ASM Vehicle Dynamics

Addendum

Complementary Description of the ASM Vehicle Dynamics Model

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dSPACE

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Contents

About This Document

Content

This document introduces you to the tire models available in ASM VehicleDynamics, its physical background, and the data required for parameterization. It also describes the coordinate systems used and how they relate to each other in the simulation of the vehicle dynamics. There are also informarion on the numerical integration method used to integrate the stiff differential equations.

Symbols

dSPACE user documentation uses the following symbols:

Symbol	Description
DANGER	Indicates a hazardous situation that, if not avoided, will result in death or serious injury.
WARNING	Indicates a hazardous situation that, if not avoided, could result in death or serious injury.
CAUTION	Indicates a hazardous situation that, if not avoided, could result in minor or moderate injury.
NOTICE	Indicates a hazard that, if not avoided, could result in property damage.
Note	Indicates important information that you should take into account to avoid malfunctions.
Tip	Indicates tips that can make your work easier.
	Indicates a link that refers to a definition in the glossary, which you can find at the end of the document unless stated otherwise.
	Precedes the document title in a link that refers to another document.

Naming conventions

dSPACE user documentation uses the following naming conventions:

%name% Names enclosed in percent signs refer to environment variables for file and path names.

< > Angle brackets contain wildcard characters or placeholders for variable file and path names, etc.

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Common Program Data folder A standard folder for application-specific configuration data that is used by all users.

%PROGRAMDATA%\dSPACE\<InstallationGUID>\<ProductName>

or

%PROGRAMDATA%\dSPACE\<ProductName>\<VersionNumber>

Documents folder A standard folder for user-specific documents.

%USERPROFILE%\Documents\dSPACE\<ProductName>\
<VersionNumber>

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<ProductName>

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To access the Web version, you must have a *mydSPACE* account.

PDF files You can access PDF files via the icon in dSPACE Help. The PDF opens on the first page.

Model Overview

Model Overview

Overview	The VehicleDynamics model simulates the dynamics of a passenger car. The vehicle is composed of engine, drivetrain, rigid vehicle body, and four wheels. The model represents the vehicle's longitudinal, lateral, and vertical dynamics and simulates in detail an elastic drivetrain, suspension kinematics and compliance, tire-road friction forces and moments, steering and brakes. The combustion process in the engine is not modeled.
Related topics	Basics Tutorials (ASM Vehicle Dynamics Model Description 🕮)

Coordinate Systems

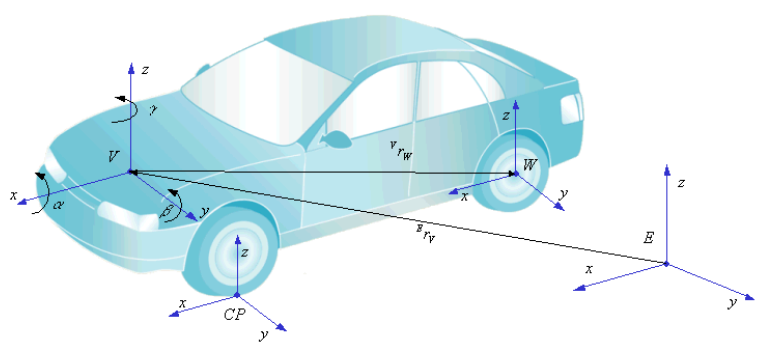
Coordinate Systems

Description

To describe the kinematics of vehicle dynamics, four coordinate systems, all rotating clockwise, were defined as a basis for modeling. The following illustration shows the system of coordinates, which are utilized in ASM VehicleDynamics Model.

Tip

In the following descriptions, the superscript of a variable indicates the coordinate system at which the variable is described. The subscript indicates the relevant body of the variable. For example, ${}^{E}r_{V}$ is the position vector of the vehicle (subscript $_V$) described in the earth coordinate system (superscript ^{E}).



As shown in the illustration, the following coordinates are used:

- Earth coordinate system, index E represents the fixed reference system.
- Vehicle reference coordinate system, index V is fixed to the vehicle body. Its origin is at zero position – configuration position – at mid-point between the front wheel centers. The x-axis is in the longitudinal direction of the vehicle and points forwards, the y-axis points towards the vehicle's left side, and the z-axis direction follows the right-hand rule and points upwards.

- Wheel coordinate system, index W is at the wheel center. Its orientation is determined by the wheel orientation, which depends on the suspension kinematics.
- Contact point coordinate system, index CP its origin is at the contact point. The x-y plane is parallel to the road local plane. For details on calculating the contact point coordinate system, refer to Contact Point Calculation on page 25.

Vehicle degrees of freedom

The position and orientation of this coordinate system is a function of the vehicle degrees of freedom, which are listed in the table below:

$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = {}^V\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_V$	The translational vehicle velocities of the origin of the coordinate system V in x, y, and z directions described in the vehicle reference coordinate system V
$\begin{bmatrix} \dot{q}_4 \\ \dot{q}_5 \\ \dot{q}_6 \end{bmatrix} = {}^V\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}_V$	The angular vehicle velocities about the x, y, and z axes of the vehicle reference coordinate system V

Note

The other vehicle degrees of freedom are not used to define the position and orientation of the vehicle coordinate system against the earth coordinate system.

From these degrees of freedom, the orientation angles roll, pitch, and yaw are calculated

$$\begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin(\beta) \\ 0 & \cos(\alpha) & \sin(\alpha)\cos(\beta) \\ 0 & -\sin(\alpha) & \cos(\alpha)\cos(\beta) \end{bmatrix}^{-1} {}^V\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}_V$$

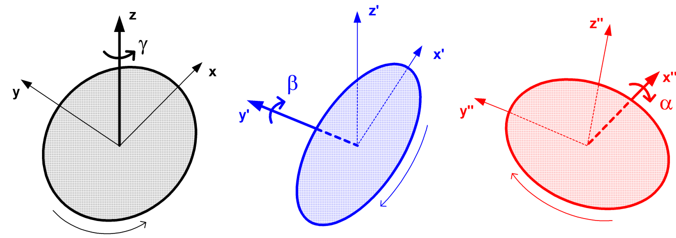
The vehicle orientation angles can then be calculated by integration where

α is the roll angle
β is the pitch angle
γ is the yaw angle

The overall rotation matrix can now be calculated by multiplying the subsequence rotation matrices as follows:

$${}^E T_V = T_z(\gamma)\, T_y(\beta)\, T_x(\alpha)$$

The following illustration shows the rotation sequence.



Using the rotation matrix, the vehicle velocity in the earth is calculated as follows:

$$ {}^{E}v_V = {}^{E}T_V \, {}^{V}v_V $$

The vehicle position can then be calculated by integration.

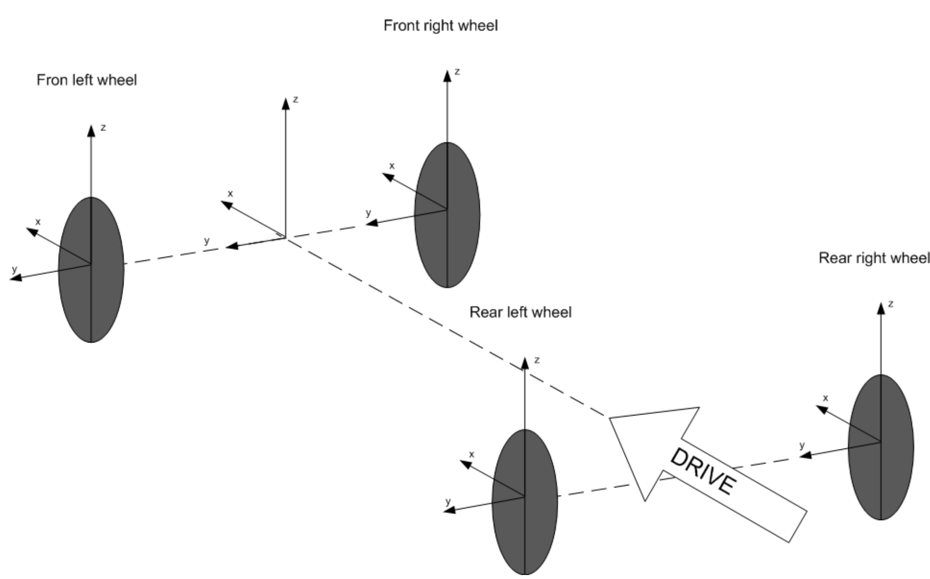
$$ {}^{E}r_V = {}^{E}\begin{bmatrix} x \\ y \\ z \end{bmatrix}_V = {}^{E}\begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}_V + \int {}^{E}\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}_V dt $$

The above equations provide the position and orientation of the vehicle reference coordinate system against the earth coordinate system.

The velocities and positions of the vehicle and wheel bodies are described in the vehicle reference coordinate system. To get the location or the velocity of a body in the earth coordinate system, a coordinate transformation must be performed.

Vehicle coordinate systems

The right-handed system of coordinates for the vehicle and four wheels can be found in the next illustration.



Related topics

Basics

Coordinate System Used by the Road Generator (ModelDesk Road Creation 📖)
Coordinate System Used in MotionDesk (MotionDesk Calculating and Streaming Motion Data 📖)

Suspension

Where to go from here

Information in this section

Sign Conventions

Introduction

The following sign conventions hold for ASM Vehicle Dynamics.

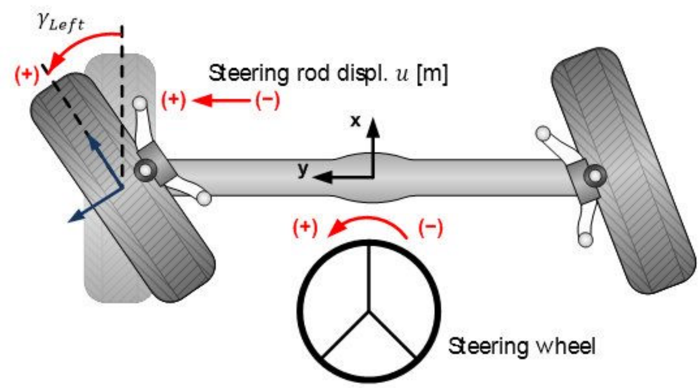
Steering and Suspension Kinematics

In ASM Vehicle Dynamics, the following sign convention is used for the steering wheel, the steering rod displacement, the left wheel rotation about the z-axis (gamma angle), and vertical wheel displacement:

Name	Positive sign (+)
Steering wheel	Counterclockwise rotation about the steering wheel vertical axis
Steering rod displacement	Movement to the left side
Gamma angle of the left wheel	Counterclockwise rotation about the z-axis
Vertical left wheel displacement	Vertical wheel movement is defined as the vertical movement of the wheel center.

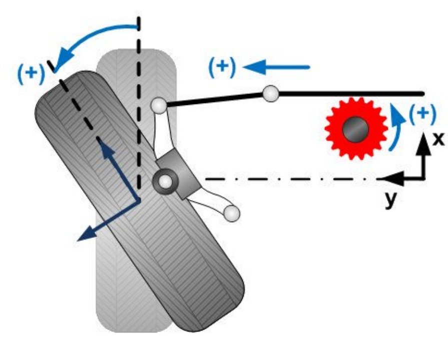
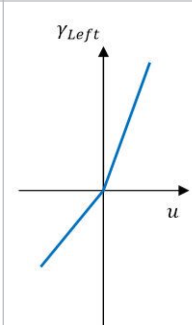
Name	Positive sign (+)
	Positive wheel movement 'displ_z' is defined as an upward movement (compression). Example: displ_z = 0 equals the wheel position in the vehicle design configuration.

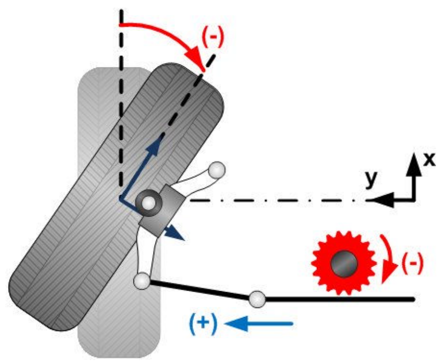
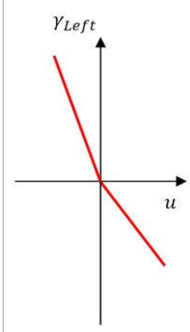
The following illustration displays this sign convention:



Different Steering Configurations

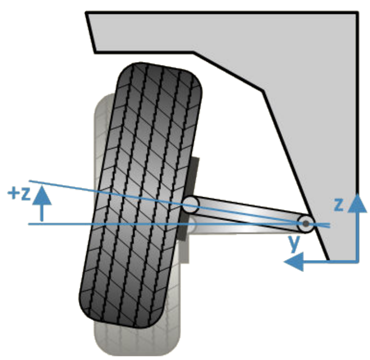
For different steering system configurations, the following table shows how to parameterize the steering gear ratio *i* and the gamma table of suspension kinematics $\gamma_{Left} = f(u)$ of the ASM Vehicle Dynamics Model. *u* is the steering degree of freedom, e.g., the steering rod displacement.

Sketch	Steering gear ratio *i*	Left gamma $\gamma_{Left} = f(u)$
	Positive (+)	

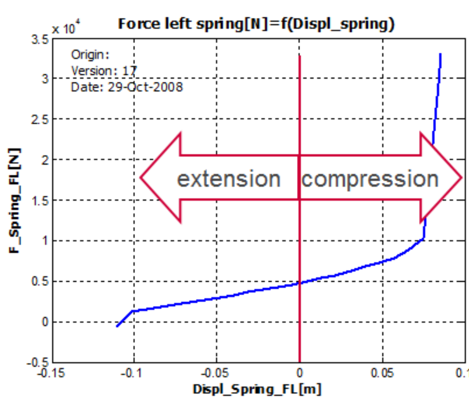
Sketch	**Steering gear ratio *i***	**Left gamma** $\gamma_{Left} = f(u)$
	Negative (-)	

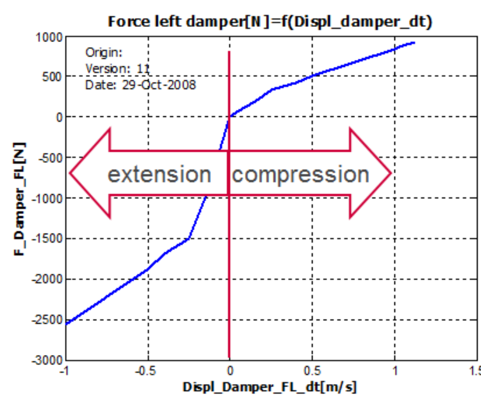
Suspension

Wheel jounce (+z in figure below) is considered to be positive, wheel rebound is negative.



From this, the spring, damper and stabilizer compressions are considered to be positive, whereas the deflections are negative. The example figures below show typical spring (left side) and damper (right side) characteristics for extension and compression.





Orientation Angles

Introduction

The camber, side-view and toe angles are used as orientation order.

Camber angle

Camber angle is the angle between the vertical axis of the wheels and the vertical axis of the vehicle when viewed from the front or rear. It is used in the design of steering and suspension. If the top of the wheel is leaned outward, it is called positive camber. If the bottom of the wheel is leaned inward, it is called negative camber.

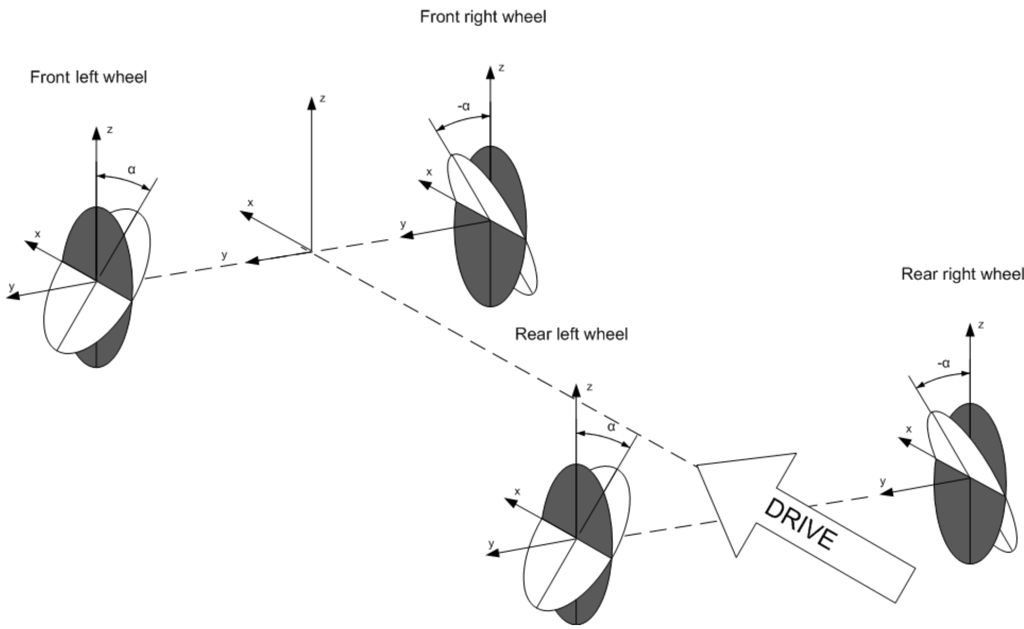
In ASM, the camber angle is defined for the left wheel within ModelDesk depending on different input values. The definition is as follows (for the left vehicle seen into the direction of drive):

- A positive camber angle that means that the upper part of the wheel has more distance to the vehicle than the lower end of the wheel is nearly equal to a negative ASM alpha angle.
- A negative camber angle that means that the upper part of the wheel is closer to the vehicle than the lower end of the wheel is nearly equal to positive ASM alpha angle.

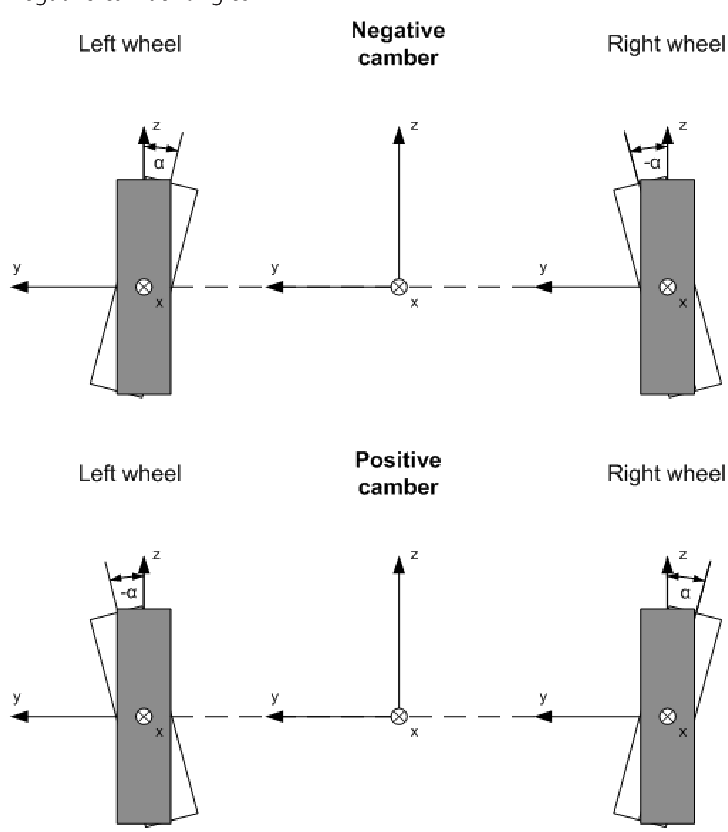
Note

Note that this definition is made for the left side of the vehicle. Due to the same orientation of the right-handed coordinate system in each wheel of an ASM vehicle the alpha angle definition for positive and negative camber angles is the other way round for the opposite site of the vehicle. This is already considered in the parameterization for a symmetric axle.

The following illustration shows the camber angle definition for ASM.



The following illustration shows the front axle as an example for positive and negative camber angles.



Side-view angle

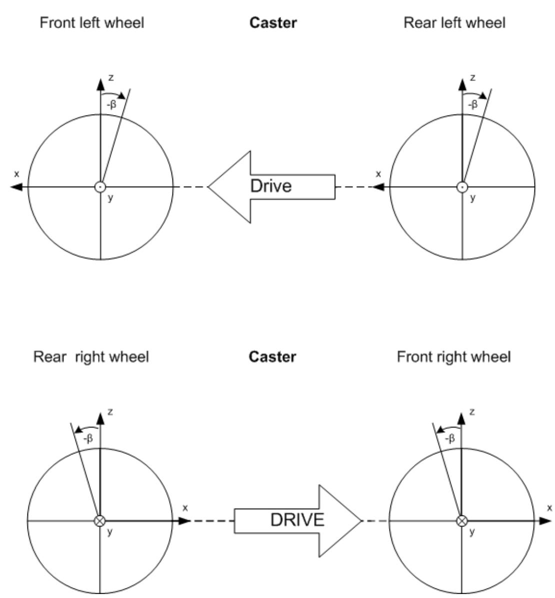
Side-view angle is the angular displacement from the vertical axis of the suspension of a steered wheel in a car, bicycle or other vehicle, measured in the longitudinal direction. It is the angle between the pivot line (in a car - an imaginary line that runs through the center of the upper ball joint to the center of the lower ball joint) and vertical.

In ASM, the side-view angle is defined for the left wheel within ModelDesk depending on different input values. The definition is as follows (for the left vehicle seen into the direction of drive):

- A positive side-view angle is nearly equal to a negative ASM beta angle.
- A negative side-view angle is nearly equal to a positive ASM beta angle.

Note

The side-view angle definition is identical for all sides of the ASM vehicle.



Toe angle

In automotive engineering, toe is the symmetric angle that each wheel makes with the longitudinal axis of the vehicle. Positive toe, or toe in, is the front of the wheel pointing in towards the centerline of the vehicle. Negative toe, or toe out, is the front of the wheel pointing away from the centerline of the vehicle.

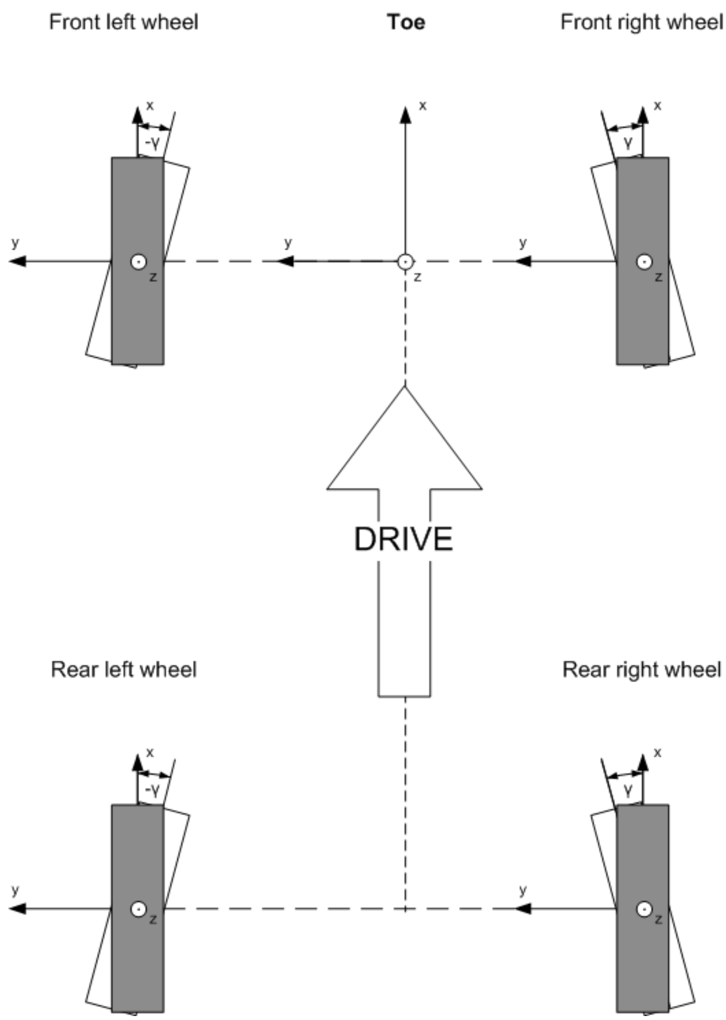
In ASM, the toe angle is defined for the left wheel within ModelDesk depending on different input values. The definition is as follows (for the left vehicle seen into the direction of travel):

- Toe in which means the front part of the wheel is closer to the vehicle than the rear part, is nearly equal to a negative ASM gamma angle.
- Toe out which means the rear part of the wheel is closer to the vehicle than the front part, is nearly equal to a positive ASM gamma angle.

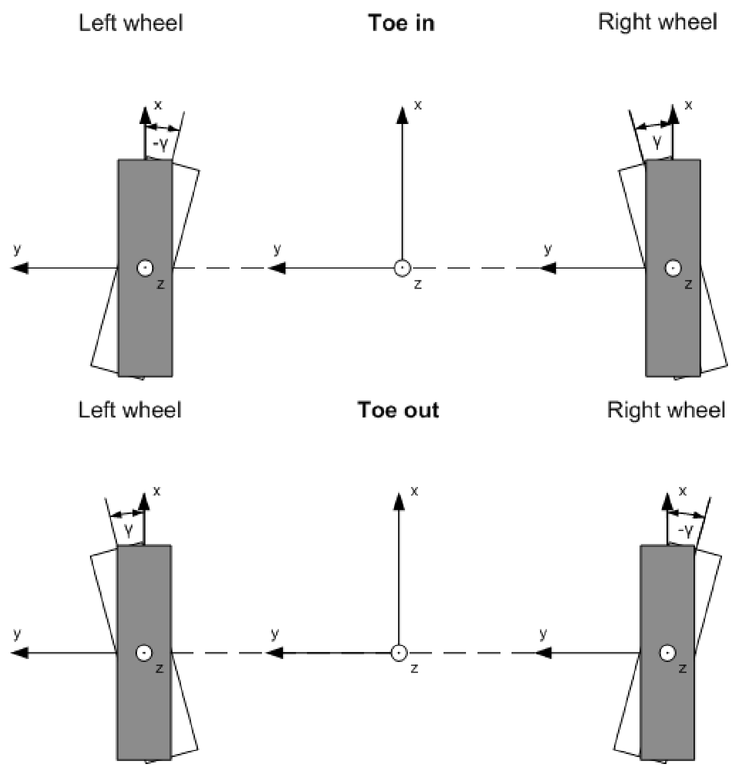
Note

Note that this definition is made for the left side of the vehicle. Due to the same orientation of the right-handed coordinate system in each wheel of an ASM vehicle, the gamma angle definition for toe in and toe out is the other way round for the opposite site. This is already considered in the parameterization for a symmetric axle.

The following illustration shows the toe angle definition for ASM.



The following illustration shows the front axle as an example for toe in and toe out.



Summary

In general, the following correlation can be approximated for the front left wheel:

Automotive Angle	**Cardan Angle**
Positive toe	Negative gamma
Positive side-view	Negative beta
Positive camber	Negative alpha

Rigid Axle with Suspension Kinematics 3DOF

Description

The following descriptions are relevant for rigid axle simulation using the Suspension Kinematics 3DOF model in the ASM library.

The rigid axle kinematics is dependent of 2 degrees of freedom (DOF):

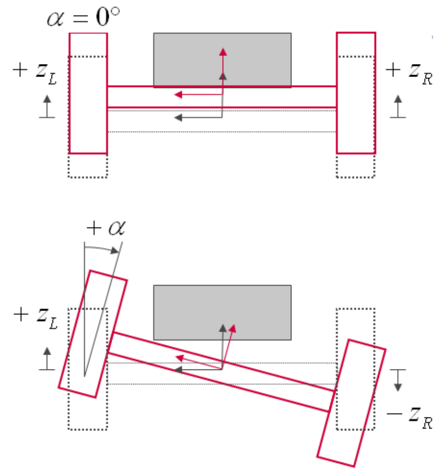
- Vertical displacement Δz of the axle center
- Roll angle $\Delta \alpha$ about the axle's longitudinal direction

If the axle is steerable, the steering rod displacement is a further DOF of the axle.

In ASM, the 3DOF suspension type is used to simulate the rigid axle kinematics:

$$x, y, \alpha, \beta, \gamma = f\left(z_{Wheel}, z_{OppositeWheel}\right) + f\left(u_{Steering}\right)$$

The transformation of the rigid axle DOF into the ASM ones can be seen from the following illustration and calculations:



$$\Delta z_L = \Delta z + \frac{L}{2} \cdot \sin\alpha \quad (1)$$

$$\Delta z_R = \Delta z - \frac{L}{2} \cdot \sin\alpha \quad (2)$$

The wheel translations (x,y) are calculated from the rigid axle ones as follows.

Set up the equations for Δz and α as functions Δz_L of and Δz_R:

$$(1) + (2): \quad \Delta z = \frac{\Delta z_L + \Delta z_R}{2}$$

$$(1) - (2): \quad \alpha = \arcsin\left(\frac{\Delta z_L - \Delta z_R}{L}\right)$$

Define value vectors for Δz_L and Δz_R, calculate the related Δz and α vectors from equations above and interpolate the given translation tables at these values:

$$\Delta x = f(\Delta z, \alpha) \Rightarrow \Delta x = f(\Delta z_L, \Delta z_R)$$

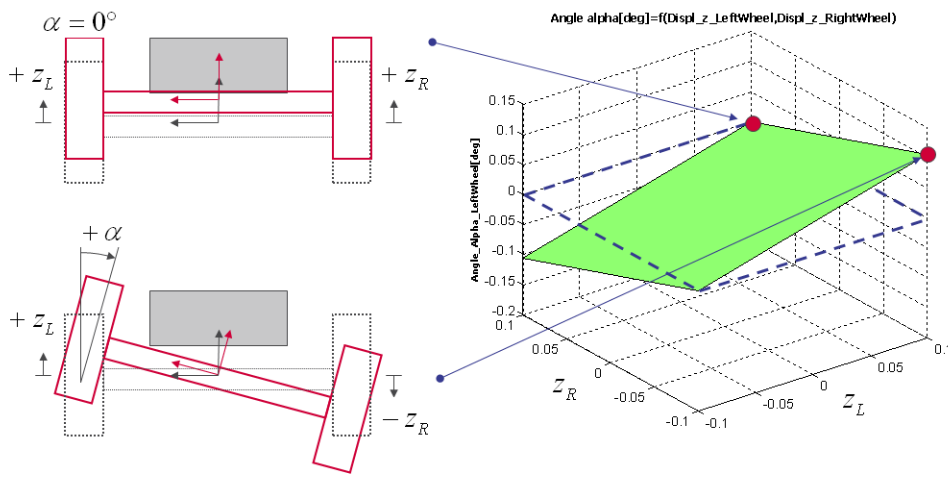
$$\Delta y = f(\Delta z, \alpha) \Rightarrow \Delta y = f(\Delta z_L, \Delta z_R)$$

The wheel orientations are calculated as follows.

Angle α is the same as the roll angle. Thus, there is no conversion needed. Define a value vector for the roll angle α. This vector describes the main diag entries of the alpha table:

$$\begin{matrix} \alpha_1 & 0 & 0 & 0 \\ 0 & \alpha_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \alpha_n \end{matrix}$$

Δz is set to zero. Δz_L and Δz_R are calculated from equations (1) and (2).



Angle β can be calculated from the given table in the same way as the translation tables:

$$\beta = f(\Delta z, \alpha) \Rightarrow \beta = f(\Delta z_L, \Delta z_R)$$

Angle γ is calculated from the self-steering ratio of the rigid axle

$$i_{SelfSteer} = \frac{\gamma[\text{deg}]}{\alpha[\text{deg}]}$$

Define a value vector for the roll angle and multiply this vector with the self-steering ratio to get the γ angle. Then, the same proceeding as for angle α holds.

Tire Models

Where to go from here

Information in this section

Basics on Tire Models

Basics on Tire Models

Basics

Several types of mathematical models have been developed over the last few decades, each of them for a specific purpose and with a different level of accuracy and complexity. Real-time applications use empirical tire models that describe measured tire characteristics by means of a table or mathematical formula and certain interpolation schemes, which reduces the computation time.

ASM VehicleDynamics offers three tire models:

- TMeasy tire model, refer to EasyToUse Tire Model on page 28
- Magic Formula tire model (earlier than version 6.1), refer to Magic Formula Tire Model (Earlier than Version 6.1) on page 39
- Magic Formula tire model 6.1, refer to Magic Formula Tire Model 6.1 on page 53

The principle of approximating measured tire forces by an interpolation curve described by various parameters is the same for both models, but the formulations of the interpolation curves are different.

ASM VehicleDynamics lets you switch between four sets of tire parameters to simulate different surface conditions which affect not only the friction coefficient but also the forces and torques curve shapes. The friction coefficient can also be changed.

In the following sections, the tire contact point calculations and wheel load are described. These calculations are the same for each model and provide the tire model input variables. Refer to Contact Point Calculation on page 25.

Then each tire model is described, along with its forces and torques calculation, and how it is parameterized.

Contact Point Calculation

Where to go from here

Information in this section

Introduction to Contact Point Calculation

Introduction

Tire models require certain wheel and tire states to calculate the tire-road friction forces and torques. These states are the standard inputs for each tire model. The wheel states are calculated in the direction of the contact point.

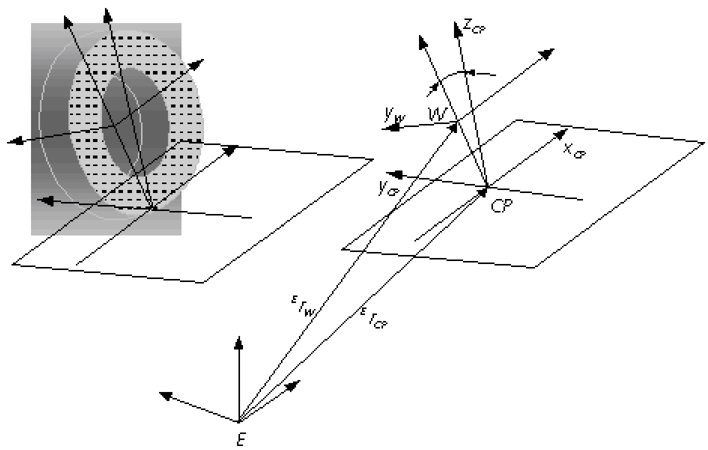
Contact point calculation has the task of:

- Calculating the tire radius according to the wheel position and contact point position
- Calculating the contact point coordinate system of each tire by calculating the coordinate system's unit vectors in the vehicle reference system
- Calculating the wheel speeds in the direction of the contact point
- Calculating the wheel bore speed (angular speed about contact point z-axis)
- Calculating the tire-road camber angle

Description of Contact Point Calculation

Contact point calculation

The tire is assumed to have a single contact point CP with the road, and the road – at the contact point – is approximated by a plane, see the following figure which shows the tire contact plane and the contact point coordinate system.



The contact point is calculated as follows:

$$^{E}r_{CPi} = {}^{E}r_{Wi} - r_{i0}\,{}^{E}e_{VZ}$$

where:

$^{E}r_{Wi}$	is the position of the wheel in the earth coordinate system
$^{E}e_{VZ}$	is the unit vector in the direction of z-axis of the vehicle coordinate system
r_{i0}	is the unloaded tire radius. It can be replaced with the current tire radius r_{Tire} of the last simulation step.

The vector $^{E}r_{CPi}$ determines only the contact point position in x- and y-direction. The z-value depends on the road definition and can be calculated from the road model for specific x and y values, i.e.:

$$^{E}r_{CPi} = \begin{bmatrix} ^{E}r_{xCPi} \\ ^{E}r_{yCPi} \\ ^{E}r_{zCPi}\left(^{E}r_{xCPi}, {}^{E}r_{yCPi}\right) \end{bmatrix}$$

Using the contact point position, the tire radius can be calculated:

$$^{E}r_{Tire} = {}^{E}r_{CP} - {}^{E}r_{W}$$

At this point, a unit vector in the direction of road normal is calculated in the road module:

$$^{E}e_{CPZ} = {}^{E}e_{CPZ}\left(^{E}r_{xCP}, {}^{E}r_{yCP}\right)$$

Then this unit vector is transformed in the vehicle coordinate system, index V.

$$^{V}e_{CPZ} = {}^{V}T_{E}{}^{E}e_{CPZ}$$

To get the other unit vectors of the contact point coordinate system, the wheel orientation is required. The unit vector $^{V}e_{CPX}$ is perpendicular to the track normal and to the unit vector normal to the wheel plane $^{V}e_{CPX}$, i.e.:

$$^{V}e_{CPX} = \frac{^{V}e_{WY} \times {^{V}e_{CPZ}}}{\left|^{V}e_{WY} \times {^{V}e_{CPZ}}\right|}$$

Now the vector $^{E}e_{CPY}$ can be calculated using the right-hand coordinate rule.

$$^{V}e_{CPY} = {^{V}e_{CPZ}} \times {^{V}e_{CPX}}$$

These direction vectors are now used to orient the wheel speeds in the contact point coordinate system. The bore speed and camber angle are also calculated.

$$^{CP}v_{\ x} = {^{V}e_{CPX}} \cdot {^{V}v_{W}}$$

$$^{CP}v_{\ y} = {^{V}e_{CPY}} \cdot {^{V}v_{W}}$$

$$^{CP}\omega_{\ z} = {^{V}e_{CPZ}} \cdot {^{V}\omega_{W}}$$

$$\gamma = \arcsin^{-1}\left({^{V}e_{WY}} \cdot {^{V}e_{CPZ}}\right)$$

EasyToUse Tire Model

Where to go from here

Information in this section

Introduction

Introduction

The TMeasy (EasyToUse) tire model was developed by Prof. G. Rill, College of Regensburg, Germany. It is a semi-empirical tire model, which describes longitudinal and lateral tire force characteristics and the self-aligning torque as functions of longitudinal and lateral slip.

The model approximates the tire characteristics and the interaction between the longitudinal and lateral forces through a simplified formula. Due to the simplified inter-extrapolation formulas used in the model, it requires less computation time than the Magic Formula tire model.

Wheel Load

Description

Wheel load means the tire force normal to the road at the contact point. It depends on the tire deflection $\Delta z \Delta z$.

$$\Delta z = (r_{Tire0} - r_{Tire}) \geq 0$$

Then the normal force can be calculated as follows:

$$F_{zdy} = (\Delta z k_z + \Delta \dot{z} d_z) \geq 0$$

where:

k_z is the equivalent tire vertical stiffness

d_z is the equivalent tire vertical damping

Slip Calculation

Description

The forces generated by the tire depend on the slip and not directly on the wheel speeds. The slip quantities must be calculated to calculate the tire forces. In TMeasy the slip in x- and in y-direction are calculated as follows:

$$S_x = \frac{-(v_{xCP} - \Omega\, r_d)}{|\Omega r_d| + v_{num}}$$

$$S_y = \frac{-v_{yCP}}{|\Omega\, r_d| + v_{num}}$$

where r_d is the dynamic rolling tire radius. It is calculated as follows:

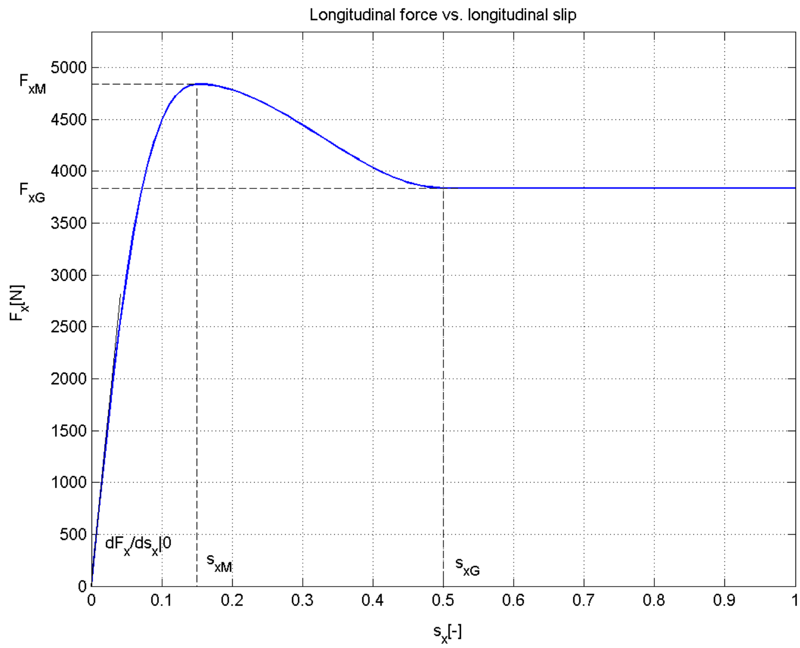
$$r_d = \frac{2}{3} r_{Tire0} + \frac{1}{3} r_{Tire}$$

The dynamic rolling tire radius represents the equivalent tire rolling radius by which the slip quantities must be calculated.

Forces Calculation

Longitudinal force characteristics

Longitudinal force is also called the traction or braking force and acts in the longitudinal direction of the tire-road contact area. The relationship between the longitudinal slip and the longitudinal force is a nonlinear one. The figure below shows the description of the longitudinal force as a function of pure longitudinal slip as approximated in TMeasy.



The longitudinal force characteristic as a function of the longitudinal slip is defined separately using the following parameters:

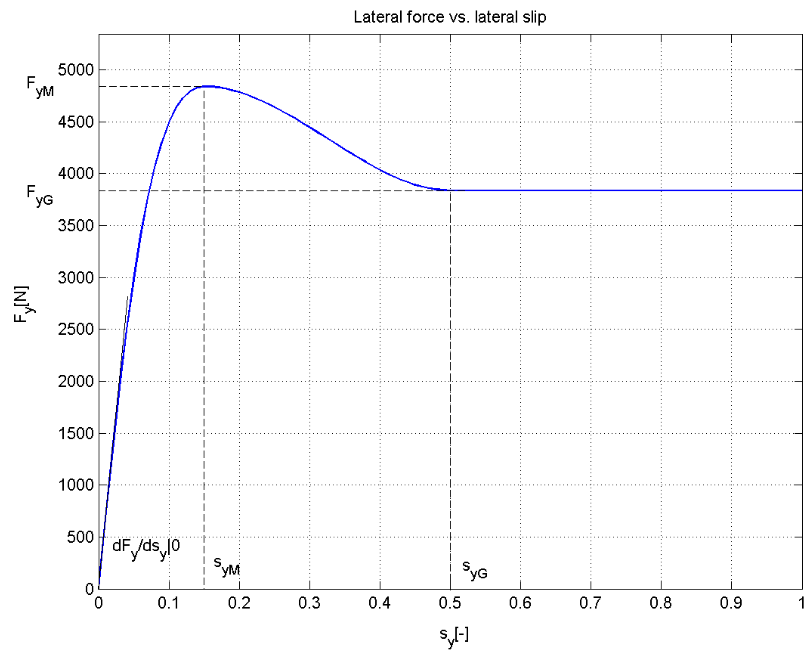
dF_x^0	is the initial slope (i.e. at zero slip)
s_x^M	is the slip at maximum forces
F_x^M	is the magnitude of the maximum forces
s_x^G	is the slip value at which the tire starts to slip
F_x^G	is the slip force

Note

These parameters must be defined for two wheel loads (normal and double normal wheel load) to approximate the wheel load effect.

Lateral force characteristics

Lateral force is also called the cornering force and acts in the lateral direction of the tire-road contact area. The relationship between the lateral slip and the lateral force is a nonlinear one. The figure below shows the description of the lateral force as a function of pure lateral slip as approximated in TMeasy.



The lateral force characteristic as a function of the lateral slip is defined separately using the following parameters:

dF_y^0	is the initial slope at zero slip
s_y^M	is the slip at maximum forces
F_y^M	is the magnitude of the maximum forces
s_y^G	is the slip value at which the tire starts to slip
F_y^G	is the slip force

Note

These parameters must be defined for two wheel loads (normal and double normal wheel load) to approximate the wheel load effect.

Generalized force calculation

The previous descriptions of the friction forces apply only to pure longitudinal and lateral slips. If the tire produces simultaneous forces in x and y directions, the situation is different as the traction used in one direction limits the available traction in the other. That generates the well-known friction circle or friction ellipse.

During general driving situations, for example, acceleration or deceleration in curves, longitudinal S_x and lateral slip S_y appear simultaneously. TMeasy uses a generalized slip quantity, which is calculated by adding the longitudinal slip S_x and the lateral slip S_y vectorial, i.e.:

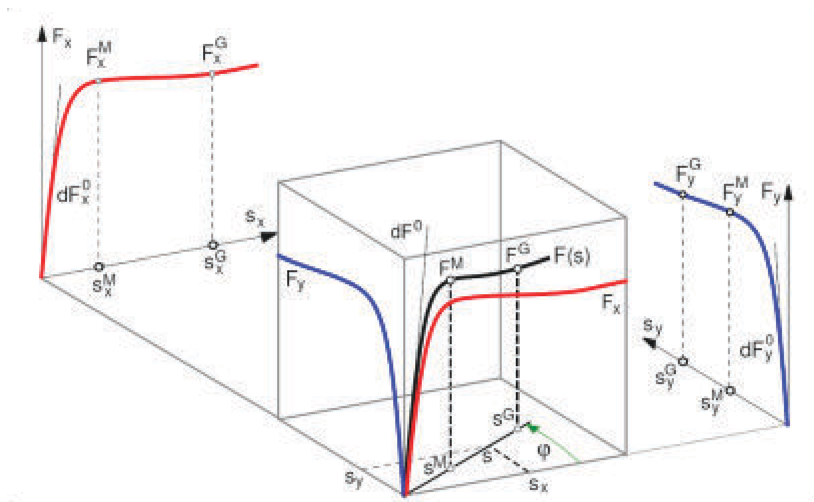
$$S = \sqrt{\left(\frac{S_x}{S_{xn}}\right)^2 + \left(\frac{S_y}{S_{yn}}\right)^2}$$

where the slips S_x and S_y were normalized to give them the same weighting in S. The normalization factors S_{xn} and S_{yn} are calculated from the force curves parameters as follows:

$$s_{xn} = \frac{s_x^M}{s_x^M + s_y^M} + \frac{\frac{F_x^M}{dF_x^0}}{\frac{F_x^M}{dF_x^0} + \frac{F_y^M}{dF_y^0}}$$

In a similar way to the curves of the longitudinal and lateral forces, the curve of the generalized tire force is defined by the characteristic parameters dF^0, s^M, F^M, s^G and F^G.

The following figure shows the generalized force parameters:



These parameters depend on the corresponding values of the longitudinal and lateral force parameters and also on the current slip values. They are calculated as follows:

$$dF^0(s_x, s_y) = \sqrt{\left(dF_x^0\, s_{xn} \cos\varphi\right)^2 + \left(dF_y^0\, s_{yn} \sin\varphi\right)^2}$$

$$F^M(s_x, s_y) = \sqrt{\left(F_x^M \cos\varphi\right)^2 + \left(F_y^M \sin\varphi\right)^2}$$

$$F^G(s_x, s_y) = \sqrt{\left(F_x^G \cos\varphi\right)^2 + \left(F_y^G \sin\varphi\right)^2}$$

$$s^M(s_x, s_y) = \sqrt{\left(\frac{s_x^M}{s_{xn}} \cos\varphi\right)^2 + \left(\frac{s_y^M}{s_{yn}} \sin\varphi\right)^2}$$

$$s^G(s_x, s_y) = \sqrt{\left(\frac{s_x^G}{s_{xn}} \cos\varphi\right)^2 + \left(\frac{s_y^G}{s_{yn}} \sin\varphi\right)^2}$$

where

$$\cos\varphi = \frac{s_x/s_{xn}}{s}\,,\ \sin\varphi = \frac{s_y/s_{yn}}{s}$$

These last equations provide a smooth transition between the characteristic curves of the longitudinal and lateral forces in the range 0° to 90°. The function $F = F(s)$ is now described in intervals by a broken rational function, a cubic polynomial and a constant F^G.

$$\left(s_x, s_y\right) = \begin{cases} s_M \cdot dF^0 \dfrac{\sigma}{1+\sigma\left(\sigma + dF^0 \dfrac{s^M}{F^M} - 2\right)}, & \sigma = \dfrac{s}{s^M}, \quad 0 \le s \le s^M \\ F^M - \left(F^M - F^G\right)\sigma^2(3 - 2\cdot\sigma)\,, & \sigma = \dfrac{s - s^M}{s^G - s^M},\ s^M \le s \le s^G \\ F^G\,, & s > s^G \end{cases}$$

Longitudinal and lateral forces result from the projections in longitudinal and lateral direction.

$$F_x = F\cos\varphi\,,\ F_y = F\sin\varphi$$

Camber angle influence

If the wheel rotation axis is inclined against the road, a lateral force occurs as a function of the inclination angle, defined as a camber angle. The force due to the camber angle is calculated as follows:

$$F_{y\alpha} = -\frac{1}{6}\frac{L\,\Omega}{|r_d\,\Omega| + v_{num}}\alpha_{cam}\,\lambda_{CFY}\,\frac{dF_y}{ds_y}$$

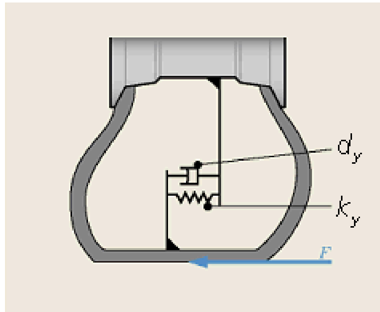
where L is the length of the contact area and λ_{CFY} is a user scaling parameter.

This force is then added to to the static lateral force, i.e.:

$$F_y = F\sin\varphi + F_{y\alpha}$$

Dynamic tire forces

From the equations described above, only the static force and torque can be approximated and calculated as a function of the corresponding slip values. Considering the tire deformation gives the ability to extend the model to calculate the dynamic tire forces and torques. TMeasy uses a first-order dynamic model by using a lumped spring k_y and damper elements d_y for the equivalent tire stiffness and damping respectively.



For instance, the lateral tire dynamic force can be calculated as follows:

$$F_{ydy} = k_y y_t + d_y \dot{y}_t$$

It can be shown that:

$$\dot{y}_t = \frac{F_y - k_y y_t}{d_y - \frac{\partial F_x}{\partial \dot{y}_t}}$$

and for the longitudinal direction:

$$\dot{x}_t = \frac{F_x - k_x x_t}{d_x - \frac{\partial F_x}{\partial \dot{x}_t}}$$

$$F_{xdy} = k_x x_t + d_x \dot{x}_t$$

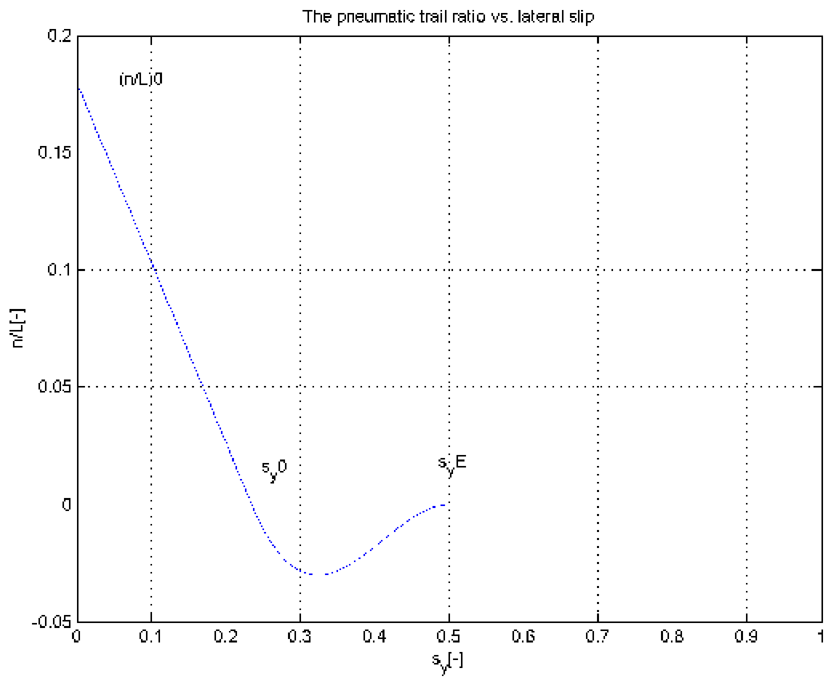
where:

y_t is the lateral tire deformation
x_t is the longitudinal tire deformation
k_y is the equivalent tire lateral stiffness
d_y is the equivalent tire lateral damping
k_x is the equivalent tire lateral stiffness
d_x is the equivalent tire lateral damping

Torques Calculation

Self-aligning torque

The distribution of the lateral force over the contact area is not symmetrical and the resulting lateral force does not act directly in the center of the contact area but shifted by an offset, known as the pneumatic trail n.



The pneumatic trail is approximated as a function of the lateral slip by a linear and a cubic polynomial. The dynamic tire offset has been normalized by the length of the contact area L. Its characteristic is described by the following parameters:

$(n/L)_0$	The pneumatic trail ratio at zero slip
s_y^0	The lateral slip at zero pneumatic trail ratio
s_y^E	The lateral slip at beginning of slipping

$$\frac{n}{L} = \begin{cases} (n/L)_0 \cdot \left(1 - \dfrac{|s_y|}{s_y^0}\right) & , |s_y| \le s_y^0 \\ -\left(\dfrac{n}{L}\right)_0 \cdot \dfrac{|s_y| - s_y^0}{s_y^0} \cdot \left(\dfrac{s_y^E - |s_y|}{s_y^E - s_y^0}\right)^2 & , s_y^0 \le |s_y| \le s_y^E \\ 0 & , |s_y| > s_y^E \end{cases}$$

where L is approximated as follows:

$$L = \sqrt{8\, r_{Tire0}\ \Delta z}$$

The self-aligning torque is calculated from the dynamic lateral force and the pneumatic trail as a torque arm:

$$M_z = -n\,F_y$$

Note

These parameters must be defined for two wheel loads (normal and double normal wheel load) to approximate the wheel load effect.

Bore torque

If the wheel angular velocity has a component in the direction of track normal, i.e. ${}^{CP}\omega_z \neq 0$, a very complicated deflection profile of the tread particles in the contact area occurs. In a simple approach, the resulting bore torque can be calculated by the parameter of the longitudinal force characteristics.

$$M_B = -\frac{1}{12}B^2\frac{{}^{CP}\omega_z}{|r_d\,\Omega| + v_{num}}\lambda_{BTMZ}\frac{dF_x}{ds_x}$$

where B is the tire contact width and λ_{BTMZ} is a user scaling parameter.

The bore torque is limited to its maximum value when slipping occurs:

$$M_{B,max} = \frac{1}{4}\,B\,F_x^G$$

Parking torque

The torque acting on tire during parking maneuver at very low or zero speed can become very large. During steering motions, the wheel rotates with angle φ_W around an axis normal to contact patch. The bore torque calculation is extended to generate a proper moment response to steering at very low or zero vehicle speed. The differential equations that govern the moment generation at standstill are as follows:

$$\left(dF_0B^2 + |r_d\Omega|d_\varphi\right)\dot{\varphi} = -dF_0B^2\dot{\varphi}_W - |r_D\Omega|c_\varphi\varphi$$

$$\dot{\varphi}_B = \left(1 - p\left|\frac{M_B}{M_{B,max}}\right|\right)\dot{\varphi}$$

$$M_B = c_\varphi\varphi_B$$

The parameter p is determined with the following equation:

$$p = 0 \text{ if } (\dot{\varphi}_W\,\varphi_B) < 0 \text{ else } p = 1$$

Overturning torque

Due to the camber angle, a torque is produced about the x-axis. This torque can be calculated as follows:

$$M_x = -\frac{1}{12}\,B^2\,k_z\,\alpha_{cam}\,\lambda_{MX}$$

where λ_{MX} is a user scaling parameter.

Rolling resistance torque

When the tire is stationary, the pressure distribution is symmetrical in the contact area. The resulting vertical force acts on the wheel center. If the tire moves, the pressure increases in the front part of the contact area. The resulting force now attacks ahead of the wheel center, which produces the rolling resistance torque.

$$M_y = -f_{roll} F_{zdyn} r_{Tire0} \, sign(\Omega)$$

where f_{roll} is the dimensionless rolling resistance coefficient.

Wheel Load Effect

Description

The resistance of a real tire against deformations has the effect that with increasing wheel load, the distribution of pressure over the contact area becomes increasingly uneven. In practice, this leads to a digressive wheel load effect on the characteristic curves of longitudinal and lateral forces and also the self-aligning torque. The forces description parameters for arbitrary wheel load are calculated by a quadratic function. For the maximum longitudinal force, it is given by:

$$F_x^M(F_z) = \frac{F_z}{F_z^N}\left[2F_x^M\left(F_z^M\right) - 0{,}5F_x^M\left(2F_z^M\right) - \left(F_x^M\left(F_z^N\right) - 0{,}5F_x^M\left(2F_z^N\right)\right)\frac{F_z}{F_z^N}\right]$$

The slip description parameters for arbitrary wheel load are defined as linear functions of the wheel load F_z. For the location of the maximum longitudinal force this results in:

$$s_x^M(F_z) = s_x^M\left(F_z^N\right) + \left(s_x^M\left(2F_z^N\right) - s_x^M\left(F_z^N\right)\left(\frac{F_z}{F_z^N} - 1\right)\right)$$

Model Parameters

Introduction

These are the model parameters for the TMeasy tire model.

Dynamic Force Parameters

k_y is the equivalent tire lateral stiffness
d_y is the equivalent tire lateral damping
k_x is the equivalent tire lateral stiffness
d_x is the equivalent tire lateral damping
k_z is the equivalent tire vertical stiffness
d_z is the equivalent tire vertical damping

Longitudinal Force Parameters	dF_x^0	is the initial slope at zero slip
	s_x^M	is the slip at maximum forces
	F_x^M	is the magnitude of the maximum forces
	s_x^G	is the slip value, at which the tire start to slip
	F_x^G	is the slip force

Note

These parameters must be defined for two wheel loads (normal and double normal wheel load) to approximate the wheel load effect.

Lateral Force Parameters	dF_y^0	is the initial slope at zero slip
	s_y^M	is the slip at maximum forces
	F_y^M	is the magnitude of the maximum forces
	s_y^G	is the slip value, at which the tire start to slip
	F_y^G	is the slip force

Note

These parameters must be defined for two wheel loads (normal and double normal wheel load) to approximate the wheel load effect.

Self-Aligning Torque Parameters	$(n/L)_0$	The pneumatic trail ratio at zero slip
	s_y^0	The lateral slip at zero pneumatic trail ratio
	s_y^E	The lateral slip at beginning of slipping

Note

These parameters must be defined for two wheel loads (normal and double normal wheel load) to approximate the wheel load effect.

Rolling Resistance Coefficient	f_{roll}	is the dimensionless rolling resistance coefficient

Magic Formula Tire Model (Earlier than Version 6.1)

Where to go from here

Information in this section

Introduction

Introduction

Note

This is the Magic Formula tire model earlier than version 6.1.

A well-known and widely used model is the Magic Formula by Prof. Pacejka, Delft University of Technology, Delft, Netherlands. It is a semi-empirical tire model, which describes longitudinal and lateral tire force characteristics and the self-aligning torque as functions of longitudinal and/or lateral slip.

The model utilizes the Magic Formula mathematical function to approximate the tire characteristics and the interaction between the longitudinal and lateral forces. The following Magic Formula implementation bases upon the book "Tire and Vehicle Dynamics", Pacejka, 2002.

The Magic Formula tire model includes the following features:

- Dynamic tire radius, used to calculate the slip quantities
- Combined slip forces calculation
- Self-aligning torque
- Scalable friction coefficient
- Longitudinal and lateral tire dynamics
- Wheel load effect.
- Rolling resistance torque and tipping torque are also calculated

The Magic Formula (Earlier than Version 6.1)

Description

The basic form of the Magic Formula reads:

$$Y(x) = D\sin[C\arctan(BX - E(BX - \arctan(BX)))] + S_V$$

$$X = x + S_H$$

The parameters can be interpreted as follows:

D	Peak factor
C	Shape factor
B	Stiffness factor
$K = BCD = dx/dy\|_{x=0}$	Slip stiffness factor
E	Curvature factor
S_V	Vertical shift factor
S_H	Horizontal shift factor

In the Magic Formula tire model, these factors are calculated from another set of parameters, which must be defined for the tire model. The parameters can be categorized as follows:

Magic Formula Hypothetical Parameters

p Parameters for defining the force curves at pure slip condition
q Parameters for defining the torque curves at pure slip condition
r Parameters for defining the force curves at combined slip condition
s Parameters for defining the torque curves at combined slip condition

User Scaling Parameters

Introduction

Magic Formula offers a set of scaling factors to control, alter, or adapt the calculation of the forces and torques. The following λ factors are available. The default value is 1.0.

Pure Slip

λ_{Fz0} is the nominal load
$\lambda_{\mu x,y}$ is the peak friction coefficient
$\lambda_{Kx\kappa}$ is the brake slip stiffness
$\lambda_{Ky\alpha}$ is the cornering stiffness
$\lambda_{Ky\alpha}$ is the shape factor
$\lambda_{Ex,y}$ is the curvature factor
$\lambda_{Hx,y}$ is the horizontal shift
$\lambda_{Vx,y}$ is the vertical shift
$\lambda_{Ky\gamma}$ is the camber force stiffness
$\lambda_{Kz\gamma}$ is the camber torque stiffness
λ_{t} is the pneumatic trail
λ_{Mr} is the residual torque

Combined Slip

$\lambda_{x\alpha}$ is the influence of the lateral slip on the longitudinal force
$\lambda_{y\kappa}$ is the influence of the longitudinal slip on the lateral force
$\lambda_{Vy\kappa}$ is the lateral force induced by longitudinal slip, ply steer
λ_{s} is the moment arm of longitudinal force, which introduces torque about z-axis

Other

λ_{Mx} is the overturning couple stiffness
λ_{My} is the rolling resistance torque

Wheel Load

Description

Wheel load means the tire force normal to the road at the contact point. It depends on the tire deflection Δz.

$$\Delta z = (r_{Tire0} - r_{Tire}) \geq 0$$

The normal force can be calculated as follows:

$$F_{zdy} = (\Delta z k_z + \Delta \dot{z} d_z) \geq 0$$

where:

k_z is the equivalent tire vertical stiffness
d_z is the equivalent tire vertical damping

To consider the wheel load change on the forces and torques, the normalized change in the vertical load is introduced

$$df_z = \frac{F_{zdy} - F'_{z0}}{F'_{z0}}$$

where

$$F'_{z0} = F_{z0}\,\lambda_{Fz0}$$

Slip Calculation

Description

The forces generated by the tire depend on the slip and not directly on the wheel speeds. The slip quantities must be calculated to calculate the tire forces. In Magic Formula, the slip in x- and in y-direction is calculated as follows:

$$\kappa = \frac{-(v_{xCP} - \Omega r_d)}{|v_{xCP}| + v_{num}}$$

$$\alpha^* = \tan(\alpha) = \frac{-v_{yCP}}{|v_{xCP}| + v_{num}}$$

$$y^* = \sin\gamma$$

where r_d is the dynamic rolling tire radius. It is calculated as follows:

$$r_d = r_{Tire0} - \Delta z_0 \left(D_{reff} \arctan\left(B_{reff} \frac{\Delta z}{\Delta z_0} \right) + F_{reff} \frac{\Delta z}{\Delta z_0} \right)$$

where Δz_0 is the nominal tire deflection at nominal load.

$$\Delta z_0 = \frac{F_{z0}}{k_z}$$

The dynamic rolling tire radius represents the effective tire rolling radius by which the slip quantities must be calculated.

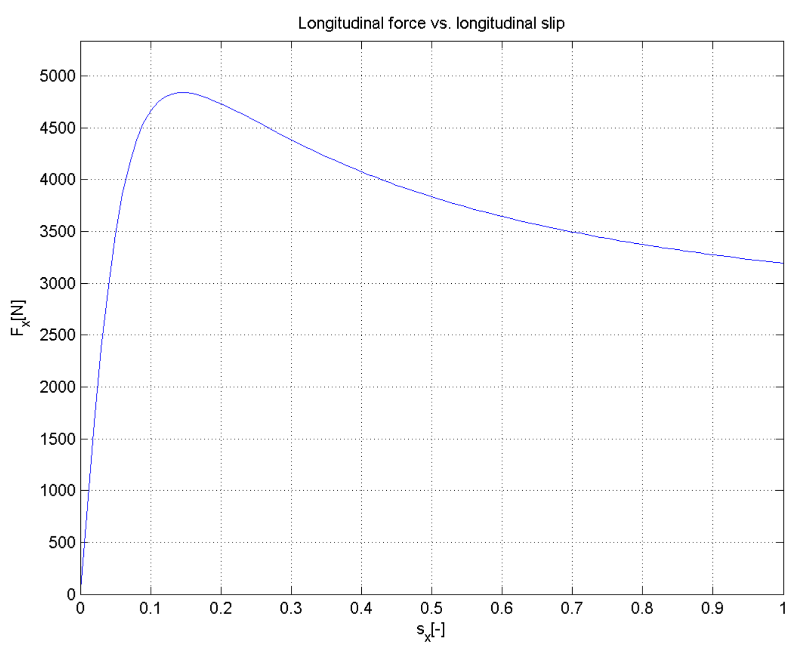
Parameters for Dynamic Tire Radius

D_{reff}	Peak value of effective rolling radius
B_{reff}	Low load stiffness effective rolling radius
F_{reff}	High load stiffness effective rolling radius

Forces Calculation

Longitudinal force characteristics

Longitudinal force is also called the traction or braking force and acts in the longitudinal direction of the tire-road contact area. The relationship between the longitudinal slip and the longitudinal force is a nonlinear one. The figure below shows the description of the longitudinal force as a function of pure longitudinal slip as approximated in the Magic Formula tire model.



The steady state longitudinal force in the case of *pure longitudinal slip* is calculated as follows:

$$F_{x0} = D_x \sin[C_x \arctan(B_x\kappa_x - E_x(B_x\kappa_x - \arctan(B_x\kappa_x)))] + S_{Vx}$$

$$\kappa_x = \kappa + s_{Hx}$$

$$C_x = p_{Cx1}\lambda_{Cx}$$

$$D_x = \mu_x F_z$$

$$\mu_x = (p_{Dx1} + p_{Dx2}df_z)\lambda_{\mu x}$$

$$E_x = \left(p_{Ex1} + p_{Ex2}df_z + p_{Ex3}df_z{}^2\right) \cdot \{1 - p_{Ex3}sgn(\kappa_x)\} \cdot \lambda_{Ex}(\leq 1)$$

$$K_{x\kappa} = F_z \cdot (p_{Kx1} + p_{Kx2} \cdot df_z) \cdot \exp(p_{Kx3} \cdot df_z) \cdot \lambda_{Kx\kappa}$$
$$\left(= B_x C_x D_x = \frac{\partial F_{x0}}{\partial \kappa_x}\, at\ \kappa_x = 0\right)$$

$$B_x = K_x/(C_x D_x)$$

$$S_{Hx} = (p_{Hx1} + p_{Hx2}df_z)\lambda_{Hx}$$

$$S_{Vx} = F_z \cdot (p_{Vx1} + p_{Vx2}df_z)\lambda_{Vx}\lambda_{\mu x}$$

Parameters for Pure Longitudinal Slip

p_{Cx1}	is the shape factor C
p_{Dx1}, p_{Dx2}	is the peak factor D
p_{Ex1}, p_{Ex2}, p_{Ex3}, p_{Ex4}	is the curvature factor E
p_{Kx1}, p_{Kx2}, p_{Kx3}	is the slip stiffness BCD
p_{Hx1}, p_{Hx2}	is the horizontal shift H
p_{Vx1}, p_{Vx2}	is the vertical shift V

In the case of *combined slip*, the longitudinal force calculation must be modified to consider the effect of lateral slip. This is achieved by calculating a weighting function $G_{x\alpha}(\alpha, \kappa, F_z)$ and multiplying it by the longitudinal force due to pure longitudinal slip F_{x0}.

$$F_x = F_{x0} \cdot G_{x\alpha}(\alpha, \kappa, F_z)$$

$$G_{x\alpha} = \cos[C_{x\alpha}\arctan\{B_{x\alpha}\alpha_s - E_{x\alpha}(B_{x\alpha}\alpha_s - \arctan(B_{x\alpha}\alpha_s))\}]/G_{x\alpha0}$$

$$G_{x\alpha0} = \cos[C_{x\alpha}\arctan\{B_{x\alpha}S_{Hx\alpha} - E_{x\alpha}(B_{x\alpha}S_{Hx\alpha} - \arctan(B_{x\alpha}S_{Hx\alpha}))\}]$$

$$\alpha_s = \alpha^* + S_{Hx\alpha}$$

$$B_{x\alpha} = r_{Bx1}\cos[\arctan\{r_{Bx2}\kappa\}] \cdot \lambda_{x\alpha}$$

$$C_{x\alpha} = r_{Cx1}$$

$$E_{x\alpha} = r_{Ex1} + r_{Ex2}df_z$$

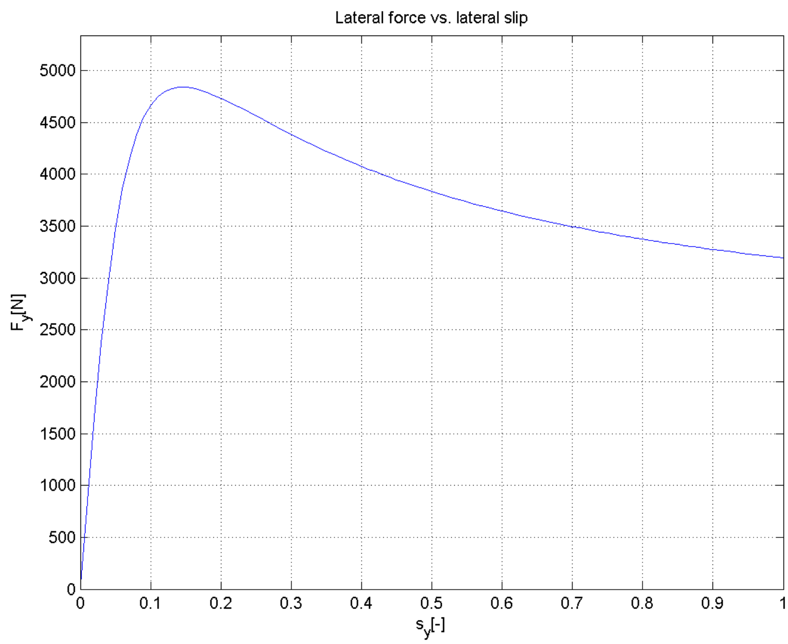
$$S_{Hx\alpha} = r_{Hx1}$$

Parameters for Combined Longitudinal and Lateral Slip (Longitudinal Force)

r_{Bx1}, r_{Bx2}	is the stiffness factor B
r_{Cx1}	is the shape factor C
r_{Hx1}	is the horizontal shift H

Lateral force characteristics

Lateral force is also called the cornering force and acts in the lateral direction of the tire-road contact area. The relationship between the lateral slip and the lateral force is a nonlinear one. The figure below shows the description of the lateral force as a function of pure lateral slip as approximated in the Magic Formula tire model.



The steady state lateral force in the case of *pure lateral slip* is calculated as follows:

$$F_{y0} = D_y \sin\left[C_y \arctan\left\{B_y\alpha_y - E_y\left(B_y\alpha_y - \arctan\left(B_y\alpha_y\right)\right)\right\}\right] + S_{Vy}$$

$$\alpha_y = \alpha^* + S_{Hy}$$

$$C_y = p_{Cy1} \cdot \lambda_{Cy}$$

$$D_y = \mu_y \cdot F_z$$

$$\mu_y = \left(p_{Dy1} + p_{Dy2} df_z\right) \cdot \lambda_{\mu y}$$

$$E_y = \left(p_{Ey1} + p_{Ey2} df_z\right) \cdot \left\{1 - \left(p_{Ey3} + p_{Ey4}\gamma^*\right) sgn\left(\alpha_y\right)\right\} \cdot \lambda_{Ey} (\leq 1)$$

$$K_{y\alpha} = p_{Ky1}F'_{z0}\sin\left[2\arctan\left\{F_z/\left(p_{Ky2}F'_{z0}\right)\right\}\right] \cdot \left(1 - p_{Ky3}{\gamma^*}^2\right) \cdot \lambda_{F_{z0}} \cdot \lambda_{Ky}$$

$$\left(= B_y C_y D_y = \frac{\partial F_{y0}}{\partial \kappa_y}\, at\ \kappa_x = 0\right)$$

$$B_y = K_y/\left(C_y D_y\right)$$

$$S_{Hy} = \left(p_{Hy1} + p_{Hy2}df_z\right) \cdot \lambda_{Hy} + p_{Hy3}\gamma^* \cdot \lambda_{\mu y}$$

Parameters for Pure Lateral Slip (Lateral Force)

p_{Cy1}	is the shape factor C
p_{Dy1}, p_{Dy2}, p_{Dy3}	is the peak factor D
p_{Ey1}, p_{Ey2}, p_{Ey3}, p_{Ey4}	is the curvature factor E
p_{Ky1}, p_{Ky2}, p_{Ky3}	is the slip stiffness BCD
p_{Hy1}, p_{Hy2}, p_{Hy3}	is the horizontal shift H
p_{Vy1}, p_{Vy2}, p_{Vy3}, p_{Vy4}	is the vertical shift V

In the case of *combined slip*, the lateral force calculation must be modified to consider the effect of longitudinal slip. This is achieved by calculating a weighting function $G_{y\kappa}(\alpha, \kappa, \gamma, F_z)$ and multiplying it by the lateral force due to pure lateral slip F_{y0}.

$$F_y = F_{y0} \cdot G_{y\kappa}(\alpha, \kappa, \gamma, F_z) + S_{Vy\kappa}$$

$$G_{y\kappa} = \cos\left[C_{y\kappa}\arctan\left\{B_{y\kappa}\kappa_S - E_{y\kappa}\left(B_{y\kappa}\kappa_S - \arctan\left(B_{y\kappa}\kappa_S\right)\right)\right\}\right]/G_{y\kappa 0}$$

$$G_{y\kappa 0} = \cos\left[C_{y\kappa}\arctan\left\{B_{y\kappa}S_{Hy\kappa} - E_{y\kappa}\right\}\right]$$

$$\kappa_S = \kappa + S_{Hy\kappa}$$

$$B_{yN} = r_{By1}\cos\left[\arctan\left\{r_{By2}\left(\alpha - r_{By3}\right)\right\}\right] \cdot \lambda_{yN}$$

$$C_{yN} = r_{Cy1}$$

$$E_{yN} = r_{Ey1} + r_{Ey2}df_z$$

$$S_{Hy\kappa} = r_{Hy1} + r_{Hy2}df_z$$

$$S_{Vy\kappa} = D_{Vy\kappa}\sin\left[r_{Vy5}\arctan\left(r_{Vy6}\kappa\right)\right] \cdot \lambda_{Vy\kappa}$$

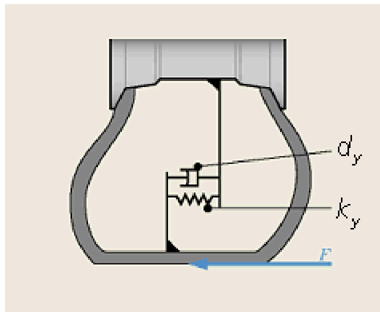
$$D_{Vy\kappa} = \mu_y F_z \cdot \left(r_{Vy1} + r_{Vy2}df_z + r_{Vy3}\gamma\right) \cdot \cos\left[\arctan\left(r_{Vy4}\alpha\right)\right]$$

Parameters for Combined Longitudinal and Lateral Slip (Lateral Force)

r_{By1}, r_{By2}, r_{By3}	is the stiffness factor B
r_{Cy1}	is the shape factor C
r_{Hy1}	is the horizontal shift H
r_{Vy1}, r_{Vy2}, r_{Vy3}, r_{Vy4}, r_{Vy5}, r_{Vy6}	is the vertical shift V

Dynamic tire forces

From the equations described above, only the static force and torque can be approximated and calculated as a function of the corresponding slip values. Considering the tire deformation gives the ability to extend the model to calculate the dynamic tire forces and torques. Magic Formula uses a first-order dynamic model by using a lumped spring k_y and damper elements d_y for the equivalent tire stiffness and damping respectively.



For instance, the lateral tire dynamic force can be calculated as follows:

$$F_{ydy} = k_y y_t + d_y \dot{y}_t$$

It can be shown that:

$$\dot{y}_t = \frac{F_y - k_y y_t}{d_y - \frac{\partial F_y}{\partial \dot{y}_t}}$$

and for the longitudinal force:

$$\dot{x}_t = \frac{F_x - k_x x_t}{d_x - \frac{\partial F_x}{\partial \dot{x}_t}}$$

$$F_{xdy} = k_x x_t + d_x \dot{x}_t$$

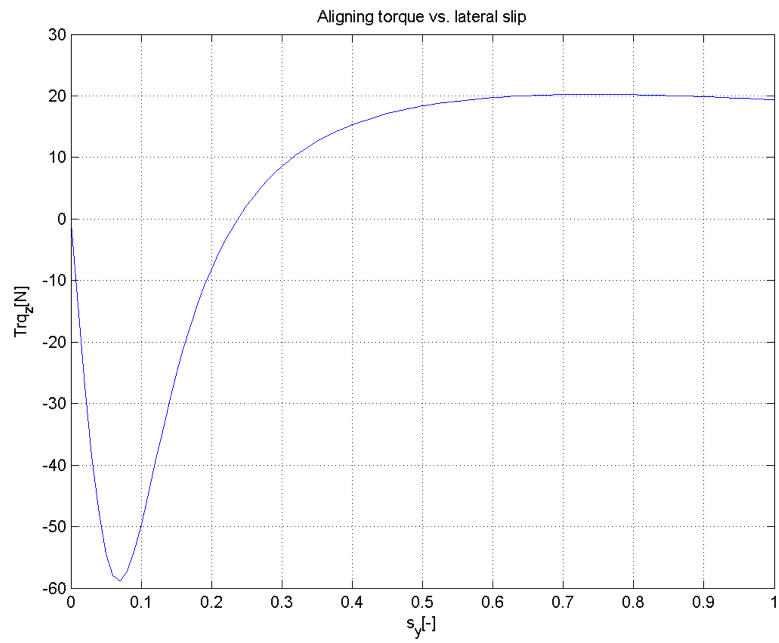
where:

y_t is the lateral tire deformation
x_t is the longitudinal tire deformation
k_y is the equivalent tire lateral stiffness
d_y is the equivalent tire lateral damping
k_x is the equivalent tire lateral stiffness
d_x is the equivalent tire lateral damping

Torques Calculation

Self-aligning torque

The torque about the z-axis in the Magic Formula tire model is composed of two components in the pure lateral slip condition. In the case of combined slip, the effect of the longitudinal force is considered. The following figure shows the self-aligning torque as approximated in the Magic Formula tire model in the case of pure lateral slip.



In the case of *pure lateral slip*, the self-aligning torque is calculated as follows:

$$M_z = M'_{zo} + M_{zro}$$

$$M'_{zo} = -t_o \cdot F_{yo}$$

$$t_o = t(\alpha_t) = D_t \cos[C_t \arctan\{B_t\alpha_t - E_t(B_t\alpha_t - \arctan(B_t\alpha_t))\}] \cdot \cos(\alpha)$$

$$\alpha_t = \alpha^* + S_{Ht}$$

$$S_{Ht} = q_{Hz1} + q_{Hz2}df_z + (q_{Hz3} + q_{Hz4}df_z)\gamma^*$$

$$M_{zro} = M_{zr}(\alpha_r) = D_r \cos[\arctan(B_r\alpha_r)]$$

$$\alpha_r = \alpha^* + S_{Hf}\left(= \alpha_f\right)$$

$$S_{Hf} = S_{Hy} + S_{Vy}/K'_{ya}$$

$$K'_{ya} = K_{ya} + \varepsilon_K$$

$$B_t = \left(q_{Bz1} + q_{Bz2}df_z + q_{Bz3}df_z^2\right) \cdot \left(1 + q_{Bz5}|\gamma^*| + q_{Bz6}\gamma^{*2}\right) \cdot \lambda_{Kya}/\lambda_{\mu y}^*$$

$$C_t = q_{Cz1}$$

$$D_t = F_z \cdot (R_0/F'_{z0}) \cdot (q_{Dz1} + q_{Dz2}df_z) \cdot \lambda_t \cdot sgnV_{cx} \cdot \left(1 + q_{Dz3}|\gamma^*| + q_{Dz4}\gamma^{*2}\right)$$

$$E_t = \left(q_{Ez1} + q_{Ez2}df_z + q_{Ez3}df_z^2\right) \cdot \left\{1 + (q_{Ez4} + q_{Ez5}\gamma^*)\frac{2}{\pi}\arctan(B_tC_t\alpha_t)\right\}(\leq 1)$$

$$B_r = q_{Bz9} \cdot \frac{\lambda_{Ky}}{\lambda^*_{\mu y}} + q_{Bz10} B_y C_y$$

$$D_r = F_z R_0 \left\{ (q_{Dz6} + q_{Dz7} df_z) \cdot \lambda_{Mr} + (q_{Dz8} + q_{Dz9} df_z) \cdot \gamma^* \lambda_{Kzy} \right\} \cdot \cos(\alpha) \cdot \lambda^*_{\mu y}$$

Parameters for Pure Cornering (Aligning Torque)

$q_{Bz1}, q_{Bz2}, q_{Bz3}, q_{Bz4}, q_{Bz5}, q_{Bz6}, q_{Bz7}, q_{Bz8}, q_{Bz9}, q_{Bz10}$	is the stiffness factor B
q_{Cz1}	is the shape factor C
$q_{Dz1}, q_{Dz2}, q_{Dz3}, q_{Dz4}, q_{Dz5}, q_{Dz6}, q_{Dz7}, q_{Dz8}, q_{Dz9}$	is the peak factor D
$q_{Ez1}, q_{Ez2}, q_{Ez3}, q_{Ez4}, q_{Ez5}$	is the curvature factor E
$q_{Hz1}, q_{Hz2}, q_{Hz3}$	is the horizontal shift H

In the case of *combined slip*, the input slip quantities are modified and the effect of the longitudinal force is considered.

$$M_z = M'_z + M_{zr} + s \cdot F_x$$

$$M'_z = -t \cdot F'_y$$

$$t = t(\alpha_{t,eq}) = D_t \cos\left[C_t \arctan\left\{B_t \alpha_{t,eq} - E_t\left(B_t \alpha_{t,eq} - \arctan(B_t \alpha_{t,eq})\right)\right\}\right] \cdot \cos(\alpha)$$

$$F'_y = F_y - S_{VyK}$$

$$M_{zr} = M_{zr}(\alpha_{r,eq}) = D_r \cos\left[\arctan(B_r \alpha_{r,eq})\right]$$

$$s = R_0 \cdot \left\{ s_{sz1} + s_{sz2}(F_y / F'_{z0}) + (s_{sz3} + s_{sz4} df_z) \cdot \gamma^* \right\} \cdot \lambda_s$$

$$\alpha_{t,eq} = \sqrt{\alpha_t^2 + \left(\frac{K_{x\kappa}}{K'_{y\alpha}}\right)^2 \kappa^2} \cdot \operatorname{sgn}(\alpha_t)$$

$$\alpha_{r,eq} = \sqrt{\alpha_r^2 + \left(\frac{K_{x\kappa}}{K'_{y\alpha}}\right)^2 \kappa^2} \cdot \operatorname{sgn}(\alpha_r)$$

Parameters for Combined Longitudinal and Lateral Slip (Aligning Torque)

$s_{sz1}, s_{sz2}, s_{sz3}, s_{sz4}$

Overturning torque

The camber angle and lateral force produce a torque about the x-axis of the contact point. This torque can be calculated as follows:

$$M_x = R_0 \cdot F_z\left(q_{sx1} - q_{sx2} \cdot \gamma^* + q_{sx3} \cdot F_y / F'_{z0}\right)\lambda_{Mx}$$

Parameters for Overturning Couple

$q_{Sx1}, q_{Sx2}, q_{Sx3}$

Rolling resistance torque

When the tire is stationary, the pressure distribution is symmetrical in the contact area. The resulting vertical force acts on the wheel center. If the tire moves, the pressure increases in the front part of the contact area. The resulting force now attacks ahead of the wheel center, which produces the rolling resistance torque.

$$M_y = -F_z R_0 \left(q_{Sy1} \arctan(\Omega r_d / v_0) + q_{Sy2} F_x / F'_{z0} \right)$$

Parameters for Rolling Resistance Torque

q_{Sy1},
q_{Sy2}

Model Parameters

Introduction

These are the parameters for the Magic Formula tire model.

Dynamic Force Parameters

k_y	is the equivalent tire lateral stiffness
d_y	is the equivalent tire lateral damping
k_x	is the equivalent tire lateral stiffness
d_x	is the equivalent tire lateral damping
k_Z	is the equivalent tire vertical stiffness
d_Z	is the equivalent tire vertical damping

Longitudinal Force Parameters

p_{Cx1}	is the shape factor C
p_{Dx1}, p_{Dx2}	is the peak factor D
$p_{Ex1}, p_{Ex2}, p_{Ex3}, p_{Ex4}$	is the curvature factor E
$p_{Kx1}, p_{Kx2}, p_{Kx3}$	is the slip stiffness BCD
p_{Hx1}, p_{Hx2}	is the horizontal shift H
p_{Vx1}, p_{Vx2}	is the vertical shift V

Parameters for Combined Longitudinal and Lateral Slip (Longitudinal Force)

r_{Bx1}, r_{Bx2}	is the stiffness factor B
r_{Cx1}	is the shape factor C
r_{Hx1}	is the horizontal shift H

Parameters for Pure Lateral Slip (Lateral Force)	p_{Cy1}	is the shape factor C
	p_{Dy1}, p_{Dy2}	is the peak factor D
	$p_{Ey1}, p_{Ey2}, p_{Ey3}, p_{Ey4}$	is the curvature factor E
	$p_{Ky1}, p_{Ky2}, p_{Ky3}$	is the slip stiffness BCD
	$p_{Hy1}, p_{Hy2}, p_{Hy3}$	is the horizontal shift H
	$p_{Vy1}, p_{Vy2}, p_{Vy3}, p_{Vy4}$	is the vertical shift V

Parameters for Combined Longitudinal and Lateral Slip (Lateral Force)	$r_{By1}, r_{By2}, r_{By3}$	is the stiffness factor B
	r_{Cy1}	is the shape factor C
	r_{Hy1}	is the horizontal shift H
	$r_{Vy1}, r_{Vy2}, r_{Vy3}, r_{Vy4}, r_{Vy5}, r_{Vy6}$	is the vertical shift V

Parameters for Pure Cornering (Aligning Torque)	$q_{Bz1}, q_{Bz2}, q_{Bz3}, q_{Bz4}, q_{Bz5}, q_{Bz6}, q_{Bz7}, q_{Bz8}, q_{Bz9}$	is the stiffness factor B
	q_{Cz1}	is the shape factor C
	$q_{Dz1}, q_{Dz2}, q_{Dz3}, q_{Dz4}, q_{Dz5}, q_{Dz6}, q_{Dz7}, q_{Dz8}, q_{Dz9}$	is the peak factor D
	$q_{Ez1}, q_{Ez2}, q_{Ez3}, q_{Ez4}, q_{Ez5}$	is the curvature factor E
	$q_{Hz1}, q_{Hz2}, q_{Hz3}$	is the horizontal shift H

Parameters for Combined Longitudinal and Lateral Slip (Aligning Torque)	$s_{sz1}, s_{sz2}, s_{sz3}, s_{sz4}$

Parameters for Overturning Couple	$q_{Sx1}, q_{Sx2}, q_{Sx3}$

Parameters for Rolling Resistance Torque	q_{Sy1}, q_{Sy2}

Using TIR Files

Description

To use Magic Formula parameters from a TIR file, the signs of the some parameters have to be inverted.

Invert the sign of the following parameters:

Pure lateral force

- P_{KY1}
- P_{HY1}
- P_{HY2}
- P_{HY3}
- P_{EY3}
- P_{EY4}

Combined longitudinal force

- R_{HX1}
- R_{BX1}

Combined lateral force

- R_{HY1}
- R_{HY2}
- R_{BY1}

Self-aligning torque

- Q_{BZ1}
- Q_{BZ2}
- Q_{BZ3}
- Q_{HZ1}
- Q_{HZ2}
- Q_{HZ3}
- Q_{HZ4}
- Q_{BZ9}

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Magic Formula Tire Model 6.1

Where to go from here

Information in this section

Introduction

Introduction

Note

This is the Magic Formula tire model in version 6.1.

A well-known and widely used model is the Magic Formula by Prof. Pacejka, Delft University of Technology, Delft, Netherlands. It is a semi-empirical tire model, that describes longitudinal and lateral tire force characteristics and the self-aligning torque as functions of longitudinal and/or lateral slip.

The model utilizes the Magic Formula mathematical function to approximate the tire characteristics and the interaction between the longitudinal and lateral forces. The following Magic Formula implementation bases upon the book "Tire and Vehicle Dynamics", Pacejka, 2012.

The Magic Formula tire model includes the following features:

- Dynamic tire radius, used to calculate the slip quantities
- Combined slip forces calculation
- Self-aligning torque
- Scalable friction coefficient
- Longitudinal and lateral tire dynamics
- Wheel load effect
- Rolling resistance torque and tipping torque are also calculated

Related topics

References

Normalized Entity

Description

In the Magic Formula, normalized load and inflation pressure are introduced:

$$df_z = \frac{F_z - F'_{zo}}{F'_{zo}}$$

$$dp_i = \frac{p_i - p_{io}}{p_{io}}$$

Where

$$F'_{zo} = \lambda_{F_{zo}} \cdot F_{zo}$$

Related topics

Basics

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The Magic Formula

Description

The basic form of the Magic Formula reads:

$$Y(x) = D\sin[C\arctan(BX - E(BX - \arctan(BX)))] + S_V$$

$$X = x + S_H$$

The parameters can be interpreted as follows:

D	Peak factor
C	Shape factor
B	Stiffness factor
$K = BCD = dx/dy\|_{x=0}$	Slip stiffness factor
E	Curvature factor
S_V	Vertical shift factor
S_H	Horizontal shift factor

In the Magic Formula tire model, these factors are calculated from another set of parameters, which must be defined for the tire model. The parameters can be categorized as follows:

Magic Formula Hypothetical Parameters

- p Parameters for defining the force curves at pure slip condition
- q Parameters for defining the torque curves at pure slip condition
- r Parameters for defining the force curves at combined slip condition
- s Parameters for defining the torque curves at combined slip condition

Related topics

Basics

References

Slip Calculation

Description

The forces generated by the tire depend on the slip and not directly on the wheel speeds. The slip quantities must be calculated to calculate the tire forces. The

Magic Formula 6.1 formulation uses ISO sign conventions. The slip in x and in y direction are calculated as follows:

$$\kappa = \frac{-(v_{xCP} - \Omega \cdot r_e)}{|v_{xCP}| + v_{add}}$$

$$\alpha^* = \tan(\alpha) = \frac{v_{yCP}}{|v_{xCP}| + v_{add}}$$

Where r_e is the effective rolling tire radius. It is calculated as follows:

$$r_e = r_\Omega - \frac{F_{zo}}{c_z}\left\{F_{reff}\frac{F_z}{F_{zo}} + D_{reff} \cdot \arctan\left(B_{reff}\frac{F_z}{F_{zo}}\right)\right\}$$

Where:

$$r_\Omega = r_0\left(q_{reo} + q_{V1}\left(\frac{r_0 \cdot \Omega}{V_0}\right)^2\right)$$

$$c_z = c_{zo}(1 + p_{Fz1} \cdot dp_i)$$

$$c_{zo} = \frac{F_{zo}}{r_o}\sqrt{q_{Fz1}^2 + 4 \cdot q_{Fz2}}$$

$$F_z = \left\{1 + q_{V2} \cdot |\Omega| \cdot \frac{r_o}{V_o} - \left(q_{Fcx1}\frac{F_x}{F_{zo}}\right)^2 - \left(q_{Fcy1}\frac{F_y}{F_{zo}}\right)^2\right\} \cdot F_{zo} \cdot \left(q_{Fz1}\frac{\rho}{r_o} + q_{Fz2}\frac{\rho^2}{r_o^2}\right) \cdot (1 + p_{Fz1} \cdot dp_i)$$

Related topics

Basics

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Model Parameters

Introduction

The Magic Formula offers a set of scaling factors to control, alter, or adapt the calculation of the forces and torques conveniently. The following λ factors are available:

Pure slip

$\lambda_{F_{zo}}$	Nominal load
$\lambda_{\mu x, y}$	Peak friction coefficient

	$\lambda_{K x \kappa}$	Brake slip stiffness
	$\lambda_{K y a}$	Cornering stiffness
	$\lambda_{C x, y}$	Shape factor
	$\lambda_{E x, y}$	Curvature factor
	$\lambda_{H x, y}$	Horizontal shift
	$\lambda_{V x, y}$	Vertical shift
	$\lambda_{K y Y}$	Camber force stiffness
	$\lambda_{K z Y}$	Camber torque stiffness
	λ_{t}	Pneumatic trail
	$\lambda_{M r}$	Residual torque
Combined slip	$\lambda_{x a}$	α influence on $F_x(\kappa)$
	$\lambda_{\mu x, y}$	κ influence on $F_y(\alpha)$
	$\lambda_{V y_\kappa}$	κ induced ply steer F_y
	λ_{s}	M_z moment arm of F_x
Other	$\lambda_{C z}$	Radial tire stiffness
	$\lambda_{M x}$	Overturning couple stiffness
	$\lambda_{V M x}$	Overturning couple vertical shift
	$\lambda_{M y}$	Rolling resistance moment
Parking torque	$\lambda_{M P}$	Peak parking torque
	$\lambda_{M P T B}$	Parking torque stiffness

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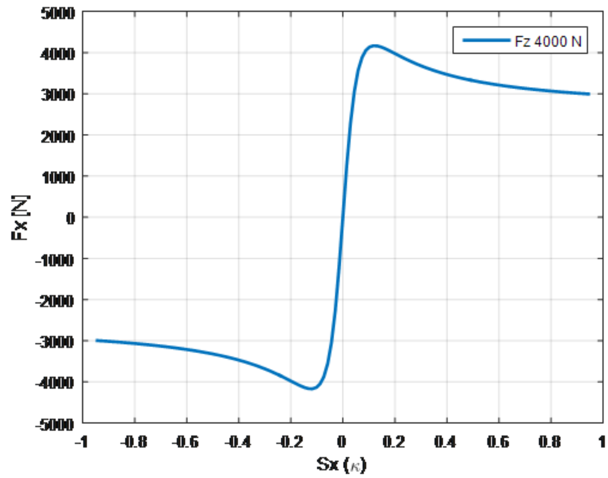
Forces Calculation

Longitudinal force characteristics

Longitudinal force is also called the traction or braking force and acts in the longitudinal direction of the tire-road contact area. The relationship between the longitudinal slip and the longitudinal force is nonlinear.

The figure below shows the description of the longitudinal force as a function of pure longitudinal slip as approximated in the Magic Formula tire model.



The steady state longitudinal force in the case of *pure longitudinal slip* is calculated as follows:

$$F_{xo} = D_x \sin[C_x \arctan\{B_x\kappa_x - E_x(B_x\kappa_x - \arctan(B_x\kappa_x))\}] + S_{Vx}$$

$$\kappa_x = \kappa + S_{Hx}$$

$$C_x = p_{Cx1} \cdot \lambda_{Cx} \;\; (>0)$$
$$D_x = \mu_x \cdot F_z \cdot \zeta_1 \;\; (>0)$$

$$\mu_x = (p_{Dx1} + p_{Dx2}df_z)\left(1 + p_{px3}dp_i + p_{px4}dp_i^2\right)\left(1 - p_{Dx3}\gamma^2\right) \cdot \lambda_{\mu x}$$

$$E_x = \left(p_{Ex1} + p_{Ex2}df_z + p_{Ex3}df_z^2\right)\{1 - p_{Ex4}\mathrm{sgn}(\kappa_x)\} \cdot \lambda_{Ex} \;\; (\leq 1)$$

$$K_{x\kappa} = F_z(p_{Kx1} + p_{Kx2}df_z)\exp(p_{Kx3}df_z)\left(1 + p_{px1}dp_i + p_{px2}dp_i^2\right)$$

$$B_x = K_{x\kappa}/(C_xD_x + \varepsilon_x)$$

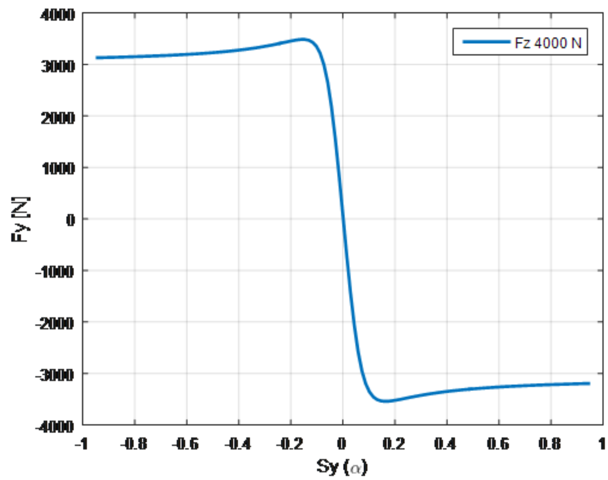
$$S_{Hx} = (p_{Hx1} + p_{Hx2}df_z) \cdot \lambda_{Hx}$$

$$S_{Vx} = F_z \cdot (p_{Vx1} + p_{Vx2}df_z) \cdot \lambda_{Vx}\lambda_{\mu x}\zeta_1$$

Lateral force characteristics

Lateral force is also called the cornering force and acts in the lateral direction of the tire-road contact area. The relationship between the lateral slip and the lateral force is nonlinear.

The figure below shows the description of the lateral force as a function of pure lateral slip as approximated in the Magic Formula tire model.



The steady state lateral force in the case of *pure lateral slip* is calculated as follows:

$$F_{yo} = D_y \sin\left[C_y \arctan\left\{B_y\alpha_y - E_y\left(B_y\alpha_y - \arctan\left(B_y\alpha_y\right)\right)\right\}\right] + S_{Vy}$$

$$\alpha_y = \alpha + S_{Hy}$$

$$C_y = p_{Cy1} \cdot \lambda_{Cy} \ (>0)$$

$$D_y = \mu_y \cdot F_z \cdot \zeta_2$$

$$\mu_y = \left(p_{Dy1} + p_{Dy2}df_z\right)\left(1 + p_{py3}dp_i + p_{py4}dp_i^2\right)\left(1 - p_{Dy3}\gamma^{*2}\right) \cdot \lambda_{\mu y}$$

$$E_y = \left(p_{Ey1} + p_{Ey2}df_z\right)\left\{1 + p_{Ey5}\gamma^{*2} - \left(p_{Ey3} + p_{Ey4}\gamma^*\right)\mathrm{sgn}\left(\alpha_y\right)\right\} \cdot \lambda_{Ey} \ (\leq 1)$$

$$K_{ya} = p_{Ky1}F'_{zo}\left(1 + p_{py1}dp_i\right)\left(1 - p_{Ky3}|\gamma^*|\right)$$

$$\cdot \sin\left[p_{Ky4}\arctan\left\{\frac{F_z/F'_{zo}}{\left(p_{Ky2} + p_{Ky5}\gamma^{*2}\right)\left(1 + p_{py2}dp_i\right)}\right\}\right] \cdot \zeta_3\lambda_{Kya}$$

$$B_y = K_{xa}/\left(C_yD_y + \varepsilon_y\right)$$

$$S_{Hy} = \left(p_{Hy1} + p_{Hy2}df_z\right) \cdot \lambda_{Hy} + \frac{K_{y\gamma o}\gamma^* - S_{Vy\gamma}}{K_{ya} + \varepsilon_K}\zeta_0 + \zeta_4 - 1$$

$$S_{Vy\gamma} = F_z \cdot \left(p_{Vy3} + p_{Vy4}df_z\right)\gamma^* \cdot \lambda_{Ky\gamma}\lambda_{\mu y}\zeta_2$$

$$S_{Vx} = F_z \cdot \left(p_{Vy1} + p_{Vy2}df_z\right) \cdot \lambda_{Vy}\lambda_{\mu y}\zeta_2 + S_{Vy\gamma}$$

$$K_{y\gamma o} = F_z \cdot \left(p_{Ky6} + p_{Ky7}df_z\right)\left(1 + p_{py5}dp_i\right) \cdot \lambda_{Ky\gamma}$$

Related topics

Basics

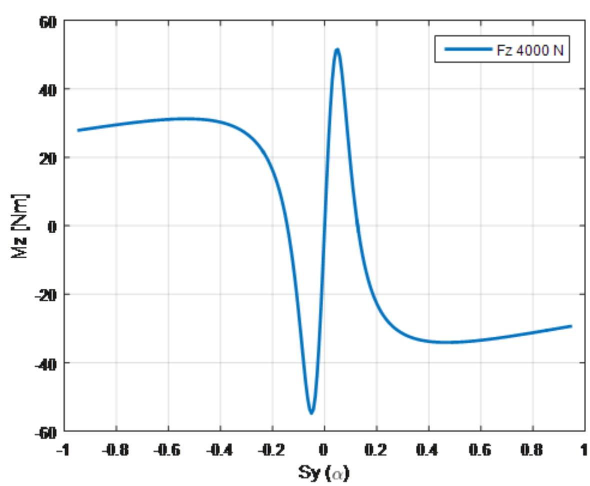
References

Torques Calculation

Self-aligning torque

The torque of the z-axis in the Magic Formula tire model is composed of two components in the pure lateral slip condition. In the case of a combined slip, the effect of the longitudinal force is considered.

The following figure shows the self-aligning torque as approximated in the Magic Formula tire model in the case of pure lateral slip.



In the case of *pure lateral slip*, the self-aligning torque is calculated as follows:

$$M_{zo} = M'_{zo} + M_{zro}$$

$$M'_{zo} = -t_o \cdot F_{yo}$$

$$t_o = t(\alpha_t) = D_t \sin[C_t \arctan\{B_t\alpha_t - E_t(B_t\alpha_t - \arctan(B_t\alpha_t))\}] \cdot \cos'\alpha$$

$$\alpha_t = \alpha^* + S_{Ht}$$

$$S_{Ht} = q_{Hz1} + q_{Hz2}df_z + (q_{Hz3} + q_{Hz4}df_z)\gamma^*$$

$$M_{zro} = M_{zr}(\alpha_r) = D_r \cos[C_r \arctan(B_r\alpha_r)] \cdot \cos'\alpha$$

$$\alpha_r = \alpha^* + S_{Hf}$$

$$S_{Hf} = S_{Hy} + S_{Vy}/K'_{ya}$$

$$K'_{ya} = K_{ya} + \varepsilon_K$$

$$B_t = \left(q_{Bz1} + q_{Bz2}df_z + q_{Bz3}df_z^2\right)\left(1 + q_{Bz5}|\gamma^*| + q_{Bz6}\gamma^{*2}\right) \cdot \lambda_{Kya}/\lambda_{\mu y}$$

$$C_t = q_{Cz1}$$

$$D_{to} = F_z \cdot \left(\frac{R_o}{F'_{zo}}\right) \cdot (q_{Dz1} + q_{Dz2}df_z)\left(1 - p_{pz1}dp_i\right) \cdot \lambda_t \cdot \operatorname{sgn} V_{cx}$$

$$D_t = D_{to} \cdot \left(1 + q_{Dz3}|\gamma^*| + q_{Dz4}\gamma^{*2}\right) \cdot \zeta_5$$

$$E_t = \left(q_{Ez1} + q_{Ez2}df_z + q_{Ez3}df_z^2\right) \cdot \left\{1 + (q_{Ez4} + q_{Ez5}\gamma^*)\frac{2}{\pi}\arctan(B_t C_t \alpha_t)\right\}$$

$$B_r = \left(q_{Bz9} \cdot \frac{\lambda_{Kya}}{\lambda_{\mu y}} + q_{Bz10}B_y C_y\right) \cdot \zeta_6$$

$$C_r = \zeta_7$$

$$D_r = F_z R_o\left[(q_{Dz6} + q_{Dz7}df_z)\lambda_{Mr}\zeta_2 + \left\{(q_{Dz8} + q_{Dz9}df_z)\left(1 + p_{pz2}dp_i\right) + (q_{Dz10} + q_{Dz11}df_z)|\gamma^*|\right\}\gamma^*\lambda_{Kzy}\zeta_0\right] \cdot \lambda_{\mu y}\operatorname{sgn} V_{cx}\cos'\alpha + \zeta_8 - 1$$

$$K_{za0} = D_{to}K_{ya}$$

$$K_{zy0} = F_z R_o(q_{Dz8} + q_{Dz9}df_z)\left(1 + p_{pz2}dp_i\right) \cdot \lambda_{Kzy}\lambda_{\mu y} - D_{to}K_{y\gamma 0}$$

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Forces and Torques Calculation for Combined Slip

Longitudinal force

$$F_x = G_{xa} \cdot F_{xo}$$

$$G_{xa} = \cos(C_{xa}\arctan\{B_{xa}\alpha_S - E_{xa}(B_{xa}\alpha_S - \arctan(B_{xa}\alpha_S))\}/G_{xao})$$

$$G_{xao} = \cos(C_{xa}\arctan\{B_{xa}S_{Hxa} - E_{xa}(B_{xa}S_{Hxa} - \arctan(B_{xa}S_{Hxa}))\})$$

$$\alpha_S = \alpha^* + S_{Hxa}$$

$$B_{xa} = \left(r_{Bx1} + r_{Bx3}\gamma^{*2}\right)\cos[\arctan(r_{Bx2})] \cdot \lambda_{xa}$$

$$C_{xa} = r_{Cx1}$$

$$E_{xa} = r_{Ex1} + r_{Ex2}df_z \ \ (\leq 1)$$

$$S_{Hxa} = r_{Hx1}$$

Lateral force

$$F_y = G_{yk} \cdot F_{yo} + S_{Vy\kappa}$$

$$G_{y\kappa} = \cos\left(C_{y\kappa}\arctan\left\{B_{y\kappa}\kappa_S - E_{y\kappa}\left(B_{y\kappa}\kappa_S - \arctan\left(B_{y\kappa}\kappa_S\right)\right)\right\}\right)/G_{y\kappa o}$$

$$G_{y\kappa o} = \cos\left(C_{y\kappa}\arctan\left\{B_{y\kappa}S_{Hy\kappa} - E_{y\kappa}\left(B_{y\kappa}S_{Hy\kappa} - \arctan\left(B_{y\kappa}S_{Hy\kappa}\right)\right)\right\}\right)$$

$$\kappa_S = \kappa + S_{Hy\kappa}$$

$$B_{y\kappa} = \left(r_{By1} + r_{By4}\gamma^{*2}\right)\cos\left[\arctan\left(r_{By2}\left(\alpha^* - r_{By3}\right)\right)\right] \cdot \lambda_{y\kappa}$$

$$C_{y\kappa} = r_{Cy1}$$

$$E_{y\kappa} = r_{Ey1} + r_{Ey2}df_z \ \ (\leq 1)$$

$$S_{Hy\kappa} = r_{Hy1} + r_{Hy2}df_z$$

$$S_{Vy\kappa} = D_{Vy\kappa}\sin\left[r_{Vy5}\arctan\left(r_{Vy6}\kappa\right)\right] \cdot \lambda_{Vy\kappa}$$

$$D_{Vy\kappa} = \mu_y F_z \cdot \left(r_{Vy1} + r_{Vy2}df_z + r_{Vy3}\gamma^*\right) \cdot \cos\left[\arctan\left(r_{Vy4}\alpha^*\right)\right] \cdot \zeta_2$$

Overturning couple

$$M_x = \quad R_o F_z \cdot \left[q_{sx1}\lambda_{VMx} - q_{sx2}\gamma\left(1 + p_{pMx1}dp_i\right) + q_{sx3}\frac{F_y}{F_{zo}} + q_{sx4}\cos\left\{q_{sx5}\arctan\left(q_{sx6}\frac{F_z}{F_{zo}}\right)^2\right\}\sin\left\{q_{sx7} + q_{sx8}\arctan\left(q_{sx9}\frac{F_y}{F_{zo}}\right)\right\} + q_{sx10}\arctan\left(q_{sx11}\frac{F_z}{F_{zo}}\right) \cdot \gamma\right] \cdot \lambda_{Mx}$$

Rolling resistance moment

$$M_y = R_o F_z \cdot \left\{q_{sy1} + q_{sy2}\frac{F_x}{F_{zo}} + q_{sy3}\left|\frac{V_x}{V_o}\right| + q_{sy4}\left(\frac{V_x}{V_o}\right)^4 + \left(q_{sy5} + q_{sy6}\frac{F_z}{F_{zo}}\right)\gamma^2\right\} \left\{\left(\frac{F_z}{F_{zo}}\right)^{q_{sy7}} \cdot \left(\frac{p_i}{p_{io}}\right)^{q_{sy8}}\right\} \cdot \lambda_{My}$$

Aligning torque

$$M_z = M_z' + M_{zr} + s \cdot F_x$$

$$M_z' = -t \cdot F_y'$$

$$t = t\left(\alpha_{t,eq}\right) = D_t\cos\left[C_t\arctan\left\{B_t\alpha_{t,eq} - E_t\left(B_t\alpha_{t,eq} - \arctan\left(B_t\alpha_{t,eq}\right)\right)\right\}\right] \cdot \cos'\alpha$$

$$F_y' = G_{y\kappa} \cdot F_{yo}$$

$$M_{zr} = M_{zr}\left(\alpha_{r,\,eq}\right) = D_r \cos\left[C_r \arctan\left(B_r \alpha_{r,\,eq}\right)\right]$$

$$s = R_o \cdot \left\{s_{sz1} + s_{sz2}\left(\frac{F_y}{F'_{zo}}\right) + (s_{sz3} + s_{sz4} df_z)\gamma^*\right\} \cdot \lambda_s$$

$$\alpha_{t,\,eq} = \sqrt{\alpha_t^2 + \left(\frac{K_{x\kappa}}{K'_{ya}}\right)^2 \kappa^2} \cdot \mathrm{sgn}(\alpha_t)$$

$$\alpha_{r,\,eq} = \sqrt{\alpha_r^2 + \left(\frac{K_{x\kappa}}{K'_{ya}}\right)^2 \kappa^2} \cdot \mathrm{sgn}(\alpha_r)$$

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Importing TIR Files

Importing TIR Files

Introduction

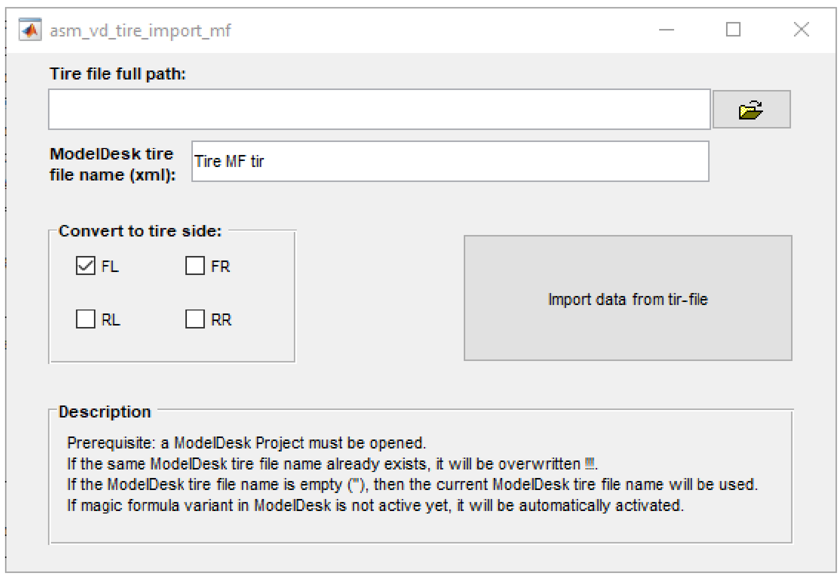
To import the Magic Formula parameters from a TIR file to the corresponding ModelDesk XML file, you can use the MATLAB script `asm_vd_tire_import_mf`.

Prerequisite

The ModelDesk project and experiment must be opened.

User interface

When you call `asm_vd_tire_import_mf`, the following dialog opens:



Arguments

The script can be called with the following parameters:

Parameter	Description
FileName	Full path file name of the TIR file.
XmlFileName	Creates an XML file with the name `<XmlFileName> + <XmlFileAppendix>`. Example: `Tire MF tir`. ▪ If `XmlFileName` is empty (`''`), the current XML file name is used. ▪ If the same XML file name already exists, the file is overwritten.

Parameter	Description
Side	An optional cell string that specifies which tire is imported. Export to one or more ModelDesk HTML pages. ▪ Default: 'FL'. ▪ Example: `Side = {'FL'};` Export only to the `Tire FL MF` html page. ▪ Example: `Side = {'Tire RL MF', 'Tire RR MF'}` Export to the `Tire RL MF` and to the `Tire RR MF` HTML page.
MainComponent	An optional string that specifies which main tire component is exported. ▪ Default: `VehicleDynamics` ▪ Example: `MainComponent = 'VehicleDynamics'` ▪ Example: `MainComponent = 'Trailer';`
XmlFileAppendix	An optional string that specifies the file appendix for the ModelDesk XML file. Default: empty String (`''`) ▪ If `XmlFileName` is empty and `XmlFileAppendix` is not empty, the current XML file name plus `XmlFileAppendix` is used. ▪ If the current XML file does not have this appendix, a new XML file with the file name `name` plus `appendix` is created. ▪ If the XML file already has the same appendix, the XML file is overwritten.

Examples

The following examples show how you can call the MATLAB script:

- `asm_vd_tire_import_mf()`
  The main user interface is called.
- `asm_vd_tire_import_mf('FileName', [pwd, '/tire.tir'])`
- `asm_vd_tire_import_mf('FileName', [pwd, '/tire.tir'], 'XmlFileName', 'Tire MF tir')`
- `asm_vd_tire_import_mf('FileName', [pwd, '/tire.tir'], 'XmlFileAppendix', '_new')`

Related topics

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Steering

Where to go from here

Information in this section

Sign Conventions

Introduction

The following sign conventions hold for ASM Vehicle Dynamics.

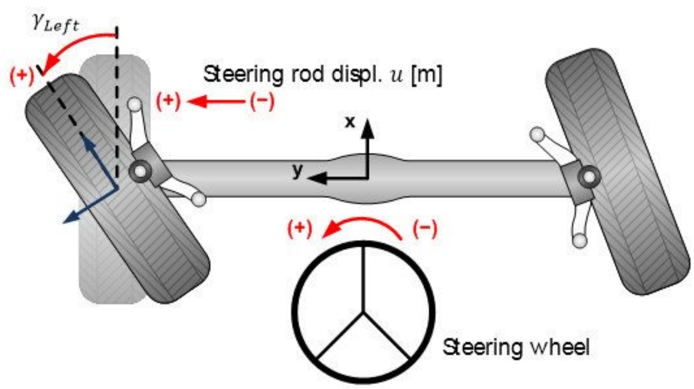
Steering and Suspension Kinematics

In ASM Vehicle Dynamics, the following sign convention is used for the steering wheel, the steering rod displacement, the left wheel rotation about the z-axis (gamma angle), and vertical wheel displacement:

Name	Positive sign (+)
Steering wheel	Counterclockwise rotation about the steering wheel vertical axis
Steering rod displacement	Movement to the left side

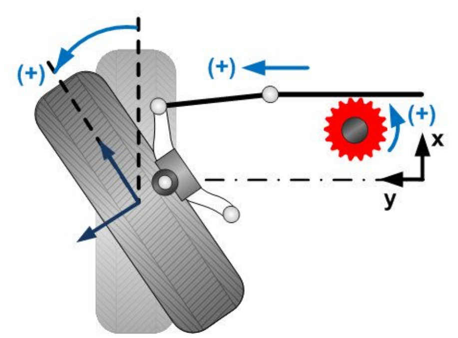
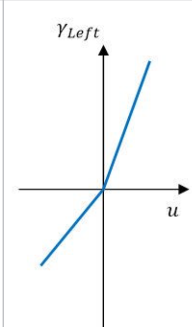
Name	Positive sign (+)
Gamma angle of the left wheel	Counterclockwise rotation about the z-axis
Vertical left wheel displacement	Vertical wheel movement is defined as the vertical movement of the wheel center. Positive wheel movement 'displ_z' is defined as an upward movement (compression). Example: displ_z = 0 equals the wheel position in the vehicle design configuration.

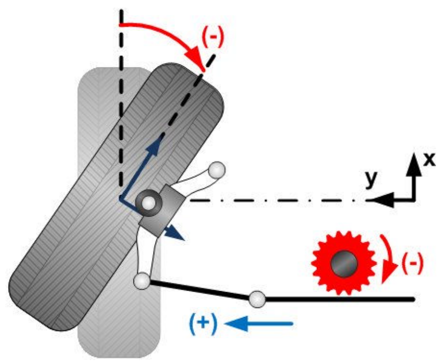
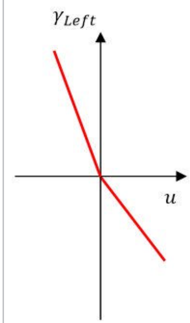
The following illustration displays this sign convention:



Different Steering Configurations

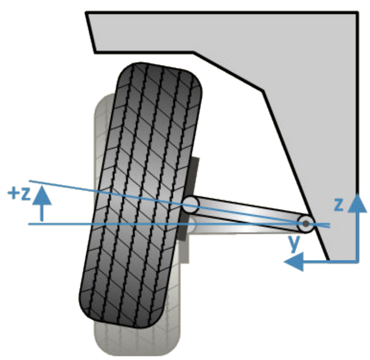
For different steering system configurations, the following table shows how to parameterize the steering gear ratio *i* and the gamma table of suspension kinematics $\gamma_{Left} = f(u)$ of the ASM Vehicle Dynamics Model. u is the steering degree of freedom, e.g., the steering rod displacement.

Sketch	Steering gear ratio ***i***	Left gamma $\gamma_{Left} = f(u)$
	Positive (+)	

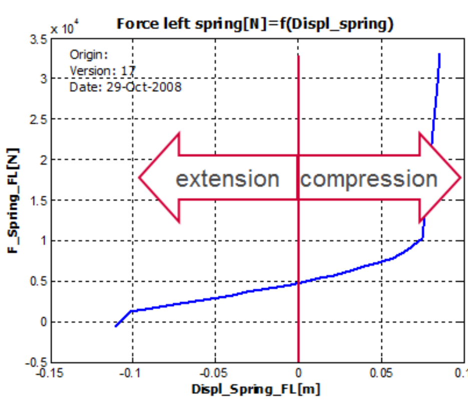
Sketch	**Steering gear ratio *i***	**Left gamma** $\gamma_{Left} = f(u)$
	Negative (-)	

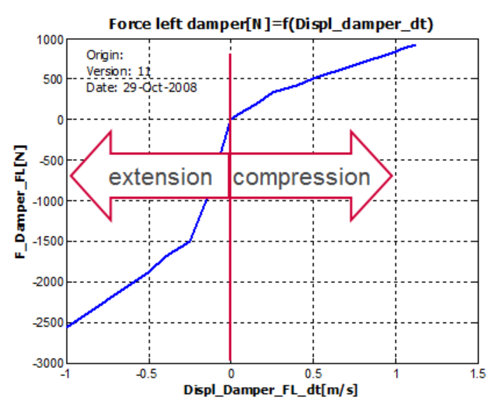
Suspension

Wheel jounce (+z in figure below) is considered to be positive, wheel rebound is negative.



From this, the spring, damper and stabilizer compressions are considered to be positive, whereas the deflections are negative. The example figures below show typical spring (left side) and damper (right side) characteristics for extension and compression.





Recirculating Ball

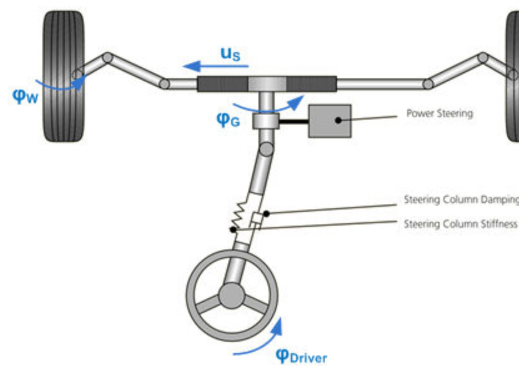
Description

Standard ASM Steering System (Rack and Pinion)

Constant ratio:
$i = \varphi_G / u_S$ [deg|m]
$\varphi_W = f(u_S, \Delta z)$

Variable ratio:
$i = f(u_S)$ [deg|m]
$\varphi_W = f(u_S, \Delta z)$

φ_W is a representative of the suspension kinematics (e.g. gamma table)

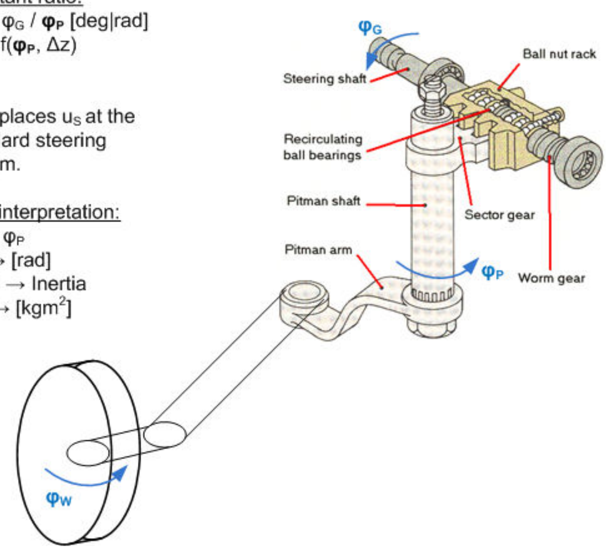


„New“ Interpretation of Standard ASM Steering System (Recirculation Ball)

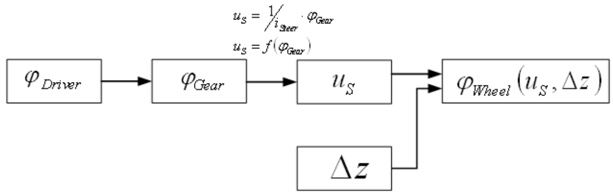
Constant ratio:
$i = \varphi_G / \boldsymbol{\varphi_P}$ [deg|rad]
$\varphi_W = f(\boldsymbol{\varphi_P}, \Delta z)$

$\boldsymbol{\varphi_P}$ replaces u_S at the standard steering system.

New interpretation:
$u_S \rightarrow \varphi_P$
[m] → [rad]
Mass → Inertia
[kg] → [kgm^2]

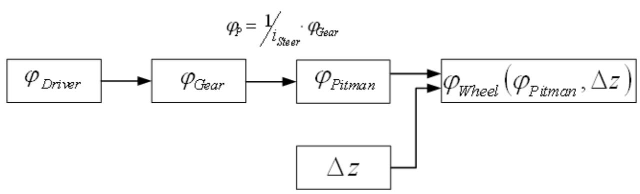


The standard ASM steering system represents a rack-and-pinion steering system:



All suspension kinematics are functions of steering rod displacement u_S and vertical wheel displacement Δz.

If a recirculating ball steering system shall be simulated, the standard ASM steering system can be kept, but has to be reinterpreted. In general, the steering rod displacement u_S is replaced with the pitman angle φ_A and the total steering mass is replaced with the total steering inertia about the pitman arm Θ_{Steer}. Then, the following information flow counts:



Note that if the implementation has not to be changed for simulating a recirculating ball steering system, the pitman arm angle φ_{Pitman} is required to be in Radian. The units of φ_{Driver}, φ_{Gear} are in degree, as the steering subsystem already includes a unit conversion for these variables. Thus, the steering ratio has to be of following unit:

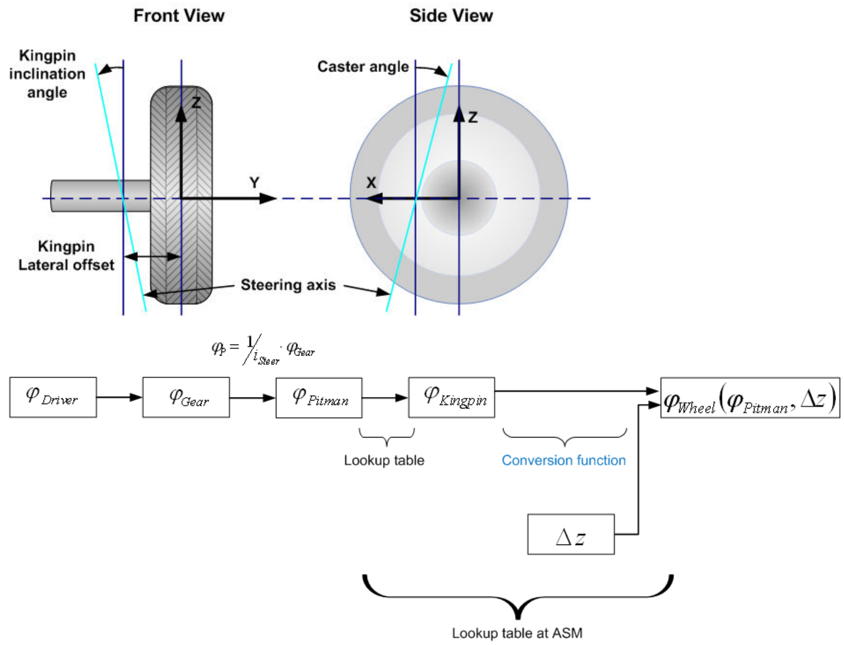
$$i_{Steer} = \frac{\varphi_{Gear}}{\varphi_{Pitman}} \frac{[deg]}{[rad]}$$

All suspension kinematics are functions of pitman arm angle φ_P and vertical displacement Δz.

Kingpin Axis

Description

If the steering system is described via the geometry of the kingpin axis and the suspension kinematics via the rotation about the kingpin axis, a conversion function is required to calculate the related ASM suspension look-up tables.



Taking the kingpin geometry into account, a rotation vector is calculated from which a transformation matrix is calculated to transform a vector from the wheel coordinate system into the vehicle one.

Another transformation matrix can be set up using the sequential rotation about the cardan angles alpha, beta and gamma.

Via comparison of the matrix elements of both rotation matrices, the cardan angles alpha, beta and gamma can be calculated depending on the rotation about the kingpin axis.

Integration Method

Introduction

To solve the model's stiff differential equations, an integration method must be used.

Euler Methods

Introduction

To solve the model's stiff differential equations, a sophisticated integration method must be used to avoid the problem of numerical instability. On the other hand, this method should not be computationally expensive. The Euler method was selected for these reasons.

Methods

There are two Euler methods, forward and backward.

For the differential equation

$$\dot{x} = f(x, t)$$

where *h* is the simulation step size, the *forward* method reads:

$$x_{n+1} = x_n + hf(x_n, t_n)$$

and for the *backward* Euler integration

$$x_{n+1} = x_n + hf(x_{n+1}, t_{n+1})$$

holds.

The backward Euler integration is absolutely stable in the left half plane of the s-domain. The forward Euler, however, has limited stability. The backward Euler was therefore used to solve the stiff differential equations. However, the backward method cannot be solved explicitly. It can only be calculated iteratively, which increases the volume of computation. The function $f(x_{n+1}, t_{n+1})$ is therefore replaced by its first two terms from Taylor series expansion. Then the function can be approximated as follows:

$$f(x_{n+1}, t_{n+1}) \cong f(x_n, t_n) + \frac{\partial f}{\partial x}(x_{n+1} - x_n) + \frac{\partial f}{\partial t}(t_{n+1} - t_n)$$

Moreover, if *f* is only a function of *x*, the following equation is obtained:

$$f(x_{n+1}, t_{n+1}) \cong f(x_n, t_n) + \frac{\partial f}{\partial x}(x_{n+1} - x_n)$$

Substituting in the backward form:

$$x_{n+1} = x_n + h\left(f(x_n, t_n) + \frac{\partial f}{\partial x}(x_{n+1} - x_n)\right)$$

and solving for x_{n+1}, yields:

$$x_{n+1} = x_n + h\left(1 - h\frac{\partial f}{\partial x}\right)^{-1} f(x_n, t_n)$$

Simulating the drivetrain dynamics including the elastic shafts leads to a stiff differential equation due to the relatively high shaft stiffness. The prescribed stabilization technique is therefore used. The drivetrain equation system can be written as follows:

$$\Theta_{[8x8]} \cdot \dot{\Omega}_{[8x1]} = T_{[8x1]}(\Omega, \theta, t)$$

where:

$\Theta_{[8x8]}$	is the inertia matrix
$\Omega_{[8x1]}$	is the angular velocity vector
$\theta_{[8x1]}$	is the angular twisting vector
$T_{[8x1]}(\Omega, \theta, t)$	is the generalized torque vector

The numerical solution can then be carried out as follows:

$$\Omega_{k+1[8x1]} = \Omega_{k[8x1]} + h \cdot \frac{T_{[8x1]}(\Omega_k, \theta_k, t)}{\left(\Theta_{[8x8]} - h \cdot (\partial T/\partial\Omega)_{k[8x88]} - h^2 \cdot (\partial T/\partial\theta)_{k[8x8]}\right)}$$

$$\theta_{k+1[8x1]} = \theta_{k[8x1]} + h \cdot \Omega_{k+1[8x1]}$$

In the vehicle dynamics system, the rebound and jounce stop in the suspension system is modeled as very stiff springs, which may lead to numerical problems. The vehicle equation of motion is therefore stabilized against the suspension spring and damper characteristics.