Solution 1

(a) There are $\binom{N-J}{K}$ ways to choose K inliers, and $\binom{N}{K}$ ways to choose K correspondences total, so the probability of getting K inliers is

$$\frac{\binom{N-J}{K}}{\binom{N}{K}} \tag{1}$$

(b) Since we know the probability of getting a set of K correspondences with no outliers, the probability of at least 1 outlier is

$$1 - \frac{\binom{N-J}{K}}{\binom{N}{K}} \tag{2}$$

The probability of getting at least one outlier in every draw of M draws is therefore

$$\left(1 - \frac{\binom{N-J}{K}}{\binom{N}{K}}\right)^M \tag{3}$$

and the probability that this doesn't happen (i.e., that we get at least one draw with no outliers), is

$$P = 1 - \left(1 - \frac{\binom{N-J}{K}}{\binom{N}{K}}\right)^M \tag{4}$$

To find the right number of draws for a fixed probability P, we just solve for M:

$$M = \frac{\ln(1-P)}{\ln\left(1 - \frac{\binom{N-J}{K}}{\binom{N}{K}}\right)} \tag{5}$$

(c) There should be $\binom{I_1}{K}$ unordered sets of K correspondences all belonging to I_1 and $\binom{I_2}{K}$ possible sets of K all belonging to I_2. (Since these are non-overlapping sets, we don't need to worry about overcounting the intersection). The probability of getting a set of K samples all from one set or the other is therefore

$$\frac{\binom{I_1}{K} + \binom{I_2}{K}}{\binom{N}{K}} \tag{6}$$

Solution 3

(a) Consider the equation for the projection coordinates in camera 1:

$$p_1 = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} f_1 & 0 & W/2 & 0 \\ 0 & f_1 & H/2 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha x' \\ \alpha y' \\ \alpha z' \\ \alpha \end{bmatrix} \tag{7}$$

$$x_1 = \frac{a}{c} = f_1 \frac{x'}{z'} + \frac{w}{2} \tag{8}$$

$$y_1 = \frac{b}{c} = f_1 \frac{y'}{z'} + \frac{H}{2} \tag{9}$$

Representing x_1 and y_1 as a vector, we can write

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \frac{f_1}{z'} \begin{bmatrix} x' \\ y' \end{bmatrix} + \begin{bmatrix} W/2 \\ H/2 \end{bmatrix} \tag{10}$$

If we solve this expression for $\begin{bmatrix} x' \\ y' \end{bmatrix}$ and plug the results into the same formula for camera 2, we get

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \frac{f_2}{z'} \left(\frac{z'}{f_1} \left(\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} - \begin{bmatrix} W/2 \\ H/2 \end{bmatrix} \right) \right) + \begin{bmatrix} W/2 \\ H/2 \end{bmatrix} \tag{11}$$

(b) As the hint suggests, suppose that all points in our image exist in a plane defined by $z = 0$. The projected coordinates for a camera then take the form:

$$p = [K|0] \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} p' \tag{12}$$

$$= \begin{bmatrix} f & 0 & W/2 & 0 \\ 0 & f & H/2 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} R_1 & R_2 & R_3 & t_1 \\ R_4 & R_5 & R_6 & t_2 \\ R_7 & R_8 & R_9 & t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha x' \\ \alpha y' \\ 0 \\ \alpha \end{bmatrix} \tag{13}$$

$$= \begin{bmatrix} f\alpha(R_1 x' + R_2 y' + t_1) + \frac{W}{2}(R_7 x' + R_8 y' + t_3) \\ f\alpha(R_4 x' + R_5 y' + t_2) + \frac{H/2}{\alpha}(R_7 x' + R_8 y' + t_3) \\ \alpha(R_7 x' + R_8 y' + t_3) \end{bmatrix} \tag{14}$$

The important point here is that by setting $z = 0$, we can express the above coordinates in terms of x', y', and α. The projection in each camera is therefore a linear transform on the 2d homogeneous coordinates of the point, which we can calculate using an invertible 3x3 transform matrix:

$$p_1 = \begin{bmatrix} \alpha x'(f_1 R_1 + W/2R_2) + \alpha y'(f_1 R_2 + W/2R_8) + \alpha(f_1 t_1 + W/2t_3) \\ \alpha x'(f_1 R_4 + H/2R_2) + \alpha y'(f_1 R_5 + H/2R_8) + \alpha(f_1 t_2 + H/2t_3) \\ \alpha x' R_7 + \alpha y' R_8 + \alpha t_3 \end{bmatrix} \tag{15}$$

$$= \begin{bmatrix} (f_1 R_1 + W/2R_2) & (f_1 R_2 + W/2R_8) & (f_1 t_1 + W/2t_3) \\ (f_1 R_4 + H/2R_2) & (f_1 R_5 + H/2R_8) & (f_1 t_2 + H/2t_3) \\ R_7 & R_8 & t_3 \end{bmatrix} \begin{bmatrix} \alpha x' \\ \alpha y' \\ \alpha \end{bmatrix} \tag{16}$$

$$= P_{2d}^{(1)} p'_{2d} \tag{17}$$

To convert from one camera's coordinates to the other, we just need to invert the transformation matrix for the one camera and multiply by the other:

$$p_2 = P^{(2d)} p_{2d} = P^{(2)}_{2d} P^{(1)-1}_{2d} p_1 \tag{18}$$

Solution 4

Here is my spliced image for part (b).

Figure 1: prob4.png

Solution 5

Disparity map for part (b).

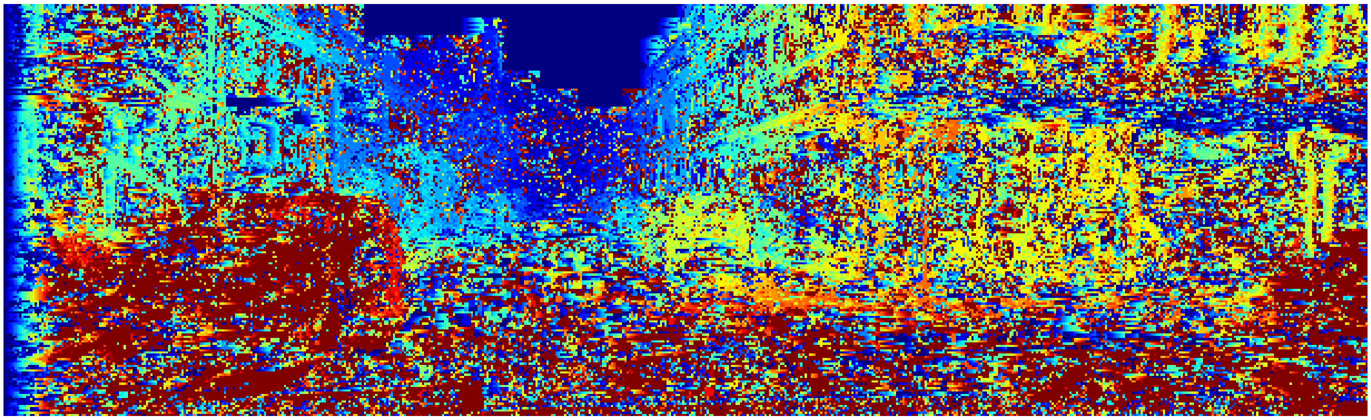

Figure 2: prob5.png

Information

This problem set took approximately 24 hours of effort.

I also got hints from the following sources:

- Various articles on problem-solving with numpy at stackoverflow.com
- Numpy documentation