Mixed Integer Linear Programming for Aircraft Carrier Deck Operations

I. Personnel

Green Maintainers: repair the aircraft
Red: load and unload weapons on the aircraft
Grape: fuel the aircraft
Brown: plane Captains
Blue: chocks and chains
Yellow: escort the aircraft from parking station to the nearest catapult available while avoiding collision
Green Checkers: checks weight of an aircraft
Green Operators: catapult repair and hold back bar installation
Yellow catapult officer

II. Equations

In the following equations i is used for indexing planes; j indexes over operations; k indexes over machines of some type K which is fixed for each operation j; J represents the phase which some operation j belongs to; and p represents the position of some operation/phase for some plane on a particular machine.

Table I describes all the operations and Table II lists all the phases and the operations that lie within each phase.

TABLE I
Job Operations

Operation ID	Operation
0	Maintain
1	Ordnance
2	Fuel
3	Chocks
4	Direct
5	Catapult operate
6	Weight check
7	Catapult Officer
8	Flying
9	Land gear
10	Landing
11	Parking
12	Chaining
13	Unloading Equipment

The constants t_K^j denote the time it takes for the j^{th} operation to be performed on machines of type K. We also define indicator constants, O_j^J which indicates that whether j^{th} operation belongs to phase J according to Table II.

$$O_j^J = \begin{cases} 1 & \text{if } j^{th} \text{ operation belongs to } J^{th} \text{ phase} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Z_{ikp}^j is the indicator variable (binary) when the j^{th} operation of plane i is performed on machine $k \in K_j$ at p^{th} position. For the catapult, captain and landing track, we similarly define $Z_{ikp}^{catapult}$, $Z_{ikp}^{captain}$ and $Z_{ikp}^{landing}$ respectively.

TABLE II
Phases

Phase ID	Phase	Operations
0	Startup	0,1,2
1	Chock-n-Chain	3
2	Director	4
3	Catapult Launch	5,6,7
4	Fly	8
5	Landing Gear	9
6	Land	10
7	Park	11
8	Chock-n-Chain	12
9	Unload Weapon	13

M_{kp} is the start time of some operation that machine $k \in K$ performs at p^{th} position. For the catapults, captains and landing track, this is denoted by $M_{kp}^{catapult}$, $M_{kp}^{captain}$, and $M_{kp}^{landing}$ respectively.

B_i^J represents the start time of the J^{th} phase for i^{th} plane.

t_{ikp}^j represents the time that k^{th} machine takes to perform j^{th} operation for i^{th} plane at p^{th} position.

y_{p_i,p_j}^{c1} is the indicator variable (binary) that the launch of a plane which is at position p_i on catapult c_1 occurs before the launch of plane at position p_j on catapult c_2, and vice versa for y_{p_i,p_j}^{c2}. We define y_{p_i,p_j}^{c3} and y_{p_i,p_j}^{c4} similarly. V is a very large number.

$$\begin{aligned} \sum_{k \in K} \sum_p Z_{ikp}^j &= 1, \qquad \forall i, j \\ \sum_{k \in K} \sum_p Z_{ikp}^{catapult} &= 1, \qquad \forall i \\ \sum_{k \in K} \sum_p Z_{ikp}^{captain} &= 1, \qquad \forall i \\ \sum_{k \in K} \sum_p Z_{ikp}^{land\_track} &= 1, \qquad \forall i \end{aligned} \tag{2}$$

$$t_{ikp}^j = Z_{ikp}^j t_K^j, \text{ where } k \text{ is a machine of type } K \quad \forall i, j, K, p \tag{3}$$

$$O_j^J t_{ikp}^j \leq B_i^{J+1} - B_i^J, \qquad \forall J \tag{4}$$

$$\sum_i Z_{ikp}^j \leq 1, \qquad \forall j, k, p \tag{5}$$

Fig. 1. Aircraft Carrier Deck

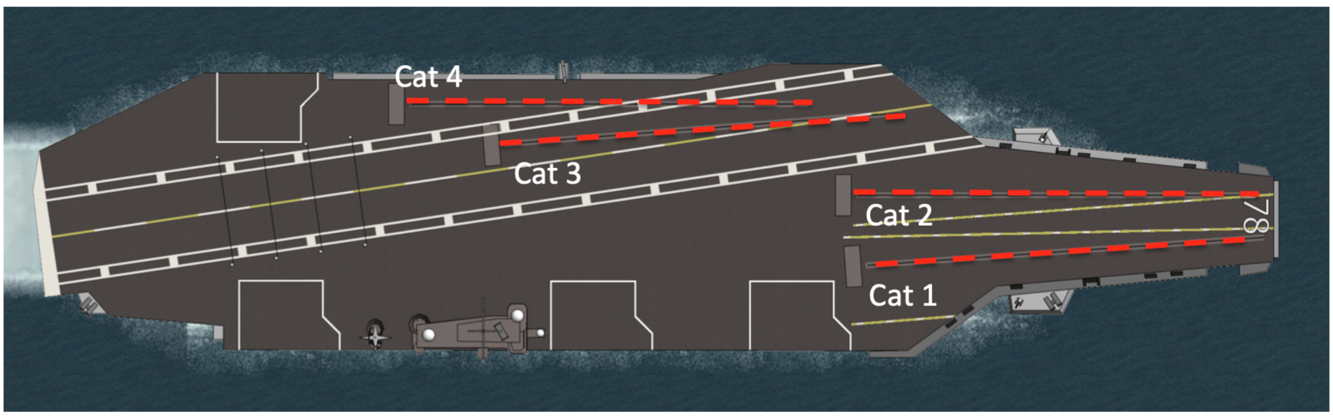


Fig. 2. Aircraft Carrier Deck catapults (taken from [2])

$$\sum_i Z^j_{ikp} \geq \sum_i Z^j_{ik(p+1)}, \qquad \forall j,k,p \tag{6}$$

We have similar constraints on $Z^{catapult}_{ikp}$, $Z^{captain}_{ikp}$ and $Z^{landing}_{ikp}$ for catapult, captain and landing track respectively.

$$t^j_{ikp} \leq M_{k(p+1)} - M_{kp}, \qquad \forall k,p \tag{7}$$

We have similar constraints for $M^{catapult}_{kp}$, $M^{captain}_{kp}$, $M^{landing}_{kp}$ for catapult, captain and landing track respectively.

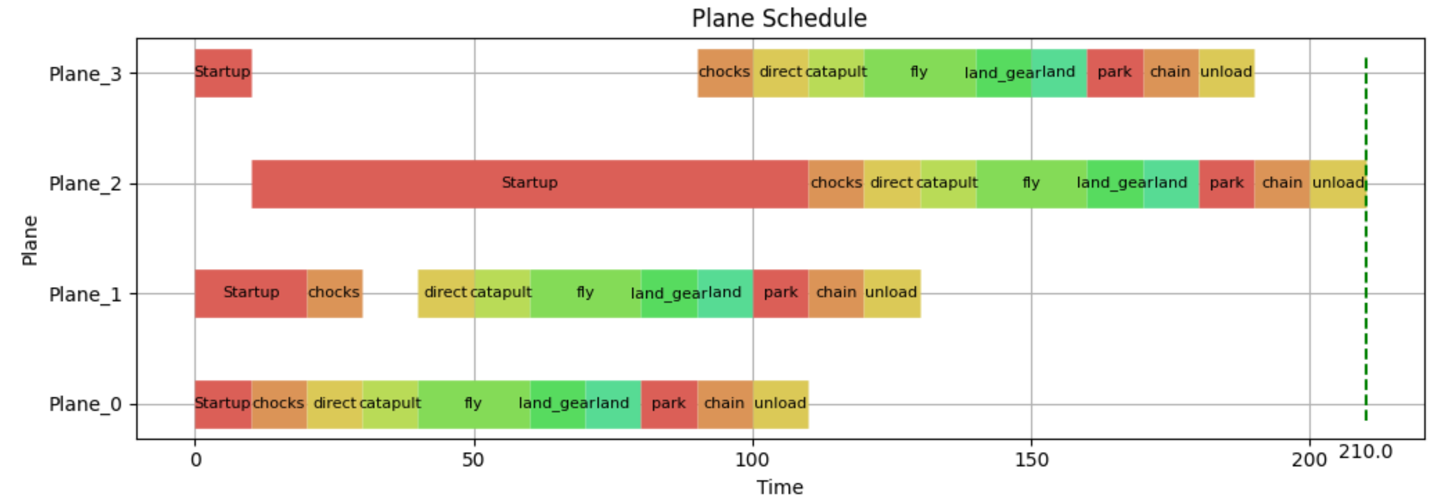
$$M_{k(p+1)} - B^J_i - t^j_{ikp} + V(1 - Z^j_{ikp}) \geq 0 \tag{8a}$$
$$\forall i,j,k,p \text{ and } J^{th} \text{ phase has the } j^{th} \text{ operation}$$
$$B^{J+1}_i - M_{kp} - t^j_{ikp} + V(1 - Z^j_{ikp}) \geq 0 \tag{8b}$$
$$\forall i,j,k,p \text{ and } J^{th} \text{ phase has the } j^{th} \text{ operation}$$

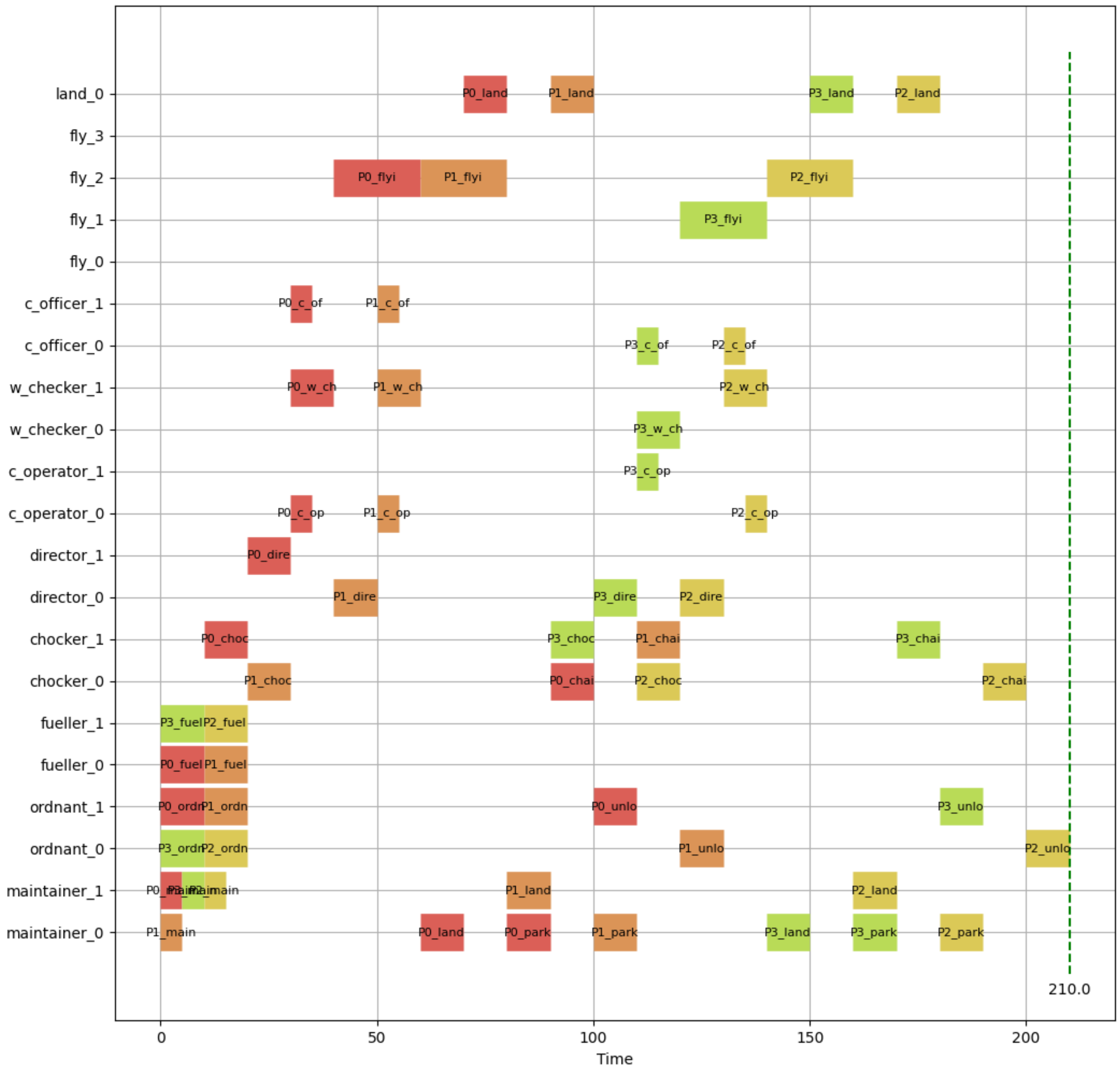
$$B^J_i - M^{catapult}_{kp} + V(1 - Z^{catapult}_{ikp}) \geq 0 \tag{9a}$$
$$\forall i,k,p \text{ and } J \text{ is the catapult phase}$$
$$M^{catapult}_{k(p+1)} - B^{J'}_i + V(1 - Z^{catapult}_{ikp}) \geq 0 \tag{9b}$$
$$\forall i,j,k,p \text{ and } J' \text{ is the phase after the catapult phase}$$

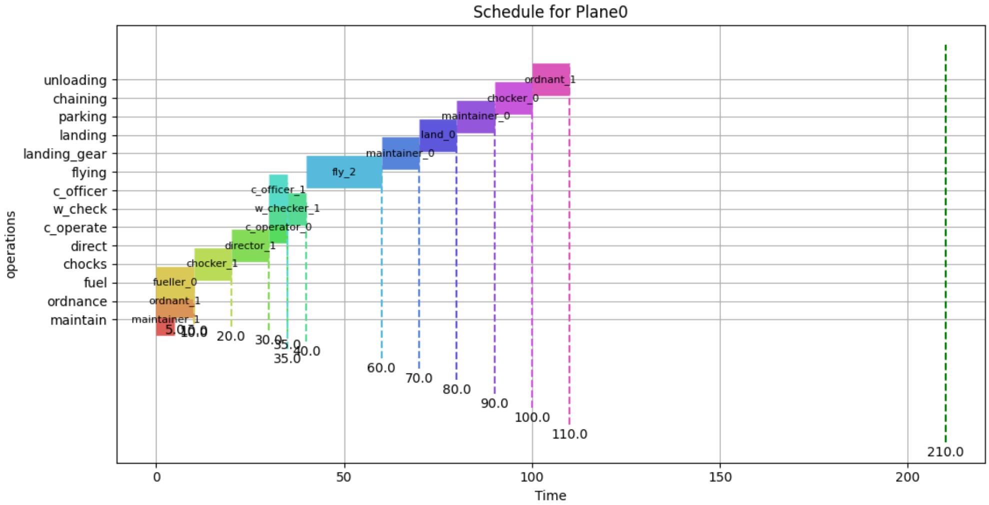
We have similar constraints on $B^{captain}_i$, $M^{captain}_{kp}$ and $B^{landing}_i$, $M^{landing}_{kp}$ for the duration of the captain, and landing track required respectively.

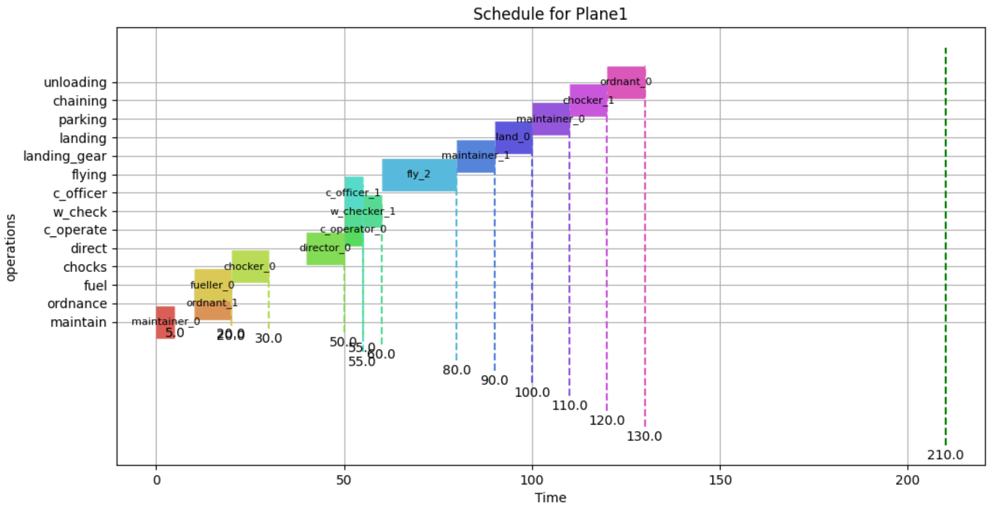
$$y^{c1}_{p_i,p_j} + y^{c2}_{p_i,p_j} = 1 \tag{10a}$$
$$M^{catapult}_{1,p_j+1} - M^{catapult}_{0,p_i} - V(1 - y^{c1}_{p_i,p_j}) \leq 0 \tag{10b}$$
$$M^{catapult}_{0,p_i+1} - M^{catapult}_{1,p_j} - V(1 - y^{c2}_{p_i,p_j}) \leq 0 \tag{10c}$$
$$\forall p_i, p_j$$

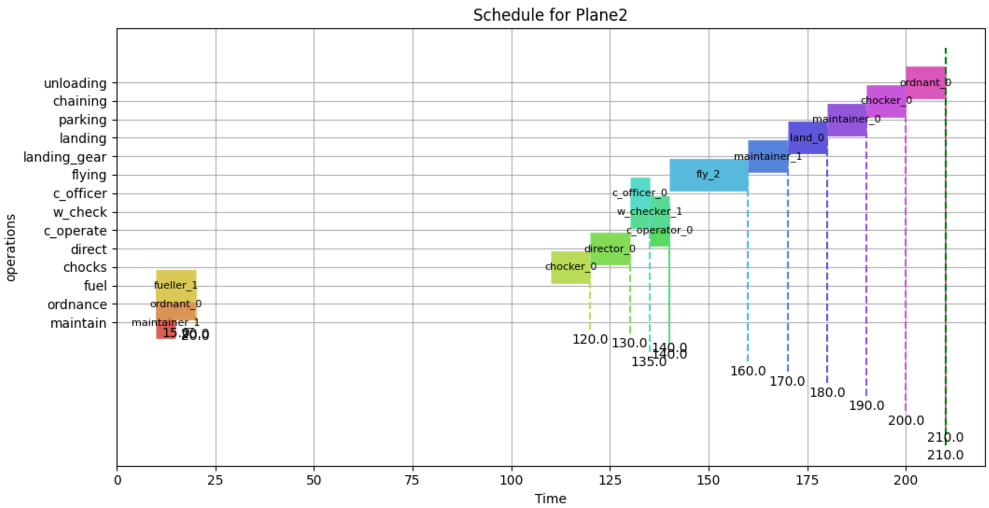
We have similar constraints for catapults 3 and 4.

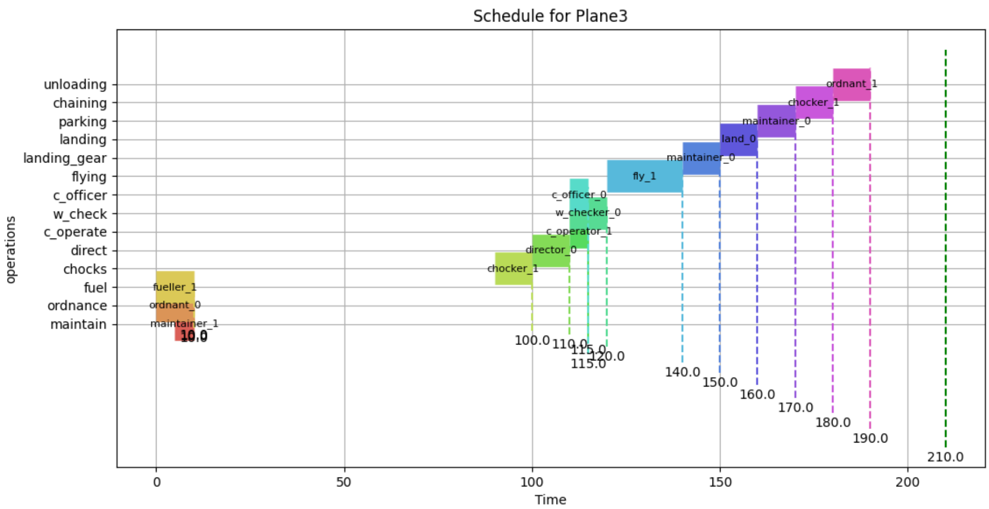

Plane Schedule
Plane_3
Plane_2
Plane_1
Plane_0
Plane
Startup
chocks
direct
catapult
fly
land_gear
land
park
chain
unload
0
50
100
150
200
210.0
Time

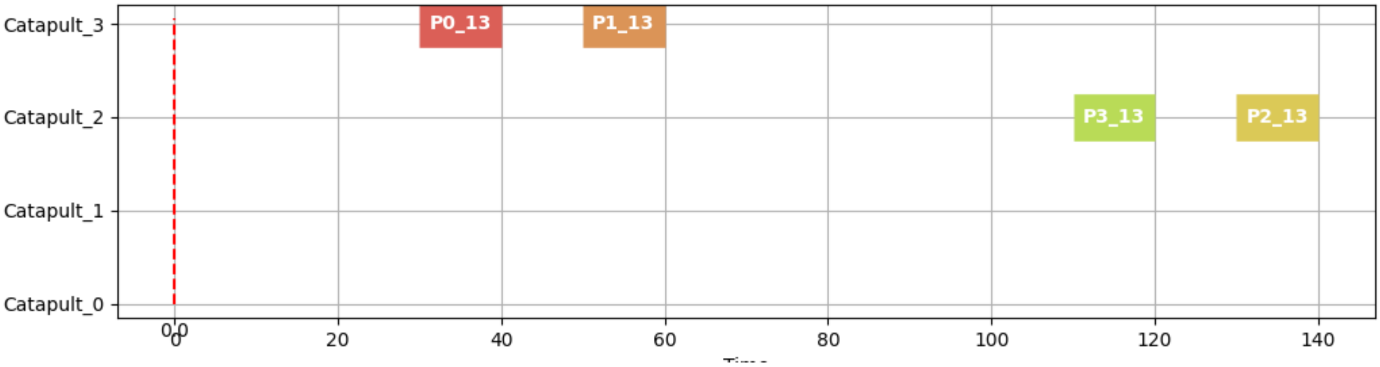

land_0
fly_3
fly_2
fly_1
fly_0
c_officer_1
c_officer_0
w_checker_1
w_checker_0
c_operator_1
c_operator_0
director_1
director_0
chocker_1
chocker_0
fueller_1
fueller_0
ordnant_1
ordnant_0
maintainer_1
maintainer_0
P0_land
P1_land
P3_land
P2_land
P0_flyi
P1_flyi
P2_flyi
P3_flyi
P0_c_of
P1_c_of
P3_c_of
P2_c_of
P0_w_ch
P1_w_ch
P2_w_ch
P3_w_ch
P3_c_op
P0_c_op
P1_c_op
P2_c_op
P0_dire
P1_dire
P3_dire
P2_dire
P0_choc
P3_choc
P1_chai
P3_chai
P1_choc
P0_chai
P2_choc
P2_chai
P3_fuel
P2_fuel
P0_fuel
P1_fuel
P0_ordn
P1_ordn
P0_unlo
P3_unlo
P3_ordn
P2_ordn
P1_unlo
P2_unlo
P1_land
P2_land
P1_main
P0_land
P0_park
P1_park
P3_land
P3_park
P2_park
210.0
0
50
100
150
200
Time


Schedule for Plane0
operations
unloading
chaining
parking
landing
landing_gear
flying
c_officer
w_check
c_operate
direct
chocks
fuel
ordnance
maintain
ordnant_1
chocker_0
maintainer_0
land_0
maintainer_0
fly_2
c_officer_1
w_checker_1
c_operator_0
director_1
chocker_1
fueller_0
ordnant_1
maintainer_1
20.0
30.0
35.0
40.0
60.0
70.0
80.0
90.0
100.0
110.0
210.0
Time
0
50
100
150
200


Schedule for Plane1
operations
unloading
chaining
parking
landing
landing_gear
flying
c_officer
w_check
c_operate
direct
chocks
fuel
ordnance
maintain
ordnant_0
chocker_1
maintainer_0
land_0
maintainer_1
fly_2
c_officer_1
w_checker_1
c_operator_0
director_0
chocker_0
fueller_0
ordnant_1
maintainer_0
5.0
30.0
50.0
55.0
60.0
80.0
90.0
100.0
110.0
120.0
130.0
210.0
Time
0
50
100
150
200


Schedule for Plane2
operations
unloading
chaining
parking
landing
landing_gear
flying
c_officer
w_check
c_operate
direct
chocks
fuel
ordnance
maintain
ordnant_0
chocker_0
maintainer_0
land_0
maintainer_1
fly_2
c_officer_0
w_checker_1
c_operator_0
director_0
chocker_0
fueller_1
ordnant_0
maintainer_1
120.0
130.0
135.0
140.0
160.0
170.0
180.0
190.0
200.0
210.0
210.0
Time
0
25
50
75
100
125
150
175
200


Schedule for Plane3
operations
unloading
chaining
parking
landing
landing_gear
flying
c_officer
w_check
c_operate
direct
chocks
fuel
ordnance
maintain
ordnant_1
chocker_1
maintainer_0
land_0
maintainer_0
fly_1
c_officer_0
w_checker_0
c_operator_1
director_0
chocker_1
fueller_1
ordnant_0
maintainer_1
100.0
110.0
115.0
120.0
140.0
150.0
160.0
170.0
180.0
190.0
210.0
Time
0
50
100
150
200



REFERENCES

[1] Ku, W. Y., & Beck, J. C. (2016). Mixed Integer Programming models for job shop scheduling: A computational analysis. *Computers & Operations Research*, *73*, 165–173. https://doi.org/10.1016/j.cor.2016.04.006

[2] Ross, W. (2016). Investigating the Tradespace between Increased Automation and Optimal Manning on Aircraft Carrier Deck. Master of Science Thesis.

[3] Aubert, M., Ross, W., Mazzari, S., Stimpson, A. J., Cummings, M. L. (2016) Interaction Design Considerations for an Aircraft Carrier Deck Agent-based Simulation. *IEEE Aerospace Conference*, pp. 1-7, doi: 10.1109/AERO.2016.7500894.