

Statistics

ALaRI Exam

27 August 2012

- Duration: 1 hour and 30 minutes
- Open book exam
- Solve all exercises

Problem 1

Five percent of the disk controllers produced by a plant are known to be defective. A sample of 15 controllers is drawn randomly from each month's production and the number X of defectives noted.

1. What is the distribution of the random variable X ?
2. What proportion of these monthly samples would have at least two defective controllers?

Problem 2

Given three components with respective reliabilities $R_1 = 0.8$, $R_2 = 0.75$ and $R_3 = 0.98$, compute the reliabilities of the system shown in Figure 1.

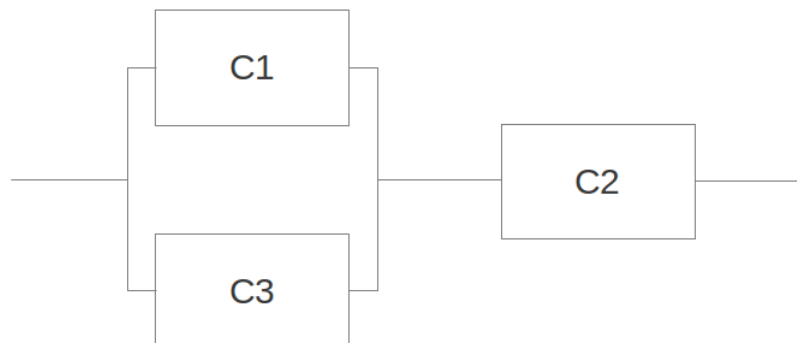


Figure 1: System reliability

Problem 3

Of all the graduate students at USI, 70% are women and 30% are men. Suppose that 20% and 25% of the female and male population, respectively, smoke cigarettes. What is the probability that a randomly selected graduate student is

1. A woman who smokes?
2. A man who smokes?
3. A smoker?

Problem 4

Let us assume that a program has two modules and that the respective module execution times X and Y are independent random variables uniformly distributed over $\{1, 2, \dots, n\}$. Find

1. $P(X \geq Y)$.
2. $P(X = Y)$.
3. The probability mass function and the probability generating function of $Z_1 = X + Y$.
4. The probability mass function of $Z_2 = \max\{X, Y\}$.
5. The probability mass function of $Z_3 = \min\{X, Y\}$.

Problem 5

Let X be exponentially distributed with density function

$$f(x) = \begin{cases} \lambda * \exp(-\lambda x), & x \geq 0, \\ 0 & \text{otherwise} \end{cases}$$

and define

$$Y = g(X) = 1 - \exp(-\lambda X).$$

1. Derive the density function of Y .
2. Compute the expected value of Y .
3. What is the distribution of Y ?
4. Compute the distribution function of Y .