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COURSE NAME: Robotics

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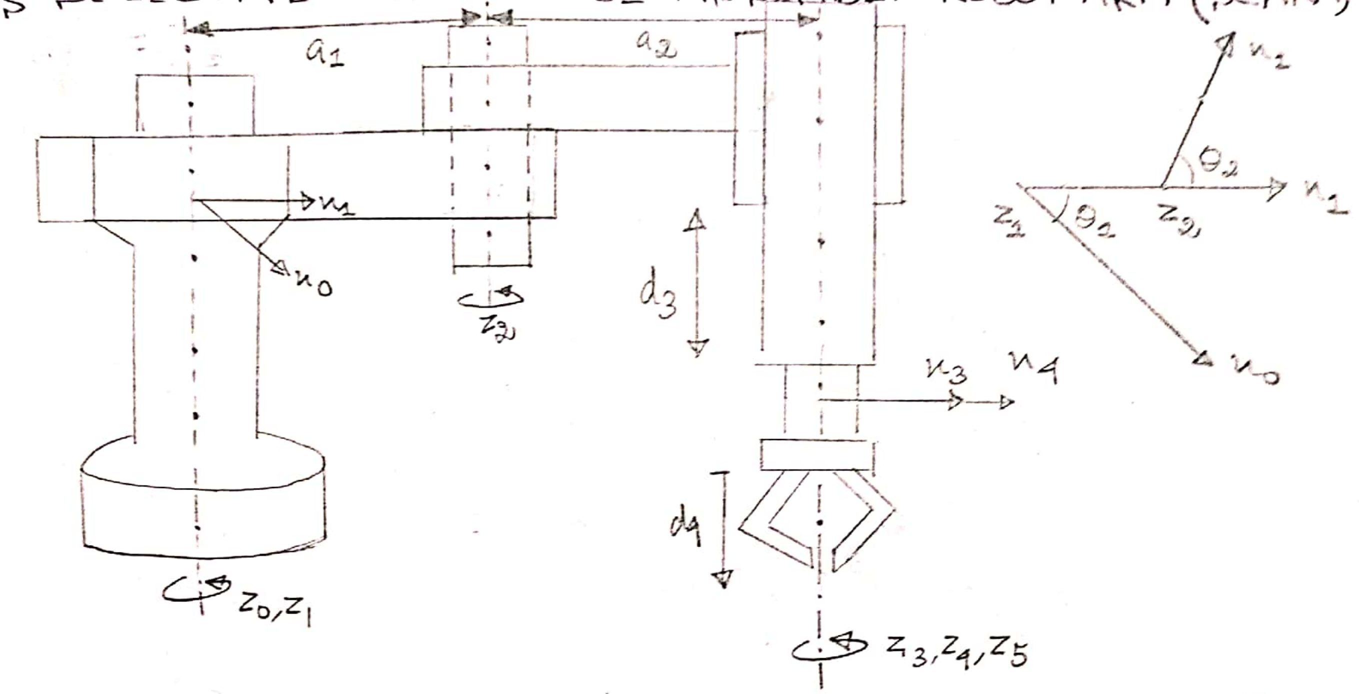
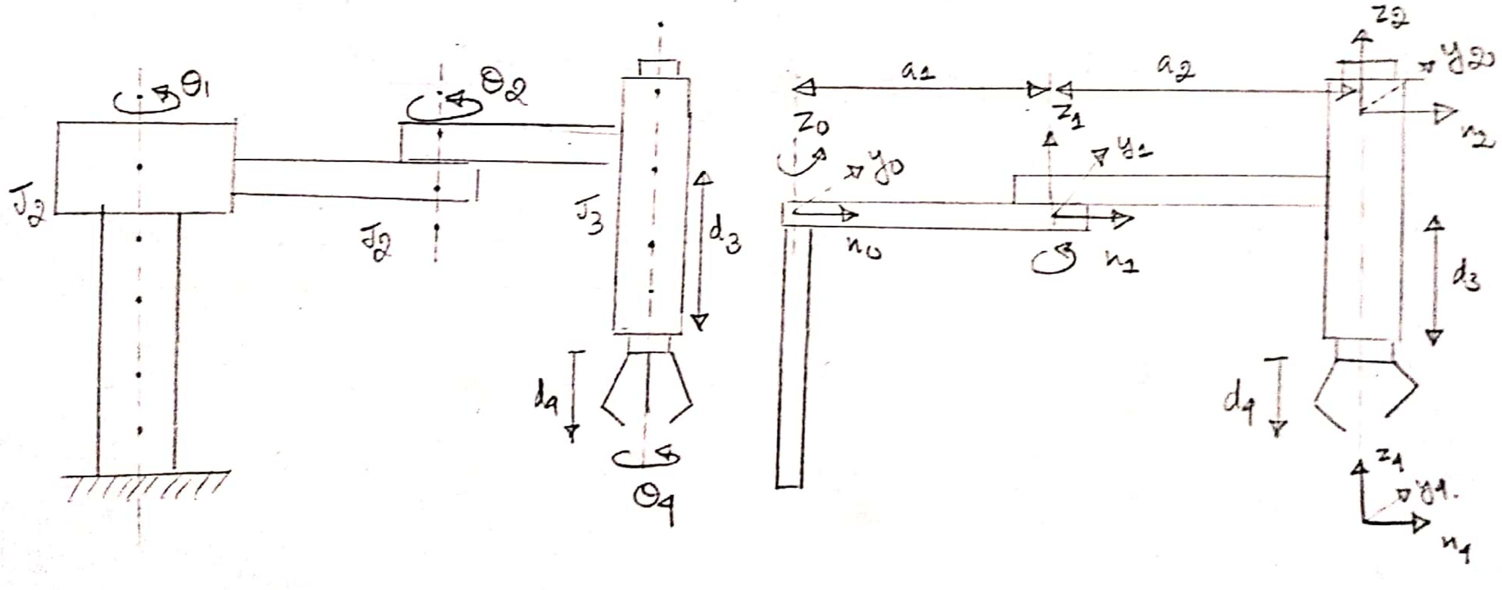
23-05-2020

4-AXIS SELECTIVE COMPLIANCE ASSEMBLY ROBOT ARM (SCARA)





DIAGRAM

4-AXIS SELECTIVE COMPLIANCE ASSEMBLY ROBOT ARM.

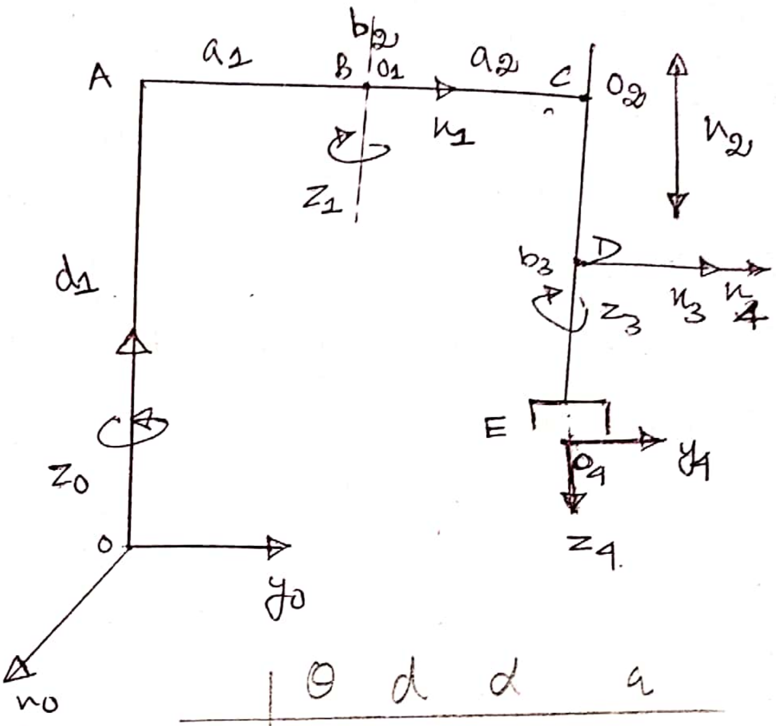
Important Notes

1. b_k is the point of intersection of x_k and z_{k-1}

2. The joint distance d_k is the distance between O_{k-1} and b_k

3. The link length a_k is the distance between O_k and b_k

4. Link Twist angle α_k is the angle between z_{k-1} and z_k rotated about z_k axis



	θ	d	α	a
1	θ_1	$d_1 = OA$	π	AB
2	θ_2	0	0	BC
3	0	$d_3 = CD$	0	0
4	θ_4	$d_4 = DE$	0	0

[1] Denavit-Hartenberg Parameters

Frame of Reference	Joint Angle (θ_i)	Link Offset (d_i)	Twist Angle (α_i)	Link Length (a_i)
1	θ_1	d_1	π	a_1
2	θ_2	0	0	a_2
3	0	d_3	0	0
4	θ_4	d_4	0	0

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12 | Forward Kinematics

We could use the basic transformation matrix or derive them from scatch :

Rotation Matrix (3x3) | Position Vector (3x1)

$$
{}^{i-1}T_i = \left[\begin{array}{ccc|c}
\cos\theta_i & -\sin\theta_i \cos\alpha_i & \sin\theta_i \sin\alpha_i & a_i \cos\theta_i \\
\sin\theta_i & \cos\theta_i \cos\alpha_i & -\cos\theta_i \sin\alpha_i & a_i \sin\theta_i \\
0 & \sin\alpha_i & \cos\alpha_i & d_i \\
\hline
0 & 0 & 0 & 1
\end{array}\right]
$$

Perspective Transformation (1x3) | Stretching or Scaling factor (1x1)

$$
{}^{0}T_1 = \begin{bmatrix}
\cos\theta_1 & \sin\theta_1 & 0 & a_1 \cos\theta_1 \\
\sin\theta_1 & -\cos\theta_1 & 0 & a_1 \sin\theta_1 \\
0 & 0 & -1 & d_1 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

$$
{}^{1}T_2 = \begin{bmatrix}
\cos\theta_2 & -\sin\theta_2 & 0 & a_2 \cos\theta_2 \\
\sin\theta_2 & \cos\theta_2 & 0 & a_2 \sin\theta_2 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

$$
{}^{2}T_3 = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & d_3 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

$d_3 \rightarrow$ variable.

$$
{}^{3}T_4 = \begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & 0 \\ \sin\theta_4 & \cos\theta_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

We can get final transformation matrix by:

$$
{}^{0}T_4 = {}^{0}T_1\,{}^{1}T_2\,{}^{2}T_3\,{}^{3}T_4
$$

Thus, using matrix multiplication:

$$
{}^{0}T_2 = \begin{bmatrix} \cos(\theta_1-\theta_2) & \sin(\theta_1-\theta_2) & 0 & a_1\cos\theta_1 + a_2\cos(\theta_1-\theta_2) \\ \sin(\theta_1-\theta_2) & -\cos(\theta_1-\theta_2) & 0 & a_1\sin\theta_1 + a_2\sin(\theta_1-\theta_2) \\ 0 & 0 & -1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}T_3 = \begin{bmatrix} \cos(\theta_1-\theta_2) & \sin(\theta_1-\theta_2) & 0 & a_1\cos\theta_1 + a_2\cos(\theta_1-\theta_2) \\ \sin(\theta_1-\theta_2) & -\cos(\theta_1-\theta_2) & 0 & a_1\sin\theta_1 + a_2\sin(\theta_1-\theta_2) \\ 0 & 0 & -1 & d_1-d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

The Resulting final transformation can help us determine the end effector position and orientation.

$$
{}^{0}T_4 = \begin{bmatrix} \cos(\theta_1-\theta_2-\theta_4) & \sin(\theta_1-\theta_2-\theta_4) & 0 & a_1\cos\theta_1 + a_2\cos(\theta_1-\theta_2) \\ \sin(\theta_1-\theta_2-\theta_4) & -\cos(\theta_1-\theta_2-\theta_4) & 0 & a_1\sin\theta_1 + a_2\sin(\theta_1-\theta_2) \\ 0 & 0 & -1 & d_1-d_3-d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

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13 Inverse Kinematics

$$x^2+y^2 = a_1^2(c_1^2+s_1^2) + a_2^2(c_2^2+s_2^2)$$
$$+ 2a_1a_2(c_1c_{12} + s_1s_{12}).$$

Task space variables

$$\begin{pmatrix} x \\ y \\ z \\ \phi \end{pmatrix} \begin{matrix} = a_1c_1 + a_2c_{12} \\ = a_1s_1 + a_2s_{12} \\ = d_3 \\ = \theta_1+\theta_2+\theta_4 \end{matrix} \begin{pmatrix} \theta_1 \\ \theta_2 \\ d_3 \\ \theta_4 \end{pmatrix}$$

$$\Rightarrow \theta_4 - \phi - \theta_1 - \theta_2$$

Rearranging terms:

$$\cos\theta_2 = \left(\frac{x^2+y^2-a_1^2-a_2^2}{2a_1a_2}\right)$$

$$\Rightarrow \theta_2 = \pm\cos^{-1}\left(\frac{x^2+y^2-a_1^2-a_2^2}{2a_1a_2}\right)$$

[2 solutions exists as left handed and Right handed solutions in Inverse Kinematics] (unless both converge) i.e. in case of singularity.

Now, we know that:

$$x = a_1c_1 + a_2(c_1c_2 - s_1s_2).$$

Segregating the known and unknown parameters

$$= (a_1+a_2c_2)c_1 - a_2s_2s_1$$

$$= k_1c_1 - k_2s_1$$

where,

$$k_1 = a_1 + a_2c_2$$

and $k_2 = a_2s_2.$

Also,

$$y = a_1s_1 + a_2(s_1c_2 + c_1s_2) = k_1s_1 + k_2c_1$$

$$\frac{x}{\sqrt{k_1^2+k_2^2}} = \underbrace{\left(\frac{k_1}{\sqrt{k_1^2+k_2^2}}\right)}_{\cos\psi}c_1 - \underbrace{\left(\frac{k_2}{\sqrt{k_1^2+k_2^2}}\right)}_{\sin\psi}s_1 = c_{\psi_1} = \cos(\psi+\theta_1)$$

$$\frac{y}{\sqrt{k_1^2+k_2^2}} = \left(\frac{k_1}{\sqrt{k_1^2+k_2^2}}\right)s_1 + \left(\frac{k_2}{\sqrt{k_1^2+k_2^2}}\right)c_1 = s_{\psi_1} = \sin(\psi+\theta_1)$$

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Dividing the above equations:

$$\tan(\psi+\theta_1) = \frac{\sin(\psi+\theta_1)}{\cos(\psi+\theta_1)} = y/x$$

Also,

$$\frac{\sin\psi}{\cos\psi} = k_2/k_1 \Rightarrow \tan\psi = k_2/k_1$$

Thus,

$$\tan(\psi+\theta_1) = y/x \Rightarrow \psi+\theta_1 = \tan^{-1}(y/x)$$

$$\Rightarrow \theta_1 = \tan^{-1}(y/x) - \psi$$

$$\boxed{\theta_1 = \tan^{-1}(y/x) - \tan^{-1}(k_2/k_1)}$$

Note use Atan2 in MATLAB function.

From here we can compute the value of θ_4 using the relation

$$\boxed{\theta_4 = \phi - \theta_1 - \theta_2.}$$

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14. Robot Dynamics

The euler-Lagrange equation for calculating the robot dynamics as derived from D'Alembert principles is:

$$\frac{d}{dt}\left(\frac{d\mathcal{L}}{\partial \ddot{\theta}_i}\right) - \left(\frac{\partial \mathcal{L}}{\partial \theta_i}\right) = \tau_i \quad ; \; i = 1,2,3,\ldots,n.$$

where $\mathcal{L} = K - P$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{bmatrix} + \begin{bmatrix} 0 & V_{12} & V_{13} \\ V_{12} & 0 & V_{23} \\ V_{13} & V_{23} & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1^2 \\ \dot{\theta}_2^2 \\ \dot{\theta}_3^2 \end{bmatrix}$$

$$+ \begin{bmatrix} 2V_{12} & 2V_{13} & 2V_{13} \\ 0 & 2V_{23} & 2V_{23} \\ -2V_{23} & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \dot{\theta}_2 \\ \dot{\theta}_1 \dot{\theta}_3 \\ \dot{\theta}_2 \dot{\theta}_3 \end{bmatrix} + \begin{bmatrix} \tau v_1 \\ \tau v_2 \\ \tau v_3 \end{bmatrix}$$

$$= M(q)\ddot{q} + h(q,\dot{q}) = \tau + \tau_0(q,\dot{q},\ddot{q})$$

where $q = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$, $\dot{q}$, $\ddot{q}$ are the vectors of joint motion, velocities and accelerations respectively

M is the inertia matrix and h is the vector due to centrifugal and Coriolis effects

τ_0 is the collection of all perturbations from rigid body dynamics

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Now,

$$I_{11} = I_1+I_2+I_3+m_1\ell_1^2+m_2(\ell_2^2+R_1^2)+m_3(\ell_3^2+R_1^2+R_2^2)$$
$$-2m_2\ell_2R_1C_E + 2m_3(\ell_3R_1C_{EW} - \ell_3\ell_2C_W - R_2R_1C_E)$$

$$I_{22} = I_2+I_3+m_2\ell_2^2+m_3(\ell_3^2+R_2^2)$$

$$I_{33} = I_3+m_3\ell_3^2$$

$$I_{12} = I_2+I_3+m_2\ell_2^2+m_3(\ell_3^2+R_2^2) - 2m_2\ell_2R_1C_E + m_3(\ell_3R_1C_{EW}$$
$$- 2\ell_3\ell_2C_W - R_2R_1C_E)$$

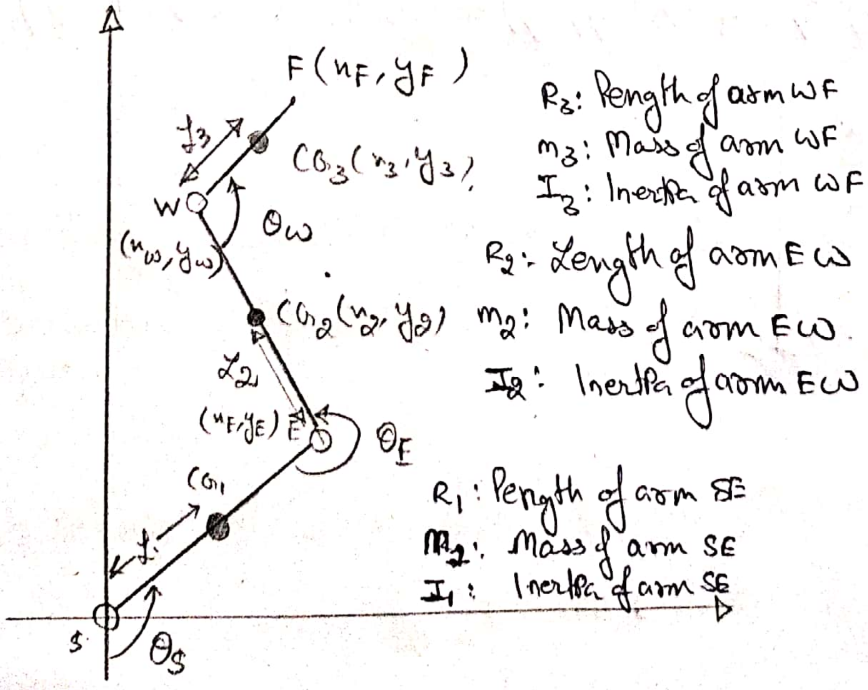
$$I_{13} = I_3+m_3\ell_3^2+m_3(\ell_3R_1C_{EW} - \ell_3\ell_2C_W)$$

$$I_{23} = I_3+m_3\ell_3^2 - m_3\ell_3R_2C_W.$$

$$V_{12} = m_2\ell_2R_1S_E + m_3(-\ell_3R_1S_{EW}+\ell_3R_2S_E).$$

$$V_{13} = m_3(-\ell_3R_1S_{EW}+\ell_3R_2S_W)$$

$$V_{23} = m_3\ell_3R_2S_W$$



where.

$S_{EW} = \sin(\theta_E+\theta_W)$

$S_E = \sin(\theta_E)$

$S_W = \sin(\theta_W)$

$C_{EW} = \cos(\theta_E+\theta_W)$

$C_E = \cos(\theta_E)$

$C_W = \cos(\theta_W)$

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5 Trajectory Planning

Consider the arm matrix of the 4-axis SCARA Robot Manipulator.

$$^{0}T_4 = \begin{bmatrix} C_{1-2-4} & S_{1-2-4} & 0 & a_1C_1 + a_2C_{1-2} \\ S_{1-2-4} & -C_{1-2-4} & 0 & a_1S_1 + a_2S_{1-2} \\ 0 & 0 & -1 & d_1 - d_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Let

$OA = d_1 = 10$

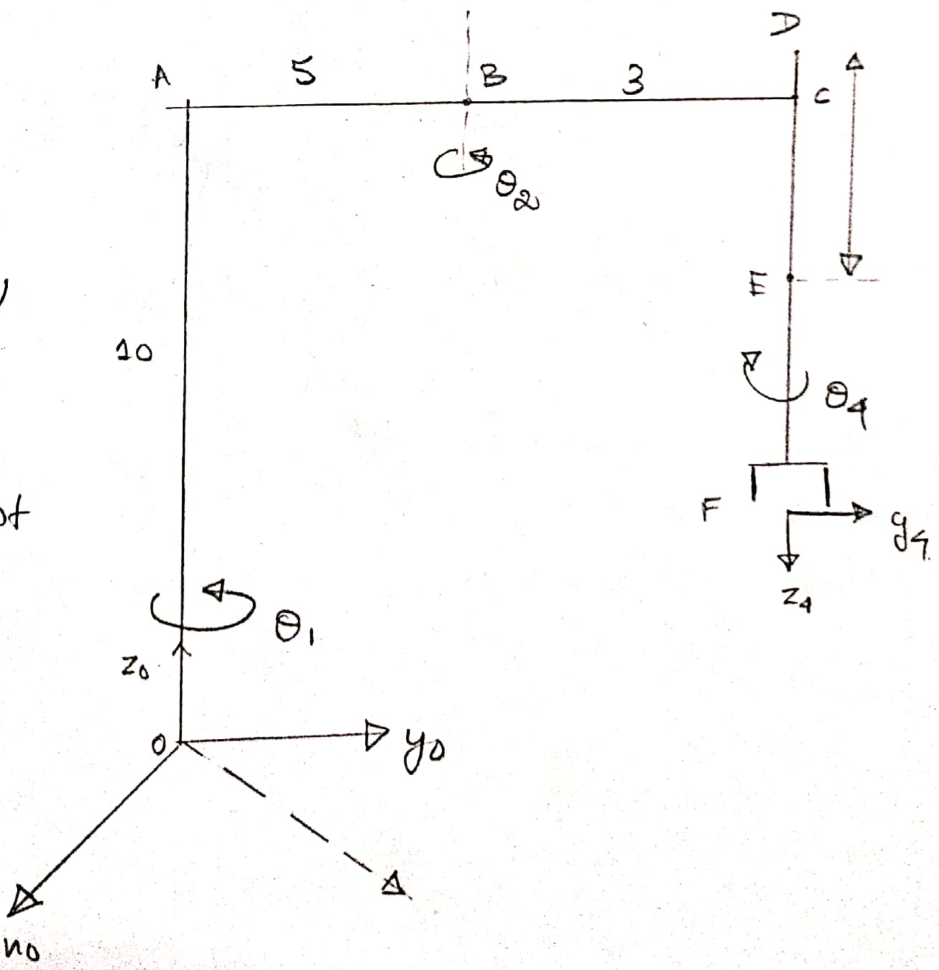
$AB = a_1 = 5$

$BC = d_2 = 3$

$CE = d_3$ (variable)

$EF = d_4 = 4.$

for our SCARA robot manipulator



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To make plan an arbitrary trajectory, let us assume that.

If

$$T(0) = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & 3 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 4 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \frac{DT}{dt}(0) = 0_{4\times4}$$

and also,

$$T(10) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 5 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \frac{DT}{dt}(10) = 0_{4\times4}$$

Fitting a cubic polynomial for the trajectory,

$$^{0}T_4(t) = T'(t) = A_0 + A_1 t + A_2 t^2 + A_3 t^3$$

$$\Rightarrow T(0) = A_0 \quad \frac{dT(0)}{dt} = A_1 = 0$$

$$T(10) = A_0 + 10A_1 + 100 A_2 + 1000 A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 5 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\dot{T}(10) = A_1 + 20 A_2 + 300 A_3 = 0$$

$$T(0) = T_0 = A_0 \quad \frac{dT}{dt} = T_1 = A_1$$

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