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The authors propose an improved model for estimating audience exposure to network television, and test this model against the best previously reported models. The model involves a new method for estimating a beta binomial distribution, which is used to model the frequency distribution of exposure. The new estimation procedure takes explicitly into account several characteristics specific to network television.

Improving the Estimation Procedure for the Beta Binomial TV Exposure Model

All advertisers face the problem of how to allocate their advertising budgets. Since the mid-sixties, several media allocation models have been developed which make these decisions more scientific (Aaker 1975; Gensch 1969, 1973; Little and Lodish 1966, 1969; Zufryden 1975). These models are designed for use with any of the common advertising media, and thus do not incorporate the special characteristics of any medium.

Such selection models can be improved by specializing their audience exposure estimation components in a particular medium and incorporating that medium's special characteristics. The purpose of our study, therefore, is to develop and test an exposure estimation model for network television which takes into account several characteristics specific to that medium.

Several good exposure estimation models have been proposed for use in evaluating magazine advertising schedules (Chandon 1976; Greene and Stock 1967; Hofmans 1966; Liebman 1971; Metheringham 1971). All of these models require duplication data. In network television, duplication data are scarce because most network viewing data come from ARB and Nielsen, which typically do not report them.

In recent years, attempts have been made using the binomial distribution to estimate the frequency distribution of exposure without requiring duplication data (Aaker 1975; Balachandra 1975).

Another distribution commonly used to model exposure is the beta binomial distribution (BBD) (Greene and Stock 1967; Hyett 1958; Metheringham 1964). The early applications of the BBD all required duplications, which made the method undesirable for television selection models. However, Teel (1976) and Headen, Klompmaker, and Teel (1976, 1977, 1979) developed a procedure using the BBD which did not require duplication data.

The exposure estimation method we propose is a refinement of the Headen-Klompmaker-Teel procedure. However, the proposed procedure estimates the beta parameters from the moments of the beta rather than the BBD, an approach which should yield better results because the shape of the underlying beta distribution is estimated directly rather than indirectly.

THE MODEL

To determine the comparative value of a schedule, we first determine the frequency distribution of exposure for that schedule. That is, we determine the distribution $f(x)$ of the proportion of the population exposed to none ($x = 0$), one ($x = 1$), two ($x = 2$), etc. of the ads in the schedule.

In the proposed model the beta binomial distribution (BBD) is used to model this distribution. Specifically, with x as the number of exposures and N as the number of advertising spots in the schedule, we have

$$(1) \qquad f(x) = \int_0^1 f_b(x|N,p^*)\beta(p^*|a,b)\,dp^*$$

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where:

(2) $$f_b(x|N,p^*) = \binom{N}{x} p^{*x}(1-p^*)^{N-x}$$

(3) $$\beta(p^*|a,b) = (\Gamma(a+b))/(\Gamma(a)\Gamma(b))(p^{*a-1}(1-p^*)^{b-1}).$$

As in the Headen-Klompmaker-Teel model, every individual is assumed to have a probability p^* which describes his or her tendency to view the advertising schedule. If N spots are in the schedule, the person's probability of being exposed to k of the N is thus assumed to be $B(k,N,p^*)$. Note the assumption that exposure to a schedule of N spots may be modeled as exposures to N insertions of a "typical" or "average" composite spot. As this assumption reduces computation considerably without much loss of fit (Teel 1976), it is an especially appropriate assumption for modeling network television.

The beta distribution serves as a probability density function for the probability of exposure to any spot in a particular advertising schedule. Because these probabilities are allowed to vary across individuals, heterogeneous viewing behavior is accounted for. In the Headen-Klompmaker-Teel model, the BBD is estimated by using the fact that the mean is known (the sum of the ratings of the spots in the schedule), and the variance of the BBD can be estimated by using characteristics of the schedule. However, the Headen-Klompmaker-Teel model fails to take advantage of the useful behavioral interpretation of the beta distribution. Though the beta binomial distribution and its variance are difficult to interpret, the underlying beta distribution has a useful behavioral interpretation as the density function of viewing probabilities across the population. Thus the variance of the beta is interpretable as the dispersion in viewing probabilities across the population. This theoretical interpretation makes it more appealing to estimate the variance of the beta than the variance of the BBD.

The question of how N, the number of spots in the schedule, is treated in the estimation must be addressed. The Headen-Klompmaker-Teel model, by including N as an independent variable in the estimation of the variance of the BBD, necessarily replaces the direct functional relationship between the beta distribution and the BBD with a linear approximation. The approximation is unnecessary, because the BBD is completely determined by the beta distribution, along with N. Thus, one would expect better estimation results from a method utilizing the moments of the beta rather than the BBD.

In estimating the moments of the beta by the proposed method, it is computationally useful to define a variable V^* such that

(4) $$V^* = V_{sup} - V$$

where V_{sup} is the upper limit of the variance, given mean μ, and V is the variance of the beta. V_{sup} may be expressed as a function of μ. This can be demonstrated by expressing V as a function of μ and a.

(5) $$V = (ab)/(a+b)^2(a+b+1) = (a/(a+b)) \cdot (b/(a+b)(a+b+1))$$

Substituting the expression for the mean,

(6) $$a/(a+b) = \mu$$

(7) $$V = \mu b/(a+b)(a+b+1).$$

Substituting

(8) $$b = a(1-\mu)/\mu$$

and

(9) $$a+b = a/\mu$$

yields

(10) $$V = a(1-\mu)/(a/\mu)((a/\mu)+1)$$

(11) $$= \mu^2(1-\mu)/(a+\mu).$$

This quantity varies inversely with a. Thus,

(12) $$V_{sup} = \lim_{a\to 0} V = \lim_{a\to 0} [\mu^2(1-\mu)/(a+\mu)] = \mu^2(1-\mu)/\mu = \mu(1-\mu).$$

As V_{sup} is a function of μ, and μ (the average rating of the spots in the schedule) is known, either V^* or V may be estimated because one determines the other. In practice, V^* is easier to estimate than V. The reason is that in some extreme cases the BBD provides a fit which is poor enough to cause V to be negative. This situation occurs whenever the observed frequencies could not have been generated from the beta binomial assumptions. Because a multiplicative regression model is to be employed, it is better to use the more stable complement of V, V^*, which given the usual range of data is always non-negative. For this reason, the proposed model obtains the parameters of the beta from the mean and V^*. Chandon (1976) has shown that parameters a and b can be estimated from the mean and variance.

(13) $$\hat{a} = (\mu/V([\mu(1-\mu)-V]$$

(14) $$\hat{b} = (\hat{a}(1-\mu))/\mu$$

Thus, substituting for V,

(15) $$V = V_{sup} - V^* = \mu(1-\mu) - V^*,$$

we have

(16) $$\hat{a} = (\mu/(\mu(1-\mu)-V^*))[\mu(1-\mu) - \mu(1-\mu)+V^*]$$

(17) $$= \mu V^*/(\mu(1-\mu)-V^*)$$

(18) $$\hat{b} = (\hat{a}(1-\mu))/\mu.$$

Specifically, the mean of the beta, which is simply the average rating across the spots in the schedule, is known:

$$\mu = \left(\sum_{v} P_v\right) \Big/ N \tag{19}$$

where P_v is the proportion of the population exposed to spot v.

Just as the mean of the beta is a measure of average rating, the variance of the beta, and thus V^*, are measures of what might be considered an indication of the extent of duplication among the spots. The proposed model makes use of this by estimating V^* using variables which have been shown empirically to influence duplication among television programs (Headen, Klompmaker, and Rust 1979). These variables are incorporated in a multiplicative regression model. This model is calibrated on a sample of typical schedules and thereafter is used predictively. The model is

$$\hat{V}^* = A(1+X_1)^{B_1}(1+X_2)^{B_2}(1+X_3)^{B_3} (1+X_4)^{B_4} \cdot X_5^{B_5} X_6^{B_6} X_7^{B_7} e^u \tag{20}$$

where:

$A, B_1, \ldots, B_7$ are coefficients,
u is the disturbance term,
X_1 = proportion of same-channel pairs,
X_2 = proportion of same-daypart pairs,
X_3 = proportion of same-program-type pairs,
X_4 = proportion of self-pairs,
X_5 = average rating across the spots in the schedule,
X_6 = variance of the ratings, and
$X_7 = N$ = number of spots in the schedule.

A systematic tendency has been noted for many exposure models to overestimate reach and underestimate frequency for schedules containing two or more spots on the same program (Lodish 1975; Teel 1976; Zufryden 1975). The proposed model's explicit inclusion of a same-program variable (X_4) is intended to alleviate this effect. The justification for the inclusion of the variance of the ratings (X_6) in the predictor equation is that a high variance of the ratings means that there are low-rated spots in the schedule. Low-rated spots cannot differentiate very much between viewers (because almost no one watches). Thus, the beta variance is smaller in that case and V^* is larger.

Given the mean, and with V^* estimated, one then can solve for beta parameter estimates $\hat{a}$ and $\hat{b}$. From these parameter estimates, and the number of spots, N, one can estimate the frequency distribution of exposure $f(x)$ using the BBD.

AN EMPIRICAL TEST OF THE MODEL

A test was developed to compare the performance of the binomial, Headen-Klompmaker-Teel, and proposed exposure estimation models. The data for this test were provided by the Simmons Market Research Bureau (formerly W. R. Simmons and Associates Research, Inc.) and are from their 1977/78 report (Simmons 1978a, b). The Simmons respondents were selected by means of a multistage area cluster sample. Television viewing data were collected from 5652 respondents in a six-week period from October 9 to November 19, 1977. The 5652 respondents were a subsample of 15,003 original respondents, all of whom answered questionnaires about demographic characteristics, product usage, and use of other media.

A preliminary random sample of television advertising schedules was developed to establish the regression coefficients for both the Headen-Klompmaker-Teel and proposed models. This sample consisted of 1200 schedules and was random except that an equal number of schedules was generated for schedules of size 3 to 12 inclusive.

In practice, it would be necessary to estimate the model once a year, as new data are obtained from a syndicated source such as Simmons. The model need not be estimated each time a schedule is evaluated.

A random sample of 400 additional schedules was drawn, as before, to test the predictive fits of the three exposure estimation models. The frequency distributions of exposure were estimated for each schedule with each model. The actual frequency distribution of exposure was "nose-counted" directly from the Simmons data tape, and each model's estimation error within each frequency class was computed.

The three models were compared on the basis of four goodness-of-fit measures. The first measure, effective error, is the absolute difference between the predicted response value and the observed response value, where response value is the frequency distribution multiplied by the response function. The response function used is

$$R(x) = x^{1/2}. \tag{21}$$

Response functions of this approximate shape have been used by Little and Lodish (1969) and many others. In this case, response refers to only the controllable response. Some response is likely to occur even without advertising, but such response is by definition unaffected by advertising.

Effective error, $E1$, is calculated as

$$E1 = \left| \sum_x \hat{f}(x)R(x) - \sum_x f(x)R(x) \right| \tag{22}$$

where $f(x)$ is the actual proportion in frequency class x, $\hat{f}(x)$ is the predicted proportion in frequency class x, $R(x)$ is the response function, and the summations are over all of the frequency classes.

For each model, for each schedule, the sum of square error, $E2$, is calculated as

(23) $$E2 = \sum_x (\hat{f}(x) - f(x))^2$$

where $\hat{f}(x)$ and $f(x)$ are defined as before.

For each model, for each schedule, the maximum frequency class error is computed as

(24) $$E3 = \max_x |\hat{f}(x) - f(x)|$$

where $\hat{f}(x)$ and $f(x)$ are defined as before.

The average frequency class error, $E4$, for each schedule is calculated for each model as

(25) $$E4 = \sum_x |\hat{f}(x) - f(x)|/N$$

where N is the number of spots in the schedule and $\hat{f}(x)$ and $f(x)$ are defined as before.

RESULTS AND DISCUSSION

Regression coefficients for the Headen-Klompmaker-Teel and proposed models were estimated from the initial subsample of 1200 schedules.

The Headen-Klompmaker-Teel procedure involved a regression on the variance of the BBD. The results of that regression are shown in Table 1.

The regression produced a very high R^2 of .98. The magnitude of this figure is perhaps not surprising if one considers that the two most influential independent variables, average rating and number of spots, are found in the expression for the variance of the BBD:

(26) $$V_{BBD} = [nab(n + a + b)] / [(a + b)^2(a + b + 1)].$$

Substituting average rating $= \mu = a/(a + b)$, number of spots $= n$, and $b = a(1 - \mu_\beta)/\mu_\beta$, we have, after simplifying

(27) $$V_{BBD} = n\mu_\beta \cdot \frac{(1 - M)(n + a + b)}{(a + b + 1)}.$$

This is evidence for a strong, if imperfect, multiplicative relationship among number of spots, average rating, and the variance of the BBD. Thus, finding a high R^2 is not altogether surprising. On an intuitive level, one would expect that the greater the number of spots in the schedule, the greater would be the opportunity for choice in viewing behavior and hence the greater the variance of the BBD. The remaining variable, number of channels, is significant, but not as important as the others.

The Headen-Klompmaker-Teel prediction equation for the variance of the BBD is

(28) $$\hat{V}_{BBD} = (.7121)(X_1^{1.1651})(X_2^{-.1005})(X_3^{.9530})$$

where:

X_1 = number of spots in the schedule,
X_2 = number of networks used, and
X_3 = average spot rating for the schedule.

One must keep in mind that though estimating the variance of the BBD is an important intermediate step in the Headen-Klompmaker-Teel procedure, ultimately the procedure must be tested on its ability to estimate the frequency distribution itself. High predictive ability in the regression does not necessarily imply high predictive accuracy in the frequency distribution, although the converse must be true.

The proposed procedure requires a regression on

(29) $$V^* = V_{sup} - V$$

where V_{sup} is the maximum possible beta variance, given mean μ, and V is the variance of the beta. The regression's results are summarized in Table 2. All of the coefficients are of the sign anticipated.

All of the correlations between the independent variables are less than .4, except for a correlation of .62 between the log of the average rating in the schedule and the log of the variance of the ratings. This is a high correlation. Nevertheless, the relative importance of the two variables is clear. Also, the fact that the model would be used primarily for prediction lessens the concern.

The correlation between the log of the average rating and the log of V^* is an amazing .99. Thus, the log of the average rating almost completely explains the variance of the log V^*. The other independent variables are all very significant (.01 level or better) yet explain only a small amount of the variance.

As in the Headen-Klompmaker-Teel regression, the high R^2 may be largely explained by looking at an expression of the dependent variable.

(30) $$V^* = V_{max} - V$$

(31) $$= \mu(1 - \mu) - V$$

The variance V is almost always very small, which permits the approximation

(32) $$V^* \simeq \mu(1 - \mu).$$

When μ is small and positive, as is the case with

Table 1
COEFFICIENT ESTIMATION REGRESSION RESULTS, HEADEN-KLOMPMAKER-TEEL ESTIMATION PROCEDURE

Independent variable		*Coefficient estimate*	F[a]
Log (number of spots in schedule)		1.1651	30,759
Log (number of networks in schedule)		−0.1005	60
Log (average rating)		0.9350	15,702
Log (constant)	−.3395		
R^2	.98		
Standard error	.0845		
F of regression	20,312		

[a]All F-values are significant at the .01 level.

Table 2
COEFFICIENT ESTIMATION REGRESSION RESULTS, PROPOSED ESTIMATION PROCEDURE

Independent variable		*Coefficient estimate*	F^a
Log (1 + the proportion of same-channel pairs)		−.0108	30.58
Log (1 + the proportion of same-daypart pairs)		−.0085	24.93
Log (1 + the proportion of same-program-type pairs)		−.0131	29.87
Log (1 + the proportion of same-program pairs)		−.0256	19.86
Log (variance of program ratings)		−.0055	339.81
Log (number of spots)		−.0086	334.52
Log (average rating)		.9355	—[b]
Log (constant)	−.2100		
R^2	.99		
Standard error	.0066		
F of regression	—[b]		

[a]All F-values significant at the .01 level.
[b]Larger than 10^5.

television ratings data, μ is correlated very highly with $\mu(1-\mu)$. This effect can be seen by observing that when $\mu = 0$, $\mu(1-\mu) = 0$, and that

$$\frac{\partial}{\partial\mu}[\mu(1-\mu)] = 1 - 2\mu \simeq 1. \tag{33}$$

Thus, $\mu(1-\mu)$, and therefore V^*, must be highly correlated with μ, which is the average rating of the spots in the schedule. The logarithms must also be highly correlated, as is borne out by the data.

The approximation in equation 33 works best when the average rating is small. Thus the very high R^2 seems to imply a low average rating for the tested schedules. This is, in fact, the case. The schedules were drawn randomly from a pool of 852 programs, consisting of 360 daytime, 360 evening, and 132 weekend sports programs. The mean of the average ratings is 4.8 and the range is 1.4 to 11.4. These ratings, like diary ratings generally, are smaller than those typically obtained by using automatic auditing. However, the question of which measurement method is right is not an easy one because the automatic audits count viewership as long as the television set is on, even if nobody is in the room. The diary methods, in contrast, may produce undercounts because of forgetfulness, but ensure a minimal level of attention to the program. Media planners concerned about this rating-size discrepancy may wish to inflate the diary counts to automatic audit levels before estimating the model.

As with the Headen-Klompmaker-Teel procedure, one must suspend judgment on the proposed procedure, in spite of its excellent regression results, until the model's ability to predict the frequency distribution can be measured.

The predictive equation arising from the regression is

$$\hat{V}^* = (.8106)(1 + X_1)^{-.0108}(1 + X_2)^{-.0085}(1 + X_3)^{-.0131} \cdot (1 + X_4)^{-.0256} X_5^{-.0055} X_6^{-.0086} X_7^{.9355} \tag{34}$$

where the variables are defined as before. The small coefficients for all except average rating confirm the relative lack of importance of the other independent variables. In fact, if a user were willing to accept slightly poorer explanation while reducing the amount of computation, he or she could simply use the following predictive equation resulting from a regression using only average rating as the independent variable.

$$\hat{V}^* = .7964(X_5^{.9446}) \tag{35}$$

If computation time is not a problem, however, one can only gain by including all of the variables (all are highly significant) in the predictive equation.

Statistics based on effective error, sum of square error, maximum frequency class error, and average frequency class error were calculated for each of the three frequency distribution estimation models. The means of each of the errors are shown in Table 3 for each of the estimation models.

The two BBD approaches estimate the frequency distribution of exposure much better than the binomial model. Of the two BBD approaches, the proposed procedure produces generally better error statistics. The significance of the difference is tested by a t-test of the form

$$t = \bar{d}/(s_d/\sqrt{n}) \tag{36}$$

where $\bar{d}$ is the mean difference, s_d is the standard deviation of the difference, n is the sample size, and there are $n - 1$ degrees of freedom. The results of this test are reported in Table 4. In two of the four error statistics, the proposed procedure is significantly better, at the 95% level.

An additional empirical test was performed to determine whether the proposed estimation procedure would alleviate the tendency of the Headen-Klompmaker-Teel model to overestimate reach for schedules including multiple insertions of the same program. Sixty schedules of six spots were generated randomly so that 20 of the schedules included two insertions of the same program, 20 included four insertions, and 20 included six insertions. For the Headen-Klompmaker-Teel procedure the mean reach overestimate was .033, whereas the proposed procedure yielded a mean overestimate of .029. A t-test for related samples shows the proposed method to be less biased, at the 99% level.

Of some concern is the fact that reach estimation tends to deteriorate as the number of insertions of a single program increases. In the extreme case of all six of the insertions being the same program, the

Table 3
MEAN ERRORS IN ESTIMATING THE FREQUENCY DISTRIBUTION OF EXPOSURE

Model	*Effective error*	*Sum of square error*[a]	*Maximum frequency class error*[b]	*Average frequency class error*[b]
Binomial	14.80	3.33	3.69	8.31
Headen-Klompmaker-Teel	7.89	1.36	2.05	4.95
Proposed	7.72	1.33	2.00	4.84

[a]Numbers $\times 10^{-3}$.
[b]Numbers $\times 10^{-2}$.

Table 4
TESTS OF DIFFERENCE BETWEEN THE PROPOSED AUDIENCE ESTIMATION PROCEDURE AND THE HEADEN-KLOMPMAKER-TEEL PROCEDURE

Error criterion	$\bar{d}$[a]	*t*	*Significance level*
Effective error	.163	1.58	.94
Sum of square error	.023	.66	.74
Maximum frequency class error	.527	2.25	.98
Average frequency class error	.118	2.27	.98

[a]All $\bar{d}$ figures $\times 10^{-3}$.

average reach overestimate is 5.3 rating points (78% of 6.7 rating points average reach), and the average effective error (using the convex response function defined before) is 38%.

These error figures are probably overstated, as the small number of weeks of data available restricts the testing of prime time multiple-insertion schedules. The main alternative, daytime, consists predominantly of serial dramas (soap operas), which have the highest reported duplications of any program type (Goodhardt, Ehrenberg, and Collins 1975; Headen, Klompmaker, and Rust 1979). Thus, the reach estimation error normally expected should be somewhat smaller.

Furthermore, one must remember that without duplication data, the estimation alternatives are limited. There are few estimation procedures which do not require duplication data. Thus, a relevant tradeoff is the one between costs of errors in estimation and costs of not having an estimate at all. Nevertheless, a media planner should be wary of the proposed procedure's apparent tendency to overestimate reach for multiple insertions of the same program.

SUMMARY

Models using the beta binomial distribution (BBD) are more accurate than models using the binomial distribution in estimating audience exposure to network television. Among the BBD models, our proposed estimation procedure is an improved method of estimating audience exposure to network television.

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