Solution 1

(a) Because $(x-y)^2 + \lambda|x|$ is discontinuous at $x = 0$, we can minimize it by finding the minima over the piecewise regions $(-\infty, 0)$ and $(0, \infty)$. Differentiating the expression with respect to x, we have

$$2(x-y) + \lambda \text{sign}(x) = 0 \tag{1}$$

which resolves to

$$x = y - \frac{1}{2}\text{sign}(x)\lambda \tag{2}$$

Note that if we plug this back into the original expression, we get

$$\text{cost} = \frac{1}{4}\lambda^2 + \lambda|x| \tag{3}$$

If we assume that $\lambda > 0$, this expression is minimized by the x with the smallest absolute value. In this case we can write

$$x = y - \frac{1}{2}\text{sign}(y)\lambda \tag{4}$$

(b) Here is the denoised image using the default initial value of λ, 0.88. I found that lower values of λ resulted in an image more similar to the input, whereas higher values of λ resulted in images closer to a uniform gray.

Figure 1: Denoised output

Solution 2

(a) White-balanced outputs:

Figure 2: Image 1

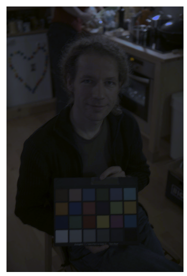

Figure 3: Image 2

Figure 4: Image 3

(b) White-balanced outputs using the top 10% of color intensities:

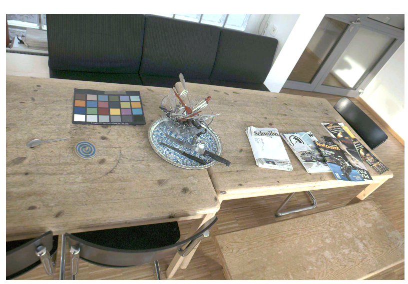

Figure 5: Image 1

Figure 6: Image 2

Figure 7: Image 3

Solution 3

Figure 8: Surface normals

(a)

Figure 9: Albedo

(b)

Solution 4

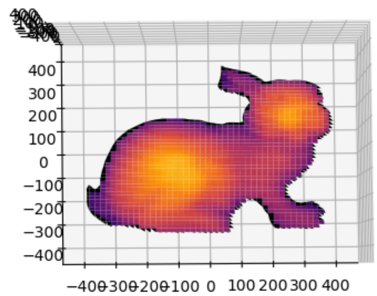

Figure 10: Depth map using Fourier transforms

Solution 5

Side note: I was curious about the magnitude of the gradient over time, so I had my program output the magnitude of the change in Z for each iteration (measured by the Euclidean norm). After 200 iterations, it dropped to about 80, and the change in the depth map was not as noticeable.

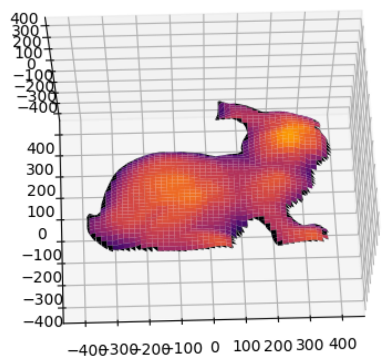

Figure 11: Depth map after 200 iterations

Information

This problem set took approximately 16 hours of effort.

I discussed this problem set with:

- Jarrett Gross

I also got hints from the following sources:

- Numpy documentation