Statistics Lecture

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Advanced Learning and Research Institute, 2012

Bernoulli Trials

Consider a random experiment with only two outcomes, “success” and “failure”, and denote the probability of the two outcomes by p and q, respectively, with $p+q=1$.

Definition

The experiment consisting in observing a sequence of n independent repetitions of the above described experiment is called a sequence of Bernoulli trials.

Examples

1. Observe n consecutive executions of an if statement, with success = “then clause is executed” and failure = ”else clause is executed”.
2. Examine components produced on an assembly line, with success = ”acceptable” and failure = ”defective”.

Bernoulli Trials

Let 0 denote failure and 1 denote success. Let S_n be the sample space of an experiment involving n Bernoulli trials

$$\begin{aligned} S_1 &= \{0,1\}, \\ S_2 &= \{(0,0),(0,1),(1,0),(1,1)\}, \\ S_3 &= \{0,1\}^n = \{2^n\, n-\text{tuples of 0s and 1s}\}. \end{aligned}$$

For all sample spaces S_n we define the $\sigma-$ algebra $\mathscr{P}(S_n)$ as the relevant $\sigma-$ algebra on which to define the probability P. On S_1 we then have $P(\{0\}) = q$ and $P(\{1\}) = p$. We wish to assign probability to points in S_n.
Define $A_i =$"success on trail i" and $\bar{A}_i =$"failure on trial i". We then have $P(A_i) = p$ and $P(\bar{A}_i) = q$. Let s be an outcome of S_n with k "1" and $n-k$ "0", i.e.

$$s = (1,1,\ldots,1,0,0,\ldots 0)$$

Bernoulli Trials

The elementary event $\{s\}$ can be written

$$\{s\} = A_1 \cap A_2 \cap \cdots \cap A_k \cap \overline{A}_{k+1} \cap \cdots \cap \bar{A}_n.$$

Because events A_i are independent we obtain

$$P(\{s\}) = P(A_1)P(A_2)\dots P(A_k)P(\bar{A}_{k+1})\dots P(\bar{A}_n)$$

so that $P(\{s\}) = p^k q^{n-k}$. Note that that we can construct $\left(\begin{array}{c} n \\ k \end{array} \right)$ different outcomes with k successes and $n-k$ failures, therefore defining $A =$"we observe exactly k successes in n trials"

$$P(A) = \left(\begin{array}{c} n \\ k \end{array} \right) p^k q^{n-k}.$$

Since by the binomial theorem $(p+q)^n = \sum_{k=0}^n \left(\begin{array}{c} n \\ k \end{array} \right) p^k q^{n-k} = 1$, P is a well defined probability law on $(S_n, \mathscr{P}(S_n))$.

Bernoulli Trials

In connection with reliability theory let us assume that a particular system with n components requires at least k components to function in order for the entire system to work correctly. Such systems are called k-out-of-n systems.

- If we let $k = n$ we have a series system.
- If we let $k = 1$ we have a system with parallel redundancy.

Assuming all components are statistically identical and function independently of each other, and denoting by R the reliability of a component ($q = 1 - R$ gives its unreliability), then the experiment of observing the statuses of n components can be thouth of as a sequence of n bernoulli trials with probability $p = R$.