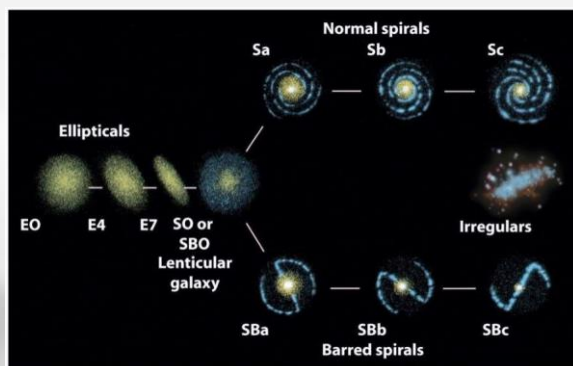


Galactic dynamics and models of galaxies

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at IF-SUT School, Phitsanulok
19th February 2023

Hubble sequence



Galaxy can be classified by **Hubble sequence** (or Hubble's tuning-fork diagram). From left to right, galaxies are identified as **early type** to **late type**.

- E = Elliptical
- S = Spiral
- SO, SB0 = Lenticular
- Irr = Irregular

cr: <https://www.physast.uga.edu/~rls/>

Elliptical galaxies

Surface brightness of elliptical galaxies along the axis can be fitted by *de Vaucouleurs law* (or $R^{1/4}$ law)

$$I(R) = I_e e^{\{-7.67[(R/R_e)^{1/4} - 1]\}}$$

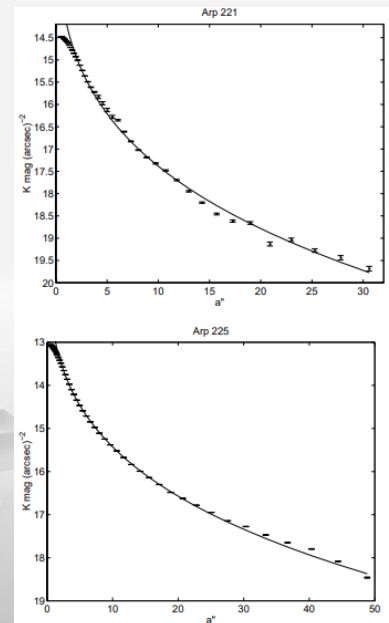
where R_e = effective radius

I_e = constant.

de Vaucouleurs profile can be generalized to *Sersic profile*

$$I(R) = I_e e^{\{-b_n[(R/R_e)^{1/n} - 1]\}}$$

where $b_n \approx 2n - \frac{1}{3}$. Note that the situation for cD or dwarf galaxies is different.



Surface brightness and de Vaucouleurs profile

cr: Chitre & Jog (2002) A&A

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Elliptical galaxies

Central velocity dispersion of massive elliptical galaxies exhibits simple scaling known as *Faber-Jackson relation*

$$L \propto \sigma_0^4$$

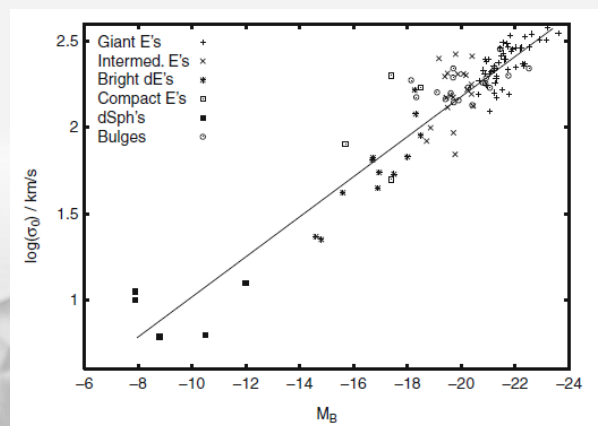
where L = luminosity

σ_0 = central velocity dispersion.

Note that

$$\mathcal{M} \propto \log(L)$$

where \mathcal{M} = absolute magnitude.



Faber-Jackson relation in various elliptical systems

cr: Bender et al. 1992, ApJ

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Elliptical galaxies

Elliptical galaxies exhibits great varieties in size, spectrum, luminosity due to different origins. Rough classification is as follow

- Normal elliptical (E or gE)
- Dwarf elliptical (dE): small mass and low luminosity
- cD elliptical: very luminous and large in the center of galaxy cluster
- Blue compact dwarf elliptical (BCD): with many young blue star.
- Dwarf spheroidal elliptical (dSph): dwarf close to spheroidal

Table 3.1 Characteristic values for early-type galaxies

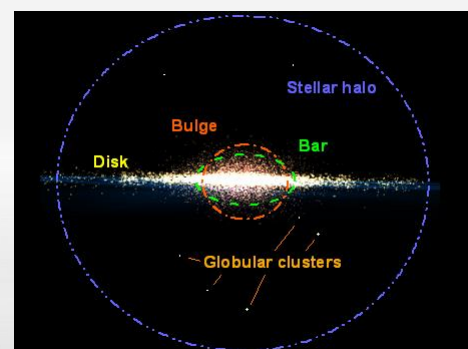
	S0	cD	E	dE	dSph	BCD
M_B	-17 to -22	-22 to -25	-15 to -23	-13 to -19	-8 to -15	-14 to -17
$M (M_\odot)$	10^{10} – 10^{12}	10^{13} – 10^{14}	10^8 – 10^{13}	10^7 – 10^9	10^7 – 10^8	$\sim 10^9$
D_{25} (kpc)	10–100	300–1000	1–200	1–10	0.1–0.5	<3
$\langle M/L_B \rangle$	~ 10	>100	10–100	1–10	5–100	0.1–10
$\langle S_N \rangle$	~ 5	~ 15	~ 5	4.8 ± 1.0	–	–

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Spiral galaxies

In general, major components of elliptical galaxies include

- Disk: flattened component where spiral arms or bar reside.
- Bulge: central ellipsoidal component.
- Halo: extended envelop.



Components of spiral (or lenticular) galaxy

cr: <https://kof.zcu.cz/st/dis/schwarzmeier/>

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Spiral galaxies

Galactic bulge follows de Vaucouleurs profile similar to elliptical galaxy, i.e.

$$I_{bulge}(R) = I_e e^{-7.67[(R/R_e)^{1/4} - 1]}$$

where R_e = effective radius

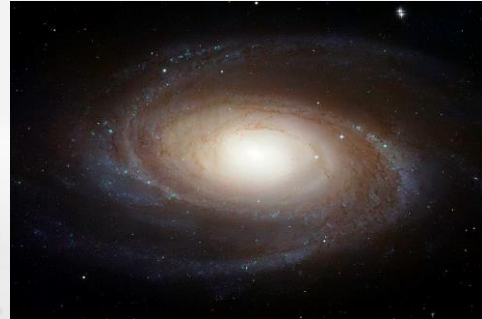
I_e = constant.

Galactic disk brightness follows exponential profile, i.e.

$$I_{disk}(R) = I_d e^{-(R/R_d)}$$

where R_d = disk scale length

I_d = constant.



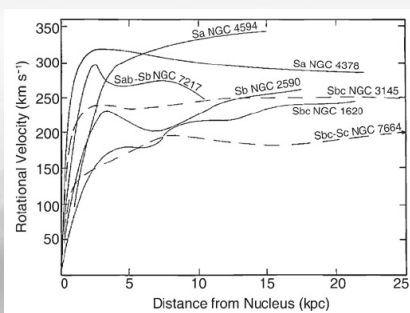
M81

cr: <https://kof.zcu.cz/st/dis/schwarzmeier/>

7

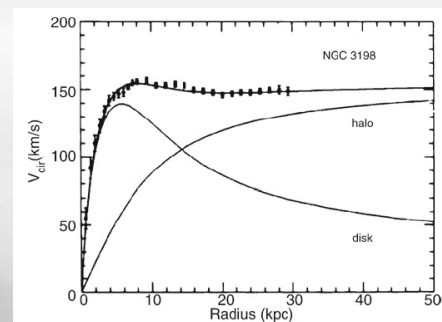
Spiral galaxies

Rotation curve of spiral galaxies is flat at large distance. This implies the embedding dark matter.



Rotation curve of spiral galaxies

cr: Rubin et al. (1978) ApJ



Reconstruction of rotation curve

cr: van Albada et al. (1985) ApJ

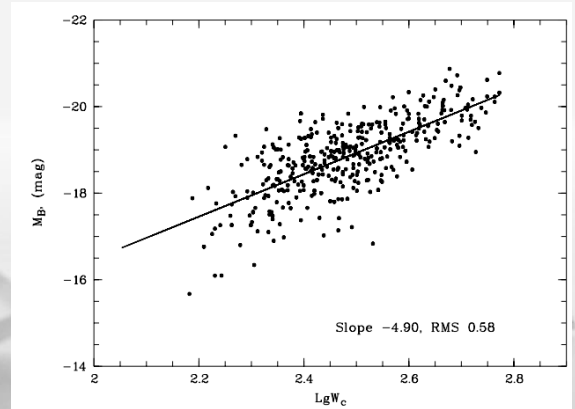
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Spiral galaxies

Maximum rotational velocity correlates with luminosity (i.e. **Tully-Fisher relation**)

$$L \propto v_{\max}^{\alpha}$$

where $\alpha \sim 4$.



Tully-Fisher relation

cr: Karachentsev et al. (2002) A&A

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Spiral galaxies

Distinction from Sa to Sc is mainly from bulge fraction and opening angle of spiral arms.

Table 3.2 Characteristic values for spiral galaxies

	Sa	Sb	Sc	Sd/Sm	Im/Ir
M_B	-17 to -23	-17 to -23	-16 to -22	-15 to -20	-13 to -18
$M (M_{\odot})$	10^9 - 10^{12}	10^9 - 10^{12}	10^9 - 10^{12}	10^8 - 10^{10}	10^8 - 10^{10}
$\langle L_{\text{bulge}}/L_{\text{tot}} \rangle_B$	0.3	0.13	0.05	—	—
Diam. (D_{25} , kpc)	5-100	5-100	5-100	0.5-50	0.5-50
$\langle M/L_B \rangle (M_{\odot}/L_{\odot})$	6.2 ± 0.6	4.5 ± 0.4	2.6 ± 0.2	~ 1	~ 1
V_{\max} range (km s^{-1})	163-367	144-330	99-304	—	50-70
Opening angle	$\sim 6^{\circ}$	$\sim 12^{\circ}$	$\sim 18^{\circ}$	—	—

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Introduction – Long-range interacting system

Given a d -dimensional pair-potential in the form

$$V \propto \frac{1}{r^\alpha}$$

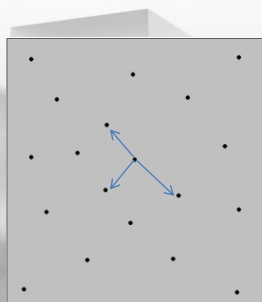
the interaction is classified as

- Short-range interaction (SRI) if $\alpha > d$
- Long-range interaction (LRI) if $\alpha \leq d$

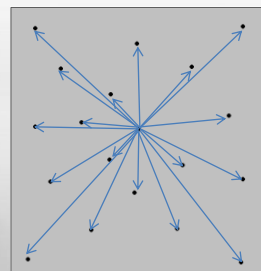
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Introduction – Long-range interacting system

Potential energy of long-range interacting system diverges as system size goes to infinity, with constant density.



short range

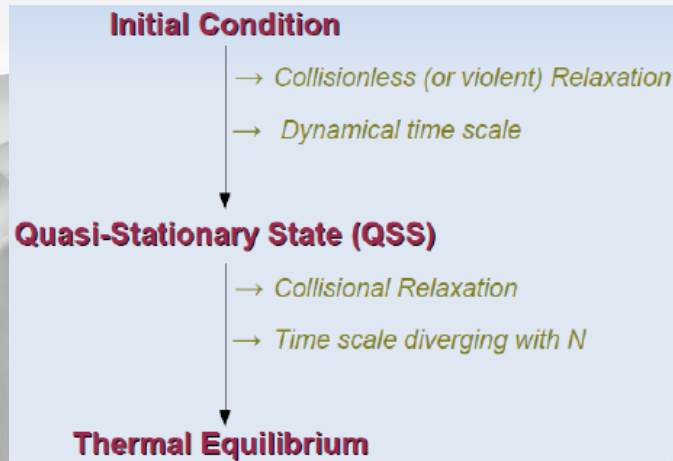


long range

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Introduction – Long-range interacting system

Current understanding suggests the following relaxation scheme for systems governed by gravity



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Introduction – Long-range interacting system

Estimate of violent relaxation time scale using the rate of mean-field fluctuation gives

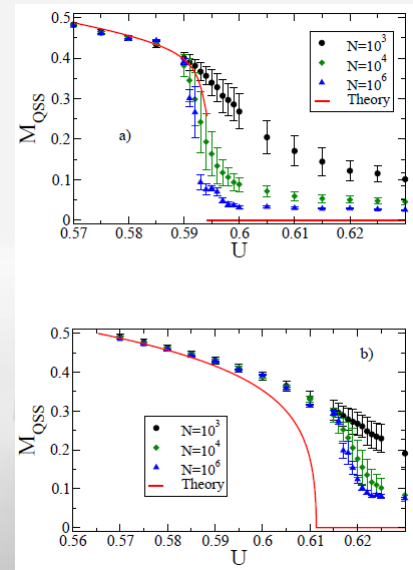
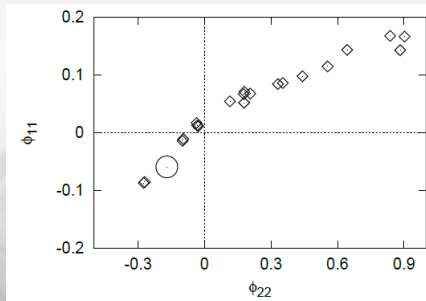
$$t_{relax} = \frac{3}{4} \left[\frac{\dot{\Phi}^2}{\Phi^2} \right]^{-\frac{1}{2}} = \frac{3}{\lambda} \sqrt{\frac{3}{32\pi G \bar{\rho}}}$$

which is of order free-fall time and does not depend on particle number, N.

Typical galaxies have free-fall time of order 10^7 - 10^8 years.

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Diversity of the QSSs



(Antoniazzi et al. 2007)

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Introduction – Long-range interacting system

Estimate of collision rate demonstrates that relaxation time scale to thermal equilibrium diverges with particle number, N , as

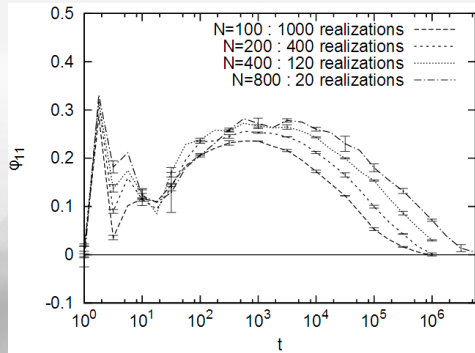
$$t_{relax} \sim \frac{N}{8 \ln N} t_{cross}$$

(see [Chandrasekhar 1943](#))

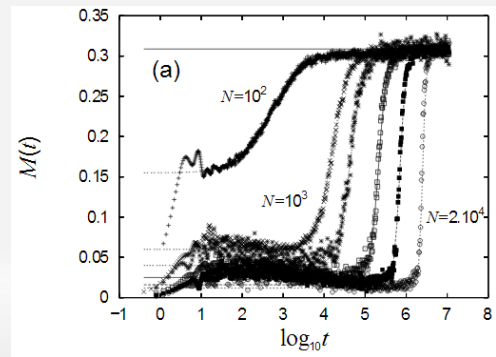
*Massive galaxies are currently in
quasi-stationary states*

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Collisional relaxation to thermal equilibrium



1D gravity
(Joyce & Worrakitpoonpon 2010)



HMF model
(Yamaguchi et al. 2004)