E-bike System Modeling and Simulation

Geethanjali Thejasree
Department of Mechanical Engineering
National Institute of Technology
Karnataka (NITK), Karanataka, India
geethanjalithejasree@gmail.com

Ranjith Maniyeri
Faculty Department of Mechanical Engineering
National Institute of Technology
Karnataka (NITK), Karnataka, India
mranji1@nitk.edu.in

***Abstract*—A mathematical model of an electric bike (e-bike) developed in the MATLAB/Simulink environment is being discussed in this paper. The subsystem models, motor (permanent magnet synchronous motor), the controller(field oriented control) and battery (lithium ion) associated with the e-bike system has been separately detailed. Vehicle dynamics associated with the bicycle has also been studied to provide the required road reaction for correct simulation. The performance of the e-bike for two different slopes and different assist levels has been checked through simulation. The results show that the motor provides required assistance as set by the rider thereby providing both the advantages of exercise as well as extra aid from the motor.**

***Index Terms*—E-bike model, Motor control FOC, Bicycle dynamics, Motor assist level**

I. INTRODUCTION

Going green and focus on health has made the pedelec system of more importance over time. The bicycle has always been very popular among different age groups with being easy to handle because of its light weight and requires less money for maintenance and fuel. Riding a bicycle also provide health benefits just the same as in any physical exercise.

Since in a normal bicycle the entire power for propulsion is provided by the human being, they tend to get tired and strained after long journeys. Also, the hilly paths are difficult to pedal due to the requirement of extra effort. The advantages of a bicycle can be combined with extra power given by the motor in an e-bike.

An e-bike is an electric bicycle with an integrated electric motor which can be used for propulsion. The motor will be used in order to give additional power to make the ride more comfortable. The e-bike system in addition to the normal bicycle consists of the motor system, which includes a controller and battery system to power it. Hence an e-bike system consists of several subsystems coordinating with each other in order to provide the overall performance [2], [9]. Here, the rider has the choice has the choice to activate the motor when their effort alone is unable to move the bike forward, as when in an uphill or strenuous long road.

The e-bikes can be classified based on how the motor provides the power. It can either be a power-on-demand type wherein the rider choose assistance by the motor by a throttle controlling the power obtained. The second type is the pedal assist type, also called the pedelec. In this, the controller obtains the pedaling speed from a cadence sensor and the motor will provide assistance only when the rider is pedaling. The assistance level can be set manually by the rider as required or in advanced systems which include a torque sensor, an automatic assist level calculation can be obtained.

This paper discusses a pedelec model with 4 levels of assist which aids the rider in pedaling over slope. The assistance levels are chosen manually. In the results provided towards the end of the paper it can be seen that additional power provided by the motor depending upon the assist level chosen helps the rider have a smooth comfortable ride.

The following section of the paper explains the subsystem models (Battery, PMSM (Permanent magnet synchronous motor) and FOC (Field oriented controller), along with vehicle dynamics) used in modeling the system and obtaining the performance through simulation.

II. DYNAMIC BATTERY MODEL

The dynamic battery model used for the system model was developed with the MATLAB/Simulink reference model [**?**]. This model provides the charging and discharging curves based on the Shepherd's equations as given below:
Discharging:

$$V_{batt} = E_0 - K\frac{Q}{Q-it} * it - R * i + Ae^{-Bt} - K\frac{Q}{Q-it} * i^* \quad (1)$$

Charging:

$$V_{batt} = E_0 - K\frac{Q}{Q-it} * it - R * i + Ae^{-Bt} - K\frac{Q}{it-0.1Q} * i^* \quad (2)$$

Where,

- V_{batt} = battery voltage (V)
- E_0 = battery constant voltage (V)
- K = polarization constant (V/Ah) or polarization resistance (Ω)
- Q = Battery capacity (Ah)
- it = $\int idt$ = actual battery charge (Ah)
- A = exponential zone amplitude (V)
- B = exponential zone time constant inverse $(Ah)^{-1}$
- R = internal resistance (Ω)
- i = battery current (A)
- i^* = filtered current (A)

This equations provide the dynamic curve as obtained from a battery while charging or discharging.In this, filtered

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current (i^*) flowing through the polarization resistance is used. By using the current filter, algebraic loop that arises while simulating can be avoided.

The parameters required for the modeling of the dynamic behavior of the battery are obtained from the manufacturer's discharge curve at steady state.The data sheet provided by the manufacturer consists of typical discharge characteristics as shown in Figure 1 [3]. The three points that are obtained

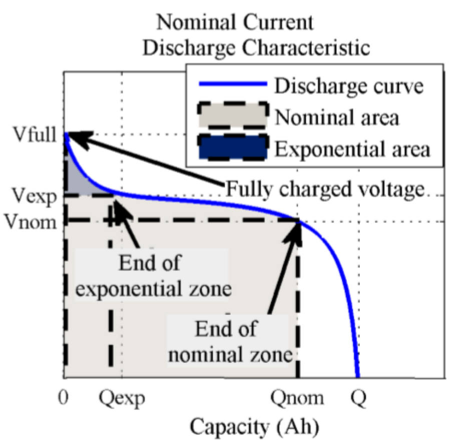


Fig. 1. Typical battery discharge curve

from the curve for parameter estimation are the fully charged voltage (V_{full}), the end of the exponential zone (Q_{exp}, V_{exp}) and the end of the nominal zone (Q_{nom}, V_{nom}) (when the voltage starts to drop abruptly). The maximum capacity (Q) and the internal resistance (R) are also provided in the data sheet. Hence by usin equation 1 and the following sets of equation (equations 3,4 and 5) parameters can be solved. The extracted charge will be zero (it =0) for a fully charged voltage and since the current step has just started the filtered current (i^*) will also be zero. The equations used for obtaining the constants [6]:

$$A = V_{full} - V_{exp} \tag{3}$$

At the end of the exponential zone, factor B can be approximated to 3/Q_{exp} since the energy of the exponential term is almost 0 (5%) after 3 time constants. The filtered current (i^*) is equal to 'i' because the current is in steady state. Therefore:

$$B = \frac{3}{Q_{exp}} \tag{4}$$

$$K = \frac{V_{full} - V_{nom} + A[e^{-BQ_{nom}} - 1][Q - Q_{nom}]}{10 * Q_{nom}} \tag{5}$$

The initial open circuit voltage

$$E_0 = V_{full} + Ki + iR - A \tag{6}$$

By simulating the model with a delay block instead of the filter the ripple frequency was obtained and then the current filter was designed with the required time constant.

The Lithium-ion (Li-ion) battery specifications: Voltage = 48V, Capacity = 6Ah.

The characteristic curve points and the calculated parameter values are tabulated and provided in Table I.

TABLE I: Important values

Points from characteristic curve	Parameter
V_{full} = 55.8714 V	A = 4.0129 V
i_{nom} = 2.6087 A	B = 10.1771 Ah^{-1}
R_i = 0.08 Ω	K = 0.04081 Ω
Q_{nom} = 5.4261 Ah	E_0 = 52.108 V
Q_{exp} = 0.29478 Ah	
V_{exp} = 51.8585 V	

The model developed as per Shepherd's equation is given in the Figure 2. For simulation the running performance of e-bike is only obtained. Hence, discharging part is only considered.

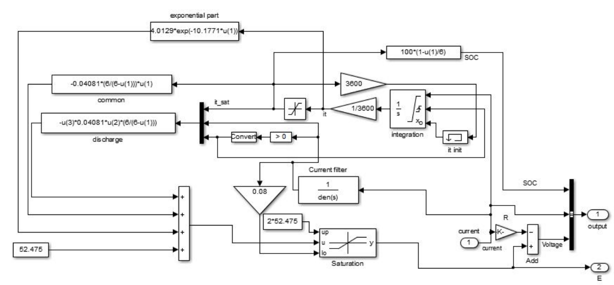


Fig. 2. Battery dynamic model

III. PMSM AND FIELD ORIENTED CONTROL

The PMSM was selected for modeling of e-bike system because of its advantages over other motors [2].The PMSM can be controlled effectively over its speed range by using various control techniques [1]. FOC being the the most popular among the vector speed control techniques of PMSM is used for the e-bike simulation.

A. *PMSM*

The Simulink PMSM block is used for system modeling [10]. The block provides the desired performance based on the motor type used and the parameter values. To build a control system over the PMSM block, the motor behaviour has to be understood.The equations that are used within the block are [10]:

$$\frac{d}{dt} i_d = \frac{1}{L_d} v_d - \frac{R}{L_d} i_d + \frac{L_q}{L_d} p\omega_m i_q \tag{7}$$

$$\frac{d}{dt} i_q = \frac{1}{L_q} v_q - \frac{R}{L_q} i_q + \frac{L_d}{L_q} p\omega_m i_d - \frac{\lambda p\omega_m}{Lq} \tag{8}$$

The electromagnetic torque equation can be hence written as:

$$T_e = 1.5p[\lambda i_q + (L_d - L_q) i_d i_q] \tag{9}$$

Where:

L_q, L_d = q and d axis inductance
R = Resistance of the stator windings
i_q, i_d = q and d axis currents
v_q, v_d = q and d axis voltages
ω_m = Angular velocity of the rotor
λ = Amplitude of the flux induced by the permanent magnets of the rotor in the stator phases
p = Number of pole pairs
T_e = Electromagnetic torque

Using a non-salient pole machine (L_d=L_q), as per the maximum torque per ampere condition when i_d is kept zero it can be seen that torque of the motor becomes directly proportional to i_q. Therefore, torque can be controlled using a PID controller taking i_q as the controllable factor (used in the model) [8], [10]. This is what is done by the FOC.

The speed of the motor will be as per the mechanical system part equation [10]:

$$\frac{d}{dt}\omega_m = \frac{1}{J}(T_e - T_f - F\omega_m - T_m) \tag{10}$$

$$\frac{d\theta}{dt} = \omega_m \tag{11}$$

Where:

J = Combined inertia of rotor and load
F = Combined viscous friction of rotor and load
θ = Rotor angular position
T_m = Shaft mechanical torque
T_f = Shaft static friction torque
ω_m = Angular velocity of the rotor (mechanical speed)

B. FOC

By decomposing the stator current into a magnetic field-generating part and a torque generating part, both can be controlled independently to obtain required value. Doing so simplifies the PMSM to a separately excited DC motor, which simplifies the control of a permanent magnet synchronous motor.

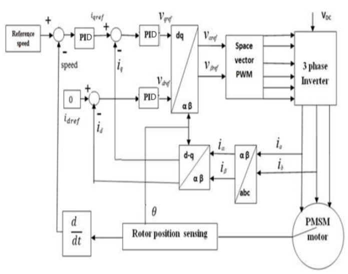


Fig. 3. FOC block diagram

The FOC can be summarised into the block diagram as shown in Figure 3 [8]. To obtain the required speed from the motor the reference speed is compared with the actual speed obtained from the feedback using PID. The error thus obtained is then translated into the orthogonal current components. This then has to be translated into AC current which requires inverse Clarke and Park transformations, so as to provide the required gate pulses by PWM (Pulse width modulation) method to the inverter. Here a second PID is used to control the current by obtaining feedback by direct Clarke and Park transformations.The inverter based on the gating pulse obtained produces the sinusoidal current thereby, rotating the motor in required speed.

Considering a three phased balanced system the current in each phase can be expressed as:

$$I_a = Isin\omega t \tag{12}$$

$$I_b = Isin(\omega t + 2\pi/3) \tag{13}$$

$$I_c = Isin(\omega t - 2\pi/3) \tag{14}$$

1) Clarke Transformation : Clarke Transformation : These three-phase quantities are transformed to the two-phase orthogonal (stationary) components using this transformation.Equations related to Clarke transformation are:

$$\begin{bmatrix} I_\alpha \\ I_\beta \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{15}$$

The inverse transformation will be:

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} I_\alpha \\ I_\beta \end{bmatrix} \tag{16}$$

2) Park transformation: The two-phase orthogonal (stationary) components are transformed into rotating reference frame quantities using Park transformation [7].The Park transformation is expressed by the following equations:

$$\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} I_\alpha \\ I_\beta \end{bmatrix} \tag{17}$$

The inverse transformation will be as

$$\begin{bmatrix} I_\alpha \\ I_\beta \end{bmatrix} = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} I_d \\ I_q \end{bmatrix} \tag{18}$$

To achieve the transformations the rotor position is required for which the PMSM has built in hall sensor to provide rotor position feedback.

The PID has been tuned by trial and error method.The values being (1- First PID and 2- next two PID): K_{p1}=100 ,K_{i1}=0.01, K_{d1}=0.005— K_{p2}=10 ,K_{i2}=0.01, K_{d2}=0.005

IV. E-BIKE DYNAMICS

For the complete system model of a pedelec the bicycle dynamics is required. Which is discussed in this section.

The different forces acting on a bicycle are represented in Figure 6 [7], the longitudinal dynamics of the e-bike hence can be written as [3], [9]:

$$M\frac{dv}{dt} = -F_w - F_{rr} - F_\alpha + F_t \tag{19}$$

where:

M = total mass of pedelec
v = pedelec velocity
t = time
F_w = aerodynamic drag force

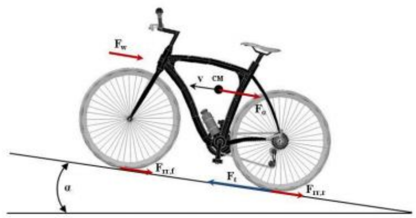


Fig. 4. Forces acting on cycle

$$F_w = \frac{1}{2}A_s C_w \rho_{air} v^2 \tag{20}$$

Where, A_s is the reference area of bicycle-rider system, C_w is the aerodynamic drag coefficient and ρ_{air} is the air density.
F_{rr}= rolling resistance

$$F_{rr} = C_{rr} M g cos\alpha \tag{21}$$

In which 'g' is the gravitational acceleration, α is the slope of the road, C_{rr} is the rolling resistance coefficient
F_α= gradient resistance force

$$F_\alpha = M g sin\alpha \tag{22}$$

Therefore the total driving torque required is:

$$T = rF_t \tag{23}$$

$$T = r[M\frac{dv}{dt} + F_w + F_{rr} + F_\alpha] \tag{24}$$

Here r is the radius of the wheel.
To reach the desired speed the total force from the motor and the pedal torque need to be equal to the load torque. Which, is given by:

$$r[M\frac{dv}{dt} + F_w + F_{rr} + F_\alpha] = T_m + T_{pedal} \tag{25}$$

In this, T_m is the motor torque and T_{pedal} is the human pedal torque. Hence giving the parameter of inertia on the PMSM block in Simulink with rotor plus load, The load torque given to the motor end is equal to:

$$T_m = r[M\frac{dv}{dt} + F_w + F_{rr} + F_\alpha] - T_{pedal} \tag{26}$$

Assuming a geared motor being used with gear ratio,

$$n = \frac{N_o}{N_i} \tag{27}$$

N_o= number of teeth in output gear
N_i= number of teeth in input gear
Transformation of motor speed to vehicle speed for the model will be:

$$v = \omega * \frac{r}{n} \tag{28}$$

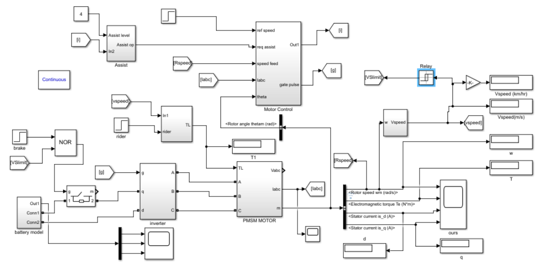


Fig. 5. Pedelec simulink model

Where, v= vehicle speed in m/s,ω= wheel speed in rad/s n= transmission ratio
The bicycle parameters are given below in Table II. The pedelec that has been modelled in this paper is made for 4 level of assist. This was achieved by controlling the maximum current that can be commanded by the controller in different assist. The complete model is provided in Figure 5.

TABLE II: Bicycle Parameters

M (mass of bike + rider)	90 Kg
ρ_{air}	1.2 Kg/m^3
A_s	0.509 m^2
C_w	0.76
C_{rr}	0.0025
g	9.8 m/s^2
n	1/10
r	33 cm

The different assist levels can be selected based on the rider request. According to the riders state a higher or lower aid from the motor can be provided. Also, the motor assist can be used for acceleration or starting the pedaling while in higher slopes [5]. The results and discussion from the simulation are provided in the next section.

V. RESULTS AND DISCUSSIONS

The system has been simulated for different slope (3^o and 4^o). The pedal force being kept the same, 10Nm in both cases. A low effort from the rider was considered to see the performance of motor assist in different cases. The system can also be made to run without any assistance providing a decent speed at higher efforts. The pedelec model that has been designed is for a 4 level assist.

The reaction forces values for the two different slopes are provided in Table III. They are calculated based on equations 21 and 22, converting them into torque and multiplying with the gear ratio. The speed, torque and the battery current

TABLE III: Reaction forces

Forces/Slopes	3^o	4^o
T_{rr}	0.072	0.072
T_α	1.52	2.03

drawn by the system are studied as shown in Figure 6,7,8,9 and 10. As can be seen from the reaction forces the system will require assist to move with a 10Nm pedal force in both gradients.

I. Slope degree = 3^o

As observed in Figure 6 it can be seen that the speed is still increasing and has not reached a steady state even after 200s. In this condition, the battery current being drawn is the maximum set for level 2 assist 3A.

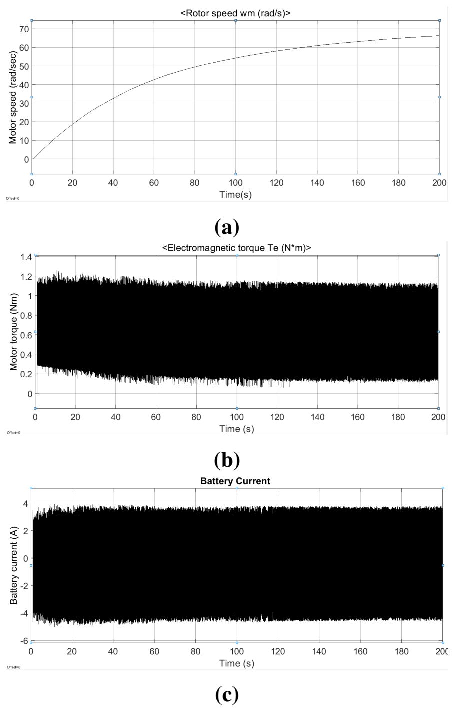


Fig. 6. (a) Speed and (b) Torque and (c) battery current for level 2 assist

By Figure 7 observation it can be seen that the speed is being settled to a value of 174.6 rad/sec (20.7 Km/hr) within a duration of 50s. It can also be seen that initially for starting and acceleration the battery current drawn is maximum set at 5A for level 3 assist. But after reaching a steady state the current drawn is lower because of the lower requirement of torque to maintain the speed. In Figure 8 it can be observed that although the speed settles to only 163 rad/sec (19.47Km/hr) it does so in very less time of 10s. Initially for acceleration, to reach the steady state speed the current drawn is maximum of 10A.

II. Slope degree = 4^o

For a 4^o slope there is slight increase the reaction forces acting on the system. Hence, with level 2 assist and pedal torque of 20Nm, the e-bike is unable to start. Hence level 3 assist is used. Wherein, the speed continues to increase without settling to a value as shown in Figure 9.

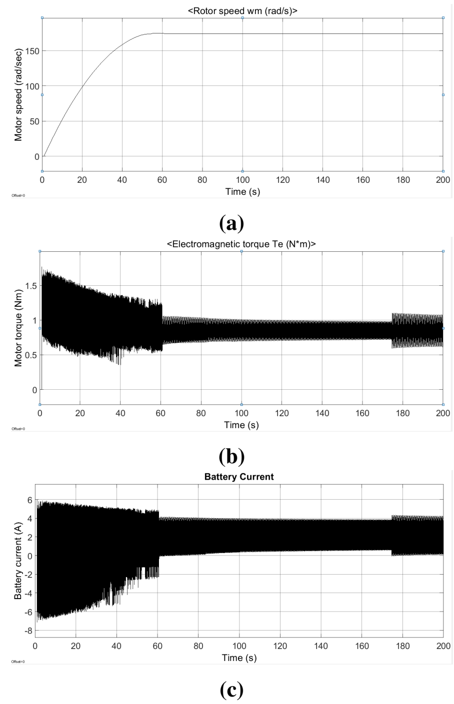


Fig. 7. (a) Speed and (b) Torque and (c) battery current for level 3 assist

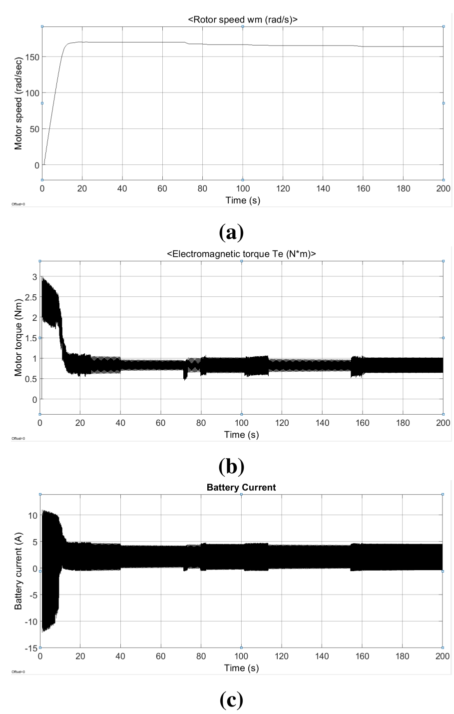
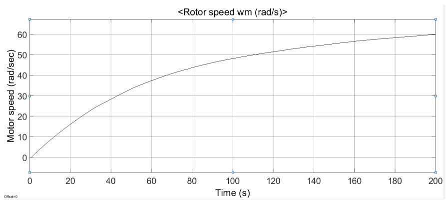
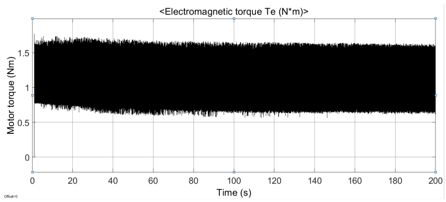


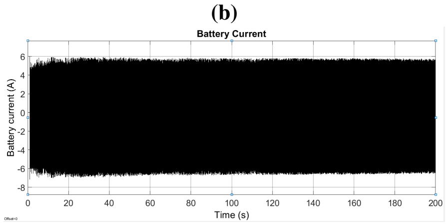
Fig. 8. (a) Speed and (b) Torque and (c) battery current for level 4 assist



(a)

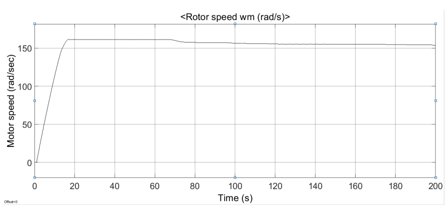


(b)

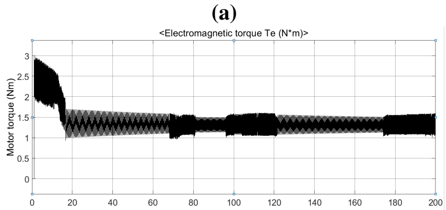


(c)

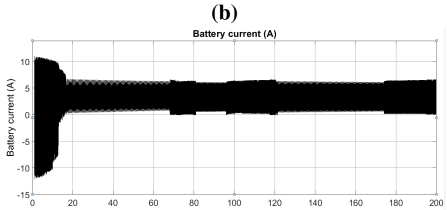
Fig. 9. (a) Speed and (b) Torque and (c) battery current for level 3 assist



(a)



(b)



(c)

Fig. 10. (a) Speed and (b) Torque and (c) battery current for level 3 assist

Here also similar to as in level 2 assist of the previous case the current drawn will be the maximum set for level 3 assistance. Observing Figure 10 it can be seen that the system settles to a speed of 155 rad/sec (18Km/hr) at about 15s. The battery current graph for level assist shows that the initial current drawn for acceleration purpose to settle down into the steady state speed is 10A after which a lower current is drawn for maintaining the speed.

VI. Conclusion

The basic e-bike system model is developed and simulated under two different slope.Performance tested for different assist levels provided under each case. It can be concluded from the results obtained that the system behaves close to a practical e-bike where the motor provides the extra torque to drive through slopes with lower pedal effort. It is also clear that higher the assistance the current drawn from the battery will be more and hence with a single charge the distance that can be traveled will reduce with increased assistance.

The motor control system developed in this model can be further improved to reduce third harmonic ripples by using SVPWM(Space vector PWM) instead of PWM.An additional cutoff limit is to be added for assistance start as in a practical e-bike

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