Statistics Lecture

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Definition

A random experiment is an experiment whose outcome can not predicted with certainty.

Examples

- Observing if a system component is functioning properly or has failed at a given point in time in the future.
- Determining the execution time of a program.
- Determining the response time of a server request.

Definition

The result of the experiment is called the outcome of the experiment. The totality of the possible outcomes of a random experiment is called the sample space of the experiment and it will be denoted by the letter S.

- The definition of the sample space S is determined by the experiment and the purpose for which the experiment is carried out. When observing the status of two components of a running system, it may be sufficient to know if zero, one or two components have failed without having to exactly identify which component has failed.
- We classify the sample spaces w.r.t. the number of elements they contain.
  - *Finite sample* space: the set of possible outcomes of the experiment is finite;
  - *Countably infinite* sample space: the outcomes of the experiment are in a one-to-one relationship with $\mathbb{N}$;
  - Otherwise the sample space is called *uncountable* or *nondenumerable*.
- A finite or countably infinite sample space is called a *discrete* sample space. *Continuous* sample spaces, such as all the points on a line or interval are examples of uncountable sample spaces.

Definition

Given a random experiment and the corresponding sample space S, a collection of certain outcomes is called an event E. E is a subset of the sample space $E \subset S$. Equivalently, any statement of conditions identifying E is called an event.

Example

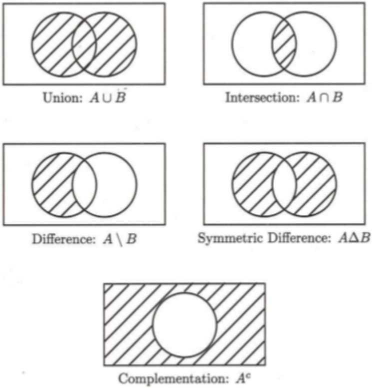
In the random experiment "Toss of a die", we define the sample space $S = \{1, \ldots, 6\}$. The event $E =$"The outcome is an even number" is equivalent to $E = \{2, 4, 6\}$.

Definition

Given a random experiment with sample space S we call a single performance of the experiment a *trial*.

Basic definitions

Basic Set Operations



Basic definitions

Sequential sample space

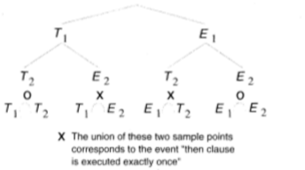


Figure 1.8. Tree diagram of a sequential sample space

The set of all leaves of the tree is the sample space of interest.

Basic definitions

Two-dimensional sample space

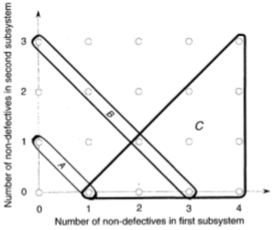


Figure 1.9. A two-dimensional sample space

1. A :=”the system has exactly one non-defective component”
2. B :=”the system has exactly three non-defective components”
3. C :=”the first subsystem has more non-defective components than the second subsystem”

- Let E be an event of S and let denote the outcome of a specific trial by s. If $s \in E$ we then say that the event E has *occurred*. Only one outcome $s \in S$ can occur on any trial. However, every event including s will occur.
- Let E, E_1 and E_2 be events. We define the event $\bar{E}$ to be *the complement* of E, i.e. $\bar{E} = \{s \in S \mid s \notin E\}$.
- Let E_1 and E_2 be two events. We define the event
    - $E_1 \cup E_2 := \{s \in S \mid s \in E_1$ or $s \in E_2\}$ *the union* of E_1 and E_2.
    - $E_1 \cap E_2 := \{s \in S \mid s \in E_1$ and $s \in E_2\}$ *the intersection* of E_1 with E_2.
    - $E_1 \setminus E_2 := \{s \in S \mid s \in E_1$ and $s \notin E_2\}$ the difference of $E_1$and E_2.
- If $E_1 \cap E_2 = \emptyset$ we say that the two events E_1 and E_2 are *mutually exclusive* or *disjoint*.
- We denote by $\mid E \mid$ *the cardinality* of E, i.e. the number of elements (outcomes) in E.

Definition

Let S denotes the sample space of a given experiment and $\mathscr{F}$ a collection of events. We say that $\mathscr{F}$ is an *algebra* over S if the following two conditions are fulfilled:

1. S must be an element of $\mathscr{F}$, i.e. $S \in \mathscr{F}$.
2. If $E_1 \in \mathscr{F}$, $E_2 \in \mathscr{F}$, the sets $E_1 \cup E_2$, $E_1 \cap E_2$ must also belong to $\mathscr{F}$.
3. If $E_1 \in \mathscr{F}$, $E_2 \in \mathscr{F}$, $E_1 \setminus E_2$ must also belong to $\mathscr{F}$.

Interpretation: an algebra $\mathscr{F}$ over S is a family of subsets of S which is closed with respect to the three binary operators $\cup, \cap$ and $\setminus$.
Remark: condition 3 of the previous definition can be replaced by the following equivalent condition:

- 3^{bis} If $E \in \mathscr{F}$ then $\overline{E} \in \mathscr{F}$.

Example

a) The collection given by $\mathscr{F} = \{S, \emptyset\}$ is an algebra (the so called *trivial* algebra over S).
b) The collection $\mathscr{F} = \{E, \bar{E}, S, \emptyset\}$ is the algebra generated by $E \subset S$.
c) The power set of S, denoted by $\mathscr{P}(S)$, is defined to be the collection consisting of all subsets of S (including the empty set $\emptyset$).

Exercise

1. *Show by mathematical induction that if S is a finite set with $\mid S \mid = n$ elements, then the power set of S contains $\mid \mathscr{P}(S) \mid = 2^n$ elements.*
2. *Show that condition 3^{bis} is equivalent to condition 3.*
3. *Given $S = \{1,2,3,4,5,6\}$ construct at least two different algebras on S.*
4. *De Morgan's law. Let A and B two events. Show that*

$$\overline{A \cup B} = \overline{A} \cap \overline{B} \text{ and } \overline{A \cap B} = \overline{A} \cup \overline{B}.$$

Exercise

1. *De Morgan's law. Let I be a non empty, possibly uncountable set and $A_i, i \in I$ a family of sets indexed by I. Show that*

$$\overline{\bigcup_{i\in I} A_i} = \bigcap_{i\in I} \overline{A}_i \text{ and } \overline{\bigcap_{i\in I} A_i} = \bigcup_{i\in I} \overline{A}_i.$$

Definition

The indicator of the set $A \subseteq S$ is the function on S given by

$$1_A(s) = \begin{cases} 1 & \text{if } s \in A \\ 0 & \text{if } s \notin A. \end{cases}$$

Interpretation: the function1_A "indicates" whether A occurs.

Exercise

Prove the following equalities:

$1_{A\cup B} = max\{1_A, 1_B\}$; $1_{A\cap B} = min\{1_A, 1_B\} = 1_A 1_B$

$1_{\overline{A}} = 1 - 1_A$; $1_{A\triangle B} = \mid 1_A - 1_B \mid$

Definition

Let S denotes the sample space of a given experiment and $\mathscr{G}$ a collection of events. We say that $\mathscr{G}$ is a $\sigma-$*algebra* over S if

1. $\mathscr{G}$ is an *algebra* over S.
2. If $E_n \in \mathscr{G}$, $n = 1, 2, \ldots$, then

$$\bigcup_{n=1}^{\infty} E_n \in \mathscr{G}.$$

Interpretation: a $\sigma-$algebra $\mathscr{G}$ over S is a family of subsets of S which is closed with respect to

- the difference operator $\setminus$ (or, equivalently, complementation),
- the *countable* union and intersection of its elements.

Theorem

Let $\mathscr{E}$ be a collection of subsets of S. Then there are a smallest algebra $\alpha(\mathscr{E})$ and a smallest $\sigma-$algebra $\sigma(\mathscr{E})$ containing all the sets that are in $\mathscr{E}$.

Proof.

$\mathscr{P}(S)$ is a $\sigma-$algebra on S. Therefore it exists at least one $\sigma-$algebra and one algebra containing $\mathscr{E}$. We define $\alpha(\mathscr{E})$ (or $\sigma(\mathscr{E})$) to consist of all sets that belong to every algebra (or $\sigma-$algebra) containing $\mathscr{E}$. It is easy to verify that this system is an algebra (or $\sigma - algebra$) and indeed the smallest. □

Examples

1. Let $\mathscr{C}$ be the family of open intervals on the real line, i.e.

$$\mathscr{C} := \{(a,b) \mid a,b \in \mathbb{R} \text{ and } a<b\}.$$

$\mathscr{B}(\mathbb{R}) := \sigma(\mathscr{C})$ is called the Borel sigma algebra over $\mathbb{R}$.

2. Let $f : S \to \mathbb{R}$ be a real valued function. The family of preimages:

$$\{f^{-1}(B) : B \in \mathscr{B}(\mathbb{R})\}$$

is a $\sigma-$algebra on S, the $\sigma-$algebra generated by f [denoted $\sigma(f)$].

Definition

The pair $(S, \mathscr{G})$ where S is the sample space and $\mathscr{G}$ a $\sigma-$algebra on S is called a measurable space and the elements of $\mathscr{G}$ are called events.

Remark: a subset A of the sample space S is an event if and only if $A \in \mathscr{G}$.

Example

Suppose we toss a die once. We choose $S = \{1,2,3,4,5,6\}$ and $\mathscr{G} := \{S, \emptyset, \{1,2,3\}, \{4,5,6\}\}$. The subset $A = \{2,4,6\}$ of S is *not* an event of $\mathscr{G}$. Let define the sigma algebra $\mathscr{H} := \sigma(\{\{1,2,3\}, A\})$ and the new measurable space $(S, \mathscr{H})$. $A \in \mathscr{H}$ so that A is now an event.

Exercise

Complete the sigma algebra $\mathscr{H}$ by enumerating its elements.

Definition

Let $(S, \mathscr{H})$ be a measurable space. A probability measure or probability law is a positive real-valued function $P : \mathscr{H} \to [0, 1]$ such that the following axioms hold

1. $P(S) = 1$.
2. For every sequence $\{E_n\}_{n\in\mathbb{N}}$ of pairwise disjoint events ($E_i \cap E_j = \emptyset$, $\forall i \neq j$) it must hold

$$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i).$$

Remark: a probability law is a function defined on a sigma algebra. The *arguments* of P are *events*, i.e. *subsets* of S and *elements* of $\mathscr{H}$.
A probability law has many useful relations, see Trivedi pp. 15-16.

Example

Probability of union of events version 1 (Trivedi, page 15).
If $A_1, A_2, \ldots, A_n$ are any events, then

$$\begin{aligned}
P(\bigcup_{i=1}^{n} A_i) &= P(A_1 \cup A_2 \cup \cdots \cup A_n) \\
&= \sum_i P(A_i) - \sum_{1 \le i < j \le n} P(A_i \cap A_j) \\
&\quad + \sum_{1 \le i < j < k \le n} P(A_i \cap A_j \cap A_k) + \ldots \\
&\quad + (-1)^{n-1} P(A_1 \cap A_2 \cap \cdots \cap A_n)
\end{aligned}$$

where the successive sums are over all possible events, pairs of events, triples of events, and so on.

Example

Probability of union of events, version 2 (Trivedi, page 16).
If $A_1, A_2, \ldots, A_n$ are any events, then

$$\begin{aligned} P(\bigcup_{i=1}^{n} A_i) &= P(A_1) + P(\bar{A}_1 \cap A_2) + P(\bar{A}_1 \cap \bar{A}_2 \cap A_3) + \ldots \\ &+ P(\bar{A}_1 \cap \bar{A}_2 \cap \cdots \cap \bar{A}_{n-1} \cap A_n). \end{aligned}$$

Exercise

Using the properties of P prove previous equality.

Definition

A probability space is a the triple $(S, \mathscr{G}, P)$ where

- S is the sample space
- $\mathscr{G}$ is a $\sigma-$algebra over S
- P is a probability measure

Remark: if the sample space is finite or countable it is possible to define a probability measure on $\mathscr{G} = \mathscr{P}(S)$. In that case every subset E of S is an event. However, if S in an uncountable sample space this is no longer true. The $\sigma-$algebra on which to define the probability law P must be smaller than $\mathscr{P}(S)$ in order to consistently define P (see Trivedi, page 17).

Exercise

1. *Let* $S = \{s_1, s_2, \ldots, s_n\}$ *be a finite sample space and* $\mathscr{G} = \mathscr{P}(S)$*. Assume that each sample point* s *is equally likely. Show that the function* P *defined for all events* $E \in \mathscr{G}$ *by*

$$P(E) = \frac{\mid E \mid}{n}$$

satisfies the axioms of a probability law.

Exercise

1. *(Trivedi, p. 19). Show that if event* B *is contained in event* A*, then* $P(B) \leq P(A)$*.*

Exercise

1. *(Trivedi, p. 19). Consider a pool of six I/O buffers. Assume that any buffer is just as likely to be available (or occupied) as any other. Compute the probabilities of the events*
    1. $A =$*"at least 2 but no more than 5 buffers occupied".*
    2. $B =$*"at least 3 but no more than 5 occupied".*
    3. $C =$*"all buffers available or an even number of buffers occupied".*

    Also determine the probability that at least one of the events A, B and C occurs.

If the sample space S of an experiment is finite, then the computation of probabilities is often simple. Assume (as it is almost always the case) that $\mathscr{G}$ contains all elementary events, i.e. $\{s_i\} \in \mathscr{G}\ \forall i = 1, \ldots, n$ and that $P(s_i) = p_i \geq 0$ and

$$\sum_{i=1}^{n} p_i = 1.$$

Because any event E consists of a certain collection of sample points, $P(E)$ can be computed by axiom 2 as the sum of the probabilities of the elementary events the union of which make up E.

Example

Let us assume that we toss a loaded die where the probabilities are given by $p_1 = 0.2$, $p_i = 0.16\ i = 2, \ldots, 6$. The probability of the event $E =$"the outcome is <4" is

$$P(E) = P(\{1\} \cup \{2\} \cup \{3\}) = 0.2 + 0.16 + 0.16 = 0.52$$

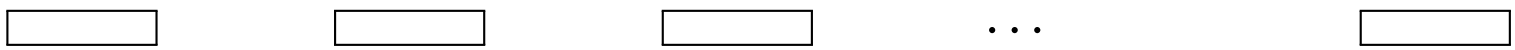
Combinatorial problems

Ordered samples of size k, with replacement

We are interested in the number of ways we can select k distinct objects from among n objects where

- the selection order is important,
- the same object can be selectet any number of times.

We call this problem a permutations with replacement problem. Alternatively, we could express the same problem as the number of ordered sequences $s_{i_1}, s_{i_2}, \ldots, s_{i_k}$, where each s_{i_r} belongs to $s_1, \ldots, s_n$.



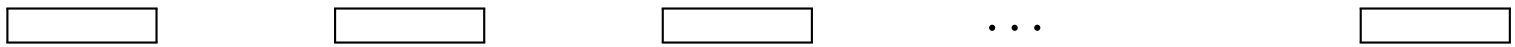
The solution is n^k. See example 1.4 at page 21.

We are interested in the number of ways we can select k distinct objects from among n objects where

- the selection order is important,
- object can be selectet at most one time.

We call this problem a permutations without replacement problem. The solution, called the number of permutations of n distinct objects taken k at a time, is denoted by $P(n,k)$ and is equal

$$P(n,k) = \frac{n!}{(n-k)!}$$



See example 1.5 at page 21.

We are interested in the number of ways we can select k distinct objects from among n objects where

- the selection order is *not* important,
- object can be selectet at most one time.

The solution is called the number of combinations of n distinct objects taken k at a time, is denoted by $C(n,k)$ or $\binom{n}{k}$ and is equal

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

See example 1.6 at page 22.

Exercise

Solve Problems 1, 3 and 5 of Trivedi, page 23.

- Probability theory models the way we should measure uncertainty.
- When we run a random experiment we only know that the outcome will belong to the sample space S. According to this information (no information) we have a probability space and a law P wich assigns probabilities to events: $(S, \mathscr{G}, P)$.
- What happens to these probabilities if we acquire new information about the outcome of the experiment?
- Suppose that we don't observe the outcome of the experiment but we know that $s \in B \in \mathscr{G}$, i.e. the event B is realised.
- Then, what is the probability of event A given that event B is realised (we simply say "given B"), denoted $P(A \mid B)$?

- Assume $B \in \mathscr{G}$ with $P(B) \neq 0$. The conditional probability of B given B is … $P(B \mid B) = 1$. There is no uncertainty regarding the realisation of event B once we know it is realised. We have *normalized* the probability of event B such that now it is equal to 1.
- Assume that S is finite and $\mathscr{G} = \mathscr{P}(S)$. If $P(B) \neq 0$ it makes then sense to define for all $s \in S$

$$P(\{s\} \mid B) = \begin{cases} \frac{P(\{s\})}{P(B)} & s \in B \\ 0 & s \notin B \end{cases}.$$

In fact, recall that $B = \bigcup_{s \in B} \{s\}$ and from axiom 2 of a (conditional) probability law

$$P(B \mid B) = P(\bigcup_{s \in B} \{s\} \mid B) = \sum_{s \in B} P(\{s\} \mid B) = \sum_{s \in B} \frac{P(\{s\})}{P(B)} = \frac{P(B)}{P(B)} = 1.$$

For any $A \in \mathscr{G}$ recall that $A = (A \setminus B) \cup (A \cap B)$.

(continued)

$$\begin{aligned} P(A \mid B) &= \sum_{s \in A} P(\{s\} \mid B) \\ &= \sum_{s \in A \setminus B} P(\{s\} \mid B) + \sum_{s \in A \cap B} P(\{s\} \mid B) \\ &= 0 + \sum_{s \in A \cap B} \frac{P(\{s\})}{P(B)} \\ &= P(A \cap B)/P(B) \end{aligned}$$

- All sample points not in B can be disregarded because the outcome of the experiment must be in B. For each event $A \in \mathscr{G}$ the relevant sample points are those common with B, i.e. the points in $A \cap B$.

Definition

The conditional probability of event A given event B is therefore defined by

$$P(A \mid B) = \begin{cases} \frac{P(A \cap B)}{P(B)} & \text{if } P(B) > 0 \\ \text{undefined} & \text{if} P(B) = 0 \end{cases}.$$

Multiplication rule (MR): rearranging the terms in the definition of $P(A \mid B)$ we obtain the following equalities

$$P(A \cap B) = \begin{cases} P(B)P(A \mid B) & \text{if } P(B) \neq 0 \\ P(A)P(B \mid A) & \text{if } P(A) \neq 0 \\ 0 & \text{otherwise} \end{cases}.$$

Exercise

1. *Solve Problem 1 Trivedi, page 25.*
2. *Let* $(S, \mathscr{G}, P)$ *be a probability space and* $P(A) \neq 0$. *Show that the set function defined on* $\mathscr{G}$
$$P_A(B) = P(B \mid A)$$
   1. *is a probability law on* $(S, \mathscr{G})$,
   2. *satisfies* $P_A(\bar{A}) = 0$.
3. *Let* P *be the uniform distribution on a finite sample space* S *and let* A *be a event on* S. *Prove that* $P(\mid A)$ *is the uniform distribution on* A.

If the probability of the occurrence of an event A does not change regardless of whether event B has occured, we are likely to conclude that the two events are independent. Formally we have the following definition.

Definition

(Independent events). We define two events A and B to be independent if and only if

$$P(A \cap B) = P(A)P(B).$$

Remarks:

1. Do not confuse indepent events with mutually exclusive (disjoint) events.
2. If A and B are mutually exclusive events, then $A \cap B = \emptyset$, which implies $P(A \cap B) = 0$. Now, if they are independent as well, then $P(A) = 0$ or $P(B) = 0$.
3. If an event A is independent of itself, that is, if A and A are independent, then $P(A) = 0$ or $P(A) = 1$. Why? Try to prove it!

Remarks (continued):

1. The relation of independence, denoted by $\perp$, is not transitive. If $A \perp B$ and $B \perp C$ it does not follow that $A \perp C$.
2. If events A and B are independent, then so are events $\bar{A}$ and B, events A and $\bar{B}$ and events $\bar{A}$ and $\bar{B}$. Prove it!

The concept of independence can be extended to a list of n events:

Definition

Independence of a set of events. A list of n events $A_1, A_2, \ldots, A_n$ is mutually independent if and only if for each set of k $(2 \leq k \leq n)$ distinct indices $i_1, i_2, \ldots, i_k$ which are elements of $\{1, 2, \ldots, n\}$ we have

$$P(A_{i_1} \cap A_{i_2} \cap \cdots \cap A_{i_k}) = P(A_{i_1})P(A_{i_2}) \cdots P(A_{i_k}).$$

Remark: It is possible to have $P(A \cap B \cap C) = P(A)P(B)P(C)$ but $P(A \cap B) \neq P(A)P(B)$, $P(A \cap C) \neq P(A)P(C)$, $P(B \cap C) \neq P(B)P(C)$. Under these conditions A, B and C are not mutually independent. (Trivedi, Example 1.10).

Let us consider an electronic system with n independent components. Define the event

$$A_i := \text{“The } i-th \text{ component is functioning properly ”.}$$

Definition

The reliability of component i is defined as

$$R_i = P(A_i),$$

i.e. it is the probability that the component is functioning properly.
A series system is a system such that the entire system fails if any one of its components fails.

Definition

A parallel system is a system such that the entire system fails only if all its components fail.

Theorem

Product law of reliabilities for series systems. The reliability of a series system decreases "quickly" with an increase in complexity (number of components).

Proof.

Let us consider the event $A =$"The system functions properly". The reliability of a series system of n components is then

$$R = P(A) = P(A_1 \cap A_2 \cdots \cap A_n) = \prod_{i=1}^{n} P(A_i)$$

$\square$

Example

Let a series system have $n = 5$ components and $P(A_i) = 0.970$ for all components.

The system reliability is then equal to

$$R = P(A_1)^5 = 0.97^5 = 0.859.$$

If we increase $n = 10$ the system reliability decreases to 0.738!

What if $n = 1'000'000$?

In order to mitigate the problem one possible solution is to implement parallel redundancy.

Example

Consider a parallel system of n independent components. The system runs correctly if at least one of its components runs properly, i.e.

$$A = (A_1 \cup A_2 \cdots \cup A_n).$$

But this means that

$$P(\overline{A}) = P(\overline{(A_1 \cup A_2 \cdots \cup A_n)}) = P(\overline{A}_1 \cap \overline{A}_2 \cdots \cap \overline{A}_n)) = \prod_{i=1}^{n} P(\overline{A}_i)$$

Example

(continued) Applying the identity $P(\overline{B}) = 1 - P(B)$ to both sides of the equality and solving w.r.t. $P(A)$, we obtain the final formula

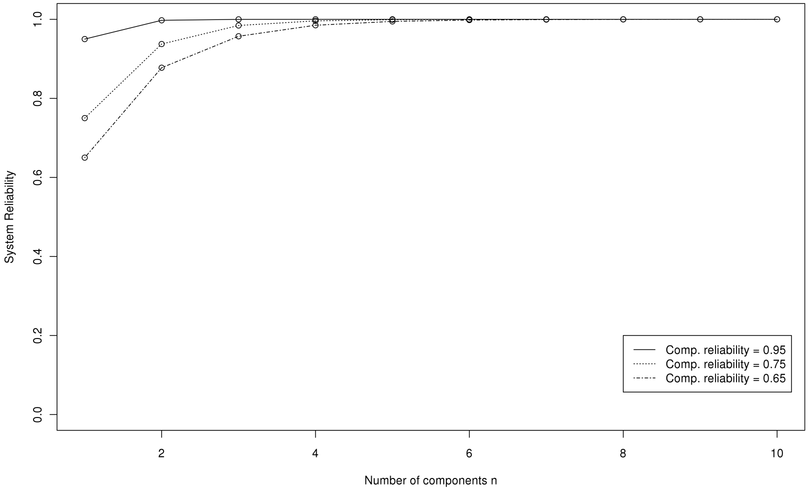
$$P(A) = 1 - \prod_{i=1}^{n}(1 - P(A_i))$$

or equivalently, in terms of R and R_i

$$R = 1 - \prod_{i=1}^{n}(1 - R_i).$$

The next picture shows the so called Product Law of Unreliabilities.

System's reliability
Product Law of Unreliabilities



From the previous picture it is evident one characteristic of parallel redundancy: the marginal increase in reliability decreases with increasing number of parallel components. This behaviour is called the **Law of Diminishing Returns**.

Definition

A system with both series and parallel parts is called a *series-parallel system*.

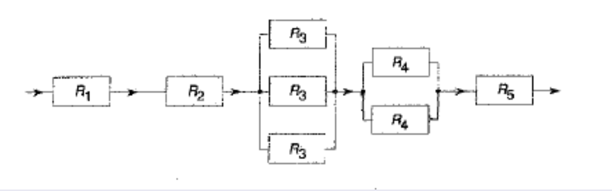


Figure: series-parallel reliability block diagram

Definition

Let X be a state vector of a system with n components so that $X = (x_1, \ldots, x_n)$ where

$$x_i = \begin{cases} 1 & \text{if component } i \text{is functioning,} \\ 0 & \text{if component } i \text{has failed.} \end{cases}$$

The structure function $\Phi(X)$ is defined by

$$\Phi(X) = \begin{cases} 1 & \text{if system is functioning,} \\ 0 & \text{if system has failed.} \end{cases}$$

The reliability of the system is then

$$R = P(\Phi(X) = 1).$$

Example

Communication network.

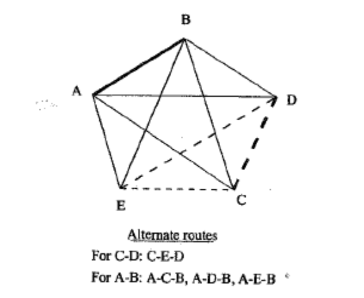


Figure: Communication network with five nodes

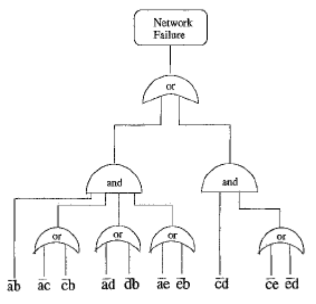


Figure: Fault tree for the communication network

Exercises: solve Problems 1,2,3,4 of Trivedi, page 35.

Bayes' Rule

Definition

A list of n events $B_1, B_2, \ldots, B_n$ that are collectively exhaustive (i.e. $\bigcup B_i = S$) and mutually exclusive (i.e. $B_i \cap B_j = \emptyset$ for $i \neq j$) form an event space $S' = \{B_1, B_2, \ldots, B_n\}$.

Let B an event and $S' = \{B, \bar{B}\}$ the corresponding event space. For any event A

$$A = (A \cap B) \cup (A \cap \bar{B}).$$

It follows (why?)

$$\begin{aligned} P(A) &= P(A \cap B) + P(A \cap \bar{B}) \\ &= P(A \mid B)P(B) + P(A \mid \bar{B})P(\bar{B}). \end{aligned}$$

The same formula can be generalized with respect to the event space $S' = \{B_1, B_2, \ldots, B_n\}$:

Bayes' Rule

Theorem

Theorem of total probability. Let $S' = \{B_1, B_2, \dots, B_n\}$ be an event space and A an event. Then

$$P(A) = \sum_{i=1}^{n} P(A \mid B_i) P(B_i).$$

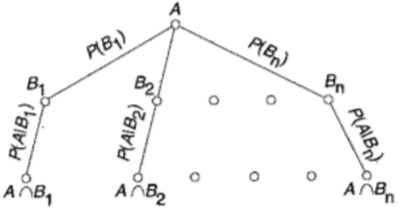


Figure 1.15. The theorem of total probability

Let us assume that the event A is known to have occured but it is not known which of the mutually exclusive and collectively exhaustive events $B_1, \ldots, B_n$ has occured. In this situation we want to use the available information in an efficent way in order to evaluate $P(B_i \mid A)$.

Theorem

Bayes' Theorem. Let $S' = \{B_1, B_2, \ldots, B_n\}$ be an event space and A and event. Then

$$\begin{aligned} P(B_i \mid A) &= \frac{P(B_i \cap A)}{P(A)} \\ &= \frac{P(A \mid B_i)P(B_i)}{\sum_{j=1}^{n} P(A \mid B_j)P(B_j)}. \end{aligned}$$

Bayes' Rule

Example

Trivedi, page 39. Measurements on a certain day indicated that the source of incoming jobs at the North Carolina Super Computing Center (NCSC) is 15% from Duke, 35% from University of North Carolina (UNC) and 50% from North Carolina State (NCS).

- Probabilities that a job initiated from these universities is a multitasking job are 0.01, 0.05 and 0.02, respectively.
- Find the probability that a job chosen at random at NCSC is a multitasking job.
- Find the probability that a randomly chosen job comes from UNC, given that it is a multitasking job.

Let $B_i =$"job is from university i" and $A =$"job uses multitasking" …

Exercises: solve Problems 1,2,3,4 of Trivedi, pp. 43-44.