

Error

If x is the exact value and \hat{x} is know approximation, therefore

$$\Delta(\hat{x}) := |x - \hat{x}|$$

$$\delta(\hat{x}) := \frac{\Delta(\hat{x})}{|\hat{x}|}$$

$$x = \hat{x} + \Delta$$

$$\hat{x} - \Delta \leq x \leq \hat{x} + \Delta$$

So $\delta(\hat{x})$ is named *relational error* and $\Delta(\hat{x})$ is called *absolute error*.

Functions (mapping)

$$f : \forall x \in X : y \in Y y = f(x)$$

or

$$f(x) := \{y \in Y | \exists x (x \in X \wedge y = f(x))\}$$

or

$$f : X \rightarrow Y$$

Mapping

- Subjective $x \rightarrow y \mid f(x) = y$
- Injective $\forall x_1 x_2 \in X \mid f(x_1) = f(x_2) \Rightarrow x_1 = x_2$
- Bijective: injective and subjective

Sequences

A function $f : \mathbb{N} \rightarrow X$ whose domain of definitions is the set of natural numbers is called a sequence

The terms of sequence is $f(n)$

$$x_n = f(n)$$

$$\{x_n\} \equiv \{x_1, x_2, \dots, x_n\}$$

Limit

A number A belongs to the set of real numbers is called **the limit of numerical sequence** if for every neighbourhood of A there exists such index N depending on neighbourhood of A , so $x_n \in V(A)$, $n > N$.

$$\lim_{n \rightarrow \infty} x_n = A := \forall V(A) \exists N \in \mathbb{N} : \forall n > N (n \in V(A))$$

$$\lim_{n \rightarrow \infty} x_n := \forall \varepsilon > 0 \exists N \in \mathbb{N} : \forall n > N \quad |x_n - A| < \varepsilon$$

Properties

- If $\exists A \wedge \exists N : x_n = A \forall n > N$, then $\{x_n\}$ is ultimately constant
- Any neighborhood of the limit of a sequence contains all but a finite number of terms of the sequence
- A convergent sequence cannot have two different limits.
- A convergent sequence is bounded.

$\{x_n\}$ and $\{y_n\}$ - numerical sequences. Then there is some product, quotient and sum. $\{x_n + y_n\}$, $\{x_n \cdot y_n\}$, $\{\frac{x_n}{y_n}\}$

Convergent

If $\lim_{n \rightarrow \infty} x_n = A$ and $x_n \rightarrow A$ as $n \rightarrow \infty$, then $\{x_n\}$ converges to A (tends to A)

If a sequence does not have a limit, it is called divergent.