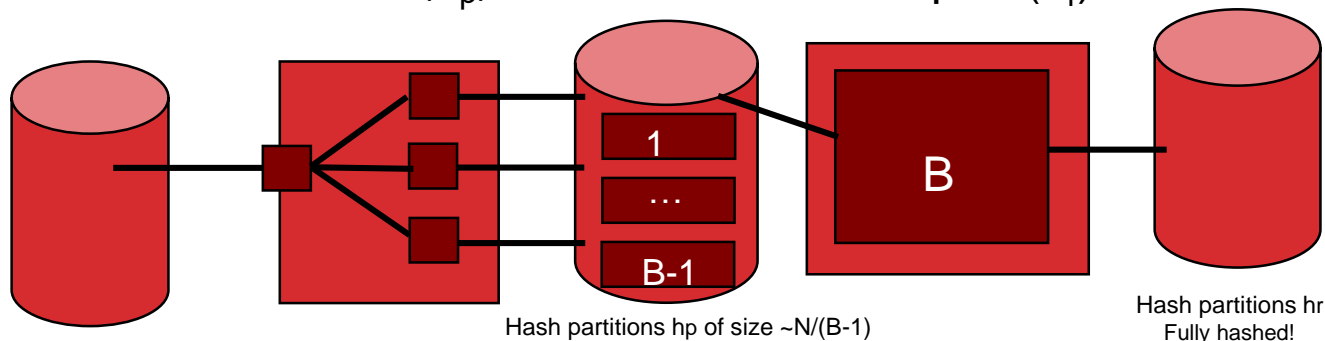
**cost =  $2 * N * (\text{\#passes}) = 4 * N$  IO' s**  
**(includes initial read, final write)**

# Memory Requirement

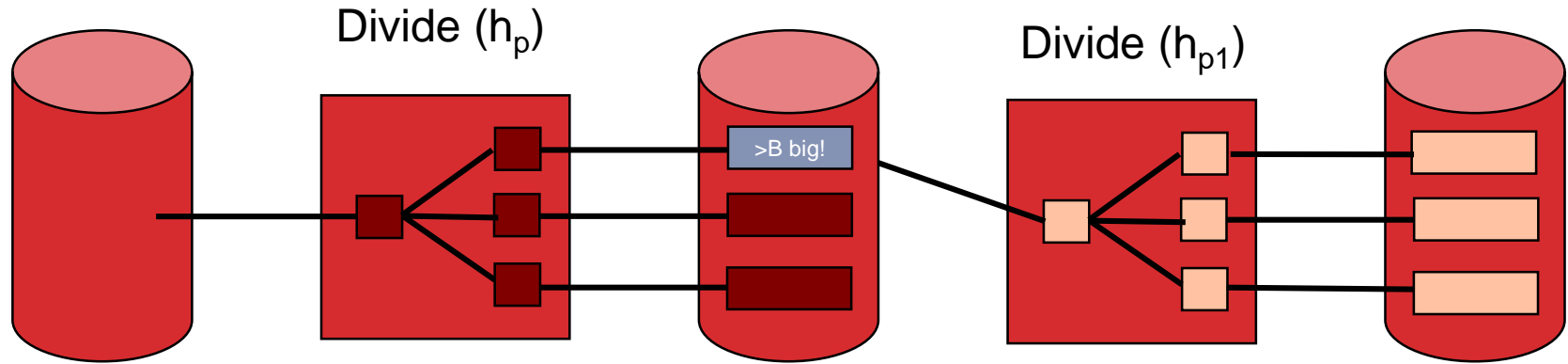
- How big of a table can we hash in two passes?
  - B-1 “partitions” result from Pass 1
  - Each should be no more than B pages in size
  - Answer:  $B(B-1)$ .
    - We can hash a table of size N pages in about  $B = \sqrt{N}$  space
  - Note: assumes hash function distributes records evenly!
- Have a bigger table? Recursive partitioning!

Divide ( $h_p$ )

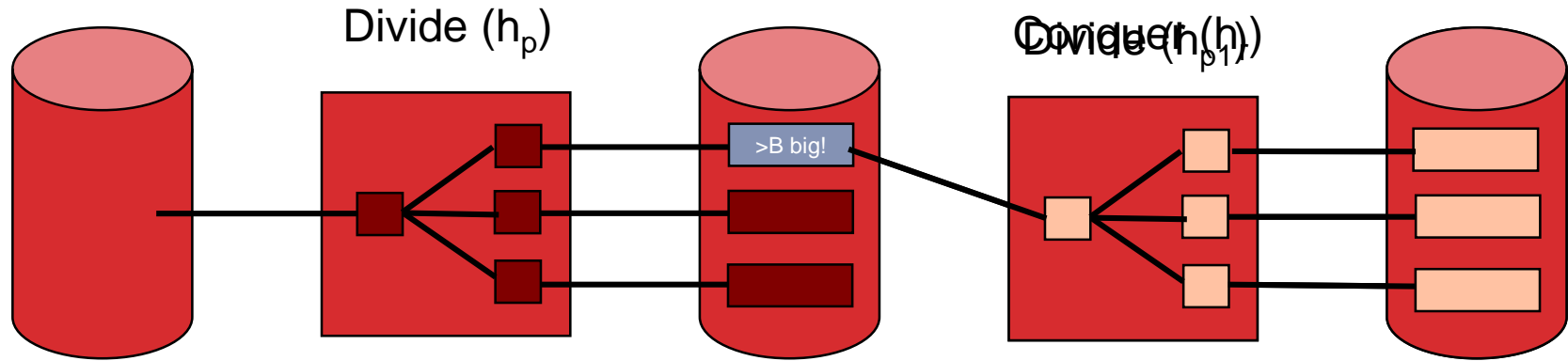
Conquer ( $h_r$ )



# Recursive Partitioning, Pt 1

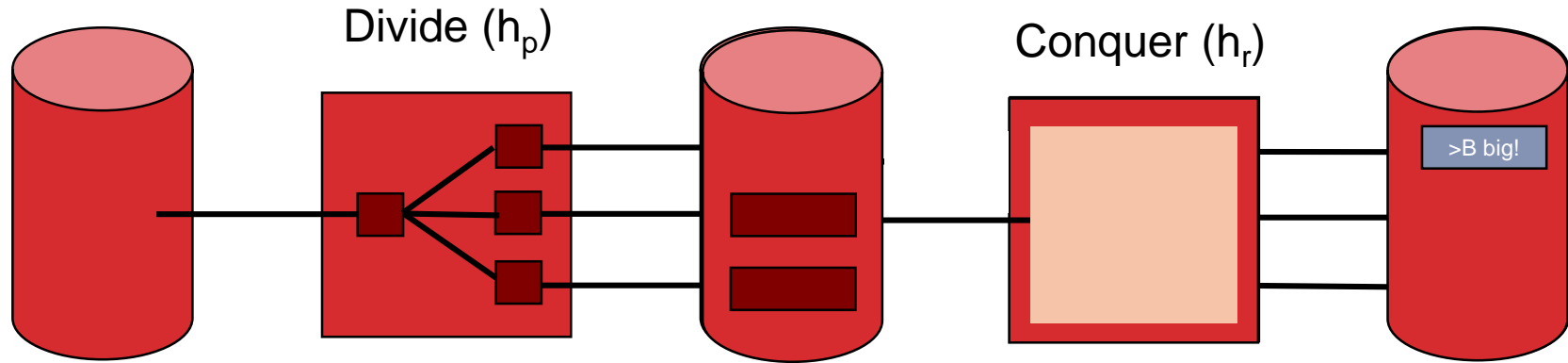


# Recursive Partitioning, Pt 2



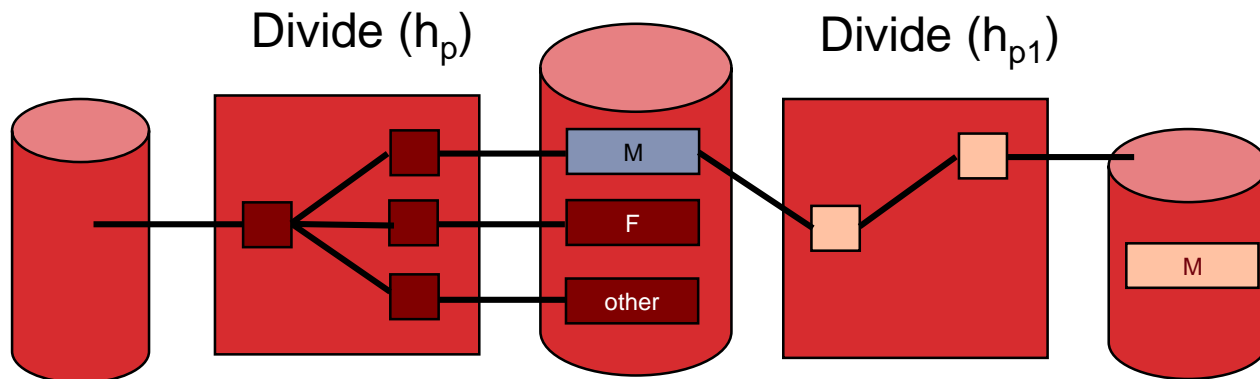


# Recursive Partitioning, Pt 3



# A Wrinkle: Duplicates

- Consider a dataset with a *very* frequent key
  - E.g. in a big table, consider the *gender* column
- What happens during recursive partitioning?

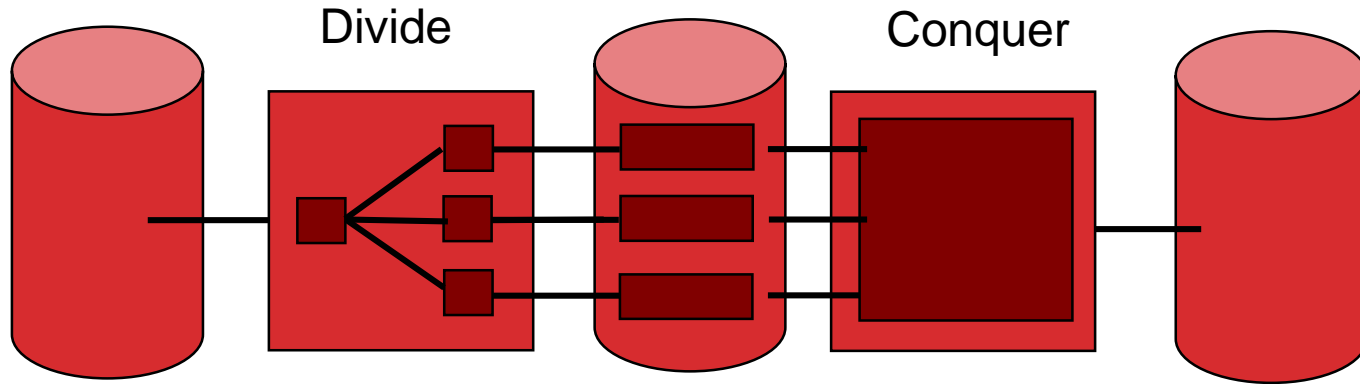


# Question...



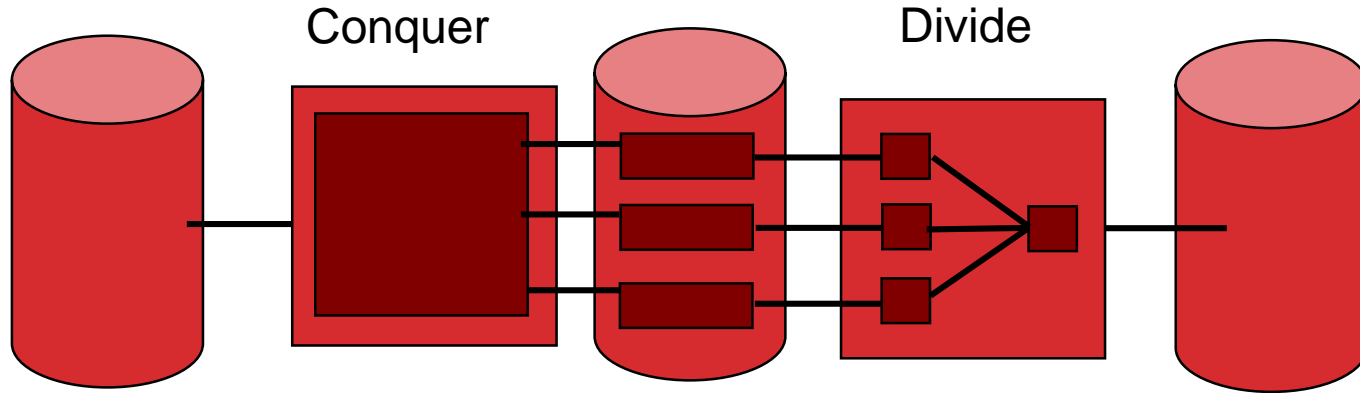
How does external hashing compare  
with external sorting?

# Cost of External Hashing



**cost =  $4 \cdot N$  IO's**  
**(including initial read, final write)**

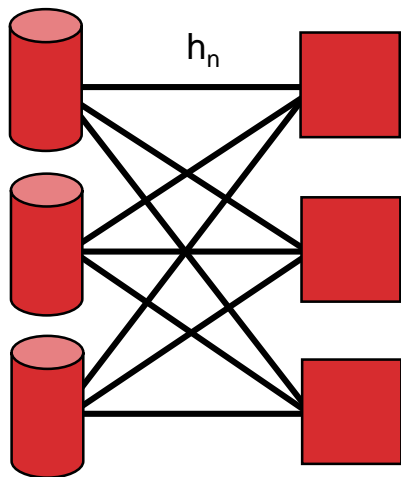
# Cost of External Sorting



**cost =  $4 \cdot N$  IO's**  
**(including initial read, final write)**

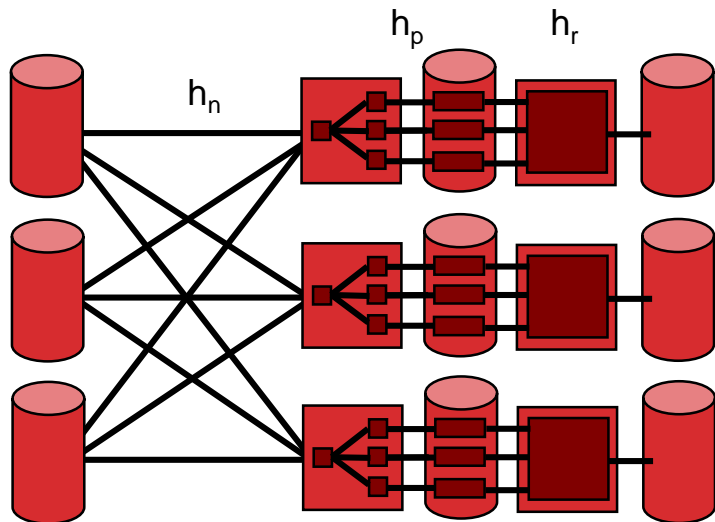
# Parallelize me! Hashing Phase 1

- Phase 1: shuffle data across machines ( $h_n$ )
  - streaming out to network as it is scanned
  - which machine for this record?  
use (yet another) independent hash function  $h_n$



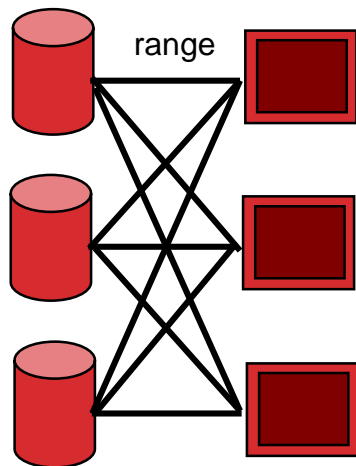
# Parallelize me! Hashing Phase 2

- Phase 1: shuffle data across machines ( $h_n$ )
- Receivers proceed with phase 1 as data streams in
  - from local disk and network



# Parallelize me! Sorting

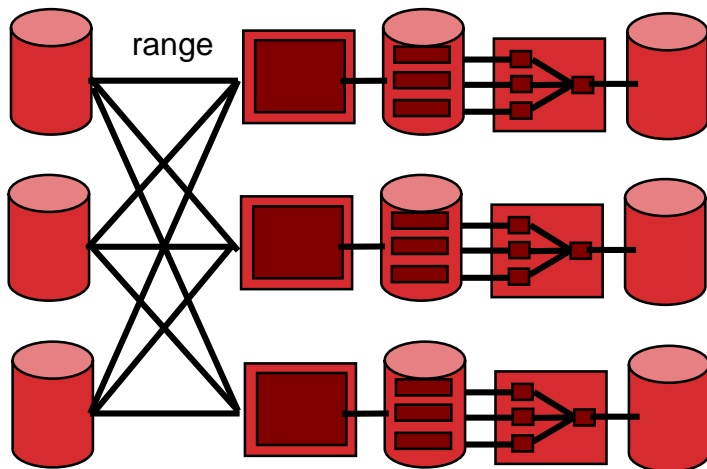
- Pass 0: shuffle data across machines
  - streaming out to network as it is scanned
  - which machine for this record?  
Split on value range (e.g.  $[-\infty, 10]$ ,  $[11, 100]$ ,  $[101, \infty]$ ).





# Parallelize me! Sorting, cont

- Pass 0: shuffle data across machines
- Receivers proceed with pass 0 as the data streams in
- A Wrinkle: How to ensure ranges are the same #pages?!
  - i.e. avoid data skew?



# So which is better ??

- Simplest analysis:
  - Same memory requirement for 2 passes
  - Same I/O cost
  - But we can dig a bit deeper...

# Sorting vs Hashing

- Hashing pros:
  - For duplicate elimination, scales with # of values
    - Delete dups in first pass while partitioning on hp
    - Vs. sort which scales with # of items!
  - Easy to shuffle equally in parallel case
- Sorting pros:
  - Great if we need output to be sorted anyway
  - Not sensitive to duplicates or “bad” hash functions

# Summary

- Sort/Hash Duality
  - Hashing is Divide & Conquer
  - Sorting is Conquer & Merge
- Sorting is overkill for rendezvous
  - But sometimes a win anyhow
- Don't forget one pass streaming and double buffering
  - Can “hide” the latency of I/O behind CPU work