PROOFS: HOMEWORK 4

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Problem 2.34

Part A
Given that a and b are nonzero polynomials, then the $\deg(a)$ and $\deg(b)$ are the highest power within the polynomials. By definition and properties of multiplying, $a \cdot b$ is the field of the product of the two nonzero polynomials which means that highest power of $a \cdot b$ is the $\deg(a) + \deg(b)$.

Part B
Assume that polynomial a has a multiplicative inverse, meaning that there exists polynomials $b, c \in F[x]$. Thus $b(x)c(x) = 1$, since b, c exist in $F[x]$, then $\deg(b) = \deg(c) = 0$. Meaning that $\deg(a) = 0$, implying that a is a constant polynomial.

Assume that the polynomial a is a constant polynomial, meaning that $\deg(a) = 0$. Let b, c be the multiplicative inverses of a such that $c(x)b(x) = 1$. Since $\deg(b) + \deg(c) = 0$ and we know that b, c are nonnegative polynomials since they are in the field F, then b, c are multiplicative inverses.

Part C

Part D

Problem 2.37

Reducing the polynomial to the form, $(x + a)(x + b)(x + c)$, we create the system equations with a, b, c

$$abc = 1$$
$$2(a + b) + c = 0$$
$$ab + 2c(a + b) = 1$$

After solving the variables we find that $a + b + c = 0$ and $abc = 1$, where there is no solution since $a, b, c > 0$. Thus the polynomial is irreducible.

Problem 2.38

1	x	x^2	$1+x$	$1+x^2$	$x+x^2$	$1+x+x^2$
0	0	0	0	0	0	0
1	x	x^2	$1+x$	$1+x^2$	$x+x^2$	$1+x+x^2$
x	x^2	$x+1$	$x+x^2$	1	x^2+x+1	x^2+1
$x+1$	x^2+x	x^2+x+1	$1+x^2$	x^2	1	x
x^2	$x+1$	$x+x^2$	$1+x+x^2$	$1+x^2$	1	x
$1+x^2$	1	x	x^2	$1+x+x^2$	$1+x$	$x+x^2$
$x+x^2$	x^2+x+1	$1+x^2$	1	$1+x$	x	x^2
$1+x+x^2$	$1+x^2$	1	x	$x+x^2$	x^2	$1+x$

Problem 2.40

Both rings hold p^e elements but $F_{p^e}[X]$ holds the coefficients of the polynomial while $Z/(p^e)Z$ holds all values from 0 to $p^e - 1$. This means all values in the modulos ring are distinct while the polynomial ring does not guarantee this.